To show that Root2 is an irrational number

⇒ method of contradiction

⇒ let $\sqrt{2}$ be rational number

then $\sqrt{2} = \frac{a}{b}$

(Here a and b are integer and $b \neq 0$)

(no common factor except 1)

⇒ $(\sqrt{2})^2 = \left(\frac{a}{b}\right)^2$

⇒ $2 = \frac{a^2}{b^2}$ [squaring both sides]

$$a^2 = 2b^2$$

$\Rightarrow$ (since multiplication of 2 with an odd or even number is even.)

$\therefore$ a^2 must be even.

$\therefore$ a must be even

$\therefore$ $\boxed{a = 2n}$ [Even number]

$$a^2 = 2b^2 \qquad a = 2n$$

$$(2n)^2 = 2b^2$$

$$4n^2 = 2b^2$$

$$b^2 = \frac{\overset{2}{\cancel{4}}n^2}{\cancel{2}} \Rightarrow b^2 = 2n^2$$

[$2n^2$ is even $\Rightarrow$ b^2 is even $\Rightarrow$ b is even]

$\therefore$ This means a and b have common factor 2.

$\therefore$ This contradicts our assumption that a and b have no common factor except 1.

$\therefore$ $\sqrt{2}$ is not an rational number.

$\therefore$ $\sqrt{2}$ is irrational number.

let $\sqrt{3}$ is an rational number

$$\sqrt{3} = \frac{a}{b}$$

(a and b are integers and $b \neq 0$)

(And having no common factor except 1)

$\therefore$ $3 = \frac{a^2}{b^2}$ [squaring both sides]

$\therefore$ $b^2 = \frac{a^2}{3}$

$\therefore$ a^2 is divisible by 3.

$\therefore$ a is divisible by 3.

$\therefore \quad a = 3c$

$\therefore \quad a^2 = 3b^2$

$(3c)^2 = 3b^2$

$3\cancel{9}c^2 = \cancel{3}b^2$

$c^2 = \dfrac{b^2}{3}$

$\therefore$ b^2 is divisible by 3.

$\therefore$ b is divisble by 3.

This contradicts to our statement that $\sqrt{3}$ has no common factor except 1.

else

This contradicts our statement that a and b are co primes

$\therefore$ $\sqrt{3}$ is not an rational number

$\therefore$ $\sqrt{3}$ is an irrational number.

(iv) $\sqrt{3} + \sqrt{5}$

Let $\sqrt{3} + \sqrt{5}$ is an rational number

$\therefore \quad \sqrt{3} + \sqrt{5} = a$

[a and b are integers and $b \neq 0$)

$\therefore$ no common factor except 1]

$$(\sqrt{3} + \sqrt{5})^2 = a$$

using $(a+b)^2 = a^2 + b^2 + 2ab$

$$(\sqrt{3})^2 + (\sqrt{5})^2 + 2(\sqrt{3})(\sqrt{5}) = a^2$$

$$8 + 2\sqrt{15} = a^2$$

$$8 + 2\sqrt{15} = a^2$$

$$2\sqrt{15} = a^2 - 8$$

$$\sqrt{15} = \frac{a^2 - 8}{2}$$

$\therefore$ Right hand side is an rational number but left hand side is irrational number.

$\therefore$ our assumption is wrong

$\therefore$ $\sqrt{3} + \sqrt{5}$ is an irrational number

$$\underset{a}{3} - \underset{b}{\sqrt{5}}$$

$$
\begin{aligned}
[a - b]^2 &= a^2 + b^2 - 2ab \\
&= (3)^2 + (\sqrt{5})^2 - 2(3)(\sqrt{5}) \\
&= 9 + 5 - 6\sqrt{5} \\
&= 14 - 6\sqrt{5}
\end{aligned}
$$

$$\sqrt[2]{3} = (3)^{1/2}$$

$$\sqrt{3 \times 3}$$

Surd $\leftarrow \sqrt[3]{9} = (9)^{1/3}$

$$\boxed{\sqrt[n]{a} = (a)^{1/n}}$$

Radicand