All Students Take Calculus
I–All pos. II–sin III–tan IV–cos

Law of Cosines
$a^2 = b^2 + c^2 - 2bc \cdot \cos A$

Difference of Cubes
$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$

Arc Length
$s = r\theta$

Heron's Formula
$A = \sqrt{s(s-a)(s-b)(s-c)}$

Change of Base
$\log_b m = \frac{\log m}{\log b}$

Choose Formula
$C(x, y) = \binom{x}{y} = \frac{x!}{y!(x-y)!}$

Law of Sines
$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Degrees to Radians
$\frac{A \cdot \pi}{180} = \theta$

Sector Area
$A = \frac{1}{2} r^2 \theta$

Area of Δ
$Area = ab \cdot \frac{1}{2} \sin C$

Polar to (x, y)
$r^2 = x^2 + y^2$
$\tan\theta = \frac{y}{x}$

$(\log_a b)(\log_c d) = (\log_a d)(\log_c b)$

$z = a + bi$
$|z| = \sqrt{a^2 + b^2}$

$z = r \operatorname{cis}\theta$
$z^n = r^n \operatorname{cis}(n\theta)$

nth roots of $z = r \operatorname{cis}\theta$
$w_k = r^{1/n} \operatorname{cis}\left(\frac{\theta + 2k\pi}{n}\right)$

$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$
$\frac{z_1}{z_2} = \frac{r_1}{r_1} \operatorname{cis}(\theta_1 - \theta_2)$

$\vec{v} = \langle a, b \rangle = a\hat{\imath} + b\hat{\jmath}$
$|\vec{v}| = \sqrt{a^2 + b^2}$

$|c\vec{u}| = |c||\vec{u}|$

Dot Product
$\vec{u} \cdot \vec{v} = a_1 a_2 + b_1 b_2$

Dot Product Theorem
$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos\theta$

θ between $\vec{u}$ & $\vec{v}$
$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}$

$\vec{u}$ and $\vec{v}$ are prependicular
$\vec{u} \cdot \vec{v} = 0$

Component of $\vec{u}$ along $\vec{v}$
$(\vec{u} \cdot \vec{v})/|\vec{v}|$

$\operatorname{proj}_{\vec{v}} \vec{u} = \left(\frac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2}\right) \vec{v}$

Work
$W = \vec{F} \cdot \vec{D}$

Trig Identities

$\sin^2 + \cos^2 = 1$

$\tan^2 + 1 = \sec^2$

$1 + \cot^2 = \csc^2$

$2 \sin u \cos u = \sin(2u)$

$\cos^2 u - \sin^2 u = \cos(2u)$

$\frac{2 \tan u}{1 - \tan^2 u} = \tan(2u)$

$\sin u \cos v \pm \cos u \sin v = \sin(u \pm v)$

$\cos u \cos v \mp \sin u \sin v = \cos(u \pm v)$

$\frac{\tan u \pm \tan v}{1 \mp \tan u \tan v} = \tan(u \pm v)$

$\cot = \frac{1}{\tan}$

$\csc = \frac{1}{\sin}$

$\sec = \frac{1}{\cos}$

$\sin\left(\frac{\pi}{2} - u\right) = \cos u$

$\tan\left(\frac{\pi}{2} - u\right) = \cot u$

$\sec\left(\frac{\pi}{2} - u\right) = \csc u$

$\cos\left(\frac{\pi}{2} - u\right) = \sin u$

$\cot\left(\frac{\pi}{2} - u\right) = \tan u$

$\csc\left(\frac{\pi}{2} - u\right) = \sec u$

$\frac{1 - \cos 2x}{2} = \sin^2 x$

$\frac{1 + \cos 2x}{2} = \cos^2 x$

$\frac{1 - \cos 2x}{1 + \cos 2x} = \tan^2 x$

$\pm\sqrt{\frac{1 - \cos u}{2}} = \sin \frac{u}{2}$

$\pm\sqrt{\frac{1 + \cos u}{2}} = \cos \frac{u}{2}$

$\frac{1 - \cos u}{\sin u} = \frac{\sin u}{1 + \cos u} = \tan \frac{u}{2}$

$2 \sin \frac{x \pm y}{2} \cos \frac{x \mp y}{2} = \sin x \pm \sin y$

$2 \cos \frac{x + y}{2} \cos \frac{x - y}{2} = \cos x + \cos y$

$-2 \sin \frac{x + y}{2} \sin \frac{x - y}{2} = \cos x - \cos y$

$\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$

$\cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$

$\cos u \cos v = \frac{1}{2}[\cos(u + v) + \cos(u - v)]$

$\sin u \sin v = \frac{1}{2}[\cos(u + v) - \cos(u - v)]$

Algebra of Functions

Let f and g be functions with domains A and B.

$$\begin{array}{rl}
(f+g)(x) = f(x)+g(x) & \text{Domain } A \cap B \\
(f-g)(x) = f(x)-g(x) & \text{Domain } A \cap B \\
(fg)(x) = f(x)g(x) & \text{Domain } A \cap B \\
\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} & \text{Domain } \{x \in A \cap B \mid g(x) \neq 0\} \\
(f \circ g)(x) = f(g(x)) & \text{Domain } \{x \in B \mid g(x) \in A\}
\end{array}$$

Polynomial Synthetic Division

$(x^3+x^2-1) \div (x+2)$

$$\begin{array}{r|rrrr}
-2 & 1 & 1 & 0 & -1 \\
 & & -2 & 2 & -4 \\
\hline
 & 1 & -1 & 2 & -5
\end{array}$$

Polynomial Long Division

$$\begin{array}{rrrrr}
 & & x^2 & -x & +2 \\
\hline
x+2) & x^3 & +x^2 & & -1 \\
 & -x^3 & -2x^2 & & \\
\hline
 & & -x^2 & & \\
 & & x^2 & +2x & \\
\hline
 & & & 2x & -1
\end{array}$$

Rational Roots Theorem

$2x^3+2x^2-3x-6$

$\pm1, \pm2 \qquad \pm1, \pm2, \pm3, \pm6$

Possible rational roots:

$\pm1, \pm\frac{1}{2}, \pm2, \pm3, \pm\frac{3}{2}, \pm6$

Decartes' Rule of Signs

Count num. of sign changes

$P(x) = 3x^6+4x^5+3x^3-x-3$

1 positive real root

$P(-x) = 3x^6-4x^5-3x^3+x-3$

1 or 3 negative real roots

Logarithm Formulas

$$\log(m \cdot n) = \log m + \log n$$

$$\log\left(\frac{m}{n}\right) = \log m - \log n$$

$$\log(m^n) = n \cdot \log m$$

$$\log_b b^x = x = b^{\log_b x}$$

Other trig stuff

$$\cot = \frac{1}{\tan}$$

$$\csc = \frac{1}{\sin}$$

$$\sec = \frac{1}{\cos}$$

Horizontal Asymptotes

$$\begin{array}{ll}
y = \dfrac{2x^2-4x+5}{x^2-2x+1} & \text{Original Equation} \\
 = \dfrac{2x^2}{x^2} & x \to \infty, \text{other terms} \to \text{tiny} \\
 = 2 & \text{Cancel, horizontal asymptote}
\end{array}$$

Slant Asymptotes

$$\begin{array}{ll}
y = \dfrac{x^2-4x-5}{x-3} & \text{Original Equation} \\
 = x-1-\dfrac{8}{x-3} & \text{Divide} \\
 = x-1 & x \to \infty, \text{other terms} \to \text{tiny}
\end{array}$$

Vertical Asymptotes

$$\begin{array}{ll}
y = \dfrac{2x^2-4x+5}{x^2-2x+1} & \text{Original Equation} \\
 = \dfrac{2x^2-4+5}{(2x-1)(x+2)} & \text{Factor demoniator} \\
x = \dfrac{1}{2} \text{ or } x = -2 & \text{Impossible}
\end{array}$$

End Behavior

$y = x^n$
n is even

$y = -x^n$
n is even

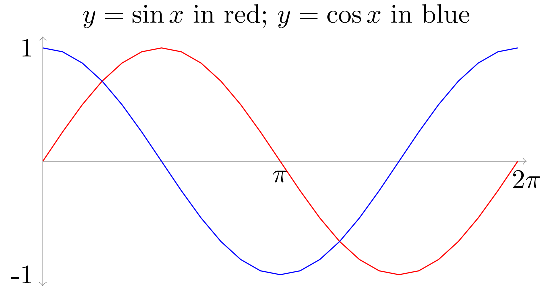
$y = x^n$
n is odd

$y = -x^n$
n is odd

$m \times n$ matrix

$$\begin{bmatrix}
a_{11} & a_{12} & \cdots & a_{1n} \\
a_{21} & a_{22} & \cdots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
a_{m1} & a_{m2} & \cdots & a_{mn}
\end{bmatrix}$$

$y = \sin x$ in red; $y = \cos x$ in blue



sin/cos Graph Properties

If in form:

$$y = a \sin k(x-b)$$

amplitude $|a|$, period $2\pi/k$, phase shift b

Allowed row operations

1. Add a multiple of one row to another
2. Multiply a row by a nonzero constant
3. Interchange two rows

Row-Echelon Form

$$\begin{bmatrix}
1 & 2 & -1 & 1 \\
0 & 1 & 4 & -7 \\
0 & 0 & 1 & -2
\end{bmatrix}$$

Reduced Row-Echelon Form

$$\begin{bmatrix}
1 & 0 & 0 & -3 \\
0 & 1 & 0 & 1 \\
0 & 0 & 1 & -2
\end{bmatrix}$$

Using matrix inverses ($AX = B \Rightarrow X = A^{-1}B$)

$$\begin{bmatrix} 2 & -5 \\ 3 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ 36 \end{bmatrix} \Longrightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 & \frac{5}{3} \\ -1 & \frac{2}{3} \end{bmatrix} \begin{bmatrix} 15 \\ 36 \end{bmatrix} = \begin{bmatrix} 30 \\ 9 \end{bmatrix}$$

Matrix Multiplication!

$$\begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 5 & 2 \\ 0 & 4 & 7 \end{bmatrix} = \begin{bmatrix} 1 \cdot (-1) + 3 \cdot 0 & 1 \cdot 5 + 3 \cdot 4 & 1 \cdot 2 + 3 \cdot 7 \\ (-1) \cdot (-1) + 0 \cdot 0 & (-1) \cdot 5 + 0 \cdot 4 & (-1) \cdot 2 + 0 \cdot 7 \end{bmatrix} = \begin{bmatrix} -1 & 17 & 23 \\ 1 & -5 & -2 \end{bmatrix}$$

2×2 Matrix Inverse

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $A^{-1} = \dfrac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$n \times n$ Matrix Inverse

$$\left[\begin{array}{rrr|rrr} 1 & -2 & -4 & 1 & 0 & 0 \\ 2 & -3 & -6 & 0 & 1 & 0 \\ -3 & 6 & 15 & 0 & 0 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{rrr|rrr} 1 & 0 & 0 & -3 & 2 & 0 \\ 0 & 1 & 0 & -4 & 1 & -\frac{2}{3} \\ 0 & 0 & 1 & 1 & 0 & \frac{1}{2} \end{array}\right]$$