Arithmetic Progression Problem

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Problem Statement

Which term of the arithmetic progression (AP): $3, 8, 13, 18, \ldots$ is 78?

Find the term (k) when the term $x(k)$ is equal to 78.

1 Input Table

Here is the input table with common difference, initial term, and a description:

$S.n$	Parameters	Description
1	$a(0)$	Initial Term of the AP
2	d	Common Difference
3	$a(k) \times u(k)$	Target Term of the AP

2 Solution

Let's solve the problem:

$$\text{Let } x(n) = [3 + (n-1)5] \times u(n)$$

$$\text{Given: } x(k) = 78$$

Substitute values into the formula:

$$\begin{aligned} 78 &= 3 + (k-1) \times 5 \\ 78 &= 3 + (k-1) \times 5 \\ 78 - 3 &= 5(k-1) \\ 75 &= 5(k-1) \\ 15 &= k - 1 \\ k &= 16 \end{aligned}$$

Therefore, the correct term number (k) when $x(k) = 78$ in the given arithmetic progression is $k = 16$.

3 Z-Transform

Let the Z-transform of $x(n)$ be $X(z)$. Let $U(z)$ be the Z-transform of $u(n)$.

$$X(z) = x(0)U(z) + dz^{-1}Z\{nu(n)\} \tag{1}$$

$$= \frac{3}{1 - z^{-1}} + \frac{5z^{-1}}{(1 - z^{-1})^2} \quad \forall \quad |z| > 1 \tag{2}$$

Using the values from the arithmetic progression problem:

$$X(z) = \frac{3}{1 - z^{-1}} + \frac{5z^{-1}}{(1 - z^{-1})^2} \quad \forall \quad |z| > 1 \tag{3}$$