LINE CONSTANTS

An ac transmission line has resistance, inductance & capacitance uniformly distributed along its length. These are known as constants of the line

1) Resistance

It's the opposition of line conductors to current flow. The resistance is distributed uniformly along the whole length of the line. The Resistance R of a line conductor having resistivity ρ, length l & area of crosssection a is given by

$$R = \frac{\rho l}{a}$$

If R_1 & R_2 are the resistances of a conductor at $t_1°C$ & $t_2°C$ ($t_2 > t_1$) & if α_1 is the temperature coefficient at $t_1°C$

then $$R_2 = R_1\left[1+\alpha_1(t_2-t_1)\right]$$

where $$\alpha = \frac{\alpha_0}{1+\alpha_0 t_1}$$

$\alpha_0 \rightarrow$ temperature coefficient at $0°C$

(i) In a single phase or 2-wired dc line, the total resistance or loop resistance is equal to double the resistance of either conductor.

(ii) In a 3ϕ transmission line, resistance per phase is the resistance of one conductor.

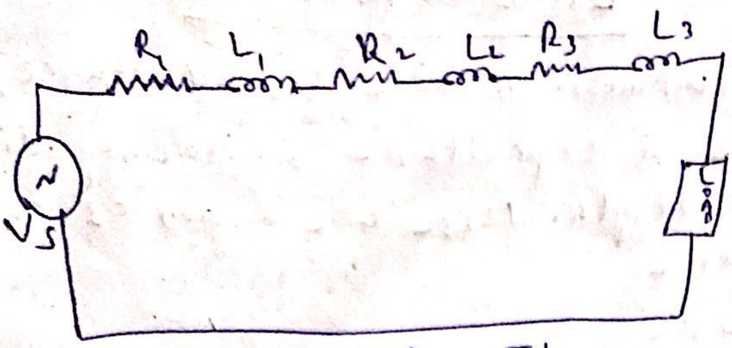
2) Inductance

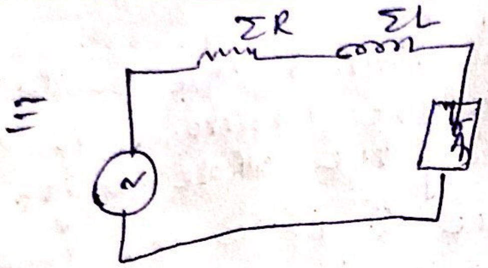
$$L = \frac{\psi}{I} \text{ henry}$$

ψ = flux linkage in weber turns

I = current in amperes

The inductance is also uniformly distributed along the length of the line.



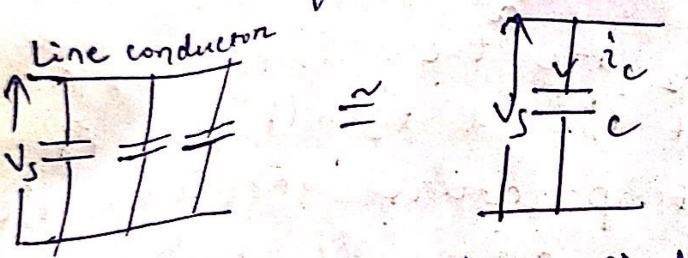


3) <u>Capacitance</u>

$$C = \frac{q}{V} \text{ farad}$$

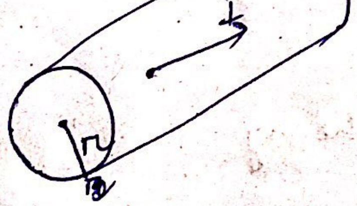
$q \rightarrow$ charge on the line

$V \rightarrow$ P.d. between conductors



Flux linkages due to a single current carrying conductor

Consider a long straight cylindrical conductor of radius r metres & carrying a current I amperes. This current will set up magnetic field. The magnetic lines of force will exist inside the conductor as well as outside the conductor. Both these fluxes will contribute to the inductance of the conductor.



① Flux linkage due to internal flux

$$H_x = \frac{I_x}{2\pi x}$$

$\oint H \cdot dl = I$ Ampere circuital law

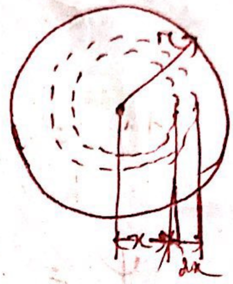
$\Rightarrow H_x 2\pi x = I_x$

$\Rightarrow H_x = \frac{I_x}{2\pi x}$

$$I_x = \frac{I}{\pi r^2} \cdot \pi x^2$$

$$H_x = \frac{I}{\cancel{\pi} r^2} \cdot \cancel{\pi} x^{\cancel{2}} \cdot \frac{1}{2\pi x}$$

$$= \frac{x}{2\pi r^2} I \quad AT/m$$



Flux density, $B_x = \mu_0 \mu_r H_x$

$$= \mu_0 H_x$$

$$= \frac{\mu_0 x}{2\pi r^2} I \quad Wb/m^2$$

Flux, $d\phi_x = B_x \cdot 1 \cdot dx$ for cylindrical shell of thickness dx & length 1 m

$$= \frac{\mu_0 x}{2\pi r^2} I \, dx \quad Wb$$

Flux linkages per metre length of the conductor

$$d\psi_x = \frac{d\phi_x}{\pi r^2} \cdot \pi x^2$$

$$= \frac{\mu_0 x}{2\pi r^2} \cdot \frac{1}{\pi r^2} \cdot \pi x^2 I \, dx$$

$$= \frac{\mu_0 x^3 I}{2\pi r^4} dx \quad Wb\text{-turns}$$

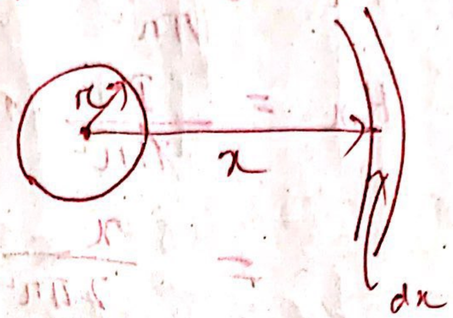
$$\psi_{int} = \int_0^r \frac{\mu_0 x^3}{2\pi r^4} dx$$

$$= \frac{\mu_0 I}{8\pi} \quad \text{Wb-turns per meter length}$$

(2) Flux linkage due to external flux

$$H_x = \frac{I}{2\pi x} \quad AT/m$$

$$B_x = \frac{\mu_0 I}{2\pi x} \quad wb/m^2$$



$$d\phi = B_x \cdot 1 \cdot dx$$

$$= \frac{\mu_0 I}{2\pi x} dx$$

$$d\psi = \frac{\mu_0 I}{2\pi x} dx$$

$$\psi_{ext} = \int_r^D \frac{\mu_0 I}{2\pi x} dx$$

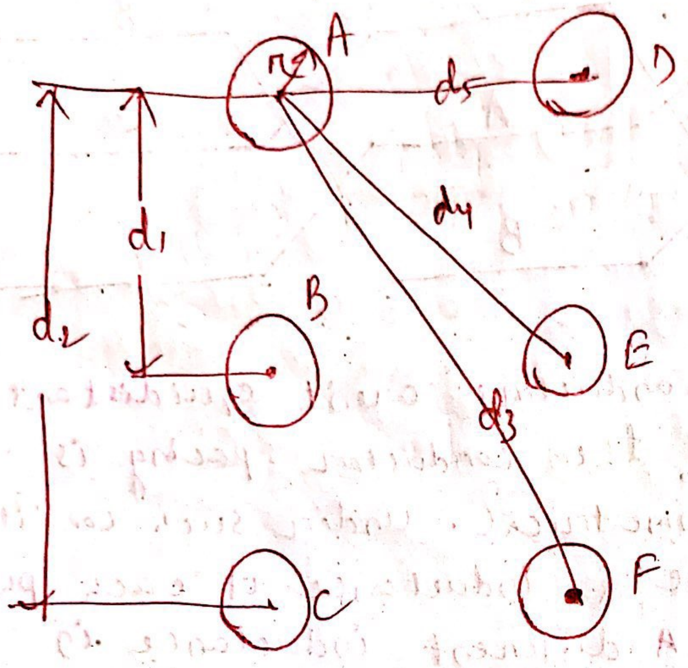
∴ Flux linkage due to single current carrying conductor

$$\therefore \psi = \psi_{int} + \psi_{ext}$$

$$\psi = \frac{\mu_0 I}{8\pi} + \int_r^D \frac{\mu_0 I}{2\pi x} dx$$

$$= \frac{\mu_0 I}{2\pi}\left[\frac{1}{4} + \int_r^D \frac{dx}{x}\right]$$

Flux linkages in parallel current carrying conductors

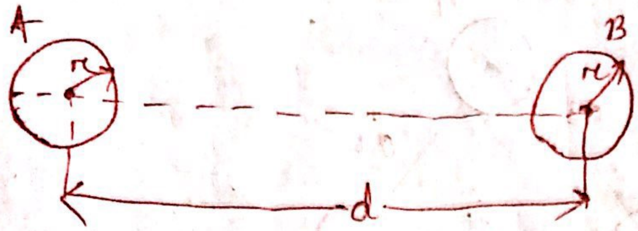


Flux linkage with conductor A

$$= \left[\frac{\mu_0 I_A}{2\pi}\left(\frac{1}{4} + \int_{r}^{\infty}\frac{dx}{x}\right)\right] + \left[\frac{\mu_0 I_B}{2\pi}\int_{d_1}^{\infty}\frac{dx}{x}\right]$$

$$+ \left[\frac{\mu_0 I_C}{2\pi}\int_{d_2}^{\infty}\frac{dx}{x}\right] + - - - - -$$

Inductance of a 1ϕ two-wire line



$$I_A + I_B = 0$$

$$\cancel{\psi_A = \frac{\mu_0 I}{8\pi} +}$$

$$\psi_A = \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} + \int_r^{\infty}\frac{dx}{x}\right] + \frac{\mu_0 I_B}{2\pi}\int_d^{\infty}\frac{dx}{x}$$

$$= \frac{\mu_0}{2\pi}\left[\left[\frac{1}{4} + (\ln\infty - \ln r)\right]I_A + I_B(\ln\infty - \ln d)\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{1}{4} + (I_A + I_B)\ln\infty - \ln r\, I_A - \ln d\, I_B\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{1}{4}I_A + I_A \ln\frac{d}{r}\right]$$

$$= \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} + \ln\frac{d}{r}\right] \text{ Wb-turns/m}$$

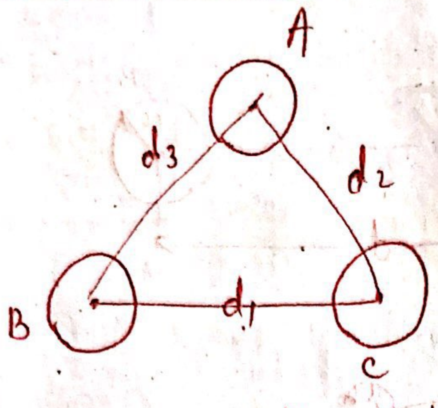
$$\therefore L_A = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \ln\frac{d}{r}\right]$$

$$= 10^{-7}\left[\frac{1}{2} + 2\log_e\frac{d}{r}\right]$$

loop inductance $= 2L_A$

$$= 10^{-7}\left[1 + 4\log_e\frac{d}{r}\right] \text{ H/m}$$

Inductance of a 3ϕ overhead line



Flux linkage with conductor A

$$\Psi_A = \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} + \int_r^{\infty}\frac{dx}{x}\right] + \frac{\mu_0 I_B}{2\pi}\int_{d_3}^{\infty}\frac{dx}{x}$$

$$+ \frac{\mu_0 I_C}{2\pi}\int_{d_2}^{\infty}\frac{dx}{x}$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} + \int_r^{\infty}\frac{dx}{x}\right) I_A + I_B\int_{d_3}^{\infty}\frac{dx}{x} + I_C\int_{d_2}^{\infty}\frac{dx}{x}\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A - I_B \log_e d_3 - I_C \log_e d_2\right]$$

Symmetrical spacing

$$d_1 = d_2 = d_3 = d$$

$$\Psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A - (I_B + I_C)\log_e d\right]$$

$$= \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} - \log_e r + \log_e d\right]$$

$$= \frac{\mu_0}{2\pi} I_A\left[\frac{1}{4} + \log_e \frac{d}{r}\right]$$

$$L_A = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \log_e \frac{d}{r}\right]$$

$$= 2\times 10^{-7}\left[\frac{1}{2} + 2\log_e \frac{d}{r}\right] \text{ H/m}$$

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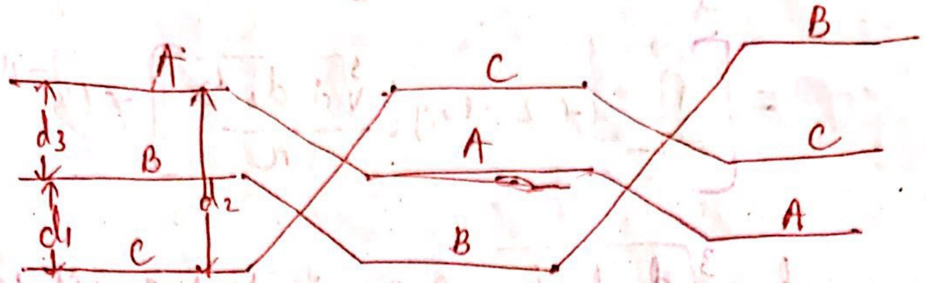
Unsymmetrical spacing



Let $I_A = I(1+j0)$

$I_B = I(-0.5 - j\,0.866)$

$I_C = I(-0.5 + j\,0.866)$

$$\psi_A = \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I_A - I_B \log_e d_3 - I_C \log_e d_2\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I - (-0.5 - j\,0.866) I \log_e d_3 - (-0.5 + j\,0.866) I \log_e d_2\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I + 0.5\, I (\log d_2 + \log d_3) + j\,0.866 \log_e d_3 - j\,0.866 \log_e d_2\right]$$

$$= \frac{\mu_0}{2\pi}\left[\left(\frac{1}{4} - \log_e r\right) I + 0.5\, I \log_e d_2 d_3 + j\,0.866 \log \frac{d_3}{d_2}\right]$$

$$= \frac{\mu_0 I}{2\pi}\left[\frac{1}{4} + \log_e \frac{\sqrt{d_2 d_3}}{r} + j\,0.866 \log \frac{d_3}{d_2}\right]$$

$$L_A = 10^{-7}\left[\frac{1}{2} + 2 \log_e \frac{\sqrt{d_2 d_3}}{r} + j\,1.732 \log \frac{d_3}{d_2}\right]$$

$$L_B = 10^{-7}\left[\frac{1}{2} + 2\log_e\frac{\sqrt{d_3 d_1}}{r} + j1.732\log_e\frac{d_1}{d_3}\right] H/m$$

$$L_C = 10^{-7}\left[\frac{1}{2} + 2\log_e\frac{\sqrt{d_1 d_2}}{r} + j1.732\log_e\frac{d_2}{d_1}\right]$$

$$L = \frac{1}{3}(L_A + L_B + L_C)$$

$$= \left[\frac{1}{2} + 2\log_e\frac{\sqrt[3]{d_1 d_2 d_3}}{r}\right] \times 10^{-7} \; H/m$$

$d = \sqrt[3]{d_1 d_2 d_3}$ → equivalent equilateral spacing

(1) A single phase line has two parallel conductors 2m apart. The diameter of each conductor is 1.2 cm. Calculate the loop inductance per km of line.

$d = 200$ cm

$r = 0.6$ cm

loop inductance per metre length of line $= 10^{-7}\left(1 + 4\log_e\frac{d}{r}\right)$

$$= 10^{-7}\left(1 + 4\log_e\frac{200}{0.6}\right)$$

$$= 24.23 \times 10^{-7} \; H$$

loop inductance per km

$$= 24.23 \times 10^{-7} \times 1000$$

$$= 24.23 \times 10^{-4} \; H$$

(2) Find the inductance per km of a 3ϕ transmission line using 1.24 cm diameter conductors when these are placed at the corners of an equilateral triangle of each side 2m.

Ans. inductance per phase / m

$$= 10^{-7}\left(0.5 + 2\log\frac{d}{r}\right)$$

$$= 12 \times 10^{-7}$$

per km $= 12 \times 10^{-7} \times 10^{3}$

(3) The three conductors of a 3 ϕ line are arranged at the corners of a triangle of sides 2m, 2.5m, & 4.5m. Calculate inductance per km of line when conductors are regularly transposed. The diameter of each conductor is 1.24 cm.

$$D_{eq} = \sqrt{D_{12}D_{23}D_{31}}$$

$$= \sqrt{2 \times 2.5 \times 4.5}$$

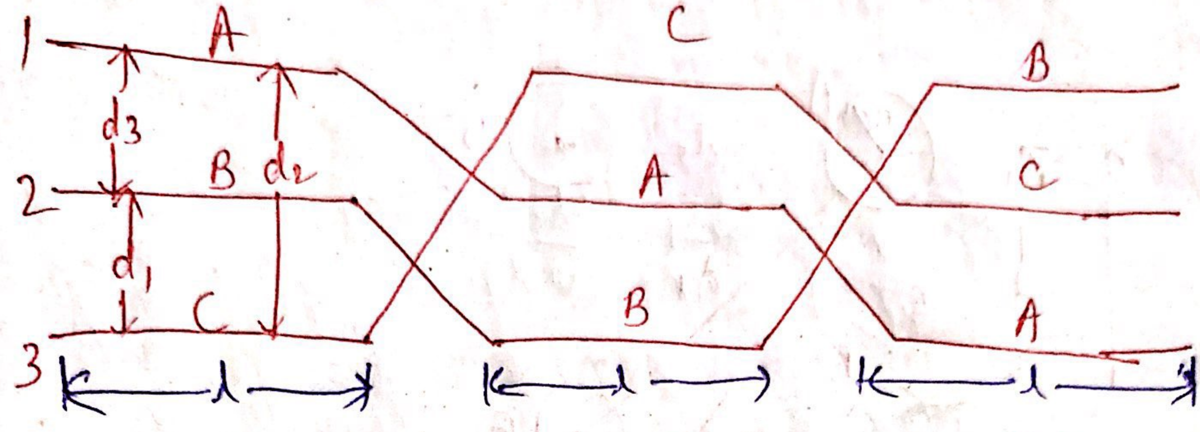
$$= 2.82 \text{ m}$$

$$= 282 \text{ cm}$$

inductance per phase / m $= \frac{1}{2} + 2 \log \frac{D_{eq}}{r}$

$$= 12.74 \times 10^{-7}$$

Transposition of lines



When 3ϕ line conductors aren't equidistant from each other, the conductor spacing is said to be unsymmetrical. Under such conditions, the flux linkage & inductance of each phase aren't the same. A different inductance in each phase results in unequal voltage drops in the three phases even of the currents are balanced. Therefore, voltage at the receiving end will not be same for all phases. To have equal voltage drop in all conductors the position of the conductors are interchanged at regular intervals along the line so that each conductor occupies the original position of every other conductor over an equal distance. Such an exchange of positions is known as <u>transposition</u>.

Inductance of 1ϕ 2wire line

$$L_A = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \ln\frac{d}{r}\right]$$

$$= 2\times10^{-7}\left[\ln e^{\frac{1}{4}} + \ln\frac{d}{r}\right]$$

$$= 2\times10^{-7} \ln\frac{d e^{\frac{1}{4}}}{r}$$

$$= 2\times10^{-7} \ln\frac{d}{r e^{-\frac{1}{4}}}$$

$$= 2\times10^{-7} \ln\frac{d}{0.7788 r}$$

$$= 2\times10^{-7} \ln\frac{d}{r'}$$

$r' = 0.7788\, r$ = GMR (Geometrical Mean Radius

or self GMD

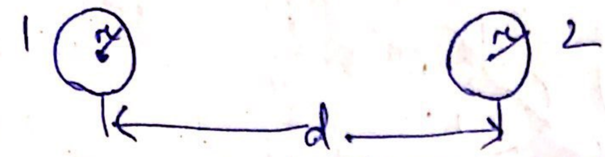
$r' \rightarrow$ effective radius upto which self flux linkage occurs.

r' can be assumed to be a fictitious conductor that has no internal flux but with the same inductance as that of a conductor with radius r.

r' is applicable to only solid round conductor.

GMD & GMR

Inductance of 1φ 2 wire line



equivalent GMR (D_s) $= \sqrt{GMR_1 \times GMR_2}$

$= \sqrt{(0.7788r)(0.7788r)}$

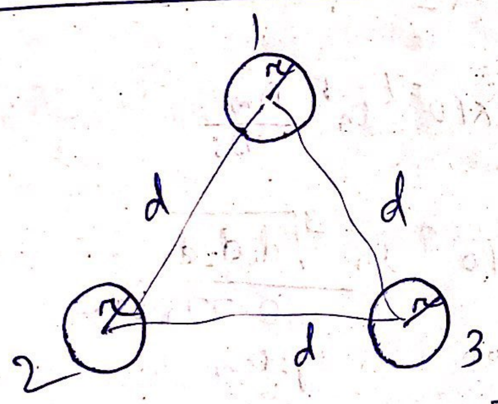
$= 0.7788r$

equivalent GMD (D_m) $= d$

Inductance $= 2\times10^{-7} \ln \frac{D_m}{D_s}$

$= 2\times10^{-7} \ln \frac{d}{0.7788r}$

3φ 3 wire line (symmetrically spaced)



equivalent GMR $(D_s) = \sqrt[3]{GMR_1 \times GMR_2 \times GMR_3}$

$= 0.7788r$

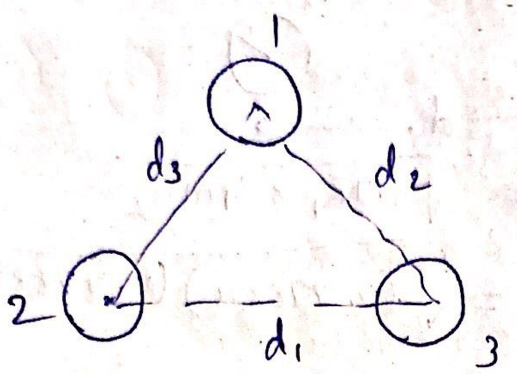
equivalent GMD $(D_m) = \sqrt[3]{D_{m_1} \times D_{m_2} \times D_{m_3}}$

$= \sqrt[3]{\sqrt{d\times d} \times \sqrt{d\times d} \times \sqrt{d\times d}}$

$= d$

Inductance $= 2\times10^{-7} \ln \frac{d}{0.7788r}$

3ϕ 3 wire line unsymmetrically spaced



equivalent GMR $(D_s) = \sqrt[3]{GMR_1 \times GMR_2 \times GMR_3}$

$= 0.7788r$

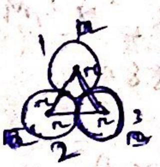
equivalent GMD$(D_m) = \sqrt[3]{\sqrt{d_1 d_2} \cdot \sqrt{d_2 d_3} \sqrt{d_3 d_1}}$

$= \sqrt[3]{d_1 d_2 d_3}$

Inductance $= 2 \times 10^{-7} \ln \frac{D_m}{D_s}$

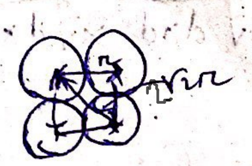
$= 2 \times 10^{-7} \ln \frac{\sqrt[3]{d_1 d_2 d_3}}{0.7788r}$

(1) find the self GMD of this system



$D_s = \sqrt[3]{D_{s1} \times D_{s2} \times D_{s3}}$

$D_{s1} = \sqrt[3]{0.7788r \times 2r \times 2r}$

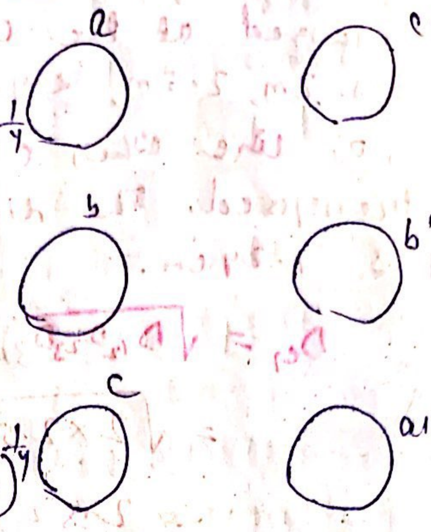


$D_s = \sqrt[4]{D_{s1} \times D_{s2} \times D_{s3} \times D_{s4}}$

$D_{s1} = \sqrt[4]{0.7788r \times 2r \times 2r \times 2\sqrt{2}r}$

$= D_{s2} = D_{s3} = D_{s4}$

for 3ϕ double circuit line



Self GMD of combination aa'

$$= D_{S_1} = \left(D_{aa} D_{aa'} D_{a'a} D_{a'a'}\right)^{\frac{1}{4}}$$

Self GMD of combination bb'

$$= D_{S_2} = \left(D_{bb} D_{bb'} D_{b'b} D_{b'b'}\right)^{\frac{1}{4}}$$

Self GMD of combination cc'

$$= D_{S_3} = \left(D_{cc} D_{c'c} D_{cc'} D_{c'c'}\right)^{\frac{1}{4}}$$

$$\boxed{\text{equivalent self GMD of one phase} = \left(D_{S_1} D_{S_2} D_{S_3}\right)^{\frac{1}{3}}}$$

Mutual GMD between phase A & B is

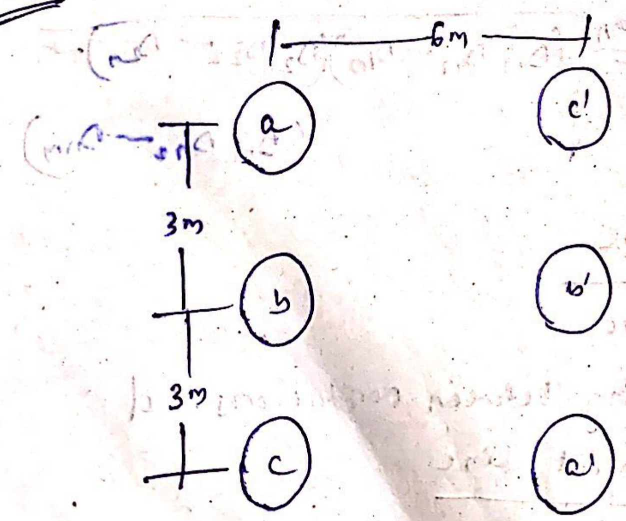
$$D_{AB} = \left(D_{ab} D_{ab'} D_{a'b} D_{a'b'}\right)^{\frac{1}{4}}$$

$$D_{BC} = \left(D_{bc} D_{bc'} D_{b'c} D_{b'c'}\right)^{\frac{1}{4}}$$

$$D_{CA} = \left(D_{ca} D_{ca'} D_{c'a} D_{c'a'}\right)^{\frac{1}{4}}$$

$$\boxed{\text{equivalent mutual GMD} = D_m = \left(D_{AB} D_{BC} D_{CA}\right)^{\frac{1}{3}}}$$

Ex-1



for a double circuit 3ϕ line, the phase sequence is ABC & the line is completely transposed. The conductor radius is 1.3 cm. Find inductance per phase per km.

Soln inductance $= 10^{-7} \cdot 2 \log \frac{D_m}{D_s}$

To find D_s

$D_s = (D_{s1} D_{s2} D_{s3})^{\frac{1}{3}}$

$D_{s1} = \left[(0.7788 \times 1.3 \times 10^{-2})^2 (6^2+6^2)\right]^{\frac{1}{4}} = 0.292\,m$

$D_{s2} = 0.246\,m$

$D_{s3} = 0.275\,m$

$D_s = (0.292 \times 0.246 \times 0.275)^{\frac{1}{3}}$

$= 0.275\,m$

To find D_m

$D_{AB} = (D_{ab} D_{ab'} D_{a'b} D_{a'b'})^{\frac{1}{4}}$

$= 4.48\,m$

$D_{BC} = 4.48\,m$

$D_{CA} = 6\,m$

$D_m = (4.48 \times 4.48 \times 6)^{\frac{1}{3}}$

$= 4.94\,m$

Inductance per phase per metre length

$= 10^{-7} \times 2 \log \frac{D_m}{D_s}$

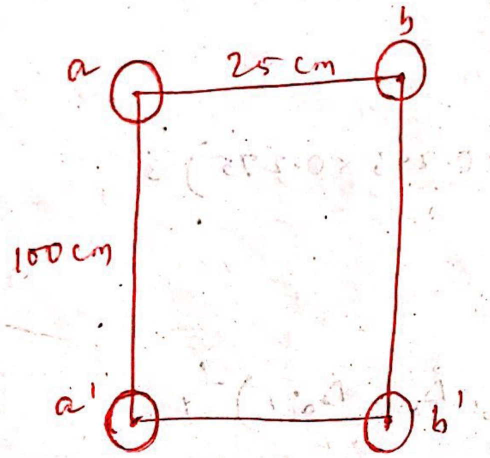
$= 5.7 \times 10^{-7}\,H$

per km $= 5.7 \times 10^{-4}\,H$

Ex-2

Two conductors of a single phase line, each of 1 cm diameter, are arranged in a vertical plane with one conductor mounted 1m above the other. A second identical line is mounted at the same height as the first & spaced horizontally 0.25 m apart from it. The two upper & the two lower conductors are connected in parallel. Determine the inductance per km of the resulting double circuit line.

Sol



$$\text{inductance} = 2\times10^{-7} \log\frac{D_m}{D_s}$$

$$D_s = \sqrt[4]{D_{aa} D_{a'a'} D_{a'a} D_{aa'}}$$

$$= 6.23 \text{ cm}$$

$$D_m = \sqrt[4]{D_{ab}\, D_{a'b'}\, D_{ab'}\, D_{a'b}}$$

$$= 50.74 \text{ cm}$$

loop inductance per km

$$= 4\times10^{-7}\times10^{3}\frac{D_m}{D_s}$$

$$= 0.84 \text{ mH}$$

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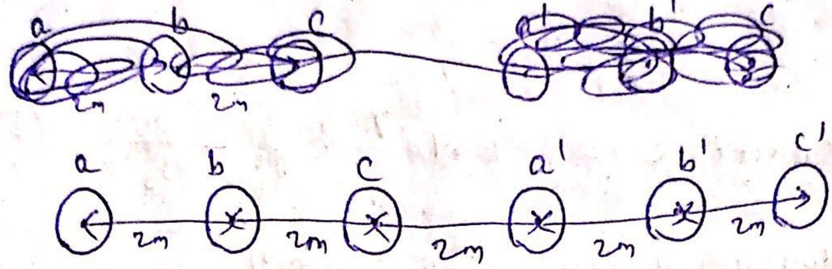
(1) There are 6 conductors in a double circuit line. Each conductor has a radius of 12 mm. The 6 conductors are arranged horizontally. The centre to centre distance between the conductors is 2m. The sequence of conductors are from left to right as follow: a, b, c, a', b', c'. Calculate the inductance per km per phase of the system.

Sol^n



$$D_s = \sqrt[3]{D_{SA}\, D_{SB}\, D_{SC}}$$

$$D_{SA} = \sqrt[4]{0.7788 \times 0.012 \times 6 \times 6 \times 0.7788 \times 0.012}$$

$$\left[\sqrt[4]{D_{aa} \times D_{aa'} \times D_{a'a} \times D_{a'a'}}\right]$$

$$= 0.2368 \text{ m}$$

$$D_{SB} = \sqrt[4]{D_{bb}\, D_{bb'}\, D_{b'b}\, D_{b'b'}}$$

$$= \sqrt[4]{0.7788 \times 0.012 \times 6 \times 6 \times 0.7788 \times 0.012}$$

$$= 0.2368 \text{ m}$$

$$D_{SC} = \sqrt[4]{D_{cc}\, D_{c'c}\, D_{cc'}\, D_{c'c'}}$$

$$= 0.2368 \text{ m}$$

$$D_s = \sqrt[3]{0.2368 \times 0.2368 \times 0.2368}$$

$$= 0.2368 \text{ m}$$

$$D_m = \sqrt[3]{D_{ab}\, D_{bc}\, D_{ca}}$$

$$D_{ab} = \sqrt[4]{d_{ab}\, d_{ab'}\, d_{a'b}\, d_{a'b'}}$$

$$= \sqrt[4]{2 \times 8 \times 4 \times 2}$$

$$= 3.364 \text{ m}$$

$$D_{bc} = \sqrt[4]{d_{bc}\, d_{b'c}\, d_{bc'}\, d_{b'c'}}$$

$$= \sqrt[4]{2 \times 8 \times 4 \times 2}$$

$$= 3.364 \text{ m}$$

$$D_{ca} = \sqrt[4]{d_{ca}\, d_{c'a}\, d_{ca'}\, d_{c'a'}}$$

$$= \sqrt[4]{4 \times 2 \times 10 \times 4}$$

$$= 4.229 \text{ m}$$

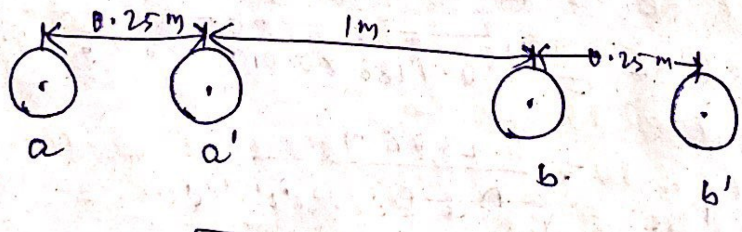
$$D_m = \sqrt[3]{3.364 \times 3.364 \times 4.229}$$

$$= 3.63 \text{ m}$$

$$\text{Inductance} = 2 \times 10^{-7} \log_e \frac{D_m}{D_s} \quad H/m$$

Inductance per km = 0.546 mH

(2) A single phase line consists of two circuits in parallel as shown in fig. Calculate the total inductance of the line per km assuming that the current is equally shared by the two parallel conductors. The diameter of each conductor is 20 mm.



Sol^n

$$D_s = \sqrt{D_{sa} D_{sb}}$$

$$D_{sa} = \sqrt[4]{D_{aa} D_{a'a} D_{aa'} D_{a'a'}}$$

$$= \sqrt[4]{0.7788 \times 0.01 \times 0.25 \times 0.25 \times 0.7788 \times 0.01}$$

$$= 4.4125 \times 10^{-2} \text{ m}$$

$$D_{sb} = \sqrt[4]{D_{bb} D_{b'b} D_{bb'} D_{b'b'}}$$

$$= 4.4125 \times 10^{-2} \text{ m}$$

$D_s = 4.4125 \times 10^{-2}$ m

$D_m = D_{ab}$

$= \sqrt[4]{d_{ab}\, d_{a'b}\, d_{ab'}\, d_{a'b'}}$

$= \sqrt[4]{1.25 \times 1.5 \times 1 \times 1.25}$

$= 1.2373$ m

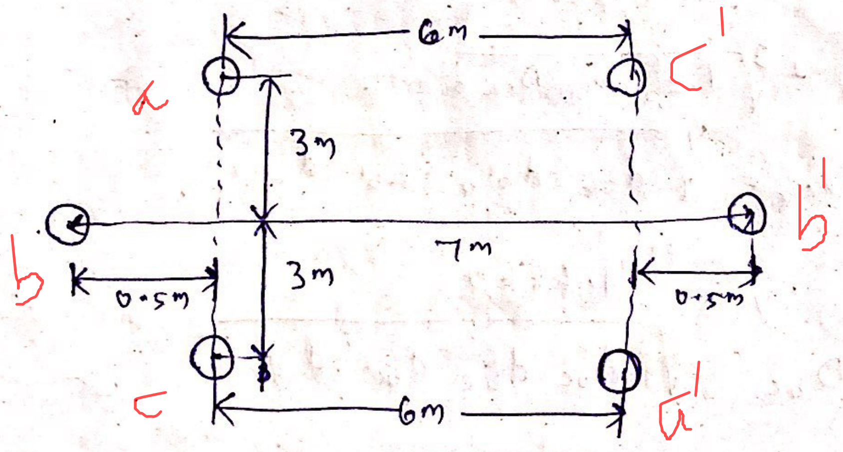
Inductance $= 2\times10^{-7} \ln \dfrac{D_m}{D_s}$ H/m

$= 2\times10^{-7} \times 10^{3} \ln \dfrac{D_m}{D_s}$ per km

Total inductance $= 2 \times 2 \times 10^{-7} \times 10^{3} \ln \dfrac{1.2373}{4.4125 \times 10^{-3}}$

$= 1.3335$ mH/km

(3) Find the inductance per phase per km of double circuit 3-phase line shown in fig. The conductors are transposed & are of radius 0.9 cm each. The phase sequence is ABC.



$r = 0.9$ cm $= 0.009$ m

$D_s = \sqrt[3]{D_{sa}\, D_{sb}\, D_{sc}}$

$$D_{sa} = \sqrt[4]{D_{aa} D_{a'a} D_{aa'} D_{a'a'}}$$

$$D_{aa} = 0.7788 \times 0.009\ m = D_{a'a'}$$

$$D_{a'a} = D_{aa'} = \sqrt{6^2+6^2} = 8.485\ m$$

$$\therefore D_{sa} = 0.2437\ m$$

similarly $D_{sb} = \sqrt[4]{D_{bb} D_{b'b} D_{bb'} D_{b'b'}}$

$$D_{bb} = D_{b'b'} = 0.7288 \times 0.009\ m$$

$$D_{b'b} = D_{bb'} = 7m$$

$$D_{sb} = \sqrt[4]{0.7788 \times 0.009 \times 7 \times 7 \times 0.7788 \times 0.009}$$

$$= 0.2214\ m$$

$$D_{sc} = \sqrt[4]{D_{cc} D_{c'c} D_{cc'} D_{c'c'}}$$

$$= \sqrt[4]{0.7288 \times 0.009 \times 8.485 \times 8.485 \times 0.7288 \times 0.009}$$

$$= 0.2437\ m$$

$$D_s = \sqrt[3]{0.2437 \times 0.2214 \times 0.2437}$$

$$= 0.236\ m$$

$$D_M = \sqrt[3]{D_{ab} D_{bc} D_{ca}}$$

$$D_{ab} = \sqrt[4]{d_{ab}\, d_{a'b}\, d_{ab'}\, d_{a'b'}}$$

$$= 4.666\ m$$

$$D_{bc} = \sqrt{d_{bc}\, d_{b'c}\, d_{bc'}\, d_{b'c'}}$$

$$d_{bc} = \sqrt{3^2+0.5^2} = 3.041\ m$$

$$d_{b'c} = \sqrt{6.5^2+3^2} = 7.159\ m$$

$$d_{bc'} = \sqrt{6.5^2+3^2} = 7.159\ m$$

$$d_{b'c'} = \sqrt{3^2+0.5^2} = 3.041\ m$$

(b) Find

$\therefore D_{bc} = \sqrt[4]{3.041 \times 7.159 \times 7.159 \times 3.041}$

$= 4.666 \text{ m}$

$D_{ca} = \sqrt[4]{d_{ca}\, d_{c'a}\, d_{ca'}\, d_{c'a'}}$

$= 6 \text{ m}$

$D_m = \sqrt[3]{D_{ab} D_{bc} D_{ca}}$

$= \sqrt[3]{4.666 \times 4.666 \times 6}$

$= 5.074 \text{ m}$

Inductance per phase per km

$= 2 \times 10^{-7} \times 10^{3} \ln \frac{D_m}{D_s}$

$= 0.6136 \text{ mH/km}$

Bundled conductors

- A bundled conductor is a conductor made up of two or more conductors, called the subconductors, per phase in close proximity compared with the spacing between phases.
- The basic difference between a composite conductor & a bundled conductor is that the subconductors of a bundled conductor are separated from each other by a constant distance varying from 0.2m. to 0.6m with the help of spacers whereas the wires of a composite conductor touch each other.
- This configuration is used when bulk power is transmitted to long distance at high voltage level.

Advantages of bundled conductors

(1) The use of bundled conductors per phase reduces the voltage gradient in the vicinity of the line & thus reduces the possibilities of corona discharge.

Reduction in communication line interference due to reduction in corona.

(2) The bundled conductors lines transmit bulk power with reduced losses, thereby giving increased transmission efficiency.

(3) Since by bundling, the GMR is increased, the inductance per phase is reduced. As a result the reactance per phase is reduced.

(4) Since the bundled conductor lines have a higher capacitance to neutral in comparison with single conductor line, therefore they have higher charging current, which helps in improving power factor.

$$C_n = \frac{2\pi\epsilon_0}{\ln\left(\frac{GMD}{GMR}\right)}$$

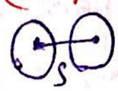
(4) Surge impedance of a line $Z_0 = \sqrt{\frac{L}{C}}$.
The bundled conductor lines have higher capacitance & lower inductance.
Therefore they have comparatively lower surge impedance with a corresponding increase in maximum power transfer capability.

(5) Reduced skin effect.

GMR of bundled conductor

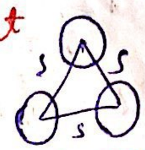
For two conductor (duplex) arrangement,

$$D_s = \sqrt{r's}$$



For triplex arrangement

$$D_s = (r's^2)^{\frac{1}{3}}$$



For 4 conductor arrangement

$$D_s = (r's^3)^{\frac{1}{4}}$$

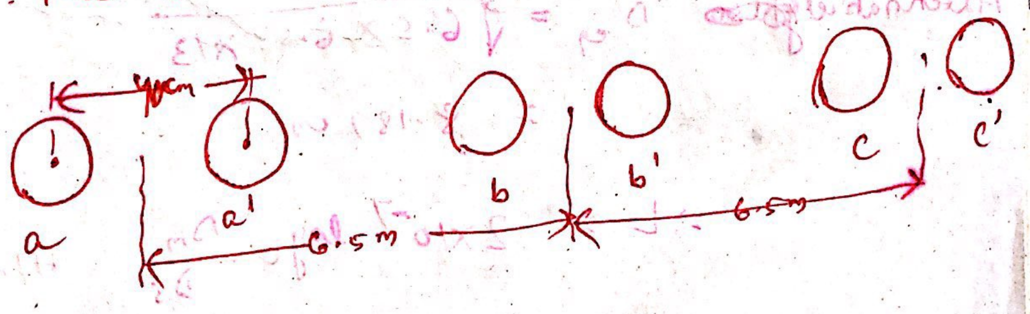
$\times 1.09$

where,

$r' \rightarrow$ GMR of each subconductor

$s \rightarrow$ spacing between subconductors of a bundle.

Ex-1 Determine the inductance per phase per km of a single circuit 460 kV line using two bundled conductors per phase as shown in the fig. The diameter of each conductor is 5 cm.



Sol^n

Radius of each subconductor,

$$r = \frac{5}{2} = 2.5 \text{ cm}$$

GMR of each subconductor,

$$r' = 0.7788\, r$$

$$= 1.947 \text{ cm}$$

$$= 1.947 \times 10^{-2} \text{ m}$$

Phase to phase separation,

$$d = 6.5 \text{ m}$$

Spacing between conductors of one phase,

$$s = 0.4 \text{ m}$$

$$\therefore \; GMR = \sqrt{r's}$$

$$= \sqrt{1.947\times10^{-2} \times 0.4}$$

$$= 0.088 \text{ m}$$

$$D_{ab} = \sqrt[4]{d_{ab}\, d_{a'b'}\, d_{ab'}\, d_{a'b}}$$

$$= \sqrt[4]{6.5 \times 6.5 \times 6.9 \times 6.1}$$

$$= 6.494 \text{ m}$$

$$D_{bc} = \sqrt[4]{d_{bc}\, d_{b'c'}\, d_{bc'}\, d_{b'c}} = 6.494 \text{ m}$$

$$D_{ca} = \sqrt[4]{d_{ca}\, d_{c'a'}\, d_{ca'}\, d_{c'a}} = 12.997 \text{ m}$$

$$\therefore \; D_m = \sqrt[3]{D_{ab} D_{bc} D_{ca}}$$

$$= 8.189 \text{ m}$$

Alternative $D_{eq} = \sqrt[3]{6.5 \times 6.5 \times 13}$

$$= 8.189 \text{ cm}$$

$$\therefore \; L = 2\times10^{-7} \log_e \frac{D_m}{D_s} \quad H/m$$

$$= 0.907 \text{ mH/km}$$

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② Determine the capacitance & charging current per km of the line of Ex-1 if the line operates at 132 kV.

Sol^n

$$GMR = \sqrt{Sr}$$

r = Radius of each subconductor

$$\therefore GMR = \sqrt{Sr} = \sqrt{2.5\times10^{-2}\times 0.4}$$
$$= 0.1\ m$$

$$D_M = 8.189\ m$$

$$C = \frac{2\pi\epsilon_0}{\log_e \frac{D_M}{D_S}}$$

$$= 12.628\times10^{-9}\ F/km$$

Charging current per km

$$= 2\pi f C V_{ph}$$

$$= 2\pi\times50\times12.628\times10^{-9}\times\frac{132}{\sqrt{3}}$$

$$= 0.3\ A/km$$

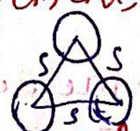
GMR of bundled conductor for Capacitance calculation

For two conductor arrangement,

$$D_s = \sqrt{rS}$$

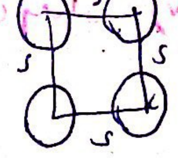
For three conductor arrangement

$$D_s = (rS^2)^{\frac{1}{3}}$$



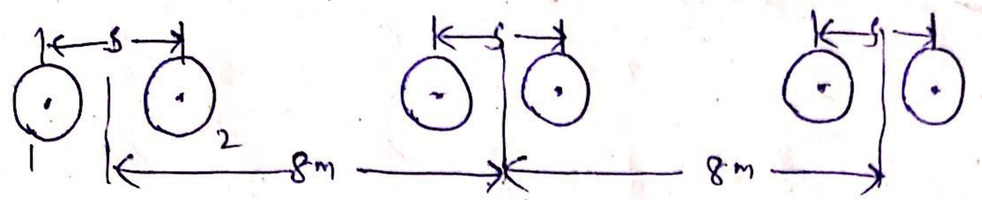
For four conductor arrangement

$$D_s = 1.09(rS^3)^{\frac{1}{4}}$$



Bundled conductor

(1) Find the inductance of the following



$r = 1.5\ cm$,

$S = 40\ cm$

$D_s = (D_{sa}\ D_{sb}\ D_{sc})^{\frac{1}{3}}$. $D_m = \sqrt[3]{D_{AB} D_{BC} D_{CA}}$

$D_{sa} = \sqrt{D_{s_1} D_{s_2}}$

$D_{s_1} = \sqrt[4]{0.7788r \times S \times S \times 0.7788r}$

$= \sqrt{S \times 0.7788\,r}$

$= \sqrt{Sr'}$

$= \sqrt{40 \times 0.7788 \times 1.5}$

$= 6.83\ cm$

$D_{s_2} = 6.83\ cm$

$D_{sa} = \sqrt{6.83 \times 6.83} = 6.83\ cm$

$D_s = (6.83 \times 6.83 \times 6.83)^{\frac{1}{3}}$

$= 6.83\ cm$

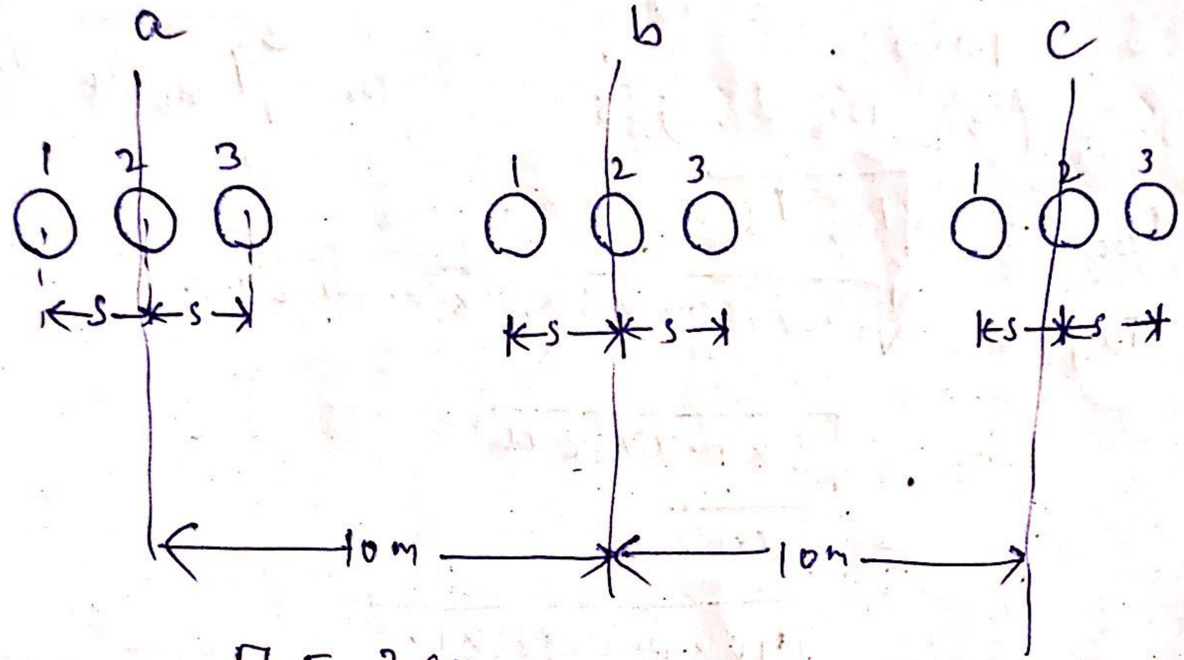
$D_m = \sqrt[3]{8 \times 8 \times 16} = 10.07\ m$

$= 1007\ cm$

$L = 2 \times 10^{-7} \log_e \frac{D_m}{D_s}$ H/m

$= 2 \times 10^{-7} \log_e \frac{1007}{6.83}$ H/m

(2)



$r = 2\,cm$

$S = 50\,cm = 0.5\,m$

$D_s = \sqrt[3]{D_{sa} \times D_{sb} \times D_{sc}}$

$D_{sa} = \sqrt[3]{D_{s_1} \times D_{s_2} \times D_{s_3}}$

$\cancel{D_{sa} = \sqrt[8]{0.7788r \times S \times 0.7788r \times S \times 0.7788r \times 2S \times 2S \times 0.7788r}}$

$\cancel{= 0.7788}$

$D_{s_1} = \sqrt[3]{0.7788r \times S \times 2S}$

$= \sqrt[3]{0.7788 \times 2 \times 10^{-2} \times 0.5 \times 2 \times 0.5}$

$= 0.1982\,m$

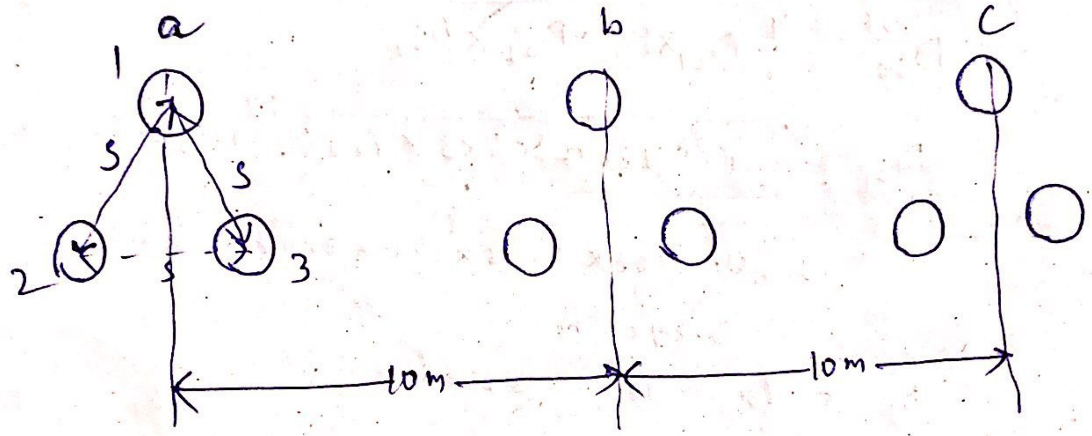
$D_{s_2} = \sqrt[3]{0.7788r \times S \times S}$

$= \sqrt[3]{0.7788 \times 0.02 \times 0.5 \times 0.5}$

$= 0.1573\,m$

$$D_{s_3} = \sqrt[3]{0.7788r \times s \times 2s}$$
$$= \sqrt[3]{0.7788 \times 0.02 \times 0.5 \times 2 \times 0.5}$$
$$= 0.1982 \text{ m}$$
$$D_{sa} = \sqrt[3]{0.1982 \times 0.1573 \times 0.1982}$$
$$= 0.1835 \text{ m} = D_{sb} = D_{sc}$$
$$D_s = \sqrt[3]{D_{sa} D_{sb} D_{sc}} = 0.1835 \text{ m}$$
$$D_m = \sqrt[3]{D_{ab} \; D_{bc} \; D_{ca}}$$
$$= \sqrt[3]{10 \times 10 \times 20}$$
$$= 12.59 \text{ m}$$
$$L = 2 \times 10^{-7} \log_e \frac{12.59}{0.1835} \text{ H/m}.$$

(3)



$$r = 2 \text{ cm} = 0.02 \text{ m}$$
$$s = 0.5 \text{ m}$$
$$D_s = \sqrt[3]{D_{sa} \; D_{sb} \; D_{sc}}$$
$$D_{sa} = \sqrt[3]{D_{s_1} \; D_{s_2} \; D_{s_3}}$$
$$D_{s_1} = \sqrt[3]{0.7788 \, r \times s \times s}$$
$$= \sqrt[3]{0.7788 \times 0.02 \times 0.5 \times 0.5}$$
$$= 0.1573 \text{ m}$$
$$D_{s_2} = \sqrt[3]{0.7788 r \times s \times s} = 0.1573 \text{ m}$$
$$D_{s_3} = 0.1573 \text{ m}$$
$$D_{sa} = \cancel{0.18} \; 0.1573 \text{ m} = D_{sb} = D_{sc}$$
$$D_s = 0.1573 \text{ m}$$
$$\therefore D_m = \sqrt[3]{10 \times 10 \times 20} = 12.59 \text{ m}$$
$$L = 2 \times 10^{-7} \log_e \frac{12.59}{0.1573} \text{ H/m} =$$

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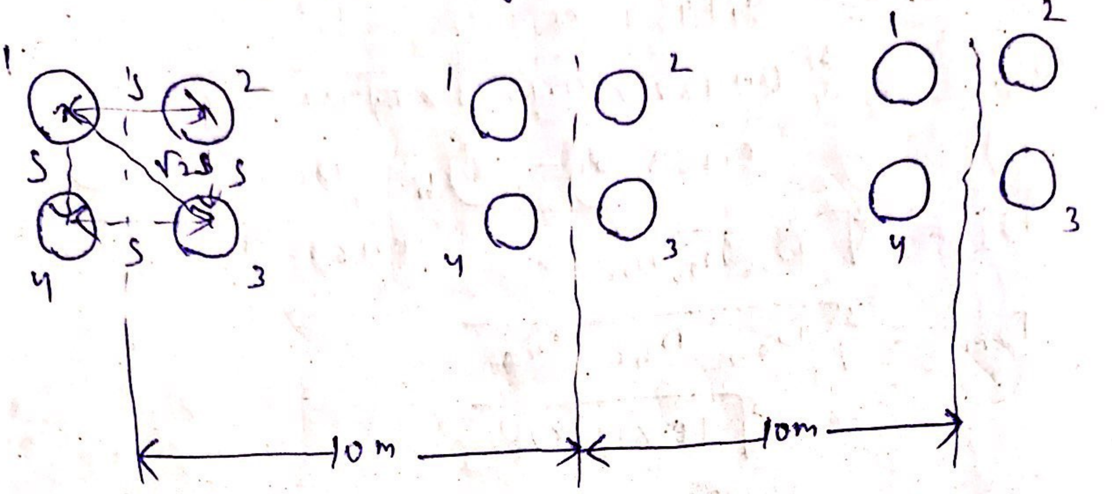
(4) Find the inductance of the following 4 sub conductor system. $r = 0.03m$, $s = 0.6m$



$$D_m = \sqrt[3]{10 \times 10 \times 20} = 12.59 m$$

$$D_s = \sqrt[3]{D_{sa} \times D_{sb} \times D_{sc}}$$

$$D_{sa} = \sqrt[4]{D_{s1} \times D_{s2} \times D_{s3} \times D_{s4}}$$

$$D_{s1} = \sqrt[4]{0.7788r \times s \times s \times \sqrt{2}s} = D_{s2} = D_{s3} = D_{s4}$$

$$= \sqrt[4]{0.7788 \times 0.03 \times 0.6 \times 0.6 \times \sqrt{2} \times 0.6}$$

$$= 0.2906 m$$

$$D_{s2} = D_{s3} = D_{s4} = 0.2906 m$$

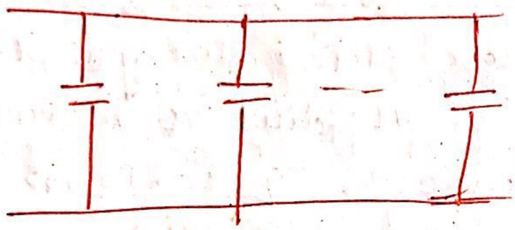
$$D_s = 0.2906 m$$

$$L = 2 \times 10^{-7} \log_e \frac{D_m}{D_s} \ H/m$$

$$= 2 \times 10^{-7} \log_e \frac{12.59}{0.2906} \ H/m$$

Capacitance of a transmission line

- In case of an overhead line, two conductors form the two plates of a capacitor & the air between the conductors behaves as the dielectric medium. Thus an overhead line can be assumed to have capacitance between conductors throughout the length of the line.



- When an alternating pd is applied across a transmission line, it draws a leading current even when supplying no load. The leading current is in quadrature with the applied voltage & is termed as charging current.

Charging current, $\boxed{I_c = V \cdot 2\pi f C}$

Advantages of transmission line having high capacitance

(i) Reduction of line losses & so increase of transmission efficiency.

(ii) Reduction in voltage drop or improvement in voltage regulation.

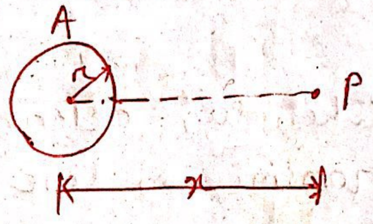
(iii) increased load capacity & improved power factor.

Electric potential

Electric potential at any point in an electric field is defined as the work done in moving a unit positive charge from infinity to that point.

Potential at a charged single conductor

Consider a long straight cylindrical conductor A of radius r metres & having a charge of q coulombs per metre length.



Electric field intensity at a distance x from the centre of conductor,

$$E = \frac{q}{2\pi \epsilon_0 \epsilon_r x} \text{ volts/m}$$

Taking air as medium,

$$E = \frac{q}{2\pi \epsilon_0 x} \text{ volts/m}$$

$$\oint \bar{E} \cdot ds = \frac{q}{\epsilon_0}$$

$$\Rightarrow E \cdot 2\pi r l = \frac{q}{\epsilon_0}$$

for $l = 1$m

$$E = \frac{q}{2\pi \epsilon_0 r}$$

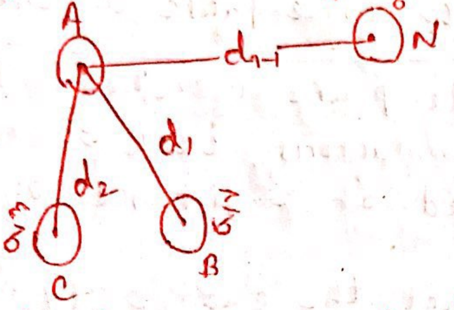
The potential difference between conductor & infinity distant neutral plane will be equal to work done in bringing a unit positive charge against E from infinity to conductor surface & is given by

$$V_A = \int_r^\infty \frac{q}{2\pi \epsilon_0 x} dx$$

$$\boxed{V_A = \frac{q}{2\pi \epsilon_0} \int_r^\infty \frac{dx}{x}}$$

potential at a charged conductor in a group of charged conductors

consider a group of long straight conductors A, B, C, D, -- N having charges of $q_1, q_2 --- q_n$ coulombs per metre length respectively



potential of conductor A due to its own charge q_1 is $= \int_r^{\infty} \frac{q_1}{2\pi\epsilon_0} \frac{dx}{x}$

Potential of conductor A due to charge q_2 is

$$= \int_{d_1}^{\infty} \frac{q_2}{2\pi\epsilon_0} \frac{dx}{x}$$

Potential of conductor A due to charge q_n is

$$= \int_{d_{n-1}}^{\infty} \frac{q_n}{2\pi\epsilon_0} \frac{dx}{x}$$

So overall potential difference between conductor A & infinite distant neutral plane

$$V_{AN} = \int_r^{\infty} \frac{q_1}{2\pi\epsilon_0} \frac{dx}{x} + \int_{d_1}^{\infty} \frac{q_2}{2\pi\epsilon_0} \frac{dx}{x} + \cdots + \int_{d_{n-1}}^{\infty} \frac{q_n}{2\pi\epsilon_0} \frac{dx}{x}$$

$$= \frac{1}{2\pi\epsilon_0} \Big[q_1 (\ln\infty - \ln r) + q_2 (\ln\infty - \ln d_1) + \cdots + q_n (\ln\infty - \ln d_{n-1}) \Big]$$

$$= \frac{1}{2\pi\epsilon_0} \Big[\ln\infty (q_1 + q_2 + \cdots q_n) + q_1 \ln\frac{1}{r} + q_2 \ln\frac{1}{d_1} + \cdots + q_n \ln\frac{1}{d_{n-1}} \Big]$$

For balanced load,

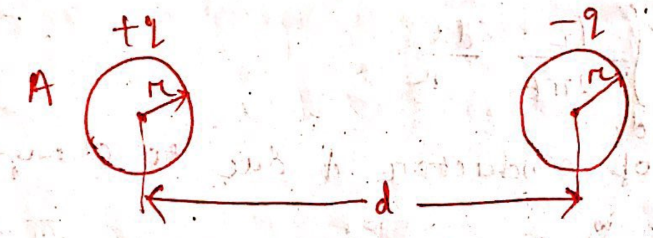
$$q_1 + q_2 + \text{---} + q_n = 0$$

$$\therefore V_{AN} = \frac{1}{2\pi\epsilon_0}\left[q_1 \ln\frac{1}{r} + q_2 \ln\frac{1}{d_1} + \text{---} + q_n \ln\frac{1}{d_{n-1}}\right]$$

Capacitance of a single-phase overhead line

Consider a single phase overhead line with two parallel conductors, each of radius r metres placed at a distance of d metres in air.

It's assumed that the charge +q coulombs on conductor A & −q coulombs on conductor B are concentrated at the centres of the two conductors which are separated from each other by d metres.



Potential difference between conductor A & neutral infinite plane,

$$V_{AN} = \int_r^{\infty} \frac{q}{2\pi\epsilon_0}\frac{dx}{x} + \int_d^{\infty} \frac{-q}{2\pi\epsilon_0}\frac{dx}{x}$$

$$= \frac{q}{2\pi\epsilon_0}\left(\ln\infty - \ln r - \ln\infty + \ln d\right)$$

$$= \frac{q}{2\pi\epsilon_0}\ln\frac{d}{r}$$

Similarly, potential difference between conductor B & neutral infinite planes,

$$V_{BN} = \int_r^{\infty} \frac{-q}{2\pi\epsilon_0}\frac{dx}{x} + \int_d^{\infty} \frac{q}{2\pi\epsilon_0}\frac{dx}{x}$$

$$= \frac{q}{2\pi\epsilon_0}\left(-\ln\infty + \ln r + \ln\infty - \ln d\right)$$

$$= \frac{-q}{2\pi\epsilon_0}\ln\frac{d}{r}$$

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PD between conductor A & B,

$$V_{AB} = V_{AN} - V_{BN}$$

$$= \frac{q}{2\pi\epsilon_0} \ln\frac{d}{r} - \frac{-q}{2\pi\epsilon_0} \ln\frac{d}{r}$$

$$= \frac{q}{\pi\epsilon_0} \ln\frac{d}{r} \text{ volts}$$

Capacitance of line,

$$C = \frac{q}{V_{AB}} = \frac{q}{\frac{q}{\pi\epsilon_0} \ln\frac{d}{r}}$$

$$\boxed{C = \frac{\pi\epsilon_0}{\ln\frac{d}{r}}} \text{ F/m}$$

capacitance between two conductors

Capacitance of each conductor C_n or phase to neutral $= \frac{2\pi\epsilon_0}{\ln\frac{d}{r}}$ F/m

Ex: Find out capacitance of a 1ϕ line 30 km long consisting of two parallel wires each 15 mm diameter & 1.5 m apart.

Ans:

$$r = \cancel{30} = \cancel{15}\ \frac{15}{2} = 7.5 \text{ mm}$$

$$d = 1.5 \text{ m} = 1500 \text{ mm}$$

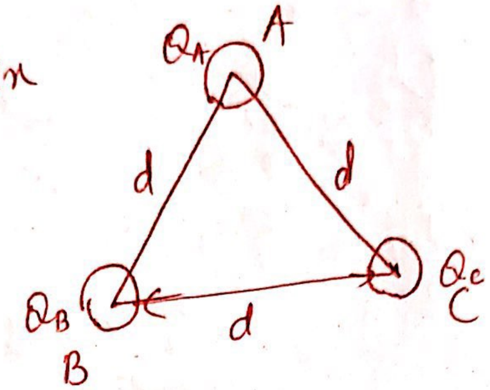
$$C = \frac{\pi\epsilon_0}{\ln\frac{d}{r}} \times 30000$$

$$= \frac{\pi \times 8.85 \times 10^{-12}}{\ln\frac{1500}{7.5}} \times 30000$$

$$= 0.1575 \ \mu F$$

Capacitance of 3ϕ overhead line

(1) Symmetrical spacing



$$V_A = \int_r^{\infty} \frac{Q_A}{2\pi \epsilon_0 x}\,dx + \int_d^{\infty} \frac{Q_B}{2\pi \epsilon_0 x}\,dx + \int_d^{\infty} \frac{Q_C}{2\pi \epsilon_0 x}\,dx$$

$$= \frac{1}{2\pi \epsilon_0}\left[Q_A \ln\frac{1}{r} + Q_B \ln\frac{1}{d} + Q_C \ln\frac{1}{d}\right]$$

$$= \frac{1}{2\pi \epsilon_0}\left[Q_A \ln\frac{1}{r} + \ln\frac{1}{d}(Q_B + Q_C)\right]$$

$$= \frac{1}{2\pi \epsilon_0} Q_A \ln\frac{d}{r}$$

Balanced load

$\because Q_A + Q_B + Q_C = 0$

$\Rightarrow Q_B + Q_C = -Q_A$

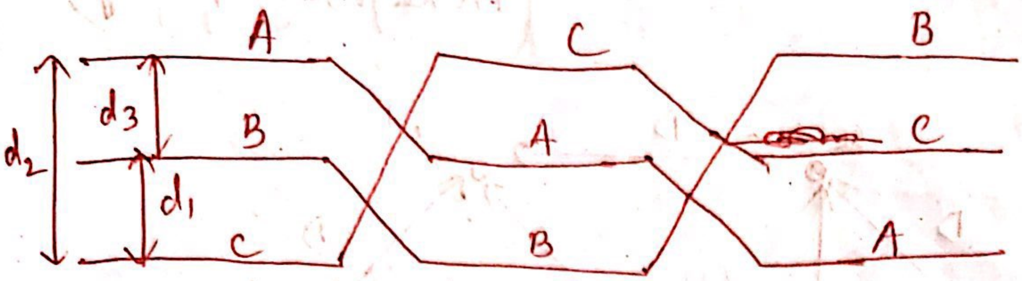
Capacitance of conductor A wrt neutral,

$$C_A = \frac{Q_A}{V_A} = \frac{Q_A}{\frac{Q_A}{2\pi\epsilon_0}\ln\frac{d}{r}}$$

$$\Rightarrow \boxed{C_A = \frac{2\pi \epsilon_0}{\ln\frac{d}{r}}}$$

Same can be calculated for conductor B & C.

(2) Unsymmetrical spacing



For conductor A

For 1st position, $V_{A_1} = \frac{1}{2\pi\epsilon_0}\left[Q_A \ln\frac{1}{r} + Q_B \ln\frac{1}{d_3} + Q_C \ln\frac{1}{d_2}\right]$

2nd position, $V_{A_2} = \frac{1}{2\pi\epsilon_0}\left[Q_A \ln\frac{1}{r} + Q_B \ln\frac{1}{d_1} + Q_C \ln\frac{1}{d_3}\right]$

3rd position, $V_{A_3} = \frac{1}{2\pi\epsilon_0}\left[Q_A \ln\frac{1}{r} + Q_B \ln\frac{1}{d_2} + Q_C \ln\frac{1}{d_1}\right]$

Average voltage on conductor A,

$$V_A = \frac{1}{3}\left(V_{A_1} + V_{A_2} + V_{A_3}\right)$$

$$= \frac{1}{3\times 2\pi\epsilon_0}\left[Q_A \ln\frac{1}{r^3} + (Q_B + Q_C)\ln\frac{1}{d_1 d_2 d_3}\right]$$

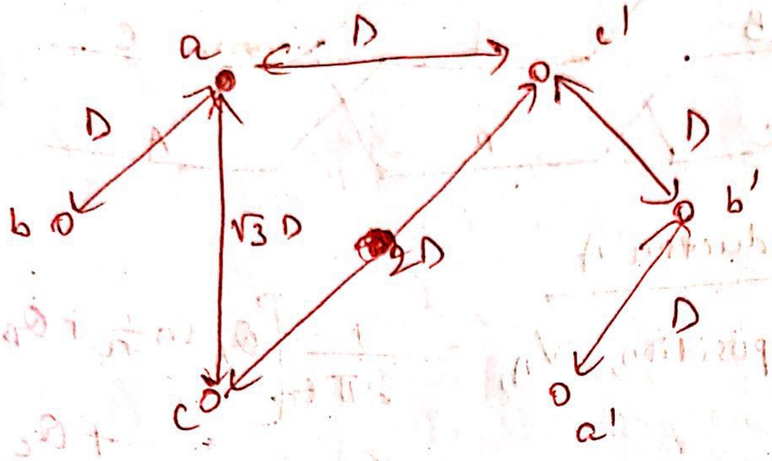
$$= \frac{Q_A}{6\pi\epsilon_0} \ln\frac{d_1 d_2 d_3}{r^3}$$

$$= \frac{Q_A}{2\pi\epsilon_0} \ln\frac{\sqrt[3]{d_1 d_2 d_3}}{r}$$

$$\Rightarrow \boxed{C_A = \frac{Q_A}{V_A} = \frac{2\pi\epsilon_0}{\ln\frac{\sqrt[3]{d_1 d_2 d_3}}{r}}}$$

Capacitance of double circuit 3ϕ line

(Hexagonal spacing)



$$V_a = \frac{q_a}{2\pi\epsilon_0}\int_r^\infty \frac{1}{x}\,dx + \frac{q_b}{2\pi\epsilon_0}\int_D^\infty \frac{1}{x}\,dx$$

$$+ \frac{q_c}{2\pi\epsilon_0}\int_{\sqrt{3}D}^\infty \frac{1}{x}\,dx + \cdots$$

Assuming balanced load,

$$q_a + q_b + q_c = 0$$

$$V_a = \frac{1}{2\pi\epsilon_0}\left\{q_a\left(\ln\frac{1}{2Dr} - \ln\frac{1}{\sqrt{3}D^2}\right)\right.$$

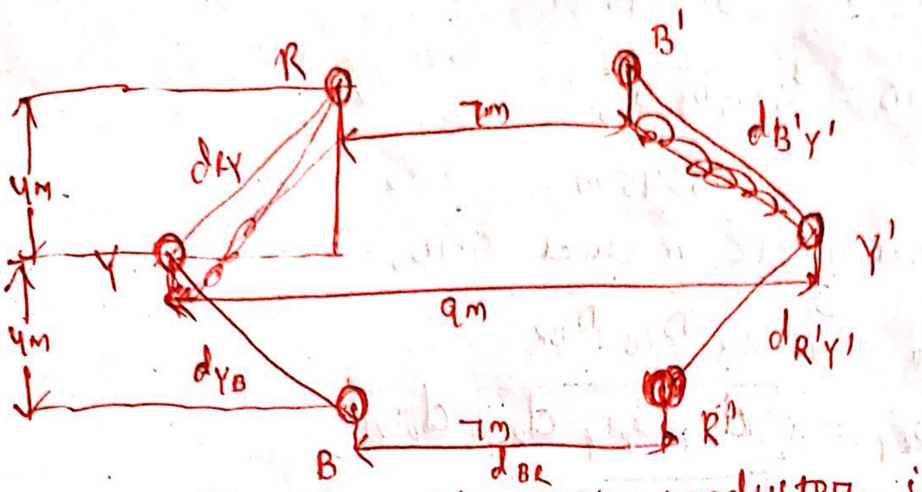
$$= \frac{q_a}{2\pi\epsilon_0}\ln\frac{\sqrt{3}D}{2r}$$

$$\Rightarrow \boxed{C_a = \frac{2\pi\epsilon_0}{\ln\frac{\sqrt{3}D}{2r}}} \text{ F/metre/conductor}$$

Per phase

$$\boxed{C_n = \frac{4\pi\epsilon_0}{\ln\frac{\sqrt{3}D}{2r}}} \text{ F/m/phase}$$

(1) Six conductors of a double circuit transmission line are arranged as shown in the figure :



The diameter of each conductor is 2.5 cm. Find the capacitive reactance to neutral & the charging current per km per phase at the 132 KV & 50 Hz, assuming that the line is regularly transposed. Neglect the effect of earth.

Sol^n

$$r = \frac{2.5}{2} = 1.25 \text{ cm}$$

GMR of each conductor = r

In place of r', r is used as GMR as the electric charge resides on the surface of the conductor unlike the magnetic flux which lies inside the conductor.

GMR of conductor, R = $D_{s_1} = \sqrt[4]{D_{RR} D_{R'R'} D_{RR'} D_{R'R}}$

$D_{RR} = D_{R'R'} = r = 1.25$ cm

$D_{RR'} = \sqrt{7^2 + (4+4)^2} = 10.63$ m

$\therefore D_{s_1} = \sqrt[4]{(1.25 \times 10^{-2})^2 \times (10.63)^2}$

$= 0.3645$ m

Similarly,

$D_{s_2} = \sqrt[4]{D_{YY} D_{Y'Y'} D_{YY'} D_{Y'Y}}$

$= 0.3354$ m

$$D_{S_3} = \sqrt[4]{D_{BB}\, D_{B'B'}\, D_{BB'}\, D_{B'B}}$$

$$= 0.3645\ m$$

$$\text{Self G.M.D} = \sqrt[3]{D_{S_1} D_{S_2} D_{S_3}}$$

$$= 0.3545\ m$$

To calculate mutual GMD,

$$D_m = \sqrt[3]{D_{RY}\, D_{YB}\, D_{BR}}$$

$$D_{RY} = \sqrt{d_{RY}\, d_{R'Y}\, d_{RY'}\, d_{R'Y'}}$$

$$d_{RY} = \sqrt{4^2+1^2}$$

$$d_{R'Y} = \sqrt{8^2+4^2}$$

$$d_{RY'} = \sqrt{8^2+4^2}$$

$$d_{R'Y'} = \sqrt{1^2+4^2}$$

$$\therefore D_{RY} = 6.072\ m$$

Similarly $D_{YB} = \sqrt[4]{d_{YB}\, d_{YB'}\, d_{Y'B}\, d_{Y'B'}}$

$$= 6.072\ m$$

$$D_{BR} = \sqrt[4]{d_{BR}\, d_{BR'}\, d_{B'R}\, d_{B'R'}}$$

$$= 7.483\ m$$

Capacitance per phase

$$= C = \frac{2\pi\epsilon_0}{\log_e \frac{D_m}{D_s}}$$

$$= 19.11 \times 10^{-9}\ F/Km$$

$$X_C = \frac{1}{2\pi f C} = 166.526\ K\Omega$$

Charging current $= 2\pi f C V_{ph}$

$$= 2\pi \times 50 \times 19.11 \times 10^{-9} \times \frac{132 \times 1000}{\sqrt{3}}$$

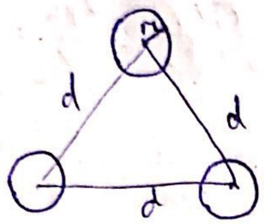
$$= 0.4575\ A/km$$

(2) Calculate the capacitance of a 100 km long 3-phase, 50 Hz overhead transmission line consisting of 3 conductors, each of diameter 2 cm & spaced 2.5 m at the corners of an equilateral triangle.

Ans

$r = 1\,cm = 0.01\,m$

$d = 2.5\,m$



$$C = \frac{2\pi\epsilon_0}{\ln \frac{d}{r}} \quad F/m$$

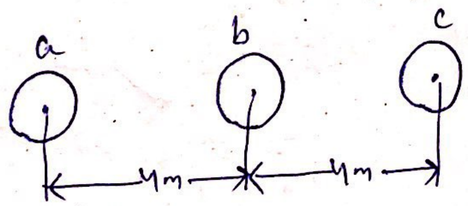
$$= \frac{2\pi \times 8.85 \times 10^{-12}}{\ln \frac{2.5}{0.01}} \quad F/m$$

$$= 10.075 \times 10^{-9} \; F/km$$

capacitance of 100 km line $= 10.075 \times 10^{-9} \times 100$

$$= 1.0075 \times 10^{-6} \; F$$

(3) A 3 phase, 50 Hz, 132 kV overhead line has conductors placed in a horizontal plane 4 m apart. conductor diameter is 2cm. If the line length is 100 km, calculate the charging current per phase assuming complete transposition



$$d_{eq} = \sqrt[3]{4 \times 4 \times 8} = 5.04\,m$$

$$r = 1\,cm = 0.01\,m$$

$$C_{ph} = \frac{2\pi\epsilon_0}{\ln \frac{d_{eq}}{r}} = \frac{2\pi \times 8.85 \times 10^{-12}}{\ln \frac{5.04}{0.01}} \; F/m$$

$$= 0.00885 \times 10^{-6} \; F/km$$

Capacitance per phase for 100 km line is

$$C_{ph} = 0.00885 \times 10^{-6} \times 100 = 0.885 \times 10^{-6}\ F$$

Phase voltage, $V_{ph} = \frac{132 \times 10^3}{\sqrt{3}} = 76210\,V$

Charging current, $I_c = \frac{V_{ph}}{X_c}$

$$= V_{ph}\,\omega C_{ph}$$

$$= 76210 \times (2\pi \times 50) \times 0.885 \times 10^{-6}$$

$$= 21.18\ A$$

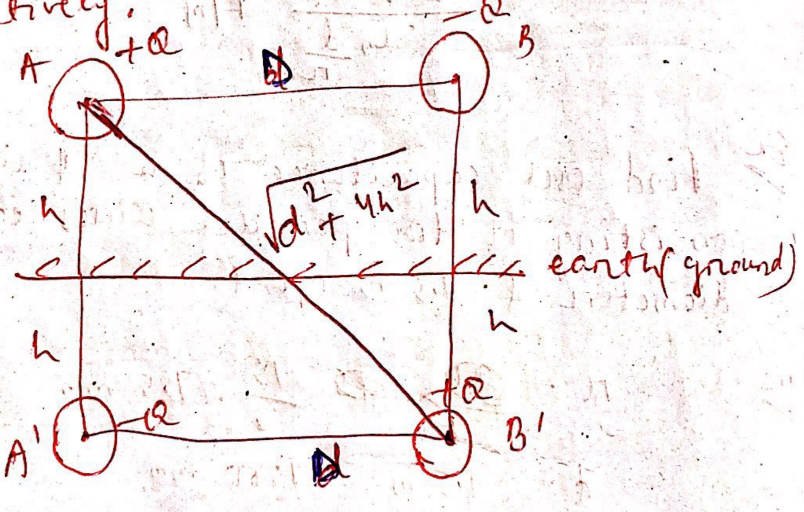
Advantages of double circuit transmission

1) More power can be transmitted over a particular distance.
$L \downarrow \quad X_L \downarrow \quad P = \frac{V_1 V_2}{X} \sin\delta \uparrow$
2) Construction cost is reduced.
3) Reliability is improved.

Effect of earth on capacitance of transmission line

- The presence of earth changes the electric field of the transmission line & consequently its capacitance is affected. Earth may be assumed to be a perfect conductor in the form of horizontal plane of infinite extent. Therefore the electric field of charged conductors is forced to conform to a presence of this equipotential surface. The effect of earth on capacitance can be taken into account by <u>method of images</u>.

- Consider a 1ϕ transmission line, assuming conductors A' & B' as image conductors of conductor A & B (each radius r) respectively.



Potential of conductor A,

$$V_A = \int_r^\infty \frac{Q}{2\pi\epsilon_0}\frac{dx}{x} + \int_D^\infty \frac{-Q}{2\pi\epsilon_0}\frac{dx}{x}$$

$$+ \int_{2h}^\infty \frac{-Q}{2\pi\epsilon_0}\frac{dx}{x} + \int_{\sqrt{4h^2+D^2}}^\infty \frac{Q}{2\pi\epsilon_0}\frac{dx}{x}$$

$$= \frac{Q}{2\pi\epsilon_0}\left[\ln\frac{D}{r} + \ln\frac{2h}{\sqrt{4h^2+D^2}}\right]$$

$$= \frac{Q}{2\pi\epsilon_0}\ln\frac{2hD}{r\sqrt{4h^2+D^2}} \text{ volts}$$

Similarly,

$$V_B = \frac{-Q}{2\pi\epsilon_0}\ln\frac{2hD}{r\sqrt{4h^2+D^2}} \text{ volts}$$

$$V_{AB} = V_A - V_B$$

$$= \frac{Q}{\pi\epsilon_0}\ln\frac{2hD}{r\sqrt{4h^2+D^2}}$$

$$= \frac{Q}{\pi\epsilon_0}\ln\frac{2hD}{2hr\sqrt{\left(\frac{D^2}{4h^2}+1\right)}}$$

$$= \frac{Q}{\pi\epsilon_0}\ln\frac{D}{r\sqrt{1+\frac{D^2}{4h^2}}}$$

$$C_{AB} = \frac{\pi\epsilon_0}{\ln\left(\frac{D}{r\sqrt{1+\frac{D^2}{4h^2}}}\right)}$$

$$C_N = \frac{2\pi\epsilon_0}{\ln\left[\frac{D}{r\sqrt{\left(1+\frac{D^2}{4h^2}\right)}}\right]} \text{ F/m to neutral}$$

$$= 2C_{AB}$$

around into

Skin effect

- The tendency of alternating current to concentrate near the surface of a conductor is known as skin effect.

- A solid conductor may be thought to be consisting of a large number of strands, each carrying a small part of current. The inductance of each strand will vary according to its position. Thus, the strands near the centre are surrounded by a greater magnetic flux & hence have larger inductance than that near the surface. The high reactance of inner strands causes the alternating current to flow near the surface of the conductor. This crowding of current near the conductor surface is the skin effect.

- Due to skin effect, the effective area of cross-section of the conductor through which current flows is ~~increased~~ reduced. Consequently, the resistance of the conductor is slightly increased, when carrying an alternating current.

- The skin effect depends upon the following factors:
  - (i) Nature of material
  - (ii) Diameter of wire → increases with the diameter
  - (iii) frequency → increases with increase in frequency.
  - (iv) Shape of wire → less for stranded conductor than solid conductor

Proximity effect

- The alternating magnetic flux in a conductor caused by the current flowing in a neighbouring conductor gives rise to circulating currents which cause an apparent increase in the resistance of a conductor. This phenomenon is called proximity effect.
- The proximity effect is pronounced in case of cables where the distance between the conductors is small whereas for overhead lines, the proximity effect is small.