Tutorial No. 1

STAT 51033

1. Let $X_1, X_2, \ldots, X_n$ is a random sample of size n from the distribution with Probability density function (p.d.f) with $f(x;\theta) = \theta\, x^{\theta-1}\,; 0 < x < 1; 0 < \theta < \infty$.
   (i). Find the method of moment estimator for the parameter θ.
   (ii). Find the maximum likelihood estimator for the parameter θ.
2. Let $X_1, X_2, \ldots, X_n$ is a random sample of size n from the exponential distribution whose pdf is $f(x;\theta) = \frac{1}{\theta} e^{-x/\theta}\,; 0 < x < \infty; 0 < \theta < \infty$.
   (i). Show that $\bar{X}$ is an unbiased estimator of θ.
   (ii). Show that the variance of $\bar{X}$ is $\frac{\overline{\theta^2}}{n}$.
   (iii). What is the good estimate of θ if a random sample of size 5 yield the sample values 3.5, 8.1, 0.9, 4.4 and 0.5?
3. Let $X_1, X_2, \ldots, X_n$ is a random sample from a Normal density $N(\theta_1, \theta_2)$ where $0 < \theta_1, \theta_2 < \infty$. That is here let $\theta_1 = \mu, and\ \theta_2 = \sigma^2$).
   (i). Show that the maximum likelihood estimators for $\theta_1\ and\ \theta_2$ are $\hat{\theta}_1 = \bar{X}, \hat{\theta}_2 = \frac{(n-1)S^2}{n}$.
   (ii). Show that the estimator $\hat{\theta}_1$ is an unbiased estimator for μ .
   (iii). Show that the estimator $\hat{\theta}_2$ is a biased estimator for σ^2.