UNIVERSITY OF COLOMBO, SRI LANKA

University of Colombo School of Computing

UCSC

BACHELOR OF SCIENCE IN COMPUTER SCIENCE

First Year Examination — Semester I— 2022

SCS 1204 – Discrete Mathematics - I

236

(Two (2) Hours)

Number of Pages = 4

Number of Questions = 4

Important Instructions to candidates:

1. Students should answer in the medium of **English language only** using the answer paper provided.
2. Note that questions appear on both sides of the paper. If a page or a part of this question paper is not printed, please inform the supervisor immediately.
3. Write your index number **CLEARLY** on each and every page of the answer paper.
4. This paper consists of **4** questions in **04** pages (including the Cover Page).
5. Answer **ALL** questions. All questions carry equal marks (**25** marks).
6. Programmable Calculators and any electronic device capable of storing and retrieving text including electronic dictionaries, smart watches and mobile phones are **not allowed**.
7. **Non-Programmable** calculators are **not allowed.**

1.

(a). Show that the compound proposition $[(p \to q) \land (q \to r)] \to (p \to r)$ is a tautology by using a truth table.

[5 Marks]

(b). Determine whether the following system specifications are consistent.
Whenever the system software is being upgraded, users cannot access the file system. If users can access the file system, then they can save new files. If users cannot save new files, then the system software is not being upgraded.

[5 Marks]

(c). Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives. Let the domain consist of all the students at UCSC.

(i). No one is perfect.
(ii). Not everyone is perfect.
(iii). All your friends are perfect.
(iv). Everyone is your friend and is perfect.
(v). At least one of your friends is perfect.

[5 Marks]

(d). Test the validity of the following argument:
No professors are ignorant. All ignorant people are vain. Therefore, no professors are vain.

[5 Marks]

(e). Determine the truth value of each of these statements in predicate logic, if the domain for all variables consists of all integers.

(i). $\forall n\, \exists m\, (n + m = 0)$
(ii). $\exists n\, \forall m\, (n < m^2)$
(iii). $\exists n\, \exists m\, (n^2 + m^2 = 7)$
(iv). $\exists n\, \exists m\, (n + m = 4 \text{ and } n - m = 1)$
(v). $\forall n\, \exists m\, (n^2 < m)$.

[5 Marks]

2. Prove or disprove each of the following statements:

(a). For all integers m and n, if m and n are odd, then mn is odd.

[4 Marks]

(b). If $n = ab$, where a, b are positive integers, then $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$.

[4 Marks]

(c). There are no solutions in integers x and y to the equation $2x^2 + 5y^2 = 14$.

[4 Marks]

(d). If A, B and C are sets, then $A - (B \cap C) = (A - B) \cap (A - C)$.

[4 Marks]

(e). There is a positive integer that can be written as the sum of cubes of positive integers in two different ways.

[4 Marks]

(f). There is a rational number x and an irrational number y such that x^y is irrational.

[5 Marks]

3.

(a). Let $A = \{0, 2, 4, 6, 8, 10\}$, $B = \{0, 1, 2, 3, 4\}$, and $C = \{0, 3, 6, 9\}$. Find

(i). $A \cap B \cap C$
(ii). $A \cup B \cup C$
(iii). $A \cap (B \cup C)$
(iv). $A - B$
(v). $(A - B) \times C$
(vi). $A \Delta B$.

[6 Marks]

(b). Let A, B, C and D be sets. Show that

(i). $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(ii). $(B - A) \cap (C - A) = (B \cap C) - A$

(iii). $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$.

[12 Marks]

(c). Let f be a function from the set A to the set B. Let S and T be subsets of A.

(i). Show that $f(S \cap T) \subseteq f(S) \cap f(T)$.

(ii). Is it true that $f(S) \cap f(T) \subseteq f(S \cap T)$? Justify your answer. **[7 Marks]**

4.

(a). Consider the binary relation R on the set of integers $(\mathbb{Z})$ defined by $(a, b) \in R$ if and only if $a + b \leq 3$ for $a, b \in \mathbb{Z}$.
Is R reflexive, symmetric, antisymmetric, and transitive? Justify your answer.

[8 Marks]

(b). A relation R is defined on the set of integers $(\mathbb{Z})$ by

$$(x, y) \in R \text{ if and only if } x - y \text{ is divisible by } 7, for\ x, y \in \mathbb{Z}.$$

(i). Show that R is an equivalence relation on $\mathbb{Z}$.

(ii). Find the equivalence class of 2 and 11.

[11 Marks]

(c). Which of the binary relations defined on $\{0, 1, 2, 3\}$ are partial order relation? Determine the properties of a partial ordering that the others lack.

(i). $R_1 = \{(0, 0), (1, 1), (1, 2), (2, 2), (3, 3)\}$

(ii). $R_2 = \{(0, 0), (1, 1), (2, 0), (2, 2), (2, 3), (3, 2), (3, 3)\}$

(iii). $R_3 = \{(0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 2), (3, 3)\}$.

[6 Marks]