Discrete Structures MT217

Lecture 02

TRUTH TABLE

1. $\sim p \wedge q$
2. $\sim p \wedge (q \vee \sim r)$
3. $(p \vee q) \wedge \sim (p \wedge q)$

TRUTH TABLE FOR

$\sim p \wedge q$

p	q	$\sim p$	$\sim p \wedge q$
T	T		
T	F		
F	T		
F	F		

TRUTH TABLE FOR

$\sim p \wedge q$

p	q	$\sim p$	$\sim p \wedge q$
T		F	
T		F	
F		T	
F		T	

TRUTH TABLE FOR

$\sim p \wedge q$

p	q	$\sim p$	$\sim p \wedge q$
	T	F	F
	F	F	F
	T	T	T
	F	T	F

TRUTH TABLE FOR

$\sim p \wedge q$

p	q	~p	~p ∧ q
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

$\sim p \wedge (q \vee \sim r)$

p	q	r	$\sim r$	$q \vee \sim r$	$\sim p$	$\sim p \wedge (q \vee \sim r)$
T	T	T				
T	T	F				
T	F	T				
T	F	F				
F	T	T				
F	T	F				
F	F	T				
F	F	F				

$\sim p \wedge (q \vee \sim r)$

p	q	r	~r	q ∨ ~ r	~p	~p ∧ (q ∨~ r)
		T	F			
		F	T			
		T	F			
		F	T			
		T	F			
		F	T			
		T	F			
		F	T			

$\sim p \wedge (q \vee \sim r)$

p	q	r	$\sim r$	$q \vee \sim r$	$\sim p$	$\sim p \wedge (q \vee \sim r)$
	T		F	T		
	T		T	T		
	F		F	F		
	F		T	T		
	T		F	T		
	T		T	T		
	F		F	F		
	F		T	T		

$\sim p \wedge (q \vee \sim r)$

p	q	r	$\sim r$	$q \vee \sim r$	$\sim p$	$\sim p \wedge (q \vee \sim r)$
				T	F	F
				T	F	F
				F	F	F
				T	F	F
				T	T	T
				T	T	T
				F	T	F
				T	T	T

$\sim p \wedge (q \vee \sim r)$

p	q	r	$\sim r$	$q \vee \sim r$	$\sim p$	$\sim p \wedge (q \vee \sim r)$
T	T	T	F	T	F	F
T	T	F	T	T	F	F
T	F	T	F	F	F	F
T	F	F	T	T	F	F
F	T	T	F	T	T	T
F	T	F	T	T	T	T
F	F	T	F	F	T	F
F	F	F	T	F	T	T

TRUTH TABLE FOR

$(p \vee q) \wedge \sim (p \wedge q)$

p	q	$p \vee q$	$p \wedge q$	$\sim p \wedge q$	$(p \vee q) \wedge \sim (p \wedge q)$
T	T	T			
T	F	T			
F	T	T			
F	F	F			

TRUTH TABLE FOR

$(p \vee q) \wedge \sim (p \wedge q)$

p	q	$p \vee q$	$p \wedge q$	$\sim p \wedge q$	$(p \vee q) \wedge \sim (p \wedge q)$
T	T		T		
T	F		F		
F	T		F		
F	F		F		

TRUTH TABLE FOR

$(p \vee q) \wedge \sim (p \wedge q)$

p	q	$p \vee q$	$p \wedge q$	$\sim p \wedge q$	$(p \vee q) \wedge \sim (p \wedge q)$
		T		F	F
		T		T	T
		T		T	T
		F		T	F

TRUTH TABLE FOR

$(p \vee q) \wedge \sim (p \wedge q)$

p	q	$p \vee q$	$p \wedge q$	$\sim p \wedge q$	$(p \vee q) \wedge \sim (p \wedge q)$
T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	T	F

LOGICAL EQUIVALENCE

Two statement forms are called logically equivalent if and only if, they have identical truth values for all possible truth values for their statement variables.

The logical equivalence of statement forms p and q is denoted by writing $p \equiv q$.

DOUBLE NEGATION

$$\sim(\sim p) \equiv p$$

p	$\sim p$	$\sim(\sim p)$
T	F	T
F	T	F

Same Truth Values

DE MORGAN'S LAWS

1) The negation of an and statement is logically equivalent to the or statement in which each component is negated.

Symbolically

$$\sim(p \wedge q) \equiv \sim p \vee \sim q$$

DE MORGAN'S LAWS

2) The negation of an or statement is logically equivalent to the and statement in which each component is negated.

Symbolically

$$\sim(p \vee q) \equiv \sim p \wedge \sim q$$

INEQUALITIES AND DEMORGAN'S LAWS

Use DeMorgan's Laws to write the negation of

$-1 < x \leq 4$

for some particular real number x.

$-1 < x \leq 4$ means $x > -1$ and $x \leq 4$

By DeMorgan's Law, the negation is:

$x > -1$ or $x \leq 4$

Which is equivalent to: $x \leq -1$ or $x > 4$

EXAMPLES

Rewrite in a simpler form:

"It is not true that I am not happy"

Solution:

Let p = "I am happy"

then $\sim p$ = "I am not happy"

and $\sim(\sim p)$ = "It is not true that I am not happy"

Since $\sim(\sim p) \equiv p$

Hence the given statement is equivalent to:

"I am happy"

PROOF

p	q	$\sim p$	$\sim q$	$p \vee q$	$\sim(p \vee q)$	$\sim p \wedge \sim q$
T	T	F	F			
T	F	F	T			
F	T	T	F			
F	F	T	T			

PROOF

$$\sim(p \vee q) \equiv \sim p \wedge \sim q$$

p	q	$\sim p$	$\sim q$	$p \vee q$	$\sim(p \vee q)$	$\sim p \wedge \sim q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

EXAMPLE

$\sim(p \wedge q)$ and $\sim p \wedge \sim q$ are not equivalent

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim(p \wedge q)$	$\sim p \wedge \sim q$
T	T	F	F			
T	F	F	T			
F	T	T	F			
F	F	T	T			

EXAMPLE

$\sim(p \wedge q)$ and $\sim p \wedge \sim q$ are not equivalent

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim(p \wedge q)$	$\sim p \wedge \sim q$
T	T			T		
T	F			F		
F	T			F		
F	F			F		

EXAMPLE

$\sim(p \wedge q)$ and $\sim p \wedge \sim q$ are not equivalent

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim(p \wedge q)$	$\sim p \wedge \sim q$
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	F	T	T

EXERCISE

Are the statements $(p \wedge q) \wedge r$ and $p \wedge (q \wedge r)$ logically equivalent ?

Are the statements $(p \wedge q) \vee r$ and $p \wedge (q \vee r)$ logically equivalent ?

TAUTOLOGY

A tautology is a statement form that is always true regardless of the truth values of the statement variables.

A tautology is represented by the symbol "t".

EXAMPLE

The statement form $p \vee \sim p$ is tautology.

p	$\sim p$	$p \vee \sim p$
T	F	T
F	T	T

$$p \vee \sim p \equiv t$$

CONTRADICTION

A contradiction is a statement form that is always false regardless of the truth values of the statement variables.

A contradiction is represented by the symbol

"c"

EXAMPLE

The statement form $p \wedge \sim p$ is contradiction.

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

$$p \wedge \sim p \equiv c$$

EXERCISE

$$(p \wedge q) \vee (\sim p \vee (p \wedge \sim q)) \equiv t$$

p	q	$p \wedge q$	$\sim p$	$\sim q$	$p \wedge \sim q$	$\sim p \vee (p \wedge \sim q)$	$(p \wedge q) \vee (\sim p \vee (p \wedge \sim q))$
T	T	T	F	F	F	F	T
T	F	F	F	T	T	T	T
F	T	F	T	F	F	T	T
F	F	F	T	T	F	T	T

EXERCISE

$$(p \wedge \sim q) \wedge (\sim p \vee q) \equiv c$$

p	q	$\sim q$	$p \wedge \sim q$	$\sim p$	$\sim p \vee q$	$(p \wedge \sim q) \wedge (\sim p \vee q)$
T	T	F	F	F	T	F
T	F	T	T	F	F	F
F	T	F	F	T	T	F
F	F	T	F	T	T	F

LAWS OF LOGIC

Given any statement variables p, q and r, a tautology t and a contradiction c, the following logical equivalences hold:

1) Commutative Laws: $p \wedge q \equiv q \wedge p$

$p \vee q \equiv q \vee p$

2) Associative Laws: $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

$(p \vee q) \vee r \equiv p \vee (q \vee r)$

LAWS OF LOGIC

3) Distributive Laws:

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (q \wedge r)$$

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (q \vee r)$$

4) Identity laws:

$$p \wedge t \equiv p$$

$$p \vee c \equiv p$$

LAWS OF LOGIC

Negation laws:	$p \vee \sim p \equiv t$
	$p \wedge \sim p \equiv c$
Double negation law:	$\sim(\sim p) \equiv p$
Idempotent laws:	$p \wedge p \equiv p$
	$p \vee p \equiv p$

LAWS OF LOGIC

DeMorgan's laws:

$$\sim(p \wedge q) \equiv \sim p \vee \sim q$$
$$\sim(p \vee q) \equiv \sim p \wedge \sim q$$

Universal bound laws:

$$p \vee t \equiv t$$
$$p \wedge c \equiv c$$

LAWS OF LOGIC

Absorption laws:

$$p \vee (p \wedge q) \equiv p$$
$$p \wedge (p \vee q) \equiv p$$

Negations of t and c:

$$\sim t \equiv c$$
$$\sim c \equiv t$$