Tracking and Estimation of moving object using Kalman filter —

Kalman filter —

state space formulation of linear dynamical systems. provides a recursive solution to the linear optimal filtering.

state space model.

$x_k = \alpha x_{k-1} + w_k$ — ① state space equation

$y_k = \cancel{h} x_k + v_k$ — ② observation equation

x_k — set of data

y_k — set of observed data

Kalman filter works in two steps

1 - Prediction step.

2 - correction step.

we have considered 1st order AR model

Markov model $s(n) = \alpha\, s(n-1) + w(n)$ — state space model.

$s(0)\ s(1)\ \cdots\ s(n-2)\ s(n-1)\ s(n)$ — states of the system.

$s(n) = \alpha\, s(n-1) + w(n)$

$f(s(n) \mid s(n-1)\ s(n-2) \cdots s(0)) = f(s(n) \mid s(n-1))$

markov process — future behaviour can not be accurately predicted from its past behaviour except the current behaviour which involves random chance or probability

$s(n) = \alpha\, s(n-1) + w(n)$ — state equation

$\uparrow$ Gaussian $\quad \uparrow$ Gaussian $\quad \uparrow$ Gaussian

$s(n-1) = N(\mu, \sigma^2)$

$w(n) = N(0, \sigma_w^2)$

$x(n) = h\, s(n) + v(n)$ — observation equation

Estimate $s(n)$ from all observations

$x(0)\ x(1)\ \cdots\ x(N-1)$

notes
phone
email
website
essential

Our Goal $\hat{s}(n) = E\{ s(n) \mid x(0)\ x(1) \cdots x(N-1)\ x(N)\}$

from $\hat{s}(n-1)$ compute $\hat{s}(n)$

$\hat{s}(n)$ depends on $\{\underbrace{\hat{s}(n-1)}_{\text{mean}},\ \underbrace{p(n-1)}_{\text{co-var}}\}$

$\hat{s}(n-1) = E\{ s(n-1) \mid x(0)\ x(1) - - x(N-1)\}$

$\hat{s}(n-1|n-1) = E\{s(n-1) \mid x(n-1)\}$

$\cancel{p(n-1)} =$

$p(n-1) = p(n-1|n-1) = E\{(s(n-1) - \hat{s}(n-1))^2\}$

Kalman filter

1- prediction step

2- correction update step

Step-I

1- Prediction step - Prediction of $s(n)$ based on the previous data

i.e. given data $x(0)\ x(1) - - x(n-1)$

$$\hat{s}(n|n-1) = E\{ s(n) \mid x(0)\, x(1) \cdots x(n-1) \}$$

$$s(n) = \alpha\, s(n-1) + w(n)$$

$$\hat{s}(n|n-1) = \alpha E\{ s(n-1) \mid x(0) \cdots x(n-1) \} + E\{ w(n) \mid x(0) \cdots x(n-1) \}$$

$$= \alpha\, \hat{s}(n-1) + 0$$

$$\hat{s}(n|n-1) = \alpha\, \hat{s}(n-1)$$

Now co-variance (error co-variance)

$$p(n|n-1) = E\{ (\hat{s}(n|n-1) - s(n))^2 \}$$

$$= E\{ (s(n) - \hat{s}(n|n-1))^2 \}$$

$$= E\{ (\alpha\, s(n-1) + w(n) - \hat{s}(n|n-1))^2 \}$$

$$p(n/n-1) = \alpha^2 p(n-1) + \sigma_w^2$$

$$s(n) \mid x(0)\, x(1) \cdots x(n-1) \sim N(\hat{s}(n|n-1),\, p(n|n-1))$$

Step-II (correction step) — Now time update step. we estimate s(n) by using $\{x(n)\ x(n-1) \cdots x(1)\}$

$x(n) = h\, s(n) + v(n)$. $\underbrace{\text{observation equation}}$

$\underbrace{\text{linear equn}}$

Now estimate $\hat{s}(n)$ i.e. $\hat{s}(n/n)$ using linear MMSE expression.

$$\hat{s}(n) = R_{sx} R_{xx}^{-1} (x - \mu_x) + \mu_s$$

$$\mu_s = \text{mean of } s(n) = \hat{s}(n/n-1)$$

$$\mu_x = \text{mean of } x(n) = \hat{x}(n/n-1) = h\,\hat{s}(n/n-1)$$

$$R_{xx} = E\{(x - \mu_x)^2\} = h^2 p(n/n-1) + \sigma_v^2$$

$$R_{sx} = E\{(x - \mu_x)(s - \mu_s)\} = h\, p(n/n-1)$$

Now

$$\hat{s}(n) = \underbrace{\frac{h\, p(n/n-1)}{h^2 p(n/n-1) + \sigma_v^2}}_{\substack{\text{weiting facter} \\ \text{or} \\ \text{Gain of Kalman}}} \underbrace{x(n) - h\,\hat{s}(n/n-1)}_{\text{error in prediction}} + \hat{s}(n/n-1)$$

Correction.

Error covariance (covariance correction update)

$$E\{(s(n)-\hat{s}(n))^2\} = R_{ss} - R_{sx} R_{xx}^{-1} R_{xs}$$

$$= p(n/n-1) - \frac{h\, p(n/n-1) \cdot h\, p(n/n-1)}{h^2 p(n/n-1) + \sigma_v^2}$$

$$= \left[1 - \frac{h^2 p(n/n-1)}{h^2 p(n/n-1) + \sigma_v^2}\right] p(n/n-1)$$

$$p(n) = [1 - h\, k(n)]\, p(n/n-1)$$

Dynamic state model in which observation equation depends on parameters that can be tuned to improve the performance

Recently several authors proposed.

for example 1- To track and estimate dynamic wireless channel

2- The parameters to be estimated are the position and velocity of a target

3 — Distributed sensor networks problem is considered where the observed signal is a linear function of the transmission gain of each sensor.

→ Here we consider the problem of optimizing the parameters (θ) in order to min the MSE