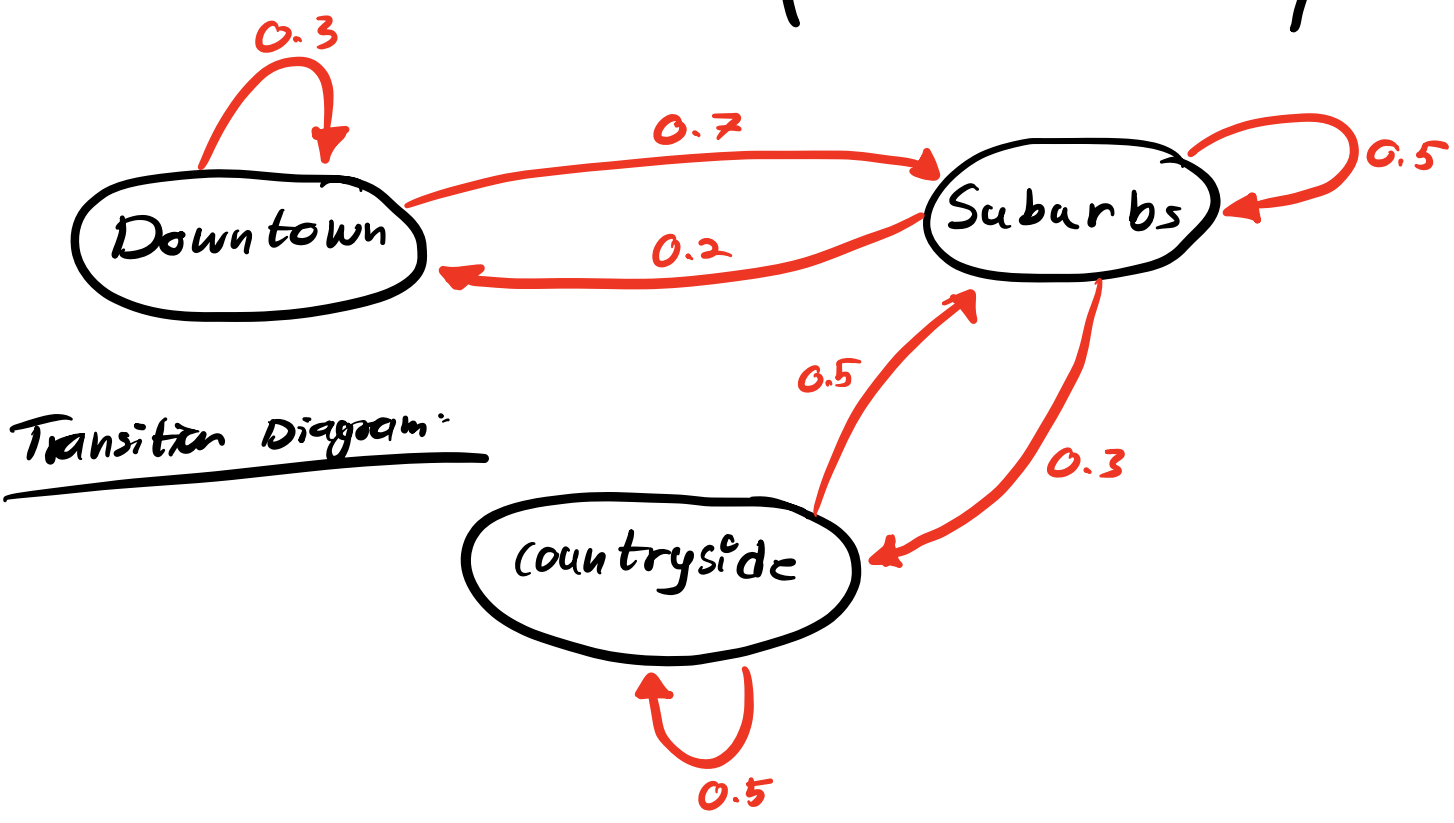
Problem ①

① 1. If a truck is currently in the downtown, what is the probability that it will be in the countryside region after 10 time steps? [2p]

Matrix $P =$

	Downtown	Suburbs	countryside
Downtown	0.3	0.7	0
Suburbs	0.2	0.5	0.3
countryside	0	0.5	0.5



Transition Diagram:

After 10 time steps, we simply want to calculate P^{10}. Then we look at the first row, 3rd column which is from Downtown to countryside. This is the answer:

=> Answer ≈ 0.32

Problem ①

② 2. If a truck is currently in the downtown, what is the probability that it will be in the countryside region the first time after three time steps or more?

Step 1: Interpret the event "first time after three time steps or more".

Let T be the first time the chain is in Countryside.

We want the probability:

$$\mathbb{P}(T \geq 3 \mid \text{Downtown})$$

This means:

- The chain must not be in countryside at step 1.
- The chain must not be in countryside at step 2.
- After that, the first time can be at step 3, 4, 5... (or later)

Step 2: Calculate probabilities:

$$\mathbb{P}(T \geq 3) = 1 - \mathbb{P}(T \leq 2)$$

Starting from Downtown:

T=1:

- This is impossible since there is no direct path from Downtown to Countryside. => Probability 0.

T=2:

- One possible solution:

Downtown -> Suburbs -> Countryside

=> $0.7 \cdot 0.3 = 0.21$

So: $P(T \geq 3) = 1 - P(T \leq 2) \Rightarrow 1 - 0.21 - 0 = 0.79$

Answer: 0.79.

Problem ①

③ 3. Is this Markov chain irreducible? Explain your answer. [3p]

- A Markov chain is irreducible if:
  - You can get from every state to every other state
  - As long as there is some path with positive probability.
  - * If matrix has no zeros at all, it is trivially irreducible, because all transitions can occur in 1 step
  - x This does not need to necessarily happen in one step

So: No zeros in matrix -> irreducible ?

Answer:

- In this case it's true since you can go from each state to each other state.

Problem ①:

④ Use code (GeneralCode.ipynb)

⑤ Same here :D

Problem ②:

In code :D