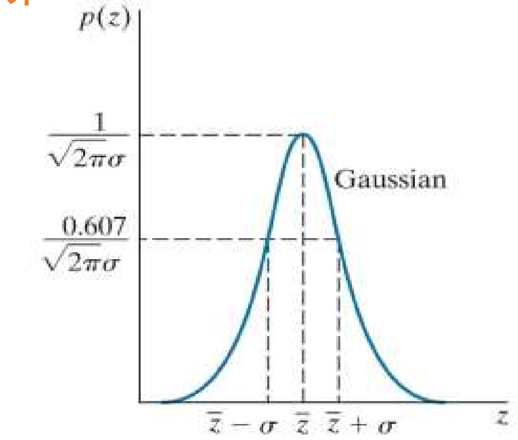
# 噪声模型

## ◆ 噪声模型

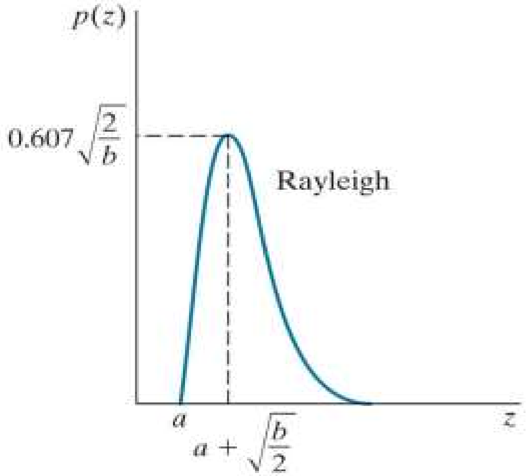



高斯噪声模型 $p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\mu)^2/2\sigma^2}$

z表示灰度值，

$\mu$表示z的平均或期望值，

$\sigma$表示z的标准差。




瑞利噪声模型

$$p(z) = \begin{cases} \dfrac{2(z-a)e^{-(z-a)^2/b}}{b} & z \geq a \\ 0, & z < a \end{cases}$$

z表示灰度值

均值$\mu = a + \sqrt{\pi b/4}$，　方差$\sigma^2 = \frac{b(4-\pi)}{4}$

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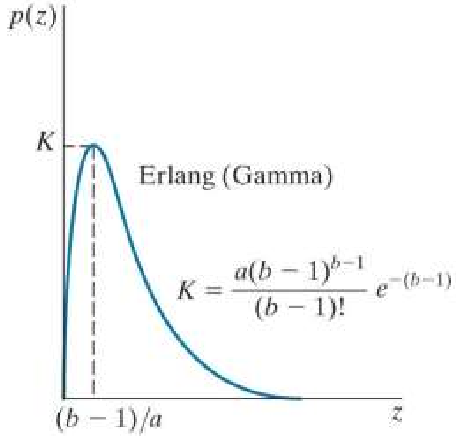
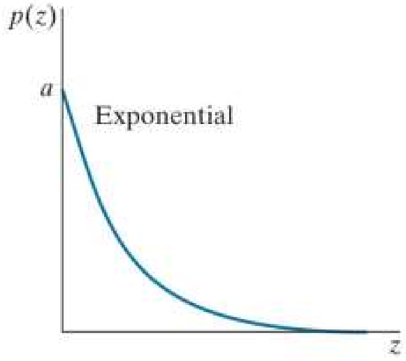



伽玛噪声模型

$$p(z)=\begin{cases}\dfrac{a^b z^{b-1}}{(b-1)!}e^{-az} & z \geq a \\ 0, & z < a\end{cases}$$

z表示灰度值,a > 0, b为正整数。

均值$\mu = b/a$, 方差$\sigma^2 = \frac{b}{a^2}$




指数分布噪声模型

$$p(z)=\begin{cases} ae^{-az} & z \geq a \\ 0, & z < a\end{cases}$$

z表示灰度值

均值$\mu = 1/a$, 方差$\sigma^2 = \frac{1}{a^2}$

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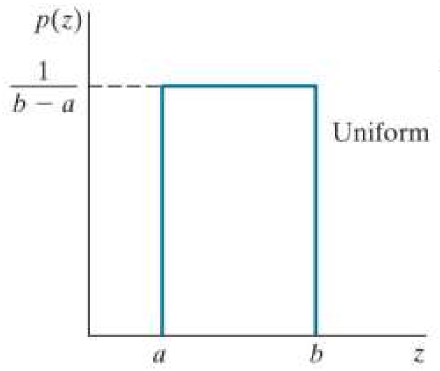
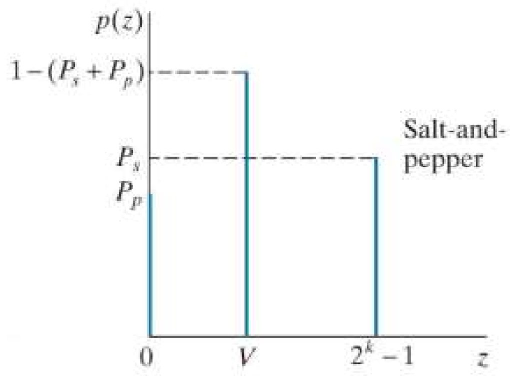



均匀分布噪声模型

$$p(z)=\begin{cases}\dfrac{1}{b-a} & a \le z \le b \\ 0, & \text{其他}\end{cases}$$

z表示灰度值

均值$\mu=\dfrac{a+b}{2}$， 方差$\sigma^2=\dfrac{(b-a)^2}{12}$




脉冲噪声模型

$$p(z)=\begin{cases}P_a & z=a \\ P_b & z=b \\ 0, & \text{其他}\end{cases}$$

z表示灰度值

# 仅含噪声的复原——空间滤波

$$g(x,y)=f(x,y)+\eta(x,y)$$

$$G(u,v)=F(u,v)+N(u,v)$$

## ◆ 均值滤波器

算术均值滤波器：$\hat{f}(x,y)=\frac{1}{mn}\sum_{(s,t)\in S_{xy}} g(s,t)$

几何均值滤波器：$\hat{f}(x,y)=\left[\prod_{(s,t)\in S_{xy}} g(s,t)\right]^{\frac{1}{mn}}$

谐波均值滤波器：$\hat{f}(x,y)=\frac{mn}{\sum_{(s,t)\in S_{xy}}\frac{1}{g(s,t)}}$

逆谐波均值滤波器：$\hat{f}(x,y)=\frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$

$Q>0$消除胡椒，$Q<0$消除盐粒

$Q=0$算数均值，$Q=-1$谐波均值

# 仅含噪声的复原——空间滤波

$$g(x,y) = f(x,y) + \eta(x,y)$$

$$G(u,v) = F(u,v) + N(u,v)$$

◆ 统计排序滤波器

中值滤波器 : $\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{\text{median}}\{g(s,t)\}$

最大值滤波器 : $\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{\max}\{g(s,t)\}$

最小值滤波器 : $\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{\min}\{g(s,t)\}$

中点滤波器 : $\hat{f}(x,y) = \frac{1}{2}\left[\underset{(s,t)\in S_{xy}}{\max}\{g(s,t)\} + \underset{(s,t)\in S_{xy}}{\min}\{g(s,t)\}\right]$

修正后阿尔法滤波器 : $\hat{f}(x,y) = \frac{1}{mn-d}\sum_{(s,t)\in S_{xy}} g_r(s,t)$

# 仅含噪声的复原——频域滤波消除周期噪声

- 陷波带阻滤波器传递函数
  中心平移到陷波中心的各个高通滤波器传递函数的乘积。
  （陷波：对称对，Q对，2Q个滤波器相乘）

$$H_{NR}(u,v) = \prod_{k=1}^{Q} H_k(u,v)H_{-k}(u,v)$$

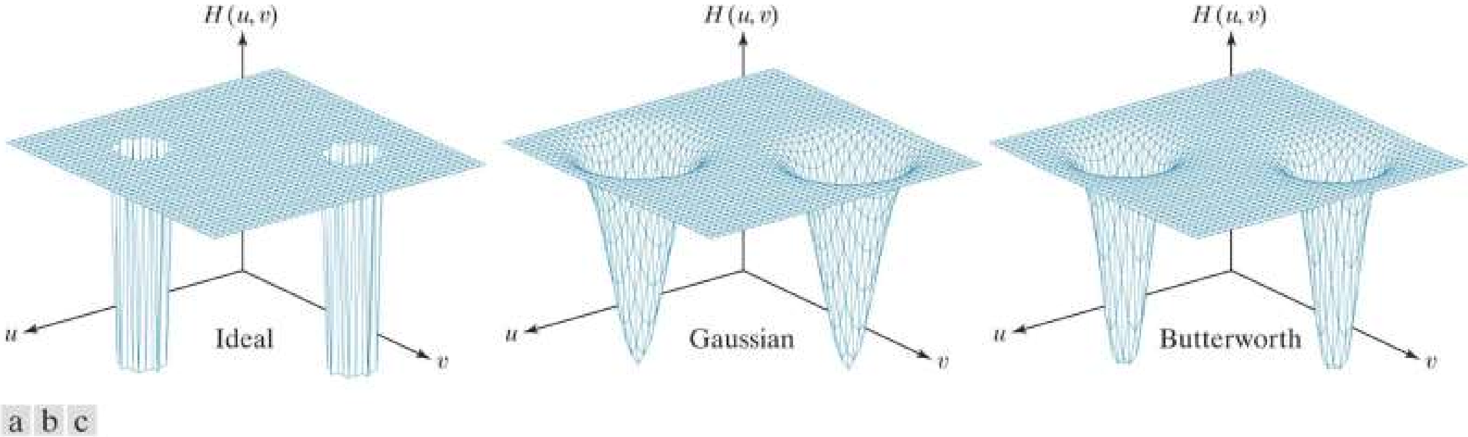



**FIGURE 5.15**
Perspective plots of (a) ideal, (b) Gaussian, and (c) Butterworth notch reject filter transfer functions.

# 仅含噪声的复原——频域滤波消除周期噪声

a b
c d

陷波带通滤波器可恢复正弦干扰模式

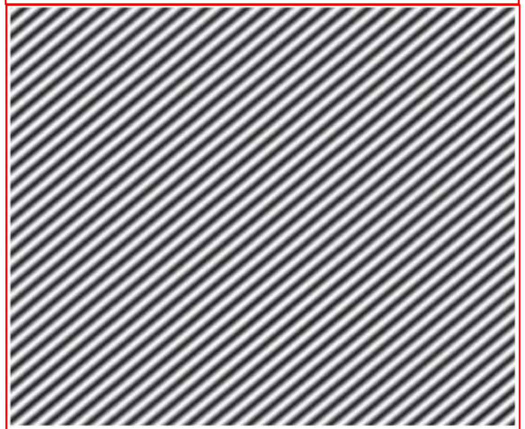


FIGURE 5.17
Sinusoidal pattern extracted from the DFT of Fig. 5.16(a) using a notch pass filter.

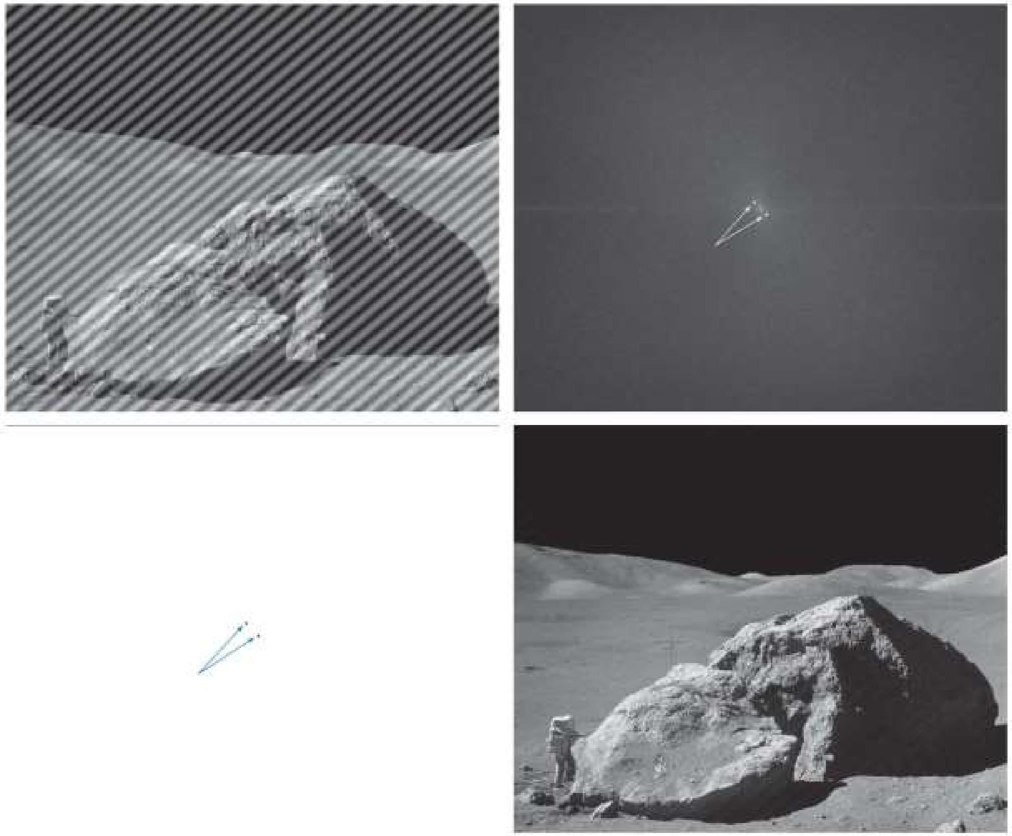


**FIGURE 5.16**
(a) Image corrupted by sinusoidal interference. (b) Spectrum showing the bursts of energy caused by the interference. (The bursts were enlarged for display purposes.) (c) Notch filter (the radius of the circles is 2 pixels) used to eliminate the energy bursts. (The thin borders are not part of the data.) (d) Result of notch reject filtering.