A Comprehensive Guide to the CASIO fx-9860GIIs

THE BETTER GRAPHING CALCULATOR FOR H2 MATHEMATICS

QUICK REFERENCE MATH INCLUDED

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Contents

C	onten	${f ts}$	i
Pı	reface	9	iii
1	The	Basics	1
	1.1	Display Modes	2
	1.2	Radian/Degree/Gradian	3
	1.3	Graphing	4
	1.4	Solving for the roots of a polynomial	6
	1.5	Solving a 1-variable Equation	7
	1.6	Solving a Multi-variable Linear System	9
	1.7	Taking Integrals	14
2	Vect	tors	17
	2.1	Entering a Vector	18
	2.2	Vector Calculations	19
	2.3	Problems involving Vectors	22

ii CONTENTS

3	Sta	tistics
	3.1	Discrete Random Variable
	3.2	Continuous Random Variable
	3.3	Use for Experimental Data/SPA
1	T in	ear Algebra
4	LIII	ear Aigeora
	4.1	Storing Matrices
	4.2	Solving Linear Systems
	43	Rest Approximations

Preface

This guide is provided as-is and was compiled by a bunch of students in 2016. We tried our best to be mathematically and technically accurate. But software versions (Ver 2.04 now) do change, and this guide may not be valid indefinitely. However, the source of this file is publically available on GitHub. Feel free to contact us.

If there are anything you want to do, but you can't seem to know how to, try Google, or RTFM. Otherwise, just experiment a bit. The CASIO interface should be intuitive enough.

Do note that some notes inside this guide were taken from the H2 Mathematics notes provided by the Hwa Chong Institution (College Section) Math Department.

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Chapter 1

The Basics

Every command you will ever need is organized neatly in the [OPTN] button on your graphing calculator (GC).

Unfortunately there are some areas where the CASIO calculator isn't very intuitive and we need some documentation.

Tip: You can just press the button that corresponds to the number/letter at the bottom right of the icons in the menu:



1.1 Display Modes

As a special mention that you probably did not notice, we shall first introduce the different display modes that the calculator is capable of. In the Set Up menu ([SHIFT] + [MENU]), there are different display options, and we summarize their corresponding functions as follows:

Mode	What it does
${\tt Fix} n$	Rounds off answer to n decimal places
Scin	Rounds off answer to n significant figures
Norm	Converts answer to scientific notation when the value x lies outside of the following range:
	Norm1: $10^{-3} > x > 10^{10}$ Norm2: $10^{-9} > x > 10^{10}$

In addition to these options, there is also a Eng mode that adds on to those above mentioned modes. The /E indicator is on display while the engineering notation is in effect (e.g. Norm1/E).

In the engineering mode, answers are displayed with standard order-of-magnitude prefixes, and the symbol that makes the mantissa a value from 1 to 1000 is automatically chosen:

E (Exa)	$\times 10^{18}$	m (milli)	$\times 10^{-3}$
P (Peta)	$\times 10^{15}$	μ (micro)	$\times 10^{-6}$
T (Tera)	$\times 10^{12}$	n (nano)	$\times 10^{-9}$
G (Giga)	$\times 10^9$	p (pico)	$\times 10^{-12}$
M (Mega)	$\times 10^6$	f (femto)	$\times 10^{-15}$
k (kilo)	$\times 10^3$		

You can also type these prefixes when doing calculations: they are available under $[\mathtt{OPTN}] > \triangleright \times 2 > \mathtt{ESYM}$ (F1). You might find these extremely useful when doing calculations for Physics.

Under that same menu, there is also the ENG and ENG which shifts the decimal place of the displayed value 3 digits to the (left/right), and (decreases/increases) the exponent by 3 respectively.

1.2 Radian/Degree/Gradian

Like the majority of other scientific calculators, this GC is able is accept input and display the magnitude of angles in either degrees, radians, or gradians. To change between the 3, enter the Set Up menu, and scroll down until you see a row labelled Angle. The 3 different units are the displayed at the bottom of the screen and can be selected by the [F1] to [F3] buttons.

Note that this setting is universal throughout your GC - it will change the units angles are handled in every mode including but not limited to usual calculations, graphing, etc, unless

you specifically specify otherwise (not covered in this section). Therefore, it is vital that you check your units before undergoing any calculation, especially before and during examinations.

1.3 Graphing

To graph a function, go to GRAPH in the main menu. Enter the functions into Y1 onwards. Press DRAW (F6) or [EXE] to graph. Most of these are pretty intuitive, just some things you should take note:

- The Y and X, etc. at the bottom of the screen is for entering the functions Y1, Y2, ... and X1, X2, For the variable x, use the $[X, \theta, T]$ button instead
- To restrict the domain of the function, type:

$$Y1 = f(x), [start,end]$$

Note: Restricting the domain disables the sketching of the inverse of a graph.

After pressing DRAW (F6 or [EXE]), you can press [AC/on] to break the plotting script.

To solve for anything, use [G-Solv] (F5).

5

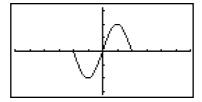


Figure 1.1: Graph of $\sin(x)$, for $-\pi \le x \le \pi$

1.3.1 Sketching

Once a graph is plotted, you can sketch additional graphs on top of the already plotted graph. This includes sketching the inverse of a function, tangent and normal to a point on the curve.

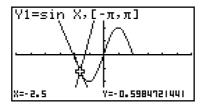


Figure 1.2: Graph of $\sin(x)$, for $-\pi \le x \le \pi$, with the normal and tangent drawn at x = -2.5

All these are accessible under the Sketch menu (SHIFT > F4).

When sketching the normal/tangent, just press the number keys if you would like to enter a specific X-value to plot the normal/tangent at.

1.3.2 Plotting a table from function

To effectively use your calculator as a function generator, you can use the TABLE mode available in the menu:

- Go to TABLE and then enter your function just like how you would in GRAPH
- 2. Go to SET (F5)

Once in table mode, you can enter values directly into the X column to generate values using the function.

This is especially useful when you are doing the same operation multiple times on different numbers, or when you are generating solving a Sequences and Series question using the Method of Differences.

1.4 Solving for the roots of a polynomial

There are mainly 2 ways to solve for the roots of a polynomial using the GC:

- Using the EQUA > Poly (F2) app
- Plot a graph and solve for roots

The EQUA > Poly is quite intuitive, and similar to the standard issue CASIO fx-95SG scientific calculator, so we will just note some limitations/features of this GC:

- The polynomial solver only accepts real coefficients
- You can change the Set Up > Complex Mode setting to a + bi for imaginary roots (but not imaginary coefficients)

To plot a graph, you go to Graph. Refer to Section 1.3 (Graphing) for more information.

1.5 Solving a 1-variable Equation

There are mainly 2 ways to solve a 1-variable equation (e.g. $e^x + 5x = 1$) using the GC:

- Using the EQUA > Solver (F3) app
- Using Solve or SolveN
- Plotting a graph
 - Move everything to one side and solve for root
 - Solve for the intersections of 2 or more graphs

1.5.1 Solving using EQUA > Solver

To use the built-in equation solver:

- 1. Go to EQUA > Solver (F3)
- 2. Enter the equation you would like to solve under Eq. Remember to put the = sign.

- Alternatively, you can use RCL (F1) to recall functions entered into GRAPH or TABLE
- 3. Press [EXE] and put your initial guess under X. An example can be as such:

Х	0
Lower	-9E+99
Upper	9E+99

4. Press Solv (F6)

Note: The initial guess isn't very important for equations with unique solutions. However, if the equation has multiple solutions, it will give the answer closer to the original guess.

For example, given the equation (x-1)(x-3) = 0, entering the following will give the corresponding answers:

X	Answer
0	1
1.5	1
2	1
2.5	3
4	3

After solving the equation, you can always access the value again by going to RUN-MAT and typing X.

1.5.2 Solving using Solve or SolveN

Alternatively, you can use the Solve or SolveN functions in RUN-MAT to obtain the solution. They can be found under [OPTN] > CALC(F4) .

Their usage is as follows, as far as we have discovered:

Solve(<equation>,<initial guess>) SolveN(<equation>)

Both Solve and SolveN will give you only 1 answer, and works similar to how the EQUA > Solver works.

Solving a Multi-variable Linear System 1.6

There are 3 types of linear systems, namely systems with:

- No solution
- 1 unique solution
- Infinitely many solutions

To solve a multi-variable system of linear equations using your GC, there are 2 main ways:

- 1. Using the EQUA > Siml (F1) app
- 2. Using matrices

1.6.1 Using the EQUA > Siml app

While you can use matrices to solve any kind of linear system, you can use the EQUA > Siml app only if the linear system has a unique solution.

Note: Should you try to use the EQUA > Siml app to solve for a linear system with no solution, or infinitely many solutions, the calculator would just throw a Ma Error at you while you stare, bemused, wondering why the calculator is so bad to you.

Anyhow, the app is intuitive enough, so I shall skip it.

1.6.2 Using Matrices

To solve a linear system the matrix way, let's first consider the following linear system:

$$\begin{cases} x + y + 2z = 9 \\ 2x + 4y - 3z = 1 \\ 3x + 6y - 5z = 0 \end{cases}$$

By taking the coefficients of the variables x, y and z, we can form the following augmented matrix:

$$\left(\begin{array}{ccc|c}
1 & 1 & 2 & 9 \\
2 & 4 & -3 & 1 \\
3 & 6 & -5 & 0
\end{array}\right)$$

To solve for x, y and z:¹

¹ Fn Keys have been left out to save space.

1.6. SOLVING A MULTI-VARIABLE LINEAR SYSTEM 11

- 1. Go to RUN·MAT > ⊳MAT
- 2. Select Mat A and enter the dimensions
 - m is the number of rows, in this case 3
 - n is the number of columns, in this case 4
- 3. Enter the augmented matrix above accordingly, putting the augmented column into the 4th column:

$$\left[\begin{array}{cccc}
1 & 1 & 2 & 9 \\
2 & 4 & -3 & 1 \\
3 & 6 & -5 & 0
\end{array}\right]$$

- 4. Press [EXIT] then [EXIT] to go back to the main RUN-MAT screen
- 5. Go to [OPTN] > MAT > > Rref
- Then ▷ × 3 > Mat, then [ALPHA] + A, giving the following:

Rref Mat A

7. Press [EXE], and then you should be able to get the following matrix:

$$\left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

This can then be rewritten as an augmented matrix:

$$\begin{pmatrix} x & y & z \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{pmatrix}$$

which can then be rewritten as:

$$\begin{cases} x = 1 \\ y = 2 \\ z = 3 \end{cases}$$

Notice that the left side of the final augmented matrix is the identity matrix. This is what you would typically see for a linear system with a unique solution.

This same procedure can be used to find the solutions of any linear system with any number of variables.

The following are possible matrices you can obtain after the Rref command for the other types of linear systems:

No solution

$$\begin{pmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{cases} x = 2 - 2t \\ y = 3 - 3t \\ 0z = 1 \end{cases}$$

where $t \in \mathbb{R}$

Since 0 = 1 is not possible, the above linear system is inconsistent and has no solutions.

• Infinitely many solutions

$$\begin{pmatrix} 1 & 0 & 2 & 2 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{cases} x = 2 - 2t \\ y = 3 - 3t \\ z = t \end{cases}$$

where $t \in \mathbb{R}$

In this case, the answer is obtained easily through backsubstitution. Rewriting the augmented matrix, we observe that:

$$\begin{cases} x + 2z = 2\\ y + 3z = 3 \end{cases}$$

By letting z be a parameter $t \ (t \in \mathbb{R})$, we observe that:

$$\begin{cases} x+2t=2\\ y+3t=3\\ z=t \end{cases} \rightarrow \begin{cases} x=2-2t\\ y=3-3t\\ z=t \end{cases}$$

Thus obtaining the general solution.

You will find this useful for both "System of Linear Equations" and "Vectors" (See Chapter 2).

For a more in-depth understanding of what is going on here, you may wish to refer to Section 4.2.

1.7 Taking Integrals

1.7.1 Definite Integrals

Your calculator can be used to solve definite integrals.

One way is to find the area under the graph after plotting the equation.

- 1. Plot the graph following the instructions from Section 1.3.
- 2. Press G-Solv (F5), followed by \triangleright (F6) and $\int dx$ (F3).
- 3. A cursor on the curve will now appear on the graph, along with x and y values. Move the cursor to the lower bound of the definite integral you're trying to find, and press EXE, followed by the upper bound, and press EXE again.
- 4. The area under the graph should now be shaded and the value of the area is displayed as $\int dx$.

This method is great if you already have the graph plotted and want to find the integral quickly without entering the equation again. If you do not need to plot the graph and simply want to find the definite integral directly, you may find the integration function in the RUN·MAT app more useful.

- 1. From the main menu, enter the RUN-MAT app (1).
- 2. Press the \mathtt{OPTN} button and select the \mathtt{CALC} option $(\mathtt{F4})$.

- 3. Under the CALC submenu, choose the \(\int \) dx option (F4).
- 4. Enter the equation as per standard Mathematical notation.

Note: You should change the variable to be integrated from whatever letter it was originally to x since the calculator forces the integral to be integrated with respect to x as can be seen by the dx at the back.

1.7.2 Indefinite Integrals

SEAB bans all calculators capable of direct symbolic integration and differentiation. In other words, you cannot enter $3x^2$ into your graphing calculator and get x^3 or vice versa. To be fair, this feature was never in the international version of the Casio anyway.

Thankfully, even in the Singaporean version of the Casio, you can plot the graph of the integral visually.

For example, we will try to plot the integral of 2x. Assuming that your GC is working properly and the Laws of Mathematics have not changed in drastic ways, the graph should be identical to that of x^2 .

- 1. From the main menu, enter the GRAPH app (5).
- 2. Under Y1, enter the function 2x.

3. Move to Y2. Enter the following function:

$$Y2 = \int_0^x Y1 \, dx$$

Note: Y2 will take a significant amount of time to plot (around 5-10 seconds). It is recommended that you deselect Y1 by pressing SEL (F1) to prevent Y1 from being plotted, saving a couple of seconds.

4. Press DRAW (F6). A parabola that identical to $y=x^2$ should be yield. You can plot $y=x^2$ on top of the integral to confirm.

While this function is unable to give you the equation of the integral directly, you can use double check your integrals using this method. Plotting the integral using this method can also be useful if you have to find multiple definite integrals (using the trace function in the graph page).

Chapter 2

Vectors

Casio added support for vectors with the 02.04.5301 firmware update. This is added on top of support for Matrices, which will be covered mainly in Chapter 4 (Linear Algebra).¹

Caution: Vector capabilities will be disabled in the new Exam Mode that ships with the Ver. 2.09 update.² However, as of present we are not aware of any requirements to activate Exam Mode during SEAB's examinations.

 $^{^1}$ While vectors are also covered in the Linear Algebra syllabus, this chapter will focus more on what is needed to tackle the H2 syllabus.

 $^{^2}$ We will not be covering this as we are covering Ver. 2.04.5301, the latest SEAB approved firmware version.

2.1 Entering a Vector

Functionally, vectors in this GC is no different from a 1 by n or a n by 1 matrix. Hence, you can use Mat and Vct interchangeably, so long as they are of the correct order.

To enter a vector into the GC:

- 1. Enter the RUN·MAT mode and select ⊳MAT (F3)
- 2. Select $M \leftrightarrow V$ (F6). The top row should now say Vector and the Vct A row should be highlighted by default
- 3. Press EXE and enter the vector's dimensions. For the following example vector, it is a 3×1 vector, so enter m=3 and n=1

$$\begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$$

- 4. Enter the values in your vector into the three rows as shown on your textbook/exam paper
- 5. Press EXIT twice to return to the main RUN·MAT screen

Alternatively, if you are in RUN·MAT and your Input/Output is set to Math in the settings, you can enter vectors simply by going to MATH (F4) > MAT and then choosing the appropriately sized matrix. These entered matrices will replace your Vct A/etc. described in the next few sections.

Note that the labelling follows the conventional $m \times n$ format, where m is the number of rows, and n is the number of columns.

2.2 Vector Calculations

2.2.1 Addition and Subtraction

To perform addition or subtraction calculations between two vectors:

- At the main RUN·MAT screen, press [OPTN], followed by MAT (F2). Press ▷ (F6) twice, then select Vct (F1).
- 2. Press the [ALPHA] button, then press the corresponding letter that represents your first vector. For example, if you entered the vector at the first row previously, press the button that corresponds to A (which is $[X, \theta, T]$).
- 3. Press the usual addition or subtraction signs at the bottom right of your calculator depending on which calculation you wish to do.
- 4. Repeat steps 1 and 2 for your second vector, then press [EXE] to calculate the result.

2.2.2 Dot Product

Vector specific calculations are slightly more complicated than addition or subtraction. To find the dot product of two vectors:

- 1. In the vector section under OPTN > MAT, select DotP (F2).
- 2. Press Vct, then press the corresponding letter that represents the first vector in your dot product expression.

- 3. Press the [,] button, then repeat step 2 but for the second vector in your dot product expression. It is good practice at this stage to press the [)] button to close the opening bracket that comes together with the DotP(function.
- 4. Press [EXE] to calculate the result.

2.2.3 Cross Product

Cross product calculations are similar to dot products. To find the cross product between two vectors:

- 1. In the vector section under OPTN > MAT, select CrsP (F3).
- 2. Press Vct, then press the corresponding letter that represents the first vector in your cross product expression.
- 3. Press the [,] button, then repeat step 2 but for the second vector in your cross product expression. It is good practice at this stage to press the [)] button to close the opening bracket that comes together with the CrossP(function.
- 4. Press [EXE] to calculate the result.

It is important to note here that the [×] button on your calculator will not work as a cross or dot product operator! Should you attempt to use it as such, your GC will return a Dimension ERROR.

21

2.2.4 Angle Between Two Vectors

Your GC is also capable of finding the angle between two vectors. To do so:

- 1. In the vector section under OPTN > MAT, select Angle (F4).
- Press Vct, then press the corresponding letter that represents the first vector.
- 3. Press the [,] button, then repeat step 2 but for the second vector. It is good practice at this stage to press the [)] button to close the opening bracket that comes together with the Angle (function.
- 4. Press [EXE] to calculate the result. Note that the angle calculated can be in radians, degrees, or gradians depending on your settings at the moment. Refer to Section 1.2 if you are not sure how to change the units.

2.2.5 Unit Vectors

If you ever need to find the unit vector of a particular vector, you can do so directly with your GC. To do so:

- 1. In the vector section under OPTN > MAT, select UntV (F5).
- 2. Press Vct, then press the corresponding letter that represents your vector. It is good practice at this stage to press the [)] button to close the opening bracket that comes together with the UnitV(function.

3. Press [EXE] to calculate the result.

2.3 Problems involving Vectors

2.3.1 Finding the line of intersection between 2 or more planes

Let's say you are given 2 planes:

$$\pi_1 : \mathbf{r} \cdot \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix} = 4$$
 and $\pi_2 : \mathbf{r} \cdot \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix} = 2$

First, convert them to linear equations:

$$\begin{cases} 3x + y + 5z = 4 \\ 5x + y + 3z = 2 \end{cases}$$

Then solve for the general solution as described in Section 1.6.2 (Using Matrices to Solve Multi-variable Linear System):

$$\begin{cases} x = t - 1 \\ y = -8t + 7 \quad \text{where } t \in \mathbb{R} \\ z = t \end{cases}$$

Separating out the t, we realize that the solution (the line of intersection) can be written as such:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{r} = \begin{pmatrix} 1 \\ -8 \\ 1 \end{pmatrix} t + \begin{pmatrix} -1 \\ 7 \\ 0 \end{pmatrix} \qquad t \in \mathbb{R}$$

Similarly, you can expand this method to find the point of intersection (unique solution).

Chapter 3

Statistics

One major use of a graphing calculator is for use in statistics. In the following chapters, we will outline the methods with which we can use our GC. Calculator functions in this section can generally be found under [OPTN] > [STAT]. Permutation and Combinations will not be covered. Calculated variables can be found under [VARS] > [STAT]

In this chapter, all random variables are represented by capital letters (usually X or Y). Small letters are reserved to represent values (e.g. x).

Note: However, please do not use Z, as it is reserved for the standard normal distribution.

3.1 Discrete Random Variable

A discrete (countable) random variable $X \in \mathbb{Z}^*$ has expectation (or expected value or mean) μ

$$E(X) = \mu = \sum_{\text{all } x} x P(X = x)$$
(3.1)

(You can think of this as the weighted sum of all the possibilities for X)

and variance $\sigma^2 = \text{stddev}^2$

$$Var(X) = \sigma^2 = E(X - \mu)^2 = E(X^2) - \mu^2$$
 (3.2)

Note: The above does not apply to continuous random variables.

The function that gives the probability that a discrete random variable is exactly equal to some value is called the *Probability Mass Function* (PMF). However, we will write PDF throughout this section instead, so as to be in-line with the curriculum/calculator functions.

3.1.1 Using Data

If given a set of histogram data (i.e. categories and its corresponding frequencies), one can calculate statistical properties of it.

For example, given the following data:

x	0	1	2	3
P(X=x)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

You can do 1-Variable Statistical calculations by:

- 1. Go into STAT and then enter the X-values into List 1 and its frequency into List 2
- 2. Go to CALC > SET and adjust the settings accordingly for 1-Var (XList:List1, Freq:List2)
- 3. [EXIT] and then choose 1VAR

If you want to further manipulate these calculated values (such as squaring σ to find variance), you can go to RUN-MAT and then <code>[VARS] > [STAT] > [X]</code> to finding the value(s) you wanted.

Refer to Section 3.3.1 (Plotting Statistics/ Visualization) for plotting statistical data.

3.1.2 Binomial Distribution

A binomial random variable X has the following characteristics:

- 1. The experiment consists of n repeated independent trials
- 2. Each trial only has 2 outcomes: "success" or "failure"

3. The probability of a 'success' p is constant in each trial

The probability P(X = x) of obtaining x successes in n trials is given by:

$$P(X = x) = \binom{n}{x} p^x (1 - p)^{n - x}, \ x \in \mathbb{Z}^*$$
 (3.3)

and has E(X) = np and Var(X) = np(1-p).

We write $X \sim B(n, p)$.

Note: the probability is only non-zero when $0 \le x \le n$ (i.e. there is an upper bound).

Binomial PDF

To obtain the probability P(X=x), we can either use the formula, or use the in-built BinomialPD function:

$${\tt BinomialPD([X],} n, p)$$

When X is not provided, the output answer will be a List.

Alternatively, we can use the STAT app to obtain the probability:

- 1. Go into STAT (2 in the menu)
- 2. Press DIST(F5) > BINM(F5) > Bpd(F1)
- 3. Set the following:

Data	Variable
x	X
Numtrial	n
p	p
Save Res	None

- 4. Press [EXE] or scroll to Execute and press CALC(F1)
- 5. You can do further manipulation of the value obtained by:
 - a) Going to RUN·MAT
 - b) Go to [VARS] > STAT(F3) > RESLT(F6) > DIST(F3)
 - c) There you can find the p value by pressing p (F1) and then [EXE]

Binomial CDF

To obtain the probability $P(X \leq x)$ (cumulative distribution function of x), we can use the in-built BinomialCD function:

$${\tt BinomialCD([X],} n, p)$$

Similarly, if X is not provided, the output answer will be a List.

Alternatively, following instructions in the previous section (Section $3.1.2 \Rightarrow$ Binomial PDF), and choosing Bcd instead.

As a general rule of thumb:

Answer in:	Use:
\mathbb{Z}	Table
\mathbb{R}	Graph

Refer to Section 1.3.2 for plotting results from a table, which might be useful for plotting the PDF/CDF of a binomial distribution, since P(X=x) is undefined when $x \notin \mathbb{Z}^*$

3.1.3 Poisson Distribution

A random variable that follows a Poisson distribution has the following characteristics:

- Events occur randomly in a fixed interval and independently of one another.
- The mean rate of occurrence of the event is **constant** in the given interval.
- The probability of more than 1 event occurring within a short interval is **negligible**.

The probability P(X = x) of obtaining x successes in a fixed interval is given by:

$$P(X = x) = e^{-\lambda} \frac{\lambda^x}{x!}, \ x \in \mathbb{Z}^*$$
 (3.4)

We write $X \sim \text{Po}(\lambda)$, where λ is the mean number of occurrence of the event in that fixed interval.

This random variable X has $E(X) = Var(X) = \lambda$.

Note:

- X ranges over an infinite number of integer values (i.e. there is no upper bound)
- $\lambda \propto t$, where t is the length of the interval

Note:

$\lambda \in \mathbb{Z}$	2 modes	
$\lambda \in \mathbb{R} \notin \mathbb{Z}$	1 mode	

Poisson PDF

To obtain the probability P(X = x), we can either use the formula, or use the in-built PoissonPD function:

$$PoissonPD(x,\lambda)$$

Alternatively, we can use the STAT app to obtain the probability:

- 1. Go into STAT (2 in the menu)
- 2. Press DIST(F5) > POISN(\triangleright F1) > Ppd(F1)
- 3. Set the following:

Data	Variable	
x	X	
μ	λ	
Save Res	None	

- 4. Press [EXE] or scroll to Execute and press CALC(F1)
- 5. You can do further manipulation of the value obtained by:
 - a) Going to RUN·MAT
 - b) Go to [VARS] > STAT(F3) > RESLT(F6) > DIST(F3)
 - c) There you can find the p value by pressing p (F1) and then [EXE]

Poisson CDF

The cumulative probability $P(X \le x)$ can be calculated by using the in-built PoissonCD function:

PoissonCD(
$$x,\lambda$$
)

Alternatively, following instructions in the previous section (Section $3.1.3 \Rightarrow \text{Poisson PDF}$), and choosing Pcd instead.

Additive Property of Poisson Random Variable

If $X \sim \text{Po}(\lambda)$ and $Y \sim \text{Po}(\mu)$, where X and Y are **independent**, then:

$$X + Y \sim \text{Po}(\lambda + \mu)$$
 (3.5)

3.2 Continuous Random Variable

The probability for a continuous variable X to fall within a particular region [a, b] is given by:

$$P(a \le X \le b) = \int_{a}^{b} f(x) \tag{3.6}$$

where f(x) is the probability density function (PDF) of X. In particular,

$$\int_{-\infty}^{\infty} f(x) = 1$$

Moreover,

$$P(a \le X \le b) = P(a \le X < b)$$
$$= P(a < X \le b)$$
$$= P(a < X < b)$$

3.2.1 Normal Distribution

A (continuous) random variable $X \in \mathbb{R}$ that follows a normal distribution with mean μ and standard deviation σ has a *probability density function* (PDF) given by:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$
(3.7)

We write $X \sim N(\mu, \sigma^2)$

Note: Z is reserved to represent the standard normal distribution. Do not use it to represent your variables (that do not follow standard normal distribution).

Normal Distribution PDF

There are one of 2 ways to plot a graph of the normal distribution PDF:

- Plot the actual equation
- Use the in-built NormPD function

However, it must be noted that the NormPD plots slower than using the actual equation. Using G-Solv is also slower.

The usage of NormPD is

$$NormPD(X, [\sigma, \mu])$$

In this case, X can be either a single value, or a List. The corresponding p-value will also be adjusted accordingly.

If σ and μ are not provided, the standard normal distribution with $\sigma = 1$ and $\mu = 0$ is assumed.

Note: You should not use this command to calculate the probability of a certain random variable P(X = x) where $x \in \mathbb{R}$. This is because for a continuous random variable, this does not make sense. It should always be a range. (i.e. use the CDF in the next section)

35

Normal Distribution CDF

CDF stands for *Cumulative Distribution Function*. This can be calculated by taking the integral of the normal PDF from $-\infty$ to x. One can take integral by plotting the graph out (refer to previous section), and then G-Solv > $\int dx$.

Alternatively, you can use the built-in NormCD:

NormCD(
$$<$$
Lower $>$, $<$ Upper $>$,[σ , μ])

If σ and μ are not provided, the standard normal distribution with $\sigma=1$ and $\mu=0$ is assumed.

To plot the Normal Distribution CDF, you can use:

$$Y = NormCD(-1E99, X, [\sigma, \mu])$$

Note: -1E99 is to simulate $-\infty$.

Alternatively, we can use the STAT app to obtain the probability:

- 1. Go into STAT (2 in the menu)
- 2. Press DIST(F5) > NORM(F1) > Ncd(F1)
- 3. Set the following:

Data	Variable
Lower	Lower
Upper	Upper
σ	σ
μ	μ
Save Res	None

- 4. Press [EXE] or scroll to Execute and press CALC(F1)
- 5. You can do further manipulation of the value obtained by:
 - a) Going to RUN·MAT
 - b) Go to [VARS] > STAT(F3) > RESLT(F6) > DIST(F3)
 - c) There you can find the p value by pressing p (F1) and then <code>[EXE]</code>

Note: You cannot use this method for plotting

Finding the value given the probability

One sometimes need to find a given P(X < a) = b where $a, b \in \mathbb{R}$ and b is the probability of X being less than a.

To do this, we need the InvNormCD function built into the calculator. The usage of InvNormCD is as follows:

InvNormCD(
$$b$$
,[σ , μ])

This will give you back a.

3.3 Use for Experimental Data/SPA

The calculator can be used to determine the equation of a best fit line/parabola etc. from a set of data from experiments. From the main menu, enter the S-SHT app (4).

In the app, simply enter each set of data into each row. Enter the data on the same axis under the same column.

For example, consider the following experimental data for an experiment involving electrical current (I) and potential difference (V):

V/V	I/A
0.7148	0.0588
0.7372	0.0633
0.7780	0.0752
1.058	0.143
1.200	0.170
1.340	0.210

Enter this set of data into the GC as follows:

SHEET	A	В
1	0.7148	0.0588
2	0.7372	0.0633
3	0.7780	0.0752
4	1.058	0.143
5	1.200	0.170
6	1.340	0.210

To plot the points onto a graph, first press the right arrow key $(\triangleright, F6)$ and select the GRPH option (F1).

Once in the GRPH menu, we will first configure the graph options.

- 1. Press SET (F6) and ensure that StatGraph1 is shown on the first line and the Graph Type is Scatter.
- 2. Move to XCellRange and enter B1:B6. This means that the values from cells B1 to B6 inclusive are that for the horizontal x-axis.
- 3. Move to YCellRange and enter A1:A6.
- 4. Finally, ensure that the Frequency option is 1.
 - You may select any Mark Type that suits your personal preferences.
- 5. Press EXIT to leave the configuration menu and return to the spreadsheet.

Now, press GPH1 (F1) . A graph with markings corresponding to the points on the spreadsheet should be yield.

You calculator can find the equation of a best fit line or curve. In this case, the data is expected to be linearly related (and it is linearly related).

- 1. Press CALC (F1).
- 2. There are now a number of options corresponding to different types of graphs and relationships such as x^2 , ln, and e^x . In this case, our data is linear, so select **X** (F2).
- 3. Select ax+b (F1).
- 4. A table with various values is yield. The value a corresponds to the gradient, and b is the y-intercept. r^2 represents how closely the data fit to the line, with the value of 1 corresponding to a perfect, ideal fit.
- 5. To view the graph visually, select DRAW (F6) . To generate y-values using the equation, select COPY (F5) , and select one of the Y1/Y2... to copy to.

Note: You will need to add an \times between the gradient and the x, or it will cause the function to error out.

The linear regression result can also be obtained using the concept of best approximations, covered in Section 4.3 under Linear Algebra.

3.3.1 Plotting Statistics/Visualization

In this section, we would explore ways we can use the calculator to help us visualize statistics, just like how it would appear in our lecture notes. The steps here are purely provided as an example, and to expose you to these functions, and you should adapt accordingly for your own use.

Binomial/Poisson Distribution

To visualize a binomial distribution, let's first generate a list of values for a certain binomial distribution. For example, let's say that $X \sim B(50, \frac{1}{3})$:

We can generate a list of probabilities easily in RUN·MAT:

BinomialPD(50,
$$\frac{1}{3}$$
) \rightarrow List 1

Thereafter, go into STAT and you'll see the probabilities all listed out for you in the List 1:

SUB	LiSt	I	LiSt 2	LiSt B	LiSt 4
5	3.9E	- В			
4	4.8E 3.8E		 549	 	 5∈-09

We can then easily graph this result:

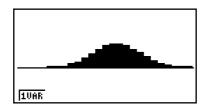
41

- 1. Go to GRPH (F1) > SET (F6)
- 2. Scroll to Graph Type and choose Bar (▷ > F3)

3. Select:	Data1	List1
	Stick Type	Length

4. Press EXIT and then GPH1 (F1)

The graph should have been plotted. You can scroll, zoom in, or out to see more:



This should look like those graphs they have in your lecture notes.

Note: the x-coordinate of 1 corresponds to P(X = 0)

A similar procedure can be followed to plot a graph of a variable that follows a Poisson distribution.

Chapter 4

Linear Algebra

Some of you might find yourself doing linear algebra, and needing to do matrix manipulations. In this chapter, I will outline some basic concepts covered in MA1101R (NUS H3 Course), and how you can use your GC to find the answer.

You may find certain concepts here useful for H2 Mathematics as well, especially in terms of matrix manipulations. I suggest you read this chapter as a complement of Chapter 2 (Vectors).

4.1 Storing Matrices

accessing individual matrix elements

4.2 Solving Linear Systems

introduce augemented matrix how to augment if 2

4.2.1 Elementary Row Operations and Row-Echelon Forms

row equiv

4.3 Best Approximations