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Velocity Dispersion in Andromeda's Stellar Disk

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DECLARATION

We Diya Mallik, Samit Uttarkar, Simran Kalra and Tejas Sonawane, hereby declare that this dissertation titled

"Velocity Dispersion in Andromeda's Stellar Disk"

Is an outcome of our own research study directed under the supervision of Prof. Jyoti Singh

St.Xavier's College (Autonomous), Mumbai

This dissertation has not formed the basis for the decoration of any degree, diploma or certificate of this institute or any other institute or university. Also we have duly acknowledged all the sources used by us in the present dissertation.

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1. Abstract

In this project we have used Python as a tool to analyse the velocity dispersion of 5800 individual stars in the Andromeda Galaxy. The data has been taken from the Panchromatic Hubble Andromeda Treasury Survey (PHAT) and Keck/DEIMOS radial velocity measurements of stars from the Spectrometric and Photometric Landscape of Andromeda's Stellar Halo Survey (SPLASH). Further we divided these stars into three groups, Main Sequence stars, Red Giant Branch and Asymptotic Giant Branch and compared the velocity dispersion of each group. For this we have referred to the paper "A Clear Age Velocity Dispersion Correlation in Andromeda's Stellar Disk". This paper has been our main inspiration for this project. Finally we tried to understand on what basis the velocity dispersion of the three groups varies.

In Section 1, we have introduced the basic concepts needed to do this project. Section 2 includes how the stars were divided in groups and the code for finding their velocity dispersion. In Section 3 we have discussed our final results.

2. Problem Statement

Andromeda is our companion galaxy of more than 1 trillion stars. In this project, we measure the velocity dispersion of star grouped into 3 regions i.e. MS (Main sequence), RGB (Reg Giant Branch) and AGB (Asymptotic Giant Branch), We are an optical Hubble space Telescope/ Advanced camera for survey photometry of 5800 stars. From the Panchromatic Hubble Andromeda Treasury (PHAT) survey and KECK/ DEIMOS radial velocity measurements of the same stars from the spectroscopic and photometric landscape of Andromeda's stellar Halo (SPHASH). This will help us understand the stellar age distribution of different star regions. We also learnt the evolution of a star in Hertzsprung Russell Diagram.

3. Introduction:

3.1 Andromeda Galaxy

The Andromeda Galaxy is the nearest large spiral galaxy approximately 2.5 light years away from the Milky Way Galaxy. It was first recorded in the book "The Constellations of Fixed Stars" by author Abu I-Hussain al-Sufi and Simon Marius. The Andromeda Galaxy is the 31st Nebulous object in the Messier catalogue which is the first systematic catalogues of non-stellar objects and hence is referred to as Messier 31 or M31. The Andromeda Galaxy is home to approximately 1 trillion stars. The galaxy has an apparent magnitude of 3.6 that makes it visible through naked eye and is inclined at 77 degrees to the Earth.. At the centre of the galaxy there is a supermassive black hole that is 30 million times heavier than the sun.

3.2 Geometry & Structure

The structure of the Andromeda Galaxy resembles the Milky Way Galaxy. It is a large barred galaxy with spiral arms and a bulging central disc. The Hubble Space Telescope of NASA discovered that the Andromeda Galaxy has a double nucleus at its centre separated by 5 light years. It looks like a flat disk with diameter more than 200,000 light years and thickness around 1000 light years.

3.3 Composition

The Andromeda Galaxy is composed of around 1 trillion stars and a supermassive black hole in its centre. It has a disc tightly packed with blue young stars and is surrounded by a ring of older red stars.

3.4 Size & Mass

The total mass of the Andromeda Galaxy lies between $8*10^{11}~M_{\odot}$ and $1.1*10^{12}~M_{\odot}$. Out of this 10- $15*10^{10}~M_{\odot}$ is that of the stellar mass of the andromeda galaxy. Nearly 30% of the mass is in the central bulge, 56% in the disc and the remaining 14% in the stellar halo.

4 The Panchromatic Hubble Andromeda Treasury Survey

The Panchromatic Hubble Andromeda Treasury Survey is a Multi Cycle Treasury Program that images a large area of the Andromeda Galaxy's bulge and disc using the Hubble Space Telescope. The telescope provides images of approximately 1/3rd of the Andromeda's star forming disc in six different filters varying from the ultraviolet (UV) to the near Infrared (NIR). Due to this broad coverage we target information like bolometric luminosity, spectral energy distribution, morphology of astrophysical objects and allows full characterization of stars, their evolved descendents and other useful background sources. On completion the PHAT survey would contain data of more than 100 million stars.

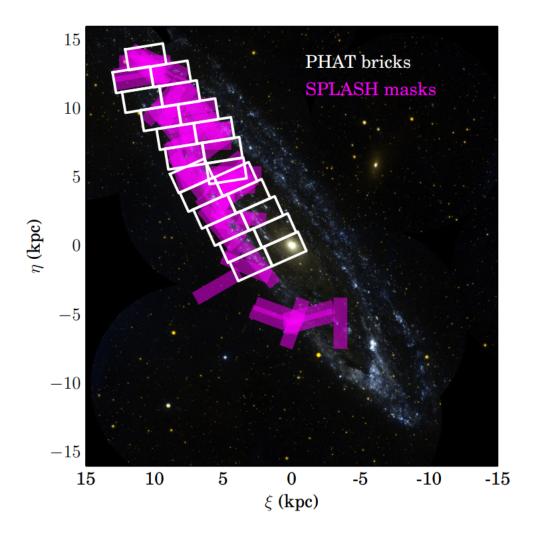


Figure 1: Spatial coverage of spectroscopic (SPLASH) and photometric (PHAT) surveys from which our data are drawn, overlaid on a GALEX UV image of M31 for reference.

Magenta regions demarcate the 47 Keck/DEIMOS spectroscopic slitmasks used in the SPLASH survey, whereas white rectangles outline the 23 PHAT "bricks" (clusters of HST pointings).Image is taken from Dorman, C. E [5]

4.1 Aerial Coverage

The Panchromatic Hubble Andromeda Treasury Survey covered a large quadrant of the Andromeda Galaxy in the northeast side. The area is chosen such that it has the largest

number of high intensity star formation, lowest number of internal extinction and least contamination from the M32 galaxy. It is a long rectangular region beginning at the centre of the galaxy and slightly bending at the middle. The area of the survey is divided into 23 sub-areas that are known as bricks arranged in two strips. The southwest end of the survey area includes part of Andromeda's bulge, the nucleus and the dense stellar population surrounding it. The northeast end of the survey includes the transition of the disk into the halo.

4.2 Choice of Filter

In the UV, F336W was the chosen filter since it was giving the highest output. It also lies in the blueward side of the Balmer break which gives the best constraint on its amplitude. The bluest filter WFC3/UVIS is not chosen due to its low efficiency. Instead F275W is chosen as the bluest and its short wavelength is able to provide the best limitation on stellar temperature for the hottest stars. It is also unaffected by variations in dust compositions.

In the optical, F814W filter was chosen since it has the highest output for stellar population studies. For the bluer optical filter F475W was chosen because it is broad and much bluer than other filters. It also provides a good colour separation from the F814W filter.

For the NIR, F110W and F160W filters are adopted since they provide the highest output throughout the Wide Field Camera 3 (WFC3)/IR camera. The only disadvantage of these filters is that their combination is the partial overlap of F110W and F814W filters

5. Keck/DEIMOS

Deep Imaging Multi-Object Spectrograph (DEIMOS) is a multi object spectrograph built for the Keck II telescope since 2002. DEIMOS was designed and built by UCO/Lick Observatory by Principal Investigator Sandra M.Faber and Project Manager Dave Cowley.

Important features of the DEIMOS are as follows:

- High Output
- High Spectral Resolution (upto $R \approx 6000$)
- Covers upto 5000 A per exposure
- generous slit length spanning 16.6 arcmin on sky (vs. 8 arcmin for LRIS)
- large 8k×8k detector mosaic featuring eight MIT/Lincoln Labs CCDs
- advanced, closed-loop flexure compensation system achieving image stability of ±0.25 px over 360° of instrument rotation
- convenient IDL-based data reduction pipeline

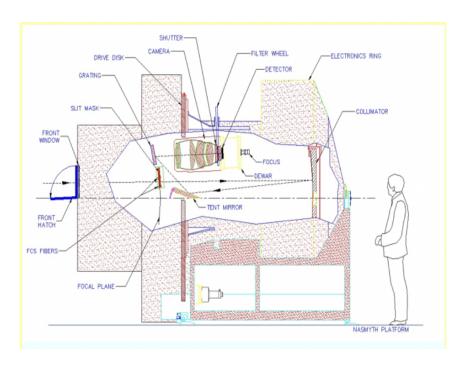


Figure 2: Side view of the optical layout

5.1 Origin of Keck/Deimos

Funding for the telescopes emanates from a giving organisation famous as the W.M. Keck Foundation. The entity was constituted in 1954 by William Myron Keck, the one who organised the Superior Oil Company, in accordance with the foundation's site. "Mr. Keck conceived a giving organisation that would supply far-reaching benefits for benevolence," the company pronounced. "By communicable a bold, imaginative approach to grant making, he forged a inheritance that the establishment boastfully upholds today."The organisation's order contains capital science, and in 1985 it accepted a \$70 heap (\$155 heap in 2014 currency) to assemble the first telescope, Keck I. During building, \$68 heap prevailed for the second compact, Keck II. The telescopes started their science in 1993 and 1996 individually, in accordance with the Keck site.

The telescopes' arsenal of implements contains various for ocular wavelengths and so forth for infrared. Keck II family's DEIMOS (Deep Extragalactic Imaging Multi-Object Spectrograph), that can gather ghostly information from 1,200 objects together. Keck I has HIRES (High Resolution Echelle Spectrometer), that can analyse the colours of starlight.

5.2 Discoveries by Keck

Keck's twin telescopes have made a host of discoveries since construction finished.

Other discoveries by Keck includes:

- Measuring the size of a distant planet whose size is equivalent to that of Uranus
- Exposing four quasars within a single system
- Discovered a galaxy almost entirely made of dark matter.
- Detection of interesting atmospheric activity on Neptune and Jupiter's volcanic moon, Io

6. SPLASH

Spectroscopic and Photometric Landscape of Andromeda's Stellar Halo. The collective data set from this large international team includes thousands of Keck/DEIMOS spectra of individual red giant branch stars, ground-based deep wide-field imaging and photometry with KPNO/Mosaic, CFHT/MegaCam, and Subaru/Suprime-Cam, and ultra-deep pencil-beam probes with HST/ACS imaging reaching below the main-sequence turnoff.

7. HR DIAGRAM

Hertzsprung-Russell diagram AKA HR diagram was first developed by Ejnar Hertzsprung and Henry Russell in the early 1900s. An HR diagram plots temperature against their luminosity, but for observational purposes colour of stars are plotted against their absolute magnitude also known as colour magnitude diagram. Initial mass of the star decides various stages of stellar evolution a star would go through. Temperature and Luminosity of a star changes as it progresses through these stages which can be seen on HR diagram as it evolves. The HR diagram is a very powerful tool to determine the internal structure of a star and evolutionary stage just by looking at its position in the HR diagram.

There are three main regions in the HR diagram:

- Main Sequence These stars dominate the HR diagram and stretch from upper left to bottom right. This region is named the main sequence as stars spend 90% of their lifetime in this region.
- Red Giant Branch The Red Giant Branch is a portion of the Hertzsprung-Russell
 Diagram where a main sequence star branches off in its course of stellar
 evolution as it turns into a Red Giant. Red Giants are much more luminous,
 larger in size, and generally cooler than the same stars in their main sequence
 phase, thereby occupying a region of the HR diagram accordingly
- Asymptotic Giant Branch The Red Giant star reaches a temperature where it
 can support the fusion of Helium. This onset of Helium fusion stops the cooling
 and expanding process, thereby leading to the increase in temperature and
 reduction in luminosity. This leads to the star moving downward and leftward

on the HR diagram in a region known as the Horizontal Branch. Eventually, the helium fusion stops, with the core now primarily composed of carbon and oxygen. This exhaustion of helium fuel leads to the star cooling and expanding again in a process similar to when it started running out of hydrogen as fuel, and thus, it moves upwards and rightwards again, in a path that almost aligns with the Red Giant Branch, hence yielding the name- Asymptotic Giant Branch.

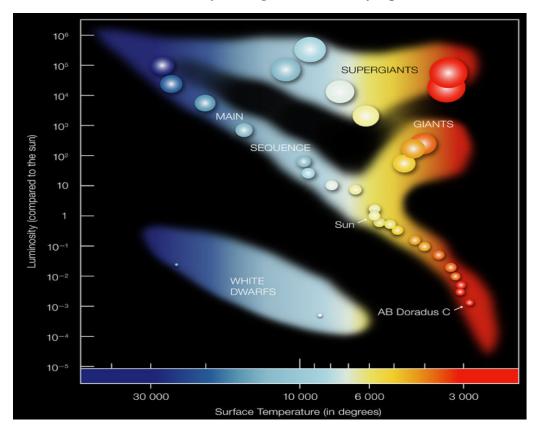


Figure 3: Hertzsprung-Russell diagram Image courtesy: European Southern
Observatory

8. Colour Magnitude Diagram

The Colour Magnitude Diagram (or CMD) is a plot of observational data which shows how a population of stars can be plotted in terms of their brightness (or luminosity) and colour (or surface temperature). The fact that we are able to interpret a star's colour as a measure of its temperature is based on the idea that stars can be considered as black-body sources, enabling us to use Wien's Law which states that the blackbody radiation curve for different temperatures will peak at different wavelengths that are inversely proportional to the temperature. It is this temperature which we can use to plot the star's spectral type on the x-axis. Colour Magnitude diagrams are the same as HR diagrams but different terms are used for representing the stars.

9. Velocity Measurements

Velocities of stars as measured by instruments on the celestial sphere can be divided into 2 types:

• Radial Velocity

This is the speed of approach or recession that is generally obtained from the Doppler shift of the source given by the formula, where all the symbols have their usual meaning.

$$v_r = (\Delta \lambda / \lambda) c$$

Proper Motion

This is the motion of the stars in the plane of the celestial sphere and usually has (units seconds of arc/year). Compared to the cyclic nature of parallax measurements, proper motions have the distinct advantage of being cumulative measurements i.e. the small angular measurements can be made over several years.

Tangential Velocity

Proper motion comes from tangential velocity i.e. linear velocity in the direction perpendicular to the line of sight. To convert the angular measure of proper motion (second of arc/year) to linear velocity (km/s) we use the following equation

$$v_t = d \sin \mu \approx \mu d$$

10. Velocity Dispersion

Velocity Dispersion refers to the statistical dispersion of velocities about the mean velocity for a group of celestial objects like galaxy clusters or globular clusters. Velocity dispersion can be estimated from the radial velocity. Higher the radial velocity, greater is the accuracy of the velocity dispersion.

11. Magnitude

In astronomy, magnitude refers to the apparent brightness of a celestial object. The origin of this scale dates back to the time of Hipparchus and Ptolemy. On this scale, the brightest star in the night sky as seen by naked eye is placed at magnitude 1, while a

less brighter star will be placed higher in magnitude. The brighter the star, the lower is the magnitude, while fainter stars sit higher on the magnitude scale.

Early photometric measurements showed that the ratio of brightness between 1st magnitude stars to the 6th magnitude stars is about 100. This resulted in adoption of a logarithmic scale of 2.512. This resulted in every interval of one magnitude getting separated by a factor of 100 in terms of brightness.

Astronomers use two main types of magnitudes, namely Apparent magnitude and Absolute magnitude.

 Apparent Magnitude: Apparent magnitude is based on the relative brightness of the star in the night sky. Mathematically, the apparent magnitude can be calculated as

$$m_2 - m_1 = -2.50 \log \left(\frac{B2}{B1}\right)$$

where m_1 and m_2 is the magnitude of the two stars and B1 and B2 are the respective brightness of the stars

Absolute Magnitude: Absolute magnitude is the apparent magnitude a star
would have if it were to place at a specific distance. Usually the distance is taken
to be 10 parsecs from earth. Since brightness of a star follows inverse square law,
it follows

$$\left(\frac{B10}{Bd}\right) = \left(\frac{d}{10}\right)^2$$

using this equation we can calculate absolute magnitude as

M - m = 2.5 log
$$\left[\left(\frac{d}{10} \right)^2 \right]$$

where M is the absolute magnitude, m is the apparent magnitude and d is the distance of the stars in parsecs.

12. Coordinate System

Celestial coordinate system is a coordinate system which is used to specify the location of a stellar object in the sky relative to earth. There are several kinds of coordinate systems described in the following subsections.

12.1 Horizontal (Altitude-Azimuth) Coordinate System

The Horizontal coordinate system, also known as altitude-azimuth system, is a local coordinate system based on an observer's location. In this coordinate system, the positions of stellar objects vary with time. This coordinate system is only good for finding the local position of celestial bodies.

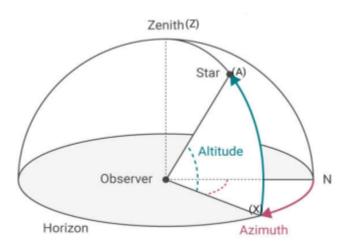


Figure 4 Horizontal coordinate system

12.2 Equatorial Coordinate System

Equatorial coordinate system is a spherical coordinate system centred at Earth's centre. In this coordinate system, the positions of the bodies are measured relative to the earth's equatorial plane and March equinox. Following figure describes how the position of the bodies are measured in the Equatorial system.

- Declination : Declination, symbolised as δ is the measure of angular distance of a body perpendicular to the celestial equator measured from the north pole.
- Right ascension: Right ascension, α is the measure of angular distance of an object eastward along the celestial equator from the vernal equinox. Vernal equinox is the point where the ecliptic meets the celestial equator. The units of right ascension are hours, minutes and seconds.

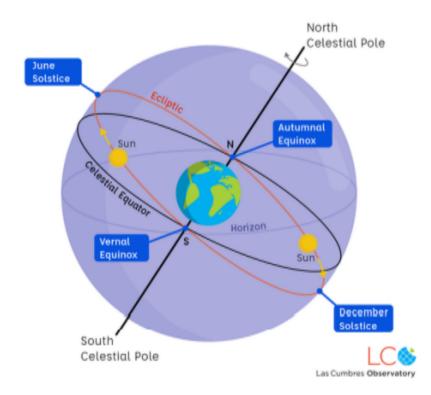


Figure 5: Equatorial coordinate system

12.3 Conversion of degrees to kiloparsecs

We have used the parallax method to convert xi and eta(Coordinates of a star wrt the centre of andromeda galaxy) from degrees to kiloparsecs. Parsec is the distance from the sun to an astronomical object which has a parallax angle of one arc second (1/3600 of a degree).

Parallax method: Parallax is a method of using two points of observation to measure the distance to an object by observing how it appears to move against a background. If we take the centre of the Andromeda galaxy as the base line, every star will be inclined with a precise angle. We will convert this angle from degrees to radians. Then we will multiply the angle in radians with 783 kpc(distance between us and andromeda galaxy). This will give us the value of the conversion factor which is equal to 13.66. We have converted degrees to kpc by multiplying the coordinates by conversion factor 13.66.

13. Procedure

13.1 Software Used

In this project we have used Python which is an interpreted high-level general-purpose programming language. The libraries of python used here are described below:

Numpy: NumPy, which stands for Numerical Python, is a library consisting of multidimensional array objects and a collection of routines for processing those arrays. Using NumPy, mathematical and logical operations on arrays can be performed.

Matplotlib: Matplotlib is a popular Python package used for data visualisation. matplotlib.pyplot is a plotting library used for 2D graphics in python programming language. It can be used in python scripts, shell, web application servers and other graphical user interface toolkits.

13.2 Data Collection

We have used 2 different types of data and collabed them into one. The radial velocity measurements are taken from the Spectroscopic and Photometric Landscape of Andromeda's Stellar Halo survey (SPLASH) using Keck/DEIMOS multi object spectrograph. The photometry of stars is from HST via the PHAT survey. In this photometry, the filters used were F475W and F814W. The collabed txt file of both data is shown below.

We import a data catalogue that includes Ra, Dec, xi, eta, filter magnitudes, velocity and error velocity(Columns from left to right) of 5800 stars.


```
File
      Edit
             View
                     Language
    #RA DEC XI ETA F475W F814W V (km/s) VERR (km/s)
    #Coords all in degrees
    10.908 41.138 0.168 -0.131 24.801 21.463 -165.868 4.393
    10.909 41.139 0.169 -0.13 24.294 20.212 -712.209 1.286
    10.911 41.141 0.17 -0.128 23.584 20.872 -339.004 2.925
    10.911 41.143 0.17 -0.126 24.09 20.381 -270.403 2.054
    10.922 41.149 0.179 -0.12 24.745 21.008 -346.327 1.589
    10.907 41.154 0.167 -0.115 23.823 21.169 -148.785 3.345
    10.924 41.155 0.18 -0.114 24.306 21.109 -451.149 1.952
    10.902 41.155 0.164 -0.114 24.493 21.279 -227.198 2.831
    10.928 41.155 0.183 -0.114 23.35 20.775 -194.203 3.614
    10.869 41.156 0.139 -0.114 24.191 20.733 -225.242 2.067
    10.891 41.156 0.155 -0.113 25.755 20.997 -491.384 3.211
    10.863 41.157 0.134 -0.112 23.59 20.659 -314.538 2.014
    10.894 41.157 0.158 -0.112 24.199 21.227 -402.357 2.77
    10.929 41.158 0.184 -0.111 24.437 21.107 -422.092 1.526
    10.866 41.161 0.136 -0.108 23.596 21.148 -395.45 7.732
    10.916 41.161 0.174 -0.108 23.835 21.103 -399.92 2.075
    10.868 41.161 0.138 -0.108 24.491 21.298 -661.258 2.95
    10.919 41.162 0.176 -0.107 25.605 20.983 -96.263 3.553
    10.848 41.164 0.123 -0.105 23.369 20.594 -392.74 2.159
    10.887 41.168 0.152 -0.101 24.079 21.811 -283.92 4.604
    10.927 41.168 0.183 -0.101 23.9 20.945 -164.04 4.004
    10.934 41.17 0.188 -0.099 23.486 21.028 -118.381 5.378
    10.944 41.172 0.195 -0.098 23.679 21.279 -497.461 5.517
    10.892 41.172 0.156 -0.097 25.639 21.183 -385.614 2.227
    10.884 41.175 0.15 -0.094 24.36 20.997 -21.365 3.505
    10.946 41.177 0.197 -0.092 24.026 21.422 -320.588 3.677
    10.922 41.18 0.179 -0.089 24.261 20.579 -228.675 4.092
    10.936 41.182 0.189 -0.087 24.61 20.879 -712.273 2.119
```

Figure 6: Txt file of collabed data with all required parameters.

We have used Jupyter Notebook as a platform to import and analyse data and also for plotting the graphs.

13.3 Importing of Data

In this project, we use only stars in the intersection of these two surveys: those with both PHAT optical photometry and reliable SPLASH-derived radial velocities. We collabed both data into a single text file using csv editor and saved that txt file in jupyter user folder. (Fig 7)

13.4 Reading data

We will use jupyter notebook root folder to access the data and numpy to read the data.



Figure 7: Saving data and separate catalogues in jupyter user folder

13.5 Plotting CMD

We will be plotting the Color-Magnitude diagram of the stars and from that we take separate the stars in the corresponding AGB, RGB and MS sequence, after which we store the name of the star, its Cartesian position, radial velocity and magnitude values into a 3 files for further reference. The absolute magnitude is the apparent magnitude

of a star it would appear if placed at a distance of 10 parsecs. Different filters can be used to observe the different wavelengths magnitudes and by subtracting these magnitudes we can observe the colour. The two filters that are used here are F814W (I), which corresponds to a wavelength of 8133.06 °A and F475W(B) corresponding to 4826.43 °A. The data is taken such that the position of the stars is relative to the centre of M31 and the distance is normalised, therefore while plotting we must multiply by a scaling factor.

13.6 Plotting Velocity map

Here we used the last 2 columns of our data viz. Radial velocities and error in radial velocity of stars. The mean of all velocities is -81 km/s which is negative. So, mostly all stars will have negative velocity. So, we have chosen minimum negative velocity as -700 which will show blue shift and 200 as maximum velocity which will show yellow shift. None of the stars will show a redshift as the velocity values are not highly positive enough. So, we chose a legend colour bar named 'Viridis' from the matplotlib module which provides a number of plotting legend colour bars. This bar provides a colorise shade from blue to yellow, which is most suitable for our plot. In the x and y axis, we plotted the position of the stars in kiloparsecs. For this, we converted the values from degrees to kiloparsec using a conversion factor of 13.67. Every degree value is then converted into kiloparsecs and a velocity map of 5800 stars of inner 20kpc andromeda is plotted.

13.7 Plotting velocity dispersion maps

Velocity dispersion is a statistical dispersion of velocities about a mean velocity of a group of astronomical objects like clusters, stars, etc. Once we have plotted the velocity

map, we can plot the same for each of the group of stars but instead of plotting the velocities through the colour bar, if we plot the velocity dispersion, that would give us information on how the stars move and where there is a higher activities taking place. To plot the velocity dispersion we need to choose a star and draw a small circle around it. Now we have to find the standard deviation of the velocities of all the stars which lie in that circle and assign the value to the corresponding star.

$$\sigma = \sqrt{\frac{\sum (xi - \mu)^2}{N}}$$

where N is the population number, μ is the population mean and xi each value from the population

However, in Python the standard deviation function can be used for this. Once we have plotted and saved the standard deviation, we will be plotting the histogram of these deviations and then print the median to understand the results better.

14. The Code

14.1 Importing all necessary modules

import numpy as np

import matplotlib.pyplot as plt

%matplotlib inline

from PIL import Image

import matplotlib as mpl

#Jupyter notebook ignores all the unnecessary warnings

import warnings

```
warnings.filterwarnings('ignore')
```

```
#Setting up font and font size

mpl.rcParams['font.family']='serif'

mpl.rcParams['font.size']=14
```

14.2 Code to initialise the plot function

#This function initialises the CMD plot and labels the axes.

```
def InitializePlot():
```

```
plt.clf()
f= plt.figure(figsize=(8,8))
plt.xlim(-1, 9)
plt.ylim(24, 18)
plt.xlabel('F475W - F814W (mag)')
plt.ylabel('F814W (mag)')
```

14.3 Code to store data into files

#This function writes a catalogue. It takes as arguments the name of the file,
#the different quantities you want to put in the catalogue, and the names of these
quantities

```
def WriteCatalog(catalogName, params, paramNameString):
    #first, create and open the text file for writing. Choose a name:
    catalog = open(catalogName, 'w')

#write first row
    catalog.write('#' + paramNameString + '\n')

#then, loop through stars and write 1 row for each star.

for i in range(len(params[0])): #loop over stars

for j in range(len(params)): #loop over parameters (ra, dec, v, verr,...)
    catalog.write(str(params[j][i]))
    catalog.write('\n')
    catalog.close()
```

14.4 Retrieving Data

Return

```
#The first thing we need to do is read in the file that contains all our data.

#This file contains the coordinates (ra, dec, xi, eta),

#the magnitudes (in two different bandpasses), and the velocities of all our stars
ra, dec, xi, eta, f475w, f814w, v, verr = np.loadtxt('../data/keck_hst_data.txt', unpack = True)
```

the color of the star is a very useful and important quantity; we define the array for "color" below

color = f475w - f814w

14.5 Plotting Color-Magnitude Diagram

#Now we can plot a color magnitude diagram of all the objects in our data

f = InitializePlot()

plt.scatter(color, f814w, c = 'grey', s = 3, edgecolors = 'none')

#x,y=(xi, eta)=(conversion of ra and dec into degrees)

mag1=f475w and mag2=f814w

x, y, mag1, mag2, v, verr = np.loadtxt('../data/keck_hst_data.txt', usecols=(2,3,4,5,6,7), unpack = True)

#to find the color of the star

color = mag1 - mag2

14.6 Code to separate stars into 3 different groups

#Where condition to isolate main sequence stars

group1 = (color < 1.75) & (mag2 < 23)

#Plot MS stars in blue on the plot and label that region

f = InitializePlot()

```
plt.scatter(color, mag2, c = 'gray', s = 3, edgecolors = 'none')
plt.scatter(color[group1], mag2[group1], c = 'blue', edgecolors = 'none', s = 5)
plt.text(-0.5, 23.5, 'Group 1', color = 'blue', size = 16)
plt.xlabel('F475W - F814W')
plt.ylabel('F814W')
#RGB(Red giant branch) stars
#We will use a parabolic equation to isolate RGB stars: a*(color-h)**2.+k
#a:Focus of parabola, (h,k)=vertex of parabola
#Reference is provided in reference section
a = 0.1 \#Inclination
h = 2.1 # on x axis
k = 20.4 \# on \ y \ axis
group2 = (color > 1.75) & (mag2 > a*(color-h)**2.+k) #setting up values for x and y
cordinates
f = InitializePlot()
#Add the Group 2 stars to the plot!
plt.scatter(color, mag2, c = 'gray', s = 3, edgecolors = 'none')
plt.scatter(color[group2], mag2[group2], c = 'red', edgecolors = 'none', s = 5)
plt.text(4, 22.8, 'Group 2', color = 'red', size = 16)
plt.xlabel('F475W - F814W')
```

plt.ylabel('F814W')

```
#Agb(Asymptotic giant branch) Stars
group3 = (color > 1.75 ) & (mag2 < a*(color-h)**2.+k)
#Add the Group 3 stars to the plot!
f = InitializePlot()
plt.scatter(color, mag2, c = 'gray', s = 3, edgecolors = 'none')
plt.scatter(color[group3], mag2[group3], c = 'green', edgecolors = 'none', s = 5)
plt.text(6.5, 19.0, 'Group 3', color = 'green', size = 16)
plt.xlabel('F475W - F814W')
plt.ylabel('F814W')</pre>
```

#Grouping all the stars groups in single plot

```
f = InitializePlot()

plt.scatter(color[group1], mag2[group1], c = 'blue', edgecolors = 'none', s = 5)

plt.text(-0.5, 23.5, 'Group 1', color = 'blue', size = 16)

plt.scatter(color[group2], mag2[group2], c = 'red', edgecolors = 'none', s = 5)

plt.text(4, 22.8, 'Group 2', color = 'red', size = 16)

plt.scatter(color[group3], mag2[group3], c = 'green', edgecolors = 'none', s = 5)

plt.text(6.5, 19.0, 'Group 3', color = 'green', size = 16)

plt.xlabel('F475W - F814W')

plt.ylabel('F814W')
```

14.7 Saving the 3 groups into 3 separate files

#Writing catalogues for all stars groups and saving them separately.

```
WriteCatalog('../data/Group1catalog.txt',
[x[group1], y[group1], mag1[group1], mag2[group1], v[group1], verr[group1]],
'X Y MAG1 MAG2 V VERR')
WriteCatalog('../data/Group2catalog.txt',
[x[group2], y[group2], mag2[group2], mag2[group2], v[group2], verr[group2]],
'X Y MAG1 MAG2 V VERR')
WriteCatalog('../data/Group3catalog.txt',
[x[group3], y[group3], mag1[group3], mag2[group3], v[group3], verr[group3]],
'X Y MAG1 MAG2 V VERR')
# Determine what fraction of the entire catalog are selected by the group criteria
nstars = len(color)
print('Fraction of stars in Group 1: {0:.3f}'.format(np.sum(group1)/nstars))
print('Fraction of stars in Group 2: {0:.3f}'.format(np.sum(group2)/nstars))
print('Fraction of stars in Group 3: {0:.3f}'.format(np.sum(group3)/nstars))
selected = group1 | group2 | group3
print('Fraction of stars not selected: {0:.3f}'.format(np.sum(~selected)/nstars))
Fraction of stars in Group 1: 0.155
```

Fraction of stars in Group 2: 0.537

Fraction of stars in Group 3: 0.149

Fraction of stars not selected: 0.160

14.8 Converting Degrees to kpc

#Conversion of degree coordinates of stars in kpc

```
x, y, mag1, mag2, v, verr = np.loadtxt('../data/keck_hst_data.txt', unpack = True,
usecols=(2,3,4,5,6,7))
#Converting degrees to kiloparsecs
scale_factor = 13.674
x *= scale_factor # x cordinate is xi of a star
y *= scale_factor # y cordinate is eta of a star
```

14.9 Plotting the velocity map

```
#Plotting the velocity map of stars of whole data
```

```
ax = plt.axes(aspect = 'equal') # equalizing the axes
ax.set_xlabel('x')
ax.set_ylabel('y')
ax.set_xlim(15, -5) #setting up values for x axis and y axis
ax.set_ylim(-10, 15)
cc = ax.scatter(x, y, c = v, s = 2, edgecolors = 'none', vmin = -700, vmax = 100,
cmap='viridis')
ax.text(-13, 12, 'Positive Velocity')
ax.text(-18, 5, 'Neither')
ax.text(-13, -9, 'Negative Velocity')
plt.colorbar(cc, label = 'Velocity')
```

14.10 Initialising plot for velocity dispersion

```
# We are introducing the function "show_map."

def show_map(x, ey, quantity, label, fileName):

ax = plt.axes(aspect = 'equal')

ax.set_xlabel('x')

ax.set_ylabel('y')

ax.set_title('Dispersion of Real Data')

tt = ax.scatter(x, y, c = quantity, edgecolors = 'none', s = 2, vmin=40, vmax=160)

ax.set_xlim(1, 0)

plt.colorbar(tt, label='Velocity Dispersion')

plt.savefig(fileName)
```

14.11 Code to calculate Dispersion

```
#This next function is where we are calculating the dispersion.(Standard deviation of
stars)
def get_dispersion(x, y, v, circleSize=300./3600.):
  #initialize dispersion to all zeros
  dispersion = np.zeros(len(x))
  #Loop over all items in list
  for i in range(len(x)):
     #get new coordinates
     this X = x[i]
     this Y = y[i]
     #calculate array of distances to this star (use the distance formula)
     dist = np.sqrt((x-thisX)^{**}2 + (y-thisY)^{**}2)
     #pick out stars that are closer than circleSize to the star
     keep = dist < circleSize
     #calculate the velocity dispersion (standard deviation) of those stars
     #assign to the ith element of the array "dispersion"
     dispersion[i] = np.std(v[keep])
  return dispersion
print()
```

14.12 Plotting velocity dispersion for 3 separate star groups

```
#Plotting velocity dispersion of group 1: MS
#Read data
x, y, v = np.loadtxt('.../data/Group1catalog.txt', unpack = <math>True, usecols = (0, 1, 4))
#Calculate dispersion
group1_sigma = get_dispersion(x, y, v)
print(np.min(group1_sigma), np.max(group1_sigma), np.mean(group1_sigma))
#Generate velocity map
show_map(x, y, group1_sigma, 'Velocity dispersion',
'../plots/Group1_dispersion_map.png')
print()
32.19338416895621 140.89330673158196 65.23077476108917
#Plotting velocity dispersion of group 2: RGB
x, y, v = np.loadtxt('.../data/Group2catalog.txt', unpack = True, usecols = (0, 1, 4))
keep=(np.abs(v) < 1000.)
x=x[keep]
y=y[keep]
v=v[keep]
#Calculate dispersion
group2_sigma = get_dispersion(x, y, v)
print (np.min(group2_sigma), np.max(group2_sigma), np.mean(group2_sigma))
```

```
#Generate velocity map
show_map(x, y, group2_sigma, 'Velocity dispersion',
'../plots/Group2_dispersion_map.png')
79.20975639795124 181.34957736129712 107.67844272054022
#Plotting velocity dispersion of group 3: AGB
#Read data
x, y, v = np.loadtxt('.../data/Group3catalog.txt', unpack = <math>True, usecols = (0, 1, 4))
keep=(np.abs(v) < 1000.)
x=x[keep]
y=y[keep]
v=v[keep]
#Calculate dispersion
group3_sigma = get_dispersion(x, y, v)
print (np.min(group3_sigma), np.max(group3_sigma), np.mean(group3_sigma))
#Generate velocity map
show_map(x, y, group3_sigma, 'Velocity dispersion',
'../plots/Group3_dispersion_map.png')
38.87868852146071 170.3387510362113 95.3792774428375
```

#Finding average value of velocity dispersion of each group of stars

```
print('Median Group 1 sigma (km/s): {0:.1f}'.format(np.median(group1_sigma)))

print('Median Group 2 sigma (km/s): {0:.1f}'.format(np.median(group2_sigma)))

print('Median Group 3 sigma (km/s): {0:.1f}'.format(np.median(group3_sigma)))

Median Group 1 sigma (km/s): 60.4

Median Group 2 sigma (km/s): 105.3

Median Group 3 sigma (km/s): 92.3
```

14.13 Plotting histogram

```
#Plotting histogram for comparison
ax = plt.axes()
__,__ = ax.hist(group1_sigma, bins=30,range=[0,180], histtype='stepfilled', alpha=0.3,
color='b', zorder=1)
__,__ = ax.hist(group1_sigma, bins=30, range=[0,180], histtype='step',
color='b', zorder=2)
ax.text(150, 200, 'Group 1', color='b')
ax.set_xlim(25,180)
ax.set_xlabel('Velocity Dispersion')
ax.set_ylabel('Number of stars')

ax = plt.axes()
```

,, = ax.hist(group2_sigma, bins=30, range=[0,180], histtype='stepfilled', alpha=0.3,

```
color='q', zorder=1)
_,_, = ax.hist(group2_sigma, bins=30, range=[0,180], histtype='step',
color='g', zorder=2)
ax.text(150, 600, 'Group 2', color='g')
ax.set xlim(25,180)
ax.set xlabel('Velocity Dispersion')
ax.set_ylabel('Number of stars')
ax = plt.axes()
_,_, = ax.hist(group3_sigma, bins=30, range=[0,180], histtype='stepfilled', alpha=0.3,
color='r', zorder=1)
_,_, = ax.hist(group3_sigma, bins=30, range=[0,180], histtype='step',
color='r', zorder=2)
ax.text(150,125, 'Group 3', color='r')
ax.set_xlim(25,180)
ax.set_xlabel('Velocity Dispersion')
ax.set_ylabel('Number of stars')
ax = plt.axes()
_,_, = ax.hist(group1_sigma, bins=30, normed=1, range=[0,180], histtype='stepfilled', alpha=0.3,
color='b', zorder=1)
_,_, = ax.hist(group2_sigma, bins=30, normed=1, range=[0,180], histtype='stepfilled', alpha=0.3,
color='g', zorder=1)
_,_, = ax.hist(group3_sigma, bins=30, normed=1, range=[0,180], histtype='stepfilled', alpha=0.3,
color='r', zorder=1)
_,_, = ax.hist(group2_sigma, bins=30, normed=1, range=[0,180], histtype='step',
color='q', zorder=2)
_,_, = ax.hist(group3_sigma, bins=30, normed=1, range=[0,180], histtype='step',
color='r', zorder=2)
```

```
__,__ = ax.hist(group1_sigma, bins=30, normed=1, range=[0,180], histtype='step', color='b', zorder=2)

ax.text(150, 0.03, 'Group 1', color='b')

ax.text(150, 0.027, 'Group 2', color='g')

ax.text(150, 0.024, 'Group 3', color='r')

ax.set_xlim(25,180)

ax.set_xlabel('Velocity Dispersion')

ax.set_ylabel('Normalized Count')
```

15. RESULTS AND DISCUSSION:

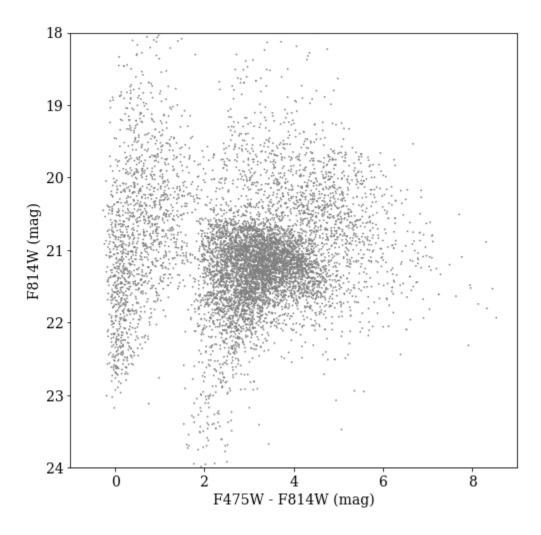


Figure 8: Grey CMD plot for used data

In the above figure, we have plotted F475W- F814W(mag) vs F814W(mag). In the Keck Deimos HST data, there are 2 columns with two spectroscopic filters. The difference between two magnitudes will give you a proxy of temperature (Higher the difference, higher the temperature value). The magnitude values in the data are in the range of 18 to 28. So, the difference between both magnitudes will range from 0 to 9. Therefore, we chose values for the x axis ranging from 0 to 8. On the other hand, the y axis will range

from 24 to 18 (Increasing range of luminosity). In this figure we can see that there are 3 different clusters of stars.

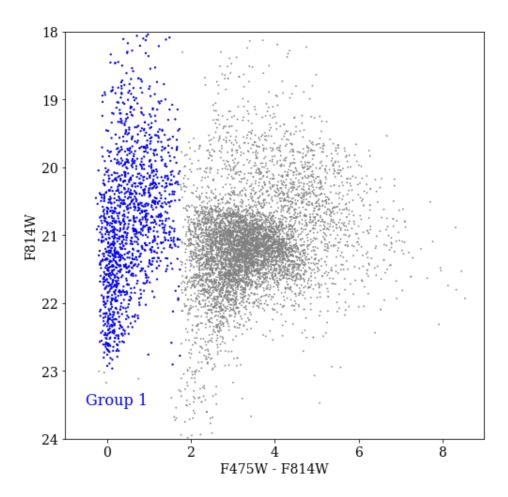


Figure 9: Highlighting MS stars

Main sequence stars follow a straight diagonal line in Hertzsprung-Russell diagram. Magnitude values of F474w and F814w are nearly the same with a very small difference. We can identify in the above figure a line pattern of stars with differences in magnitudes near around 0. Those are the main sequence stars. It also satisfies the condition where the MS sequence stars lie under RGB and AGB stars in the HR diagram. In our chosen data of 5800 stars of the particular region of Phat survey of inner 20kpc of andromeda galaxy, there are no blue giants and main sequence stars with higher temperature

values because Andromeda is a very old galaxy in which most of the main sequence stars have gone to the RGB region.

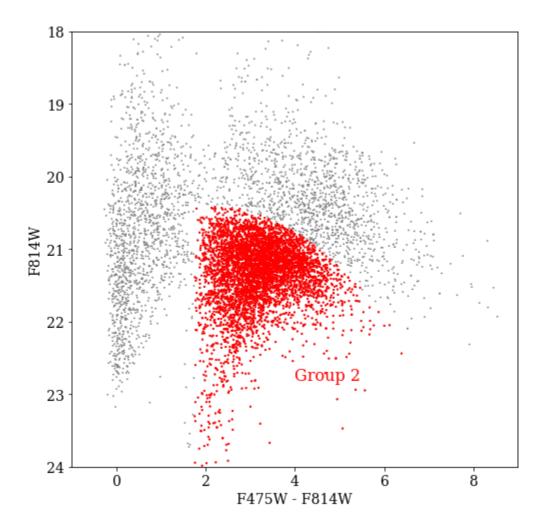


Figure 10: Highlighting RGB stars

This is the region where most of the stars are present. RGB stars are luminous and have low temperatures. RGB are red in colour so it corresponds to a specific wavelength of light which is closer to infrared wavelengths. F814w filter value is closest to infrared wavelength. So, RGB stars have higher magnitude values in F814w than F475w. So, they will have some difference in values. We chose in the figure those values to be greater

than 1.75 after the Main sequence stars. We separated AGB and RGB using a parabolic equation and highlighted them in red colour.

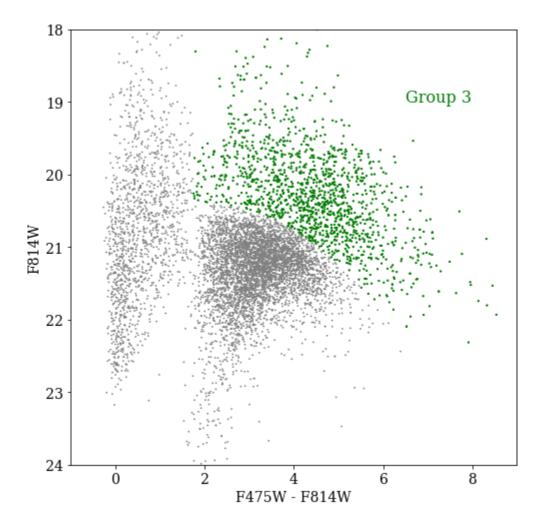


Figure 11: Highlighting AGB stars

Now, AGB is the only one group of stars remaining. The filter will have different values because AGB corresponds to wavelength near blue light as they are very luminous. So the F475w filter will have higher values as compared to F814w values. The remaining

stars will be AGB stars which lie above the RGB stars. They are color coded in green color.

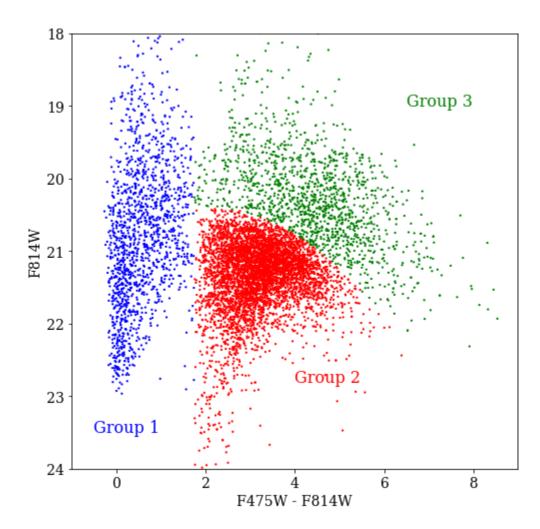


Figure 12: Plotting CMD with 3 colour groups

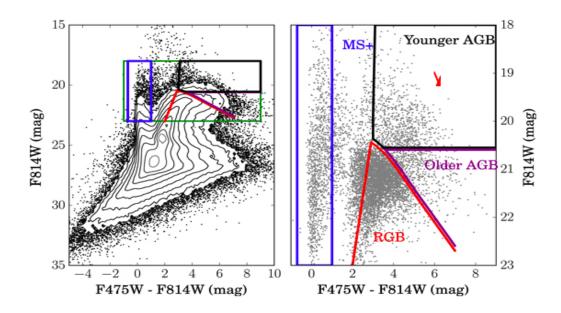


Figure 13: Hess Diagram indicating the PHAT survey stars and the region we have plotted in our CMD.

MWFG dwarfs can lie in the same magnitude window as our M31 spectroscopic targets. In figure 14, we can see that the stars in the mosaic (blurred) range are MWFG stars. These stars were removed in the data itself by pre- and post-spectroscopy. (More details in 3.1.1 in Dorman, C. E[5])

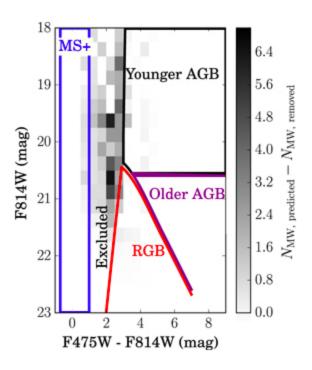


Figure 14: CMD with excluded region in the mosaic filter.

Image is taken from Dorman, C. E [5]

The MS, younger AGB, elder AGB, and RGB regions are delineated by the blue, black, violet, and red lines respectively. The most contaminated portion of the CMD—brighter than $m_{\rm F814W}=21$ and between $1 < m_{\rm F475W}-m_{\rm F814W} < 2$ is excluded from the analysis. After exclusion, MWFG stars should account for less than 0.1 percent of the stars in the sample.

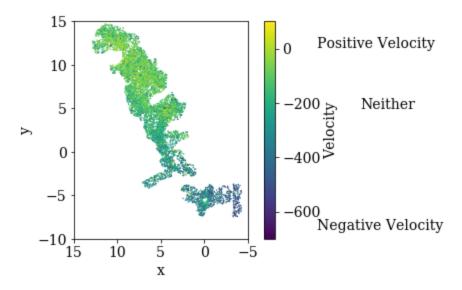


Figure 15: Velocity map of stars of used data in andromeda galaxy

Here we have plotted the graph of the velocity map where the x and y axis are xi and eta coordinates. From the graph we can observe that the stars closer to the centre of the galaxy have more negative velocity as compared to the top part of M31. When we calculated the mean net velocity with error, we found out it is equal to -81 km/s with net error of 10 km/s. From the velocity map we can observe that the maximum relative velocity of the stars are higher towards the negative side than the positive, and this means that Andromeda is moving towards the Milky way galaxy. From the graph we can also observe that some sections at the top have positive velocity indicating that the stars are moving away from the Milky Way. Thus proving that the galaxy is moving in an Anti-Clockwise direction and the section that we have selected is moving away from us a while with respect to the centre of M31. If we would have plotted the same graph but for the opposite section of M31 we would have got a velocity much more towards the negative side as the opposite part would be moving toward us rather than moving away from us.

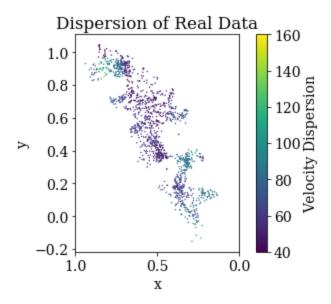


Figure 16: Velocity dispersion plot of MS stars

The Stars are observed to have less dispersion between 80-60 as seen from the graph. Main sequence stars are younger and less violent than the rest of the stars.

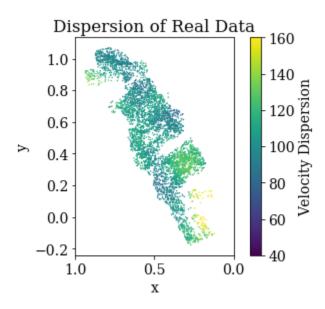


Figure 17: Velocity dispersion plot of RGB stars

We can see that RGB has the maximum number of star from the clusters that has been formed and we can also observe that RGB has the highest velocity dispersion.

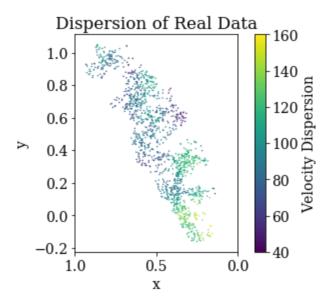


Figure 18: Velocity dispersion of AGB stars

Here velocity dispersion values of AGB stars are higher than the Main sequence stars and lower than the RGB stars. Stellar age of a star is proportional to its velocity dispersion value. From the different velocity dispersion graphs we can see the difference. RGB being in larger numbers in our set, we can see more stars and this is an indicator that this part of Andromeda is relatively older. RGB has a higher dispersion with median dispersion of 105.3 km/sec. AGB has relatively smaller dispersion with a median 92.3 km/sec; the reason here is that we have taken younger AGB stars and the number of these stars in our data set is comparatively less. MS with a median dispersion of 60.4 has the lowest radial velocity dispersion. The possible explanation for these different velocity values is the history of the andromeda. Billion of years ago, the Andromeda galaxy collided with the M32 galaxy and more collisions in the past. This collision resulted in the instability and caused the velocity dispersion of RGB and AGB stars to be higher than the MS. However, the difference between AGB and RGB velocity dispersion values is because of the size and number of RGB stars becoming white dwarfs.

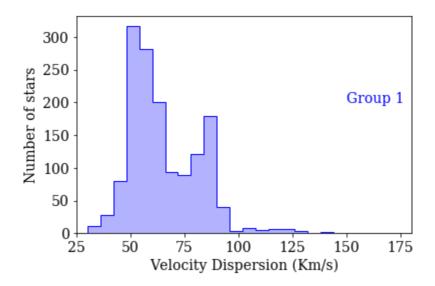


Figure 19: Histogram of velocity dispersion of Main sequence group with number of stars

In the x axis, we have chosen velocity dispersion values ranging from 25 to 175 because of the mean values we got for each group of stars. In the Y axis, we have taken the number of stars. Here most of the stars in main sequence has peak velocity dispersion values at around 50 km/s

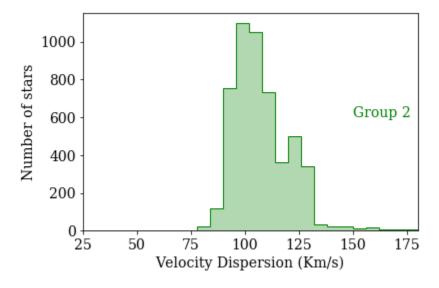


Figure 20: Histogram of velocity dispersion of RGB stars with number of stars

RGB contains the most number of stars with very high velocity dispersion values. The peak velocity dispersion value is around 100 km/s for more than 1000 stars.

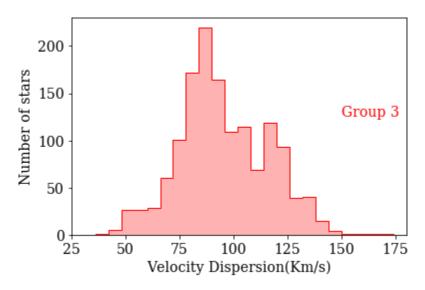


Figure 21:In this histogram, AGB peak is achieved at nearly 80 km/s for around 200 stars.

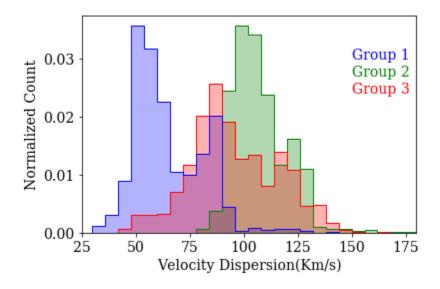


Figure 22: Histogram of velocity dispersion of all three groups with normalised count Here we combined all three groups and plotted the histogram with a normalised count at Y axis. We normalised the values under 1. Here the difference is clearly seen that RGB has highest dispersion values followed by AGB then MS stars. This with relation to stellar age tells us that RGB is the oldest region of the galaxy containing the maximum

number of stars with older age than MS and AGB stars. The age of RGB stars is nearly 4GYr. The age of AGB stars is 2GYr and 30-40 MYr for MS stars. So, we can directly correlate with velocity dispersion values we got in results. So, velocity dispersion and stellar are directly proportional. Older stars show higher velocity dispersion than younger stars.

16. Conclusion:-

For the above project, we have imported data from SPLASH and KECK/DEIMOS analysed the data for 5800 stars in M31 galaxy and we have divided the stars into Main sequence, Red Giant and Asymptotic Giant sequence. We plotted the colour-magnitude diagram and segregated the stars into 3 age bins. There was a clear distinction between MS, AGB and RGB. We further imported data of position for the 3 age bins into 3 different catalogues.

We used radial velocity and xi, eta coordinates columns from the data to plot the velocity map of 5800 stars from andromeda's inner 20 kpc disk. We observed that most of the stars have negative velocity. We can then conclude that this part of Andromeda galaxy is coming towards us by observation. Further study of the velocity map also tells us that the whole andromeda is moving towards us.

Further we used those catalogues to make velocity dispersion of separate age bins of stars by drawing a circle around a star and comparing its velocity to its all neighbouring star's velocities which lie inside the circle. We found the velocity dispersion of RGB is highest, and the Main sequence has the lowest. This result is due to age and stellar velocity dispersion correlation.

For further studies we can analyse the stars of some different galaxies in the Local Group, divide them into MS, RGB and AGB and see how the velocity dispersion of these groups differ. Further we can compare our result from M31 with that of the other galaxy and check if there is any relation between them.

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