

Discrete Assignment

SAMMETA SAIPOORNA
EE23BTECH11055

Question (11.9.3.14) The sum of first three terms of a G.P. is 16 and the sum of next three terms is 128. Determine the first term, the common ratio, and the sum to n terms of the G.P.

Answer

Parameter	Description	Value
$x(0)$	First term of AP	–
r	Common ratio	–
$x(n)$	General term of given AP	$x(0)r^n u(n)$
$x(0) + x(1) + x(2)$	sum of 1st,2nd and 3rd terms	16
$x(3) + x(4) + x(5)$	sum of 3rd,4th and 5th terms	128

$$y(n) = x(n) * u(n) \quad (1)$$

$$Y(z) = X(z)U(z) \quad (2)$$

Applying inverse Z-transform:

$$y(n) = x(0) \left[\frac{r^n - 1}{r - 1} \right] \quad (3)$$

For $y(3)$:

$$y(3) = x(0) \left[\frac{r^3 - 1}{r - 1} \right] \quad (4)$$

For $y(6) - y(3)$:

$$y(6) - y(3) = x(0) \left[\frac{r^6 - 1}{r - 1} - \frac{r^3 - 1}{r - 1} \right] \quad (5)$$

$$128 = x(0) \left[\frac{r^6 - 1}{r - 1} - \frac{r^3 - 1}{r - 1} \right] \quad (6)$$

$$128 = x(0)r^3(r^3 - 1) \quad (7)$$

$$16 = x(0) \left[\frac{r^3 - 1}{r - 1} \right] \quad (8)$$

Divide equation (7) by equation (8):

$$\frac{128}{16} = r^3 \quad (9)$$

$$r^3 = 8 \tag{10}$$

$$r = 2 \tag{11}$$

So, the solution for r is 2. Substituting this value back into the expression for $x(0)$, we get:

$$x(0) = \frac{16}{2^2 + 2 + 1} \tag{12}$$

$$= \frac{16}{7} \tag{13}$$

So, $r = 2$ and $x(0) = \frac{16}{7}$.