Arithmetic Progression Problem

SAMMETA SAIPOORNA EE23BTECH11055

Which term of the arithmetic progression (AP): 3, 8, 13, 18, ... is 78? Find the term (k) when the term x(k) is equal to 78.

1 Input Table:

Here is the input table with common difference, initial term, and a description:

n	Parameters	Description
1	x(0)	Initial Term
2	d	Common Difference
3	$x(k) \times u(k)$	Target Term

2 Solution:

Let's solve the problem:

Let
$$x(n) = [3 + (n-1)5] \times u(n)$$

Given: $x(k) = 78$

Substitute values into the formula:

$$78 = 3 + (k - 1) \times 5$$

$$78 = 3 + (k - 1) \times 5$$

$$78 - 3 = 5(k - 1)$$

$$75 = 5(k - 1)$$

$$15 = k - 1$$

$$k = 16$$

Therefore, the correct term number (k) when x(k) = 78 in the given arithmetic progression is k = 16.

Z-Transform: 3

Let the Z-transform of x(n) be X(z). Let U(z) be the Z-transform of u(n).

$$X(z) = x(0)U(z) + dz^{-1}Z\{nu(n)\}$$
(1)

$$= \frac{3}{1 - z^{-1}} + \frac{5z^{-1}}{(1 - z^{-1})^2} \tag{2}$$

$$X(z) = x(0)U(z) + dz^{-1}Z\{nu(n)\}$$

$$= \frac{3}{1 - z^{-1}} + \frac{5z^{-1}}{(1 - z^{-1})^2}$$

$$= \frac{3(1 - z^{-1}) + 5z^{-1}}{(1 - z^{-1})^2}$$
(2)
$$= \frac{3(1 - z^{-1}) + 5z^{-1}}{(1 - z^{-1})^2}$$
(3)

$$=\frac{3-3z^{-1}+5z^{-1}}{(1-z^{-1})^2}\tag{4}$$

$$= \frac{3 + (5-3)z^{-1}}{(1-z^{-1})^2} \quad \text{for } |z| > 1$$
 (5)