EXERCISE

Define a binary relation E on the set of the integers Z, as follows:

$m, n \in Z$, $m \mathrel{E} n \Leftrightarrow m - n$ is even.

ELEMENTS RELATED UNDER RELATION E

Two elements are related under E if their diffrence is an **Even Number**.

$2\ E\ 4$ because $2-4=-2$(**Even Number**)

Even Numbers are related with **Even Numbers**
Odd Numbers are related with **Odd Numbers**

Define a binary relation E on the set of the integers Z, as follows:

$m,n \in Z$, $m\ E\ n \Leftrightarrow m - n$ is even.

a. Is $0E0$?

  Is $5E2$?

  Does $(6,6) \in E$?

  Does $(-1,7) \in E$?

b. Prove that for any even integer n, $nE0$.

SOLUTION

b.   For any **even integer**, n, we have
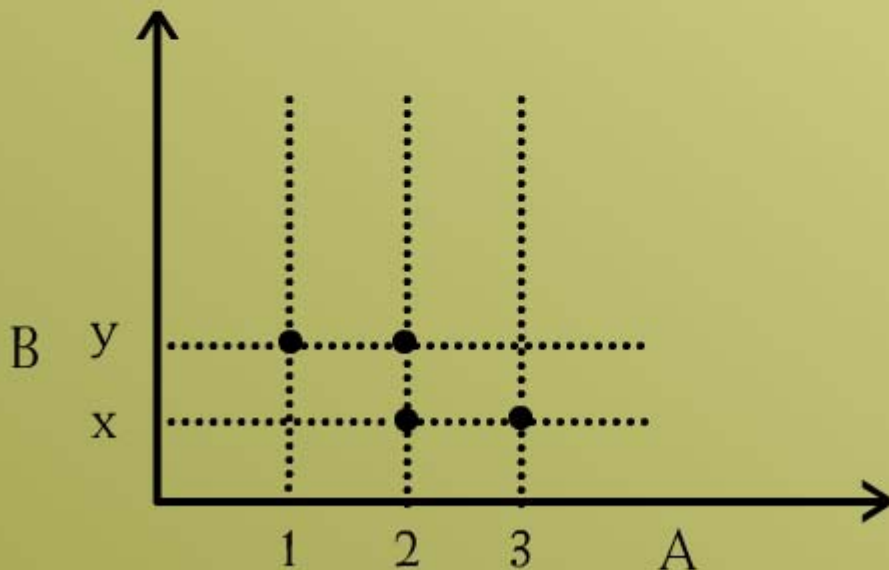
$$n - 0 = n, \quad \text{an even integer}$$

so $(n, 0) \in E$

or   equivalently $n \, E \, 0$

DIAGRAM OF A RELATION

Let $A = \{1, 2, 3\}$ and $B = \{x, y\}$

$$R = \{(1, y), (2, x), (2, y), (3, x)\}$$

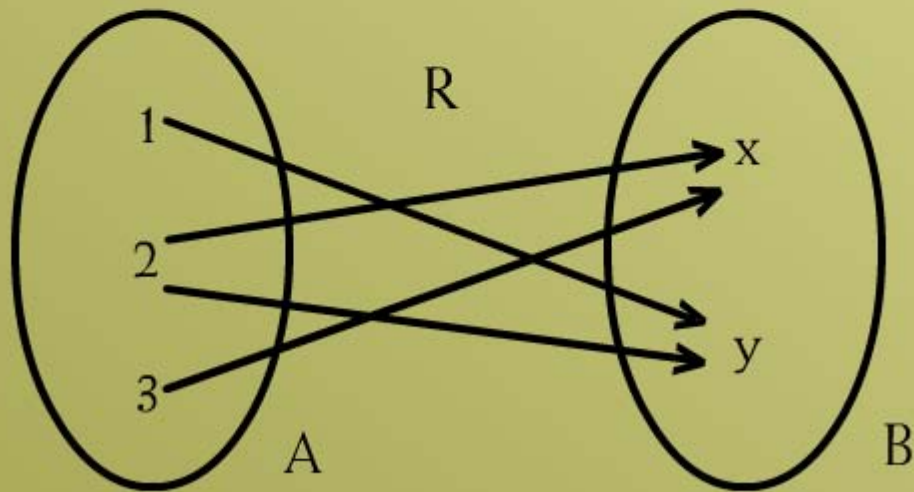# ARROW DIAGRAM OF A RELATION

Let A = {1, 2, 3}   B = {x, y}
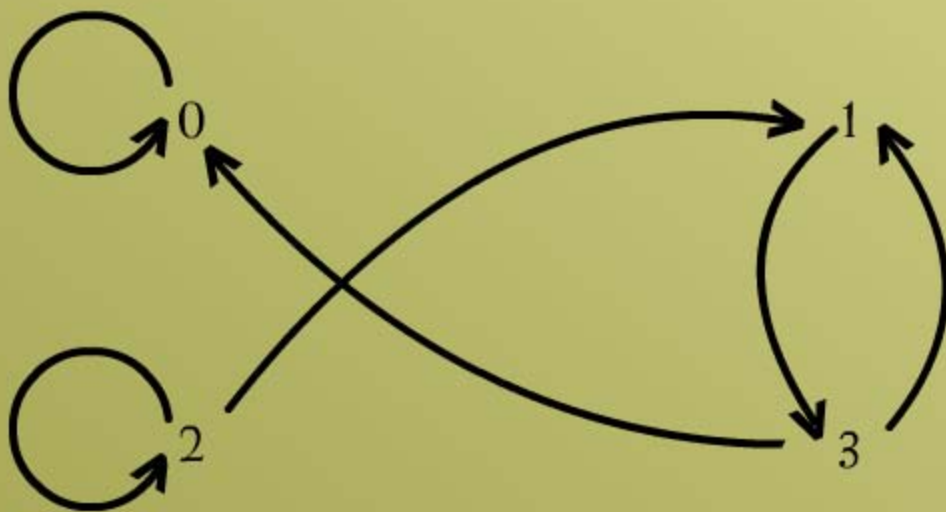
R = {1,y), (2,x), (2,y), (3,x)}

DIRECTED GRAPH OF A RELATION

Let A = {0, 1, 2, 3}

R = {(0,0), (1,3), (2,1), (2,2), (3,0), (3,1)}

MATRIX REPRESENTATION OF A RELATION

Let $A = \{a_1, a_2, \ldots, a_n\}$

$B = \{b_1, b_2, \ldots, b_m\}$

Let R be a relation from A to B.

Define the matrix M of order $n \times m$ by

$$m_{(i,j)} = \begin{cases} 1 & \text{if } (a_i, b_i) \in R \\ 0 & \text{if } (a_i, b_i) \notin R \end{cases}$$

for $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$

EXAMPLE

Let A = {1, 2, 3}   and   B = {x, y}

R = {(1,y), (2,x), (2,y), (3,x)}

Order of matrix = 3×2

$$M = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{cc} x & y \\ \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix} \end{array}_{3 \times 2}$$

EXAMPLE

For the **relation matrix**.

$$M = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{ccc} 1 & 2 & 3 \\ \left[\begin{array}{ccc} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{array}\right] \end{array}$$

1. List the set of **ordered pairs** represented by **M**.

2. Draw the **directed graph** of the relation.

Solution contd…

1. R = {(1,1), (1,3), (2,1), (3,1), (3,2), (3,3)}
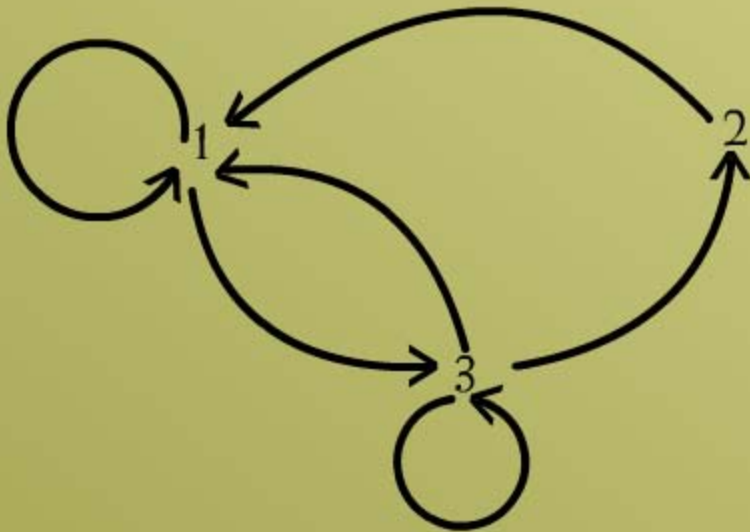
EXERCISE

Let A = {2, 4} and B = {6, 8, 10}
   and define relations R and S from A to
   B as follows:

$$R = \{(x,y) \in A \times B /, x\, R\, y \Leftrightarrow x \mid y\}$$

$$S = \{(x,y) \in A \times B /, x\, S\, y \Leftrightarrow y - 4 = x\}$$

State explicitly which ordered pairs are in
A × B, R, S, R∪S and R∩S

SOLUTION

$A \times B$

$= \{(2,6), (2,8), (2,10), (4,6), (4,8), (4,10)\}$

$R = \{(2,6), (2,8), (2,10), (4,8)\}$

$S = \{(2,6), (4,8)\}$

$S \subseteq R$

$R \cup S = \{(2,6), (2,8), (2,10), (4,8)\} = R$

$R \cap S = \{(2,6), (4,8)\} = S$