INVERSE OF A RELATION

Let R be a relation from A to B. The **inverse relation** R^{-1} from B to A is defined as:

$$R^{-1} = \{(b,a) \in B \times A \mid (a,b) \in R\}$$

More simply, the **inverse relation** R^{-1} of R is obtained by **interchanging** the **elements** of all the **ordered pairs** in R.

EXAMPLE

Let $A = \{2, 3, 4\}$, $B = \{2,6,8\}$ and let R be the "divides" relation from A to B

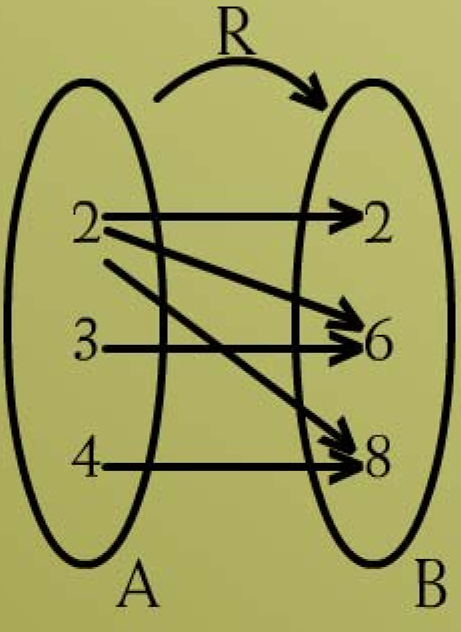
i.e. for all $(a,b) \in A \times B$, $a \, R \, b \Leftrightarrow a \mid b$
(a divides b)

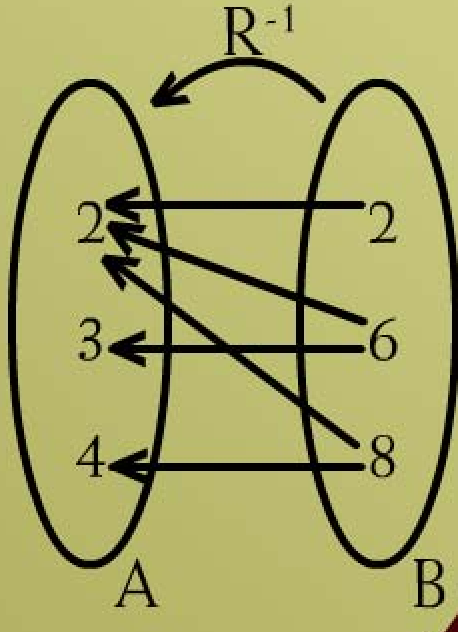
$$R = \{(2,2), (2,6), (2,8), (3,6), (4,8)\}$$

$$R^{-1} = \{(2,2), (6,2), (8,2), (6,3), (8,4)\}$$

ARROW DIAGRAM OF AN INVERSE RELATION

$R = \{(2,2), (2,6), (2,8), (3,6), (4,8)\}$





MATRIX REPRESENTATION OF INVERSE RELATION

R = {(2, 2), (2, 6), (2, 8), (3, 6), (4, 8)} from
A = {2, 3,4} to B = {2, 6, 8}

$$M = \begin{array}{c} \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{ccc} 2 & 6 & 8 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \qquad M^t = \begin{array}{c} \\ 2 \\ 6 \\ 8 \end{array}\begin{array}{c} \begin{array}{ccc} 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$$

COMPLEMENTRY RELATION

Let R be a **relation** from a set A to a set B. The **complementry relation** $\overline{R}$ of R is the set of all those **ordered pairs** in A×B that do not belong to R.

Symbolically:

$$\begin{aligned}\overline{R} &= A \times B - R \\ &= \{(a,b) \in A \times B \mid (a,b) \notin R\}\end{aligned}$$

EXAMPLE

Let

$$A = \{1,2,3\}$$

$$A \times A = \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)\}$$

$$R = \{(1,1),\ (1,3),\ (2,2),\ (2,3),\ (3,1)\}$$

Then

$$\overline{R} = \{(1,2),\ (2,1),\ (3,2),\ (3,3)\}$$

COMPOSITE RELATION

Let R be a relation from a set A to a set B and S a relation from B to a set C. The composite of R and S denoted $S \circ R$ is the relation from A to C, consisting of ordered pairs (a,c) where $a \in A$, $c \in C$, and for which there exists an element $b \in B$ such that $(a,b) \in R$ and $(b,c) \in S$.

Symbolically:

$$S \circ R = \{(a,c) \mid a \in A,\ c \in C, \exists\, b \in B,\ (a,b) \in R \text{ and } (b,c) \in S\}$$

EXAMPLE

Let $A = \{a,b,c\}$

$B = \{1,2,3,4\}$

$C = \{x,y,z\}$

$$R = \{(a,1), (a,4), (b,3), (c,1), (c,4)\}$$

$$S = \{(1,x), (2,x), (3,y), (3,z)\}$$

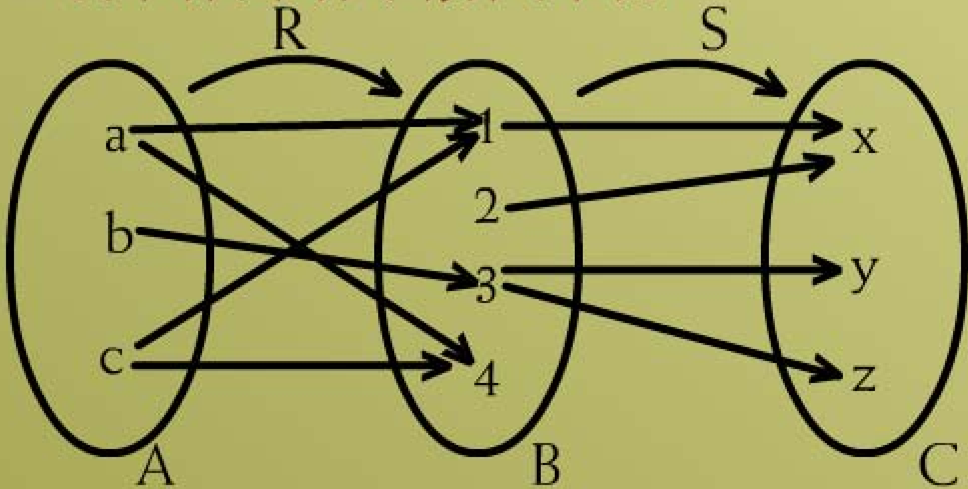
$$S \circ R = \{(a,x), (b,y), (b,z), (c,x)\}$$

COMPOSITE RELATION FROM ARROW DIAGRAM

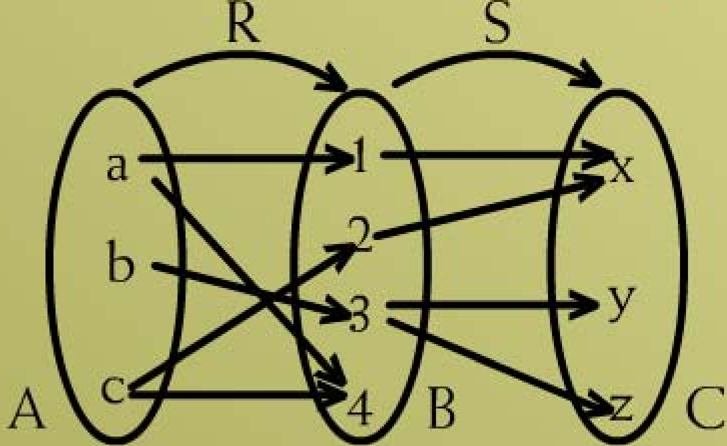
Let A = {a,b,c}   B = {1,2,3,4}   C = {x,y,z}

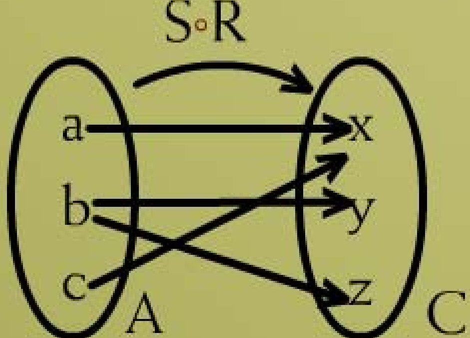
R = {(a,1), (a,4), (b,3),(c,1), (c,4)}

S = {(1,x),(2,x), (3,y), (3,z)}



$A = \{a,b,c\}$ $C = \{x,y,z\}$





MATRIX REPRESENTATION OF COMPOSITE RELATION

The matrix representation of the composite relation can be found using the Boolean product of the matrices for the relations. Thus if M_R and M_S are the matrices for relations R (from A to B) and S (from B to C), then

$$M_{SoR} = M_R \odot M_S$$

is the matrix for the composite relation SoR from A to C.

BOOLEAN ALGEBRA

BOOLEAN ADDITION	BOOLEAN MULTIPLICATION
(a) $1 + 1 = 1$	(a) $1 . 1 = 1$
(b) $1 + 0 = 1$	(b) $1 . 0 = 0$
(c) $0 + 0 = 0$	(c) $0 . 0 = 0$

EXERCISE

We are given relations R and S in matrix form as.

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad M_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

SOLUTION

$$M_{S \circ R} = M_R \odot M_S = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$