Tracking and Estimation using Kalman filter and convex-optimization.

1) optimal power allocation for parameter tracking
2) minimize ECM
3) multi-missile Kalman filter

1) optimal power allocation for parameter tracking —

a) min error under power constraints

b) min the sum transmit power

system model.

θ_n = complex-valued dynamic parameter as a first-order Gauss-Markov Process

$$\theta_n = \alpha \theta_{n-1} + u_n$$

The measurement for ith sensor at time n

$$s_{i,n} = \theta_n + v_{i,n}$$

where n denotes the time step α is correlation parameter and u_n is process noise zero mean with σ_u^2 var.

θ_n is stationary process with zero mean and var $\sigma_\theta^2 = \frac{\sigma_u^2}{1-|\alpha|^2}$

$v_n \rightarrow$ measurement noise $CN(0, \sigma_{v_i}^2)$

DECEMBER							2017
Wk	M	T	W	T	F	S	S
49					1	2	3
50	4	5	6	7	8	9	10
51	11	12	13	14	15	16	17
52	18	19	20	21	22	23	24
53	25	26	27	28	29	30	31

JANUARY							2018
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01	1	2	3	4	5	6	7
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Now each sensor multiplies its observation by a complex gain factor and transmit the result over a wireless channel to the fusion centre (FC)

Received signal at FC from all N nodes

$$y_n = \sum_{i=1}^{N} h_{i,n} a_{i,n} s_{i,n} + w_n$$

$$= \sum_{i=1}^{N} (h_{i,n} a_{i,n} \theta_n + h_{i,n} a_{i,n} v_{i,n}) + w_n$$

$h_{i,n} \Rightarrow$ gain of the channel.

$a_{i,n} \Rightarrow$ complex transmission gain.

$w_n \Rightarrow$ noise $CN(0, \sigma_w^2)$

$$y_n = a_n^H h_n \theta_n + a_n^H H_n v_n + w_n$$

Now we have used the standard Kalman filter to track the parameter θ_n

Prediction step - $\hat{\theta}_{n|n-1} = \alpha \hat{\theta}_{n-1|n-1}$

Prediction MSE - $P_{n|n-1} = \alpha^2 P_{n-1|n-1} + \sigma_u^2$

Kalman Gain - $K_n = \dfrac{P_{n|n-1} h_n^H a_n}{a_n^H H_n V H_n^H a_n + P_{n|n-1} a_n^H h_n h_n^H a_n + \sigma_w^2}$

where $h_n = [h_{1,n} \cdots h_{N,n}]^T$

$a_n = [a_{1,n} \cdots a_{N,n}]^H$

$H_n = \text{diag}\{h_{1,n} \cdots h_{N,n}\}$

$v_n = [v_{1,n} \cdots v_{N,n}]^T \Rightarrow$ var $V = E\{v_n v_n^H\} = \text{diag}\{\sigma_{v,1}^2 \cdots \sigma_{v,N}^2\}$

Measurement update -

$$\hat{\theta}_{n|n} = \hat{\theta}_{n|n-1} + K_n (y_n - a_n^H h_n \hat{\theta}_{n|n-1})$$

Filtered MSE -

$$P_{n|n} = (1 - K_n a_n^H h_n) P_{n|n-1} \quad \text{— (A)}$$

our goal is to determine 'an' optimal choice for the gain 'an' that minimizes the filtered MSE under a power constraint OR minimizes the power consumed in transmitting the data to the FC under MSE constraint.

i) minimize MSE under sum power constraint

Now we can write the optimization problem

$$\min_{a_n} P_{n|n}$$
$$\text{s.t.} \; a_n^H D_n a_n \leq P_T \quad \text{— (1)}$$

$a_n^H D a_n$ = actual power

P_T = Total transmit power

$$\min_{a_n} (1 - K_n a_n^H h_n) P_{n|n-1}$$
$$\text{s.t.} \; a_n^H D a_n \leq P_T$$

$$D = \mathrm{diag}(\sigma_\theta^2 + \sigma_{v,1}^2, \ldots, \sigma_\theta^2 + \sigma_{v,N}^2)$$

from equ^n ① min. the MSE $P_{n|n}$ is equivalent to max

$$\max_{a_n} K_n a_n^H h_n$$

$$\text{s.t} \quad a_n^H D a_n \leq P_T$$

$$\max_{a_n} \frac{P_{n|n-1} a_n^H h_n h_n^H a_n}{a_n^H H_n V H_n^H a_n + P_{n|n-1} a_n^H h_n h_n^H a_n + \sigma_w^2}$$

$$\text{s.t} \quad a_n^H D a_n \leq P_T$$

we can rewrite the above optimization problem.

$$\max_{a_n} \frac{a_n^H h_n h_n^H a_n}{a_n^H H_n V H_n^H a_n + \sigma_w^2} \quad -(2)$$

$$\text{s.t} \quad a_n^H D a_n \leq P_T$$

Now solve the equ^n(2) ~~using~~ optimization problem. using lagrangian method.

equ^n (2) is the QCQP problem. we can write its lagrangian. $L(a_n, \lambda)$

$$L(a_n, \lambda) = \frac{a_n^H h_n h_n^H a_n}{a_n^H H_n V H_n^H a_n + \sigma_w^2} + \lambda\left(a_n^H D a_n - P_T\right) \quad -(3)$$

notes
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essential

The optimum solution of (3) as a_n^* using lagrangian and its dual.

in eqn (2) objective function is monotonicaly increasing function in the norm of a_n. and for the optimal solution, the sum transmit power constraint should hold the equality ie. $a_n^{*H} D a_n^* = P_T$

the closed form solution of eqn (3) is possible.

The optimal solution is given by

$$a_n^* = \sqrt{\frac{P_T}{h_n^H B^{-1} D B^{-1} h_n}}\, B^{-1} h_n$$

$$B = H_n V H_n^H + \frac{\sigma_w^2}{P_T} D$$

Now the max value of the objective function in (2) can be expressed as

$$\frac{a_n^{*H} h_n h_n^H a_n^*}{a_n^{*H}\left(H_n V H_n^H + \frac{\sigma_w^2}{P_T} D\right) a_n^*} = h_n^H B^{-1} h_n$$

given that $h_n^H B^{-1} h_n \overset{(a)}{<} h_n^H (H V H^H)^{-1} h_n$ ——— (4)

$$= \sum_{i=1}^{N} \frac{1}{\sigma_{v,i}^2}$$

17 February

SATURDAY

WEEK - 7 / DAY (048-317)

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plugging (G) into (A) a lower bound on MSE can be expressed

$$P_{n|n} > \left(1 - \frac{1}{1 + \frac{1}{\left(\sum_{i=1}^{N} \frac{1}{\sigma_{v,i}^2}\right) P_{n|n-1}}}\right) P_{n|n-1}$$

$$= \frac{P_{n|n-1}}{1 + \left(\sum_{i=1}^{N} \frac{1}{\sigma_{v,i}^2}\right) P_{n|n-1}}$$

18 Sunday

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