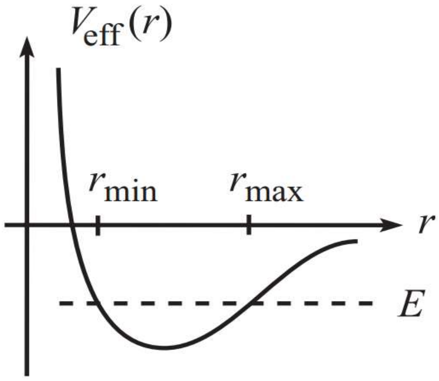
MILITARY INSTITUTE OF SCIENCE AND TECHNOLOGY (MIST)

Department of Naval Architecture and Marine Engineering (NAME)

Course Title: Numerical Methods Sessional *Course Code: NAME 364*

<u>Final Assignment</u>

1. Consider the following graph:



The points $\boldsymbol{r_{min}}$ and $\boldsymbol{r_{max}}$ are the positions when the radial kinetic energy of the satellite becomes $\mathbf{0}$ and only the effective potential remains. So, at those two points, the equation of energy becomes:

$$\boldsymbol{E = V_{eff} \Rightarrow E - V_{eff} = 0}$$

Write a function called root_eqn() that will solve the above equation using False Position Method. Follow the instructions given below:

- The function root_eqn() takes one input, the initial guess of the solution ($\boldsymbol{x_0}$)
- Take the initial guess for $\boldsymbol{r_{min}}$ to be $\mathbf{2.9 \times 10^7}$ and call the function to obtain the value of $\boldsymbol{r_{min}}$
- Take the initial guess for $\boldsymbol{r_{max}}$ to be $\mathbf{7 \times 10^7}$ and call the function to obtain the value of $\boldsymbol{r_{max}}$

2. Viscosity, μ , is a property of gases and fluids that characterizes their resistance to flow. For most materials, viscosity is highly sensitive to temperature. Below is a table that gives the viscosity of SAE 10 W oil at different temperatures (data from B.R. Munson, D.P. Young, and T.H. Okiishi, Fundamentals of Fluid Mechanics, 4th ed., John Wiley and Sons, 2002). Determine an equation that can be best fitted to the data.

T(^{o}C)	-20	0	20	40	60	80	100	120
μ (Ns/m^2) (x10^{-5})	4	0.38	0.095	0.032	0.015	0.0078	0.0045	0.0032

3. The widths of a deep tank bulkhead at equal intervals of 1.2 m commencing at the top, are 8.0, 7.5, 6.5, 5.7, 4.7, 3.8 and 3.0 m. Calculate the load on the bulkhead and the position of the centre of pressure, if the bulkhead is flooded to the top edge with sea water on one side only.

4. Write a MATLAB code to solve $2x\cos 2x - (x - 2)^2 = 0$ for $2 \le x \le 3$ and $3 \le x \le 4$ by using Newton-Raphson Method.

5. The following data come from a table that was measured with high precision. Use the best numerical method in MATLAB to determine y at x=3.5.

x	0	1.8	5	6	8.2	9.2	12
y	26	16.415	5.375	3.5	2.015	2.54	8

6. The radial displacement u of a pressurized hollow thick cylinder (inner radius = 5 ″, outer radius = 8″) is given at different radial locations.

Radius (in)	**Radial Displacement (in)**
5.0	0.0038731
5.6	0.0036165
6.2	0.0034222
6.8	0.0032743
7.4	0.0031618
8.0	0.0030769

The maximum normal stress, in psi, on the cylinder is given by

$$\sigma_{\max} = 3.2967\times 10^{6}\left(\frac{u(5)}{5} + 0.3\frac{du}{dr}(5)\right)$$

Determine, using MATLAB, the maximum stress in psi.

7. An important problem in structural engineering is that of finding the forces in a statically determinate truss (Figure 1). This type of structure can be described as a system of coupled linear algebraic equations derived from force balances. The sum of the forces in both horizontal and vertical directions must be zero at each node, because the system is at rest. Therefore, for node 1:

$$\sum F_H = 0 = -F_1\cos 30^0 + F_3\cos 60^0 + F_{1,h}$$
$$\sum F_V = 0 = -F_1\sin 30^0 - F_3\sin 60^0 + F_{1,v}$$

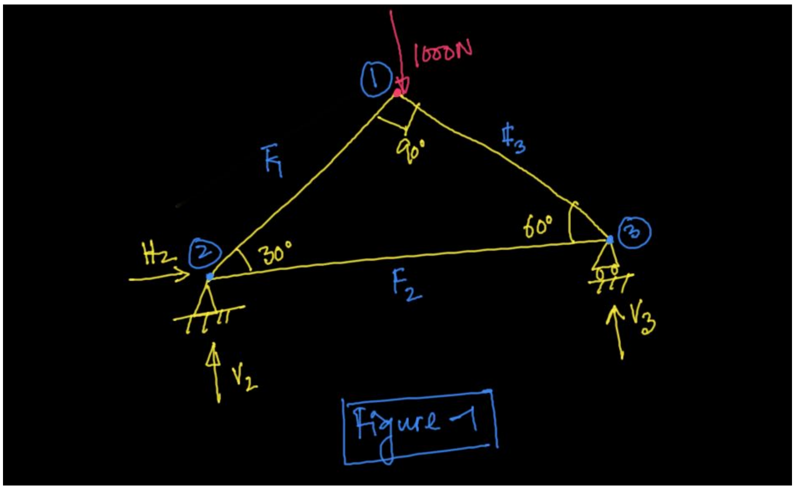
For node 2:

$$\sum F_H = 0 = F_2 + F_1\cos 30^\circ + F_{2,h} + H_2$$
$$\sum F_V = 0 = F_1\sin 30^\circ + F_{2,v} + V_2$$

For node 3:

$$\sum F_H = 0 = -F_2 - F_3\cos 60^\circ + F_{3,h}$$
$$\sum F_V = 0 = F_3\sin 60^\circ + F_{3,v} + V_3$$

Where $F_{i,h}$ is the external horizontal force applied to node i (where a positive force is from left to right) and $F_{i,v}$ is the external vertical force applied to node i (where a positive force is upward). Thus, in this problem, the 2000-N downward force on node 1 corresponds to $F_{i,v}$ = -2000. For this case all other $F_{i,v}$'s and $F_{i,h}$'sare zero. Express this set of linear algebraic equations in matrix form and then solve with Gauss-Jordan algorithm with partial pivoting in MATLAB to find out the unknowns.



8. Use zero through fourth order Taylor series expansion to predict f(2) for f(x) = ln(x) using a base point at x=1. Compute the true percent relative error ε_t for each approximation. Discuss the meaning of the results.

9. To calculate a planet's space coordinates, we have to solve the function:

f(x)=x - 1 – 0.5sin(x)

Let the base point be a = $x_i = \frac{\pi}{2}$ on the interval [0, π]. Determine the highest order Taylor series expansion resulting in a maximum error of 0.015 on the specified interval. The error is equal to the absolute value of the difference between the given function and the specific Taylor series expansion.

10. How to return the last column of a sample matrix with MATLAB code? Give example.

11. Fill in the blanks:

i) Approximate result contains ___________.

ii) Absolute error is the _______________ between the _________ of the number or result and the ___________ used to represent the number.

iii) Rounding off is the process of _________ some ___________ from the actual representation of the number following an ____________________ process.

12. Use the Gauss-Seidel method to solve the following system until the percent relative error falls below ε_s = 5%

$$\begin{bmatrix} 0.8 & -0.4 & \\ -0.4 & 0.8 & -0.4 \\ & -0.4 & 0.8 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 41 \\ 25 \\ 105 \end{Bmatrix}$$

13. Figure-2 depicts a chemical exchange process consisting of a series of reactors in which a gas flowing from left to right is passed over a liquid flowing from right to left. The transfer of a chemical from the gas into the liquid occurs at a rate that is proportional to the difference between the gas and liquid concentrations in each reactor. At steady state, a mass balance for the first reactor can be written for the gas as:

$$Q_G C_{G0} - Q_G C_{G1} + D(C_{L1} - C_{G1}) = 0$$

and for the liquid as:

$$Q_L C_{L2} - Q_L C_{L1} + D(C_{G1} - C_{L1}) = 0$$

Where Q_G and Q_L are the gas and liquid flow rates, respectively, and D = the gas-liquid exchange rate. Similar balances can be written for the other reactors. Use Gauss-Seidel method (with MATLAB code) to solve for the concentrations given the following values: Q_G=2, Q_L=1, D=0.8, c_{G0}=100, c_{L6}=10