Arithmetic Progression Problem

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Which term of the arithmetic progression (AP): $3, 8, 13, 18, \ldots$ is 78?
Find the term (k) when the term $x(k)$ is equal to 78.

1 Input Table:

n	Parameters	Description
1	$x(0)$	Initial Term
2	d	Common Difference
3	$x(k) \times u(k)$	Target Term
4	$X(z)$	Z-Transform

2 Solution:

Let's solve the problem:

$$\text{Let } x(n) = [3 + (n-1)5] \times u(n)$$
$$\text{Given: } x(k) = 78$$

Substitute values into the formula:

$$78 = 3 + (k-1) \times 5$$
$$75 = (k-1) \times 5$$
$$k = 16$$

Therefore, the correct term number (k) when $x(k) = 78$ in the given AP is $k = 16$.

3 Z-Transform:

Let the Z-transform of $x(n)$ be $X(z)$. Let $U(z)$ be the Z-transform of $u(n)$.

$$X(z) = x(0)U(z) + dz^{-1}Z\{nu(n)\} \tag{1}$$

$$= \frac{3}{1 - z^{-1}} + \frac{5z^{-1}}{(1 - z^{-1})^2} \tag{2}$$

$$= \frac{3(1 - z^{-1}) + 5z^{-1}}{(1 - z^{-1})^2} \tag{3}$$

$$= \frac{3 - 3z^{-1} + 5z^{-1}}{(1 - z^{-1})^2} \tag{4}$$

$$= \frac{3 + (5 - 3)z^{-1}}{(1 - z^{-1})^2} \quad \text{for all } |z| > 1 \tag{5}$$

The general form of an AP is given by $x(n) = a + (n-1)d$, The Z-transform of the general AP $x(n)$ is given by:

$$X(z) = \mathcal{Z}\{x(n)\}$$

$$= A(z) + \frac{D(z)}{z - 1}$$

where $A(z)$ and $D(z)$ are the Z-transforms of a and d, respectively.

Substitute the values for the given AP $3, 8, 13, 18, \ldots$ into this general Z-transform expression:

$$a = 3$$
$$d = 5$$

Substituting these values:

$$A(z) = \frac{3}{1 - z^{-1}}$$

$$D(z) = \frac{5}{1 - z^{-1}}$$

Now, substitute these into the general Z-transform expression:

$$X(z) = \frac{3}{1 - z^{-1}} + \frac{5}{z - 1}$$

$$X(z) = \frac{3z}{z - 1} + \frac{5}{z - 1}$$