

# Discrete Assignment

SAMMETA SAIPOORNA  
EE23BTECH11055

**Question (11.9.3.14)** The sum of first three terms of a G.P. is 16 and the sum of next three terms is 128. Determine the first term, the common ratio, and the sum to  $n$  terms of the G.P.

**Answer**

Parameter	Description	Value
$x(0)$	First term of AP	–
$r$	Common ratio	–
$x(n)$	General term of given AP	$x(0)r^n$
$x(0) + x(1) + x(2)$	sum of 1st, 2nd and 3rd terms	16
$x(3) + x(4) + x(5)$	sum of 3rd, 4th and 5th terms	128

Given:

$$x(0) + x(0)r + x(0)r^2 = 16 \quad (1)$$

$$x(0)r^3 + x(0)r^4 + x(0)r^5 = 128 \quad (2)$$

From Equation (1):

$$x(0)(1 + r + r^2) = 16 \quad (3)$$

$$x(0) = \frac{16}{1 + r + r^2} \quad (4)$$

$$x(0)r^3 = \frac{128}{1 + r + r^2} \quad (5)$$

$$r^3 = 8 \quad (6)$$

$$r = 2 \quad (7)$$

$$x(0) = \frac{16}{7} \quad (8)$$

Using the z-transform approach:

$$(x(0) * u)(z) = X(z)U(z) \quad (9)$$

Given  $X(z)$ :

$$X(z) = \frac{1}{1 - \frac{1}{2}z^{-1}} \quad (10)$$

Therefore,

$$(x(0) * u)(z) = \frac{U(z)}{1 - \frac{1}{2}z^{-1}} \quad (11)$$

Using the inverse z-transform:

$$(x(0) * u)_n = U(z) \cdot \left(\frac{1}{2}\right)^n u(n) \quad (12)$$

$$= \frac{2^{n+1} - 1}{2 - 1} \quad (13)$$

$$= 2^{n+1} - 1 \quad (14)$$

So, the sum of the first  $n$  terms of the sequence  $x(n) = 2^n$  is  $2^{n+1} - 1$ .

$$S_n = \frac{16}{7}(2^n - 1) \quad (15)$$

$$= \frac{16}{7} \cdot 2^n - \frac{16}{7} \quad (16)$$

$$= \frac{16}{7} \cdot (a * b)_n - \frac{16}{7} \quad (17)$$

$$= \frac{16}{7} \cdot (2^{n+1} - 1) - \frac{16}{7} \quad (18)$$

The Z-transform will be:

$$X(z) = \frac{\frac{16}{7}}{1 - 2z^{-1}} \quad (19)$$