Discrete Structures MT217

Lecture 03

APPLICATION

Simplify:

$$p \vee [\sim(\sim p \wedge q)]$$

Solution:

$p \vee [\sim(\sim p \wedge q)]$	
$\equiv p \vee [\sim(\sim p) \vee (\sim q)]$	DeMorgan'sLaw
$\equiv p \vee [p \vee (\sim q)]$	Double Negative Law
$\equiv [p \vee p] \vee (\sim q)$	Associative Law for $\vee$
$\equiv p \vee (\sim q)$	Indempotent Law

Which is the simplified statement form.

EXAMPLE

Verify :

$$\sim(\sim p \wedge q) \wedge (p \vee q) \equiv p$$

$\sim(\sim p \wedge q) \wedge (p \vee q)$	
$\equiv (\sim(\sim p) \vee \sim q) \wedge (p \vee q)$	DeMorgan's Law
$\equiv (p \vee \sim q) \wedge (p \vee q)$	Double Negative Law
$\equiv p \vee (\sim q \wedge q)$	Distributive Law in Reverse
$\equiv p \vee c$	Negation Law
$\equiv p$	Identity Law

SIMPLIFYING A STATEMENT

"You will get an A if you are hardworking and the sun shines, or you are hardworking and it rains."

Solution:

Let

p = "You are hardworking'

q = "The sun shines"

r = "It rains"

The condition is then $(p \wedge q) \vee (p \wedge r)$

$(p \wedge q) \vee (p \wedge r)$

$\equiv p \wedge (q \vee r)$ Distributive Law in Reverse

Putting $p \wedge (q \vee r)$ back into English, we can rephrase the given sentence as

"You will get an A if you are hardworking and the sun shines or it rains.

EXERCISE

Use Logical Equivalence to rewrite each of the following sentences more simply.

1) It is not true that I am tired and you are smart.

2) It is not true that I am tired or you are smart.

EXERCISE

3) I forgot my pen or my bag and I forgot my pen or my glasses.

4) It is raining and I have forgotten my umbrella, or it is raining and I have forgotten my hat.

CONDITIONAL STATEMENTS

"If you earn an A in Math, then I'll buy you a computer."

p: "You earn an A in Math,"

and

q: "I will buy you a computer."

CONDITIONAL STATEMENTS

The original statement is then saying :

If p is true, then q is true

Or

If p, then q

We can also phrase this as p implies q, and we write $p \rightarrow q$.

CONDITIONAL STATEMENTS OR IMPLICATIONS

If p and q are statement variables, the conditional of q by p is "If p then q" or "p implies q" and is denoted $p \rightarrow q$.

CONDITIONAL STATEMENTS OR IMPLICATIONS

The arrow "→" is the conditional operator

p is called the hypothesis (or antecedent)

q is called the conclusion (or consequent)

TRUTH TABLE FOR

$p \rightarrow q$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

EXAMPLE

STATEMENTS	TRUTH VALUES
1. "If $1 = 1$, then $3 = 3$."	TRUE
2. "If $1 = 1$, then $2 = 3$."	FALSE
3. "If $1 = 0$, then $3 = 3$."	TRUE
4. "If $1 = 2$, then $2 = 3$."	TRUE
5. "If $1 = 1$,then $1 = 2$ and $2 = 3$."	FALSE
6. "If $1 = 3$ or $1 = 2$ then $3 = 3$."	TRUE

ALTERNATIVE WAYS OF EXPRESSING IMPLICATIONS

- "if p then q"
- "p implies q"
- "if p, q"
- "p only if q"
- "p is sufficient for q"

ALTERNATIVE WAYS OF EXPRESSING IMPLICATIONS

- "not p unless q"
- "q follows from p"
- "q if p"
- "q whenever p"
- "q is necessary for p"

EXERCISE

a) Your guarantee is good only if you bought your CD less than 90 days ago.

If your guarantee is good, then you must have bought your CD player less than 90 days ago.

b) To get tenure as a professor, it is sufficient to be world-famous.

If you are world-famous, then you will get tenure as a professor.

EXERCISE

c) That you get the job implies that you have the best credentials.

If you get the job, then you have the best credentials.

d) It is necessary to walk 8 miles to get to the top of the Peak.

If you get to the top of the peak, then you must have walked 8 miles.

TRANSLATING ENGLISH SENTENCES TO SYMBOLS

Let p and q be propositions:

p = "you get an A on the final exam"

q = "you do every exercise in this book"

r = "you get an A in this class"

TRANSLATING ENGLISH SENTENCES TO SYMBOLS

To get an A in this class it is necessary for you to get an A on the final.

SOLUTION:

$$p \to r$$

TRANSLATING ENGLISH SENTENCES TO SYMBOLS

You do every exercise in this book; You get an A on the final, implies, you get an A in the class.

SOLUTION:

$$p \wedge q \rightarrow r$$

TRANSLATING ENGLISH SENTENCES TO SYMBOLS

Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.

SOLUTION:

$$p \wedge q \rightarrow r$$

TRANSLATING SYMBOLIC PROPOSITIONS TO ENGLISH

Let p, q, and r be the propositions:

p = "you have the flu"
q = "you miss the final exam"
r = "you pass the course"

TRANSLATING SYMBOLIC PROPOSITIONS TO ENGLISH

$p \to q$

If you have flu, then you will miss the final exam.

$\sim q \to r$

If you don't miss the final exam, then you will pass the course.

TRANSLATING SYMBOLIC PROPOSITIONS TO ENGLISH

$\sim p \wedge \sim q \rightarrow r$

If you neither have flu nor miss the final exam, then you will pass the course.

HIERARCHY OF OPERATIONS FOR LOGICAL CONNECTIVES

1) $\sim$ (negation)

2) $\wedge$ (conjunction), $\vee$ (disjunction)

3) $\rightarrow$ (conditional)

TRUTH TABLE FOR

$$p \vee \sim q \rightarrow \sim p$$

$p \vee \sim q \rightarrow \sim p$ means $(p \vee (\sim q)) \rightarrow (\sim p)$

p	q	$\sim q$	$\sim p$	$p \vee \sim q$	$p \vee \sim q \rightarrow \sim p$
T	T	F	F		
T	F	T	F		
F	T	F	T		
F	F	T	T		

TRUTH TABLE FOR

$$p \vee \sim q \rightarrow \sim p$$

$p \vee \sim q \rightarrow \sim p$ means $(p \vee (\sim q)) \rightarrow (\sim p)$

p	q	$\sim q$	$\sim p$	$p \vee \sim q$	$p \vee \sim q \rightarrow \sim p$
T		F		T	
T		T		T	
F		F		F	
F		T		T	

TRUTH TABLE FOR

$p \vee \sim q \rightarrow \sim p$

$p \vee \sim q \rightarrow \sim p$ means $(p \vee (\sim q)) \rightarrow (\sim p)$

p	q	$\sim q$	$\sim p$	$p \vee \sim q$	$p \vee \sim q \rightarrow \sim p$
			F	T	F
			F	T	F
			T	F	T
			T	T	T

$$(p \to q) \wedge (\sim p \to r)$$

p	q	r	$p \to q$	$\sim p$	$\sim p \to r$	$(p \to q) \wedge (\sim p \to r)$
T	T	T	T			
T	T	F	T			
T	F	T	F			
T	F	F	F			
F	T	T	T			
F	T	F	T			
F	F	T	T			
F	F	F	T			

$(p \to q) \wedge (\sim p \to r)$

p	q	r	$p \to q$	$\sim p$	$\sim p \to r$	$(p \to q) \wedge (\sim p \to r)$
		T		F	T	
		F		F	T	
		T		F	T	
		F		F	T	
		T		T	T	
		F		T	F	
		T		T	T	
		F		T	F	

$(p \rightarrow q) \wedge (\sim p \rightarrow r)$

p	q	r	$p \rightarrow q$	$\sim p$	$\sim p \rightarrow r$	$(p \rightarrow q) \wedge (\sim p \rightarrow r)$
T	T	T	T	F	T	T
T	T	F	T	F	T	T
T	F	T	F	F	T	F
T	F	F	F	F	T	F
F	T	T	T	T	T	T
F	T	F	T	T	F	F
F	F	T	T	T	T	T
F	F	F	T	T	F	F

$$p \to q \equiv \sim q \to \sim p$$

p	q	$\sim q$	$\sim p$	$p \to q$	$\sim q \to \sim p$
T	T	F	F	T	T
T	F	T	F	F	F
F	T	F	T	T	T
F	F	T	T	T	T

IMPLICATION LAW

$$p \to q \equiv \sim p \vee q$$

p	q	$p \to q$	$\sim p$	$\sim p \vee q$
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

NEGATION OF A CONDITIONAL STATEMENT

Since $p \to q \equiv \sim p \vee q$ therefore

$$\sim (p \to q) \equiv \sim (\sim p \vee q)$$

$\equiv \sim (\sim p) \wedge (\sim q)$ De Morgan's law

$\equiv p \wedge \sim q$ Double Negative law

INVERSE OF A CONDITIONAL STATEMENT

The inverse of the conditional statement $p \rightarrow q$ is $\sim p \rightarrow \sim q$

$p \to q$ is not equivalent to $\sim p \to \sim q$

p	q	$p \to q$	$\sim p$	$\sim q$	$\sim p \to \sim q$
T	T	T	F	F	T
T	F	F	F	T	T
F	T	T	T	F	F
F	F	T	T	T	T

WRITING INVERSE

1. If today is Friday, then $2 + 3 = 5$.
   If today is not Friday, then $2 + 3 \neq 5$.

2. If it snows today, I will ski tomorrow.
   If it does not snow today I will not ski tomorrow.

WRITING INVERSE

3. If P is a square, then P is a rectangle.
   If P is not a square then P is not a rectangle.

4. If my car is in the repair shop, then I cannot get to class.
   If my car is not in the repair shop, then I shall get to the class.

CONVERSE OF A CONDITIONAL STATEMENT

The converse of the conditional statement $p \to q$ is $q \to p$

p	q	$p \to q$	$q \to p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

WRITING CONVERSE

1. If today is Friday, then $2 + 3 = 5$.
   If $2 + 3 = 5$, then today is Friday.

2. If it snows today, I will ski tomorrow.
   I will ski tomorrow only if it snows today.

WRITING CONVERSE

3. If P is a square, then P is a rectangle.
   If P is a rectangle then P is a square.

4. If my car is in the repair shop, then I cannot get to class.
   If I cannot get to the class, then my car is in the repair shop.

CONTRAPOSITIVE OF A CONDITIONAL STATEMENT

The contrapositive of the conditional statement $p \rightarrow q$ is

$$\sim q \rightarrow \sim p$$

A conditional and its contrapositive are equivalent. Symbolically,

$$p \rightarrow q \equiv \sim q \rightarrow \sim p$$

WRITING CONTRAPOSITIVITY

1. If today is Friday, then $2 + 3 = 5$.
   If $2 + 3 \neq 5$, then today is not Friday.

2. If it snows today, I will ski tomorrow.
   I will ski tomorrow only if it does not snow today.

WRITING CONTRAPOSITIVITY

3. If P is a square, then P is a rectangle.
   If P is not a rectangle then P is not a square.

4. If my car is in the repair shop, then I cannot get to class.
   If I get to the class, then my car is not in the repair shop.