UNION

Let A and B be subsets of a universal set U. The union of sets A and B is the set of all elements in U that belong to A or to B or to both, and is denoted $A \cup B$.

Symbolically:

$$A \cup B = \{x \in U \mid x \in A \text{ or } x \in B\}$$

UNION

EXAMPLE:

Let

$$U = \{a, b, c, d, e, f, g\}$$

$$A = \{a, c, e, g\}$$

$$B = \{d, e, f, g\}$$

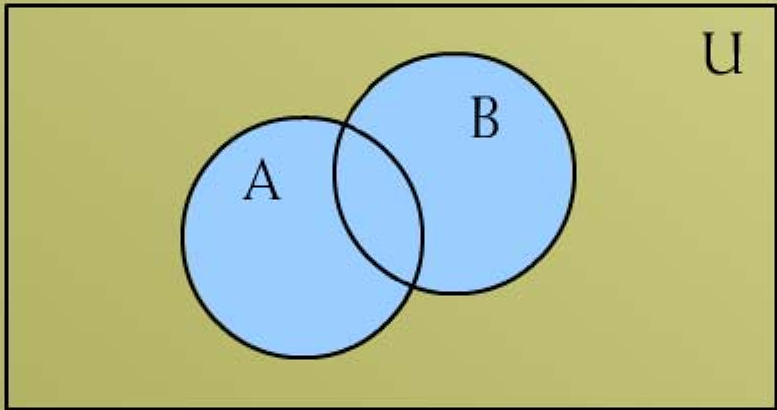
Then

$$A \cup B = \{a, c, e, g\} \cup \{d, e, f, g\}$$

$$= \{a, c, d, e, f, g\}$$

VENN DIAGRAM FOR

$A \cup B$



REMARK

1. $A \cup B = B \cup A$
2. $A \subseteq A \cup B$ and $B \subseteq A \cup B$

MEMBERSHIP TABLE FOR

$A \cup B$

A	B	$A \cup B$
1	1	1
1	0	1
0	1	1
0	0	0

INTERSECTION

Let A and B subsets of a universal set U. The intersection of sets A and B is the set of all elements in U that belong to both A and B and is denoted $A \cap B$.

Symbolically:

$$A \cap B = \{x \in U \mid x \in A \text{ and } x \in B\}$$

INTERSECTION

EXMAPLE

Let $U = \{a, b, c, d, e, f, g\}$

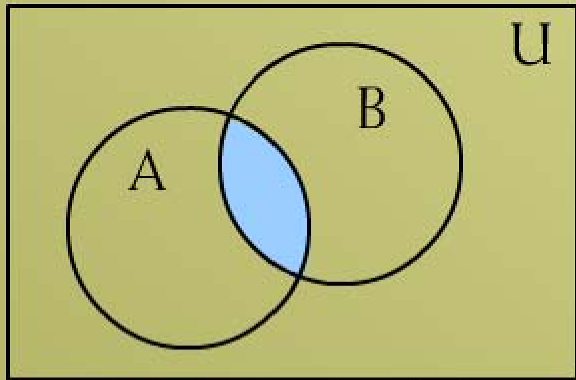
$A = \{a, c, e, g\}$

$B = \{d, e, f, g\}$

Then

$$A \cap B = \{a, c, e, g\} \cap \{d, e, f, g\}$$
$$= \{e, g\}$$

VENN DIAGRAM



$A \cap B$ is shaded

REMARK

1. $A \cap B = B \cap A$
2. $A \cap B \subseteq A$ and $A \cap B \subseteq B$
3. If $A \cap B = \varnothing$

   then A & B are called disjoint sets.

MEMBERSHIP TABLE FOR

$A \cap B$

A	B	$A \cap B$
1	1	1
1	0	0
0	1	0
0	0	0

SET DIFFERENCE

Let A and B be subsets of a universal set U. The difference of "A and B" (or relative complement of B in A) is the set of all elements in U that belong to A but not to B, and is denoted $A - B$ or $A \setminus B$.

Symbolically:

$$A - B = \{x \in U \mid x \in A \text{ and } x \notin B\}$$

SET DIFFERENCE

EXMAPLE

Let $U = \{a, b, c, d, e, f, g\}$

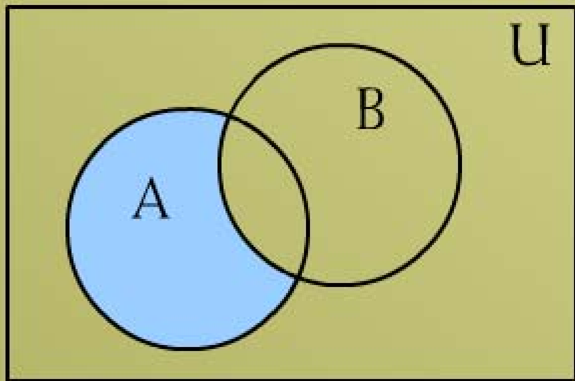
$A = \{a, c, e, g\}$

$B = \{d, e, f, g\}$

Then

$$A - B = \{a, c, e, g\} - \{d, e, f, g\}$$
$$= \{a, c\}$$

VENN DIAGRAM



A - B is shaded

REMARKS:

1. $A - B \neq B - A$
2. $A - B \subseteq A$
3. $A - B$, $A \cap B$ and $B - A$ are mutually disjoint sets.

MEMBERSHIP TABLE FOR

A – B

A	B	A – B
1	1	0
1	0	1
0	1	0
0	0	0

COMPLEMENT

Let A be a subset of universal set U. The complement of A is the set of all element in U that do not belong to A, and is denoted A^c, $\overline{A}$ or A'

Symbolically:

$$A' = \{x \in U \mid x \notin A\}$$

COMPLEMENT

EXMAPLE

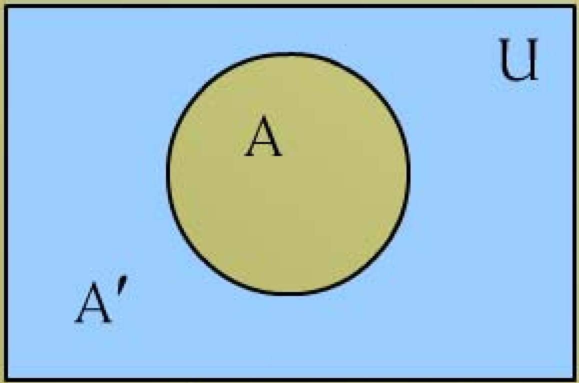
Let $U = \{a, b, c, d, e, f, g\}$

$A = \{a, c, e, g\}$

Then

$$
\begin{aligned}
A' &= \{a, b, c, d, e, f, g\} - \{a, c, e, g\} \\
&= \{b, d, f\}
\end{aligned}
$$

VENN DIAGRAM



A′ is shaded

REMARKS:

1. $A' = U - A$
2. $A \cap A' = \varnothing$
3. $A \cup A' = U$

MEMBERSHIP TABLE FOR

A'

A	A'
1	0
0	1

EXERCISE

Let $U = \{1, 2, 3, \ldots, 10\}$

$X = \{1, 2, 3, 4, 5\}$

$Y = \{y \mid y = 2x, x \in X\}$

$Z = \{z \mid z^2 - 9z + 14 = 0\}$

Enumerate:

(i) $X \cap Y$ (ii) $Y \cup Z$

(iii) $X - Z$ (iv) Y'

(v) $X' - Z'$ (vi) $(X - Z)'$

SOLUTION

Given

$$U = \{1, 2, 3, \ldots, 10\}$$
$$X = \{1, 2, 3, 4, 5\}$$

$$Y = \{y \in U \mid y = 2x, x \in X\}$$
$$= \{2, 4, 6, 8, 10\}$$

$$Z = \{z \in U \mid z^2 - 9z + 14 = 0\}$$
$$= \{2, 7\}$$

SOLUTION

(i) $X \cap Y = \{1, 2, 3, 4, 5\} \cap \{2, 4, 6, 8, 10\}$

$= \{2, 4\}$

(ii) $Y \cup Z = \{2, 4, 6, 8, 10\} \cup \{2, 7\}$

$= \{2, 4, 6, 7, 8, 10\}$

(iii) $X - Z = \{1, 2, 3, 4, 5\} - \{2, 7\}$

$= \{1, 3, 4, 5\}$

SOLUTION

(iv) $Y' = U - Y$

$= \{1, 2, 3, \ldots, 10\} - \{2, 4, 6, 8, 10\}$

$= \{1, 3, 5, 7, 9\}$

(v) $X' - Z'$

$= \{6, 7, 8, 9, 10\} - \{1, 3, 4, 5, 6, 8, 9, 10\}$

$= \{7\}$

(vi) $(X - Z)'$

$= U - (X - Z)$

$= \{1, 2, 3, \ldots, 10\} - \{1, 3, 4, 5\}$

$= \{2, 6, 7, 8, 9, 10\}$

EXERCISE

$$U = \{x \in U \mid x \in \mathbb{Z}, 0 \le x \le 10\}$$

$$P = \{x \in U \mid x \text{ is a prime number}\}$$

$$Q = \{x \in U \mid x^2 < 70\}$$

(i) Draw a Venn diagram for the above

(ii) List the elements in $P^c \cap Q$

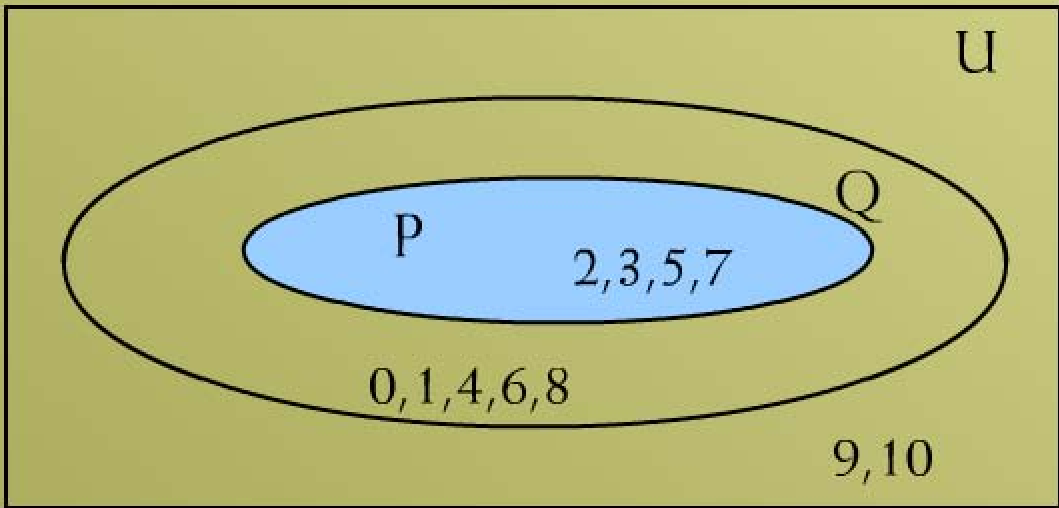
SOLUTION

$$U = \{x \in U \mid x \in Z, 0 \leq x \leq 10\}$$
$$= \{0, 1, 2, 3, \ldots, 10\}$$

$$P = \{x \in U \mid x \text{ is a prime number}\}$$
$$= \{2, 3, 5, 7\}$$

$$Q = \{x \in U \mid x^2 < 70\}$$
$$= \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$$

VENN DIAGRAM



ELEMENTS OF

(ii) $P' \cap Q$

$$P' = U - P$$
$$= \{0, 1, 2, 3, \ldots, 10\} - \{2, 3, 5, 7\}$$
$$= \{0, 1, 4, 6, 8, 9, 10\}$$

and

$$P' \cap Q$$
$$= \{0, 1, 4, 6, 8, 9, 10\} \cap \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$$
$$= \{0, 1, 4, 6, 8\}$$

EXERCISE

Let $U = \{1, 2, 3, 4, 5\}$ $C = \{1, 3\}$

Where A and B are non empty sets. Find A in each of the following:

(i) $A \cup B = U$ $A \cap B = \varnothing$ and $B = \{1\}$

EXERCISE

(ii) $A \subset B$ and $A \cup B = \{4, 5\}$

(iii) $A \cap B = \{3\}$ $A \cup B = \{2, 3, 4\}$
and $B \cup C = \{1,2,3\}$

(iv) A and B are disjoint, B and C are disjoint, and the union of A and B is the set $\{1, 2\}$.

SOLUTION

(i) $A \cup B = U \quad A \cap B = \varnothing$ and $\quad B = \{1\}$

SOLUTION:

$$\text{Since } A \cup B = U$$
$$= \{1, 2, 3, 4, 5\}$$
$$\text{and } \quad A \cap B = \varnothing$$

Therefore $\quad A = B'$

$$= \{1\}'$$
$$= \{2, 3, 4, 5\}$$

SOLUTION

(ii) $A \subset B$ and $A \cup B = \{4, 5\}$ also $C = \{1, 3\}$

SOLUTION:

When $A \subset B$

then $A \cup B = B$

$= \{4, 5\}$

Also A being a proper subset of B implies

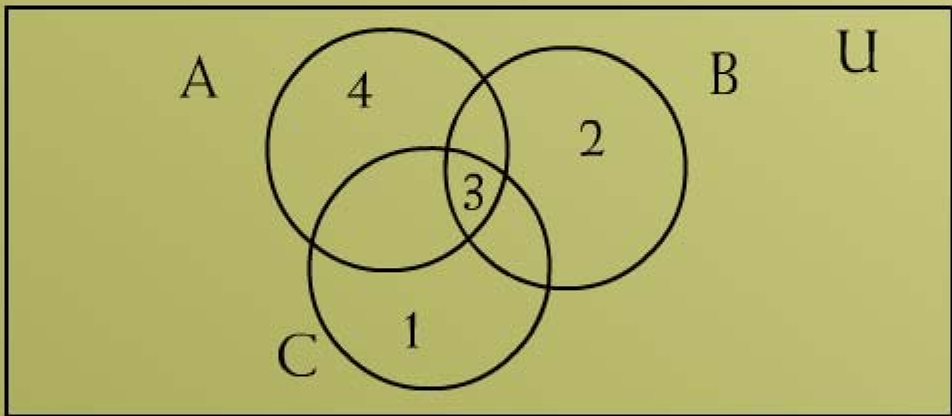
$A = \{4\}$ or $A = \{5\}$

Solution contd...

(iii) $A \cap B = \{3\}$ $A \cup B = \{2, 3, 4\}$

and

$B \cup C = \{1,2,3\}$ Also $C = \{1, 3\}$

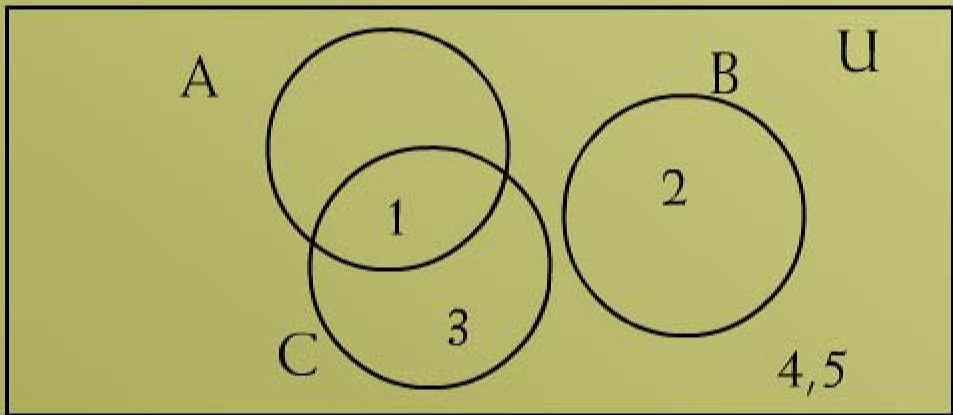


$A = \{3, 4\}$ $B = \{2, 3\}$

Solution contd...

(iv) $A \cap B = \varnothing \qquad B \cap C = \varnothing$

$A \cup B = \{1, 2\}$ Also $C = \{1, 3\}$

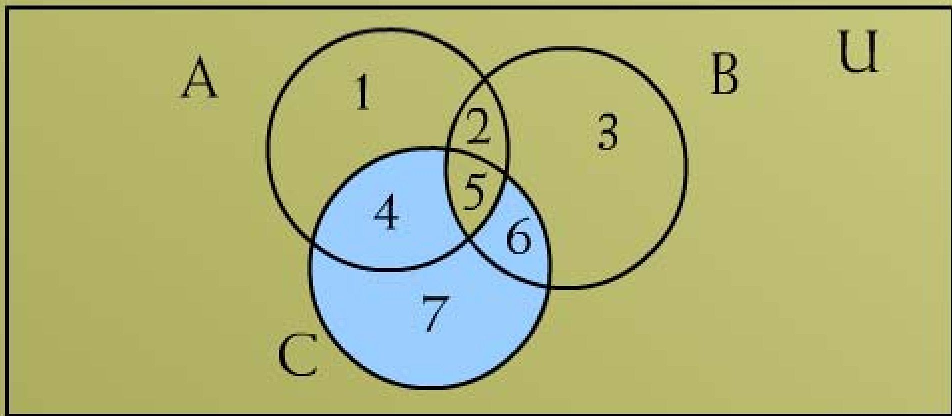


$A = \{1\}$

EXERCISE

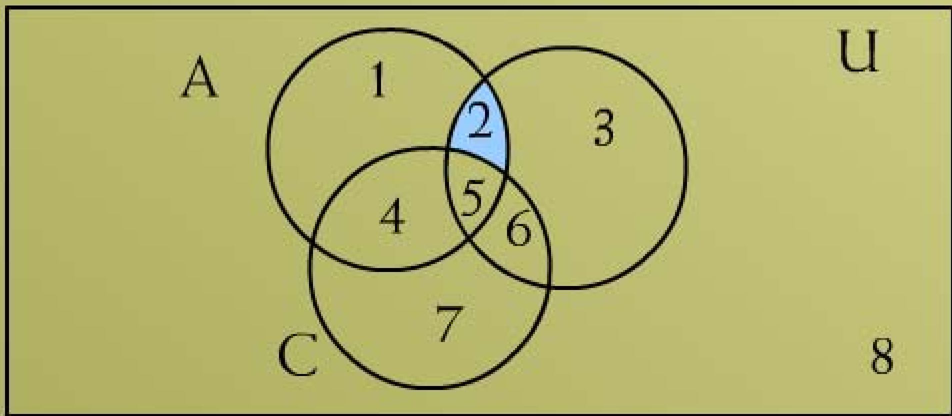
(i) $(A \cap B) \cap C'$ (ii) $A' \cup (B \cup C)$

(iii) $(A - B) \cap C$ (iv) $(A \cap B') \cup C'$



VENN DIAGRAM FOR

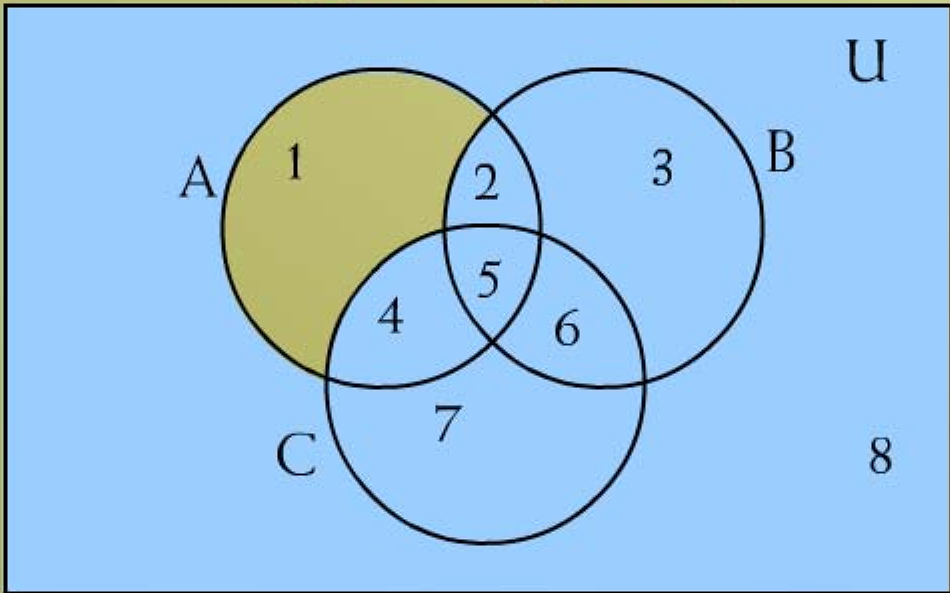
(i) $(A \cap B) \cap C'$



$(A \cap B) \cap C' = \{2\}$

VENN DIAGRAM FOR

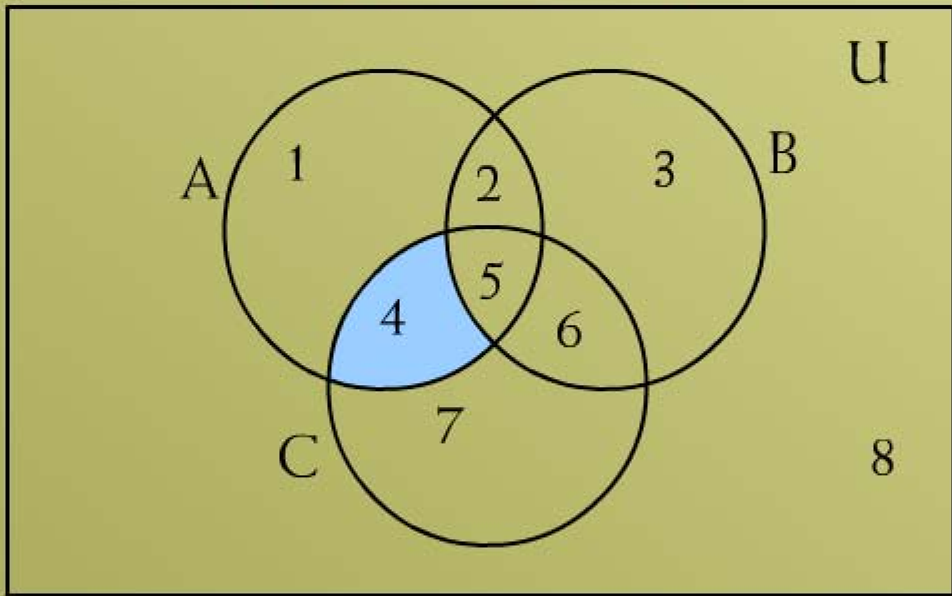
(ii) $A' \cup (B \cup C)$



$A' \cup (B \cup C) = \{2, 3, 4, 5, 6, 7, 8\}$

VENN DIAGRAM FOR

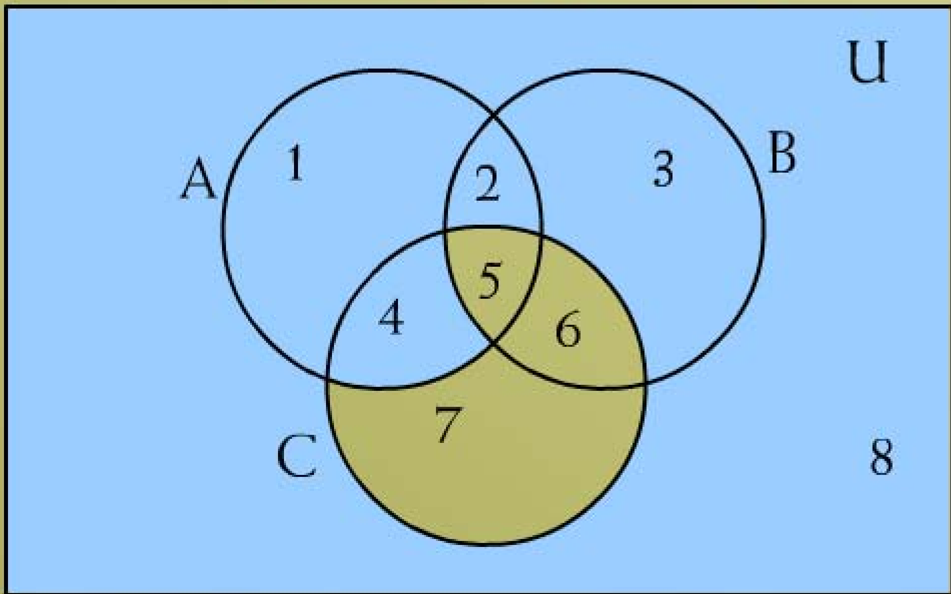
(iii) (A – B) ∩ C



$(A - B) \cap C = \{4\}$

VENN DIAGRAM FOR

(iv) $(A \cap B') \cup C'$



$(A \cap B') \cup C' = \{1,2,3,4,8\}$