Properties of Inequations

(i) (Add)

$x - 2 < 4$

$x - 2 + 2 < 4 + 2$

$x < 6$

(ii) Subtract

$x - 2 < 4$

$x - 2 - 1 < 4 - 1$

$x - 3 < 3$

(iii) If multiply and divide

sign → reverse

③ $9 \leq 1 - 2x$

$9 - 1 \leq -2x$

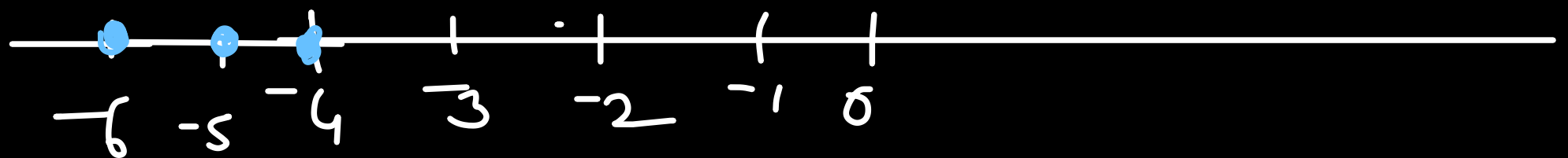
$8 \leq -2x$

$\frac{8}{-2} \leq x$ (negative number sign reverses)

$-4 \geqslant x$

$\therefore x \leq -4$

$\therefore$ Solution set is $-4, -5, -6$



If $a > b$

then

$a - c > b - c \longrightarrow T$

$a + c > b + c \longrightarrow T$

$\frac{a}{2} > \frac{b}{2} \longrightarrow T$

$\frac{a}{-2} < \frac{b}{-2} \longrightarrow T$

$-2a > -2b \longrightarrow F$

(ii) If $a < b$ then $ac < bc$ → T

(iii) If $a > b$ then $\frac{a}{c} > \frac{b}{c}$ → T → only when $c > 0$

(iv) If $a - c < b \underline{=} d$

then

$a + \underline{d} < b + c$ → T

$=$

Ex ③:

$$12 + 1\frac{5}{6}x \leq 5 + 3x, \quad x \in \mathbb{R}$$

$$\frac{11}{6}x - \frac{3x}{1} \times 6 \leq 5 - 12$$

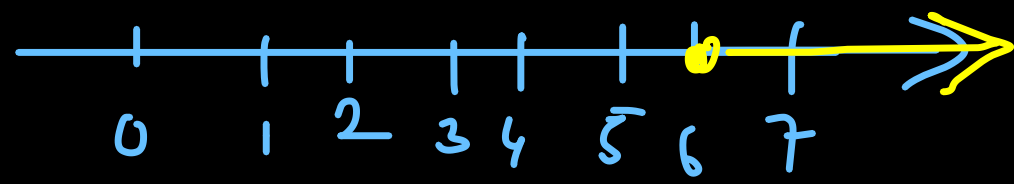
$$\frac{11x - 18x}{6} \leq -7$$

$$-\frac{7x}{6} \leq -7$$

$$x \geq -42$$
$$= -7$$

$$\boxed{x \geq 6}$$

∴ Solution set: $\{x \in R : x \geq 6\}$



Q) $-3 < -\frac{1}{2} - \frac{2x}{3} \leq \frac{5}{6}$; $x \in R$

$-3 < -\frac{1}{2} - \frac{2x}{3}$ and $-\frac{1}{2} - \frac{2x}{3} \leq \frac{5}{6}$

$-3 < \frac{-3-4x}{6}$

$-18 < -3-4x$

$-18+3 < -4x$

$-15 < -4x$

$\frac{-15}{-4} > x$

$\therefore \boxed{x < \frac{15}{4}}$

$-\frac{2x}{3} \leq \frac{5}{6} + \frac{1}{2}$

$-\frac{2x}{3} \leq \frac{10+6}{12}$

$-\frac{2x}{3} \leq \frac{16}{12}$ $\frac{4}{3}$

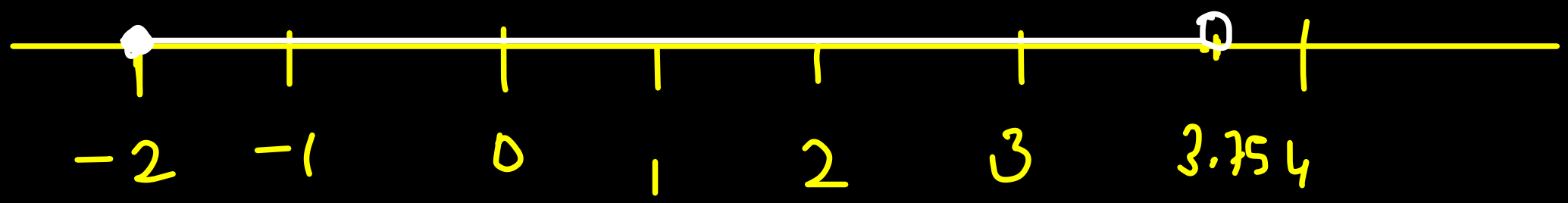
$-2x \leq 4 \times 3$

$x \geq \frac{4}{-2}$

$\boxed{x \geq -2}$

$x \geq -2$ $\quad$ $x < \frac{15}{4}$

$x \geq -2$ $\quad$ $x < 3.75$



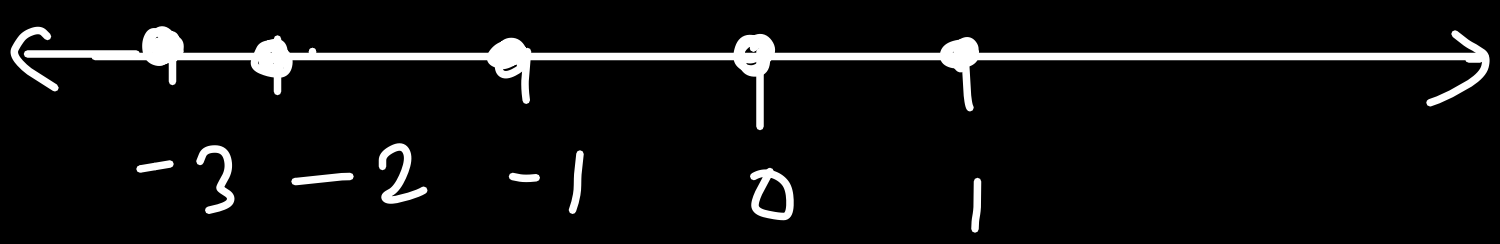
Ex: 5 $2x - 5 \leq 5x + 4 < 11$, $x \in I$

$-5 - 4 \leq 5x - 2x$ and $5x + 4 < 11$

$-9 \leq 3x$ $\qquad 5x < 7$

$-3 \leq x$ $\qquad x < \frac{7}{5}$

$x < 1.4$



solution set : $\{-3, -2, -1, 0, 1\}$