Discrete Assignment

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Question (11.9.3.14) The sum of first three terms of a G.P. is 16 and the sum of next three terms is 128. Determine the first term , the common ratio, and the sum to n terms of the G.P.

Answer

Parameter	Description	Value
$x(0)$	First term of AP	–
r	Common ratio	–
$x(n)$	General term of given AP	$x(0)r^n u(n)$
$x(0) + x(1) + x(2)$	sum of 1st,2nd and 3rd terms	16
$x(3) + x(4) + x(5)$	sum of 3rd,4th and 5th terms	128

Given:

$$x(0) + x(0)r + x(0)r^2 = 16 \tag{1}$$

$$x(0)r^3 + x(0)r^4 + x(0)r^5 = 128 \tag{2}$$

From Equation (1):

$$x(0)(1 + r + r^2) = 16 \tag{3}$$

$$x(0) = \frac{16}{1 + r + r^2} \tag{4}$$

$$x(0)r^3 = \frac{128}{1 + r + r^2} \tag{5}$$

$$r^3 = 8 \tag{6}$$

$$r = 2 \tag{7}$$

$$x(0) = \frac{16}{7} \tag{8}$$

Solving for r in terms of $x(0)$:

$$(x(0) * b)_n = \sum_{k=0}^{n} 2^k \cdot 1 \tag{9}$$

$$= \frac{2^{n+1} - 1}{2 - 1} \tag{10}$$

$$= 2^{n+1} - 1 \tag{11}$$

[where b is a constant representing the coefficient associated with the initial condition $x(0)$ in the convolution operation.]

$$= \frac{2^{n+1} - 1}{2 - 1} \tag{12}$$

$$= 2^{n+1} - 1 \tag{13}$$

So, the sum of the first n terms of the sequence $x(n) = 2^n$ is $2^{n+1} - 1$.

$$S_n = \frac{16}{7}(2^n - 1) \tag{14}$$

$$= \frac{16}{7} \cdot 2^n - \frac{16}{7} \tag{15}$$

$$= \frac{16}{7} \cdot (a * b)_n - \frac{16}{7} \tag{16}$$

$$= \frac{16}{7} \cdot (2^{n+1} - 1) - \frac{16}{7} \tag{17}$$

The Z-transform will be:

$$X(z) = \frac{\frac{16}{7}}{1 - 2z^{-1}} \tag{18}$$