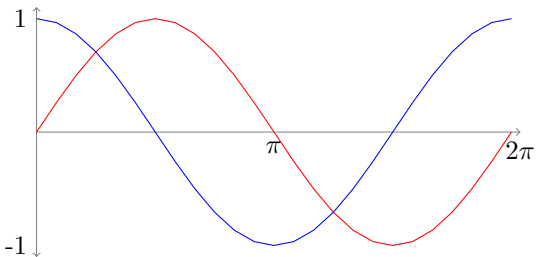**Positive Trigonometric Functions**
I–All pos. II–sin III–tan IV–cos

Law of Cosines
$a^2 = b^2 + c^2 - 2bc \cdot \cos A$

Difference of Cubes
$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$

Arc Length
$s = r\theta$

Heron's Formula
$A = \sqrt{s(s-a)(s-b)(s-c)}$

Change of Base
$\log_b m = \dfrac{\log m}{\log b}$

Choose Formula
$C(x,y) = \dbinom{x}{y} = \dfrac{x!}{y!(x-y)!}$

Law of Sines
$\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

Degrees to Radians
$\dfrac{A \cdot \pi}{180} = \theta$

Sector Area
$A = \dfrac{1}{2}r^2\theta$

Area of Δ
$Area = ab \cdot \dfrac{1}{2}\sin C$

Polar to (x,y)
$r^2 = x^2 + y^2$
$\tan\theta = \dfrac{y}{x}$

$(\log_a b)(\log_c d) = (\log_a d)(\log_c b)$

$z = a + bi$
$|z| = \sqrt{a^2 + b^2}$

$z = r\operatorname{cis}\theta$
$z^n = r^n \operatorname{cis}(n\theta)$

nth roots of $z = r\operatorname{cis}\theta$
$w_k = r^{1/n}\operatorname{cis}\left(\dfrac{\theta + 2k\pi}{n}\right)$

$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$
$\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\operatorname{cis}(\theta_1 - \theta_2)$

$\vec{v} = \langle a, b\rangle = a\hat{\imath} + b\hat{\jmath}$
$|\vec{v}| = \sqrt{a^2 + b^2}$

$|c\vec{u}| = |c||\vec{u}|$

Dot Product
$\vec{u} \cdot \vec{v} = a_1 a_2 + b_1 b_2$

Dot Product Theorem
$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}|\cos\theta$

θ between $\vec{u}$ & $\vec{v}$
$\cos\theta = \dfrac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}$

$\vec{u}$ and $\vec{v}$ are prependicular
$\vec{u} \cdot \vec{v} = 0$

Component of $\vec{u}$ along $\vec{v}$
$(\vec{u} \cdot \vec{v})/|\vec{v}|$

$\operatorname{proj}_{\vec{v}}\vec{u} = \left(\dfrac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2}\right)\vec{v}$

Work
$W = \vec{F} \cdot \vec{D}$

Trig Identities

$\sin^2 + \cos^2 = 1$ | $\tan^2 + 1 = \sec^2$ | $1 + \cot^2 = \csc^2$ | $2\sin u \cos u = \sin(2u)$ | $\cos^2 u - \sin^2 u = \cos(2u)$ | $\dfrac{2\tan u}{1 - \tan^2 u} = \tan(2u)$

$\sin u \cos v \pm \cos u \sin v = \sin(u \pm v)$ | $\cos u \cos v \mp \sin u \sin v = \cos(u \pm v)$ | $\dfrac{\tan u \pm \tan v}{1 \mp \tan u \tan v} = \tan(u \pm v)$ | $\cot = \dfrac{1}{\tan}$

$\csc = \dfrac{1}{\sin}$ | $\sec = \dfrac{1}{\cos}$ | $\sin\left(\dfrac{\pi}{2} - u\right) = \cos u$ | $\tan\left(\dfrac{\pi}{2} - u\right) = \cot u$ | $\sec\left(\dfrac{\pi}{2} - u\right) = \csc u$ | $\cos\left(\dfrac{\pi}{2} - u\right) = \sin u$

$\cot\left(\dfrac{\pi}{2} - u\right) = \tan u$ | $\csc\left(\dfrac{\pi}{2} - u\right) = \sec u$ | $\dfrac{1 - \cos 2x}{2} = \sin^2 x$ | $\dfrac{1 + \cos 2x}{2} = \cos^2 x$ | $\dfrac{1 - \cos 2x}{1 + \cos 2x} = \tan^2 x$ | $\pm\sqrt{\dfrac{1 - \cos u}{2}} = \sin\dfrac{u}{2}$

$\pm\sqrt{\dfrac{1 + \cos u}{2}} = \cos\dfrac{u}{2}$ | $\dfrac{1 - \cos u}{\sin u} = \dfrac{\sin u}{1 + \cos u} = \tan\dfrac{u}{2}$ | $2\sin\dfrac{x \pm y}{2}\cos\dfrac{x \mp y}{2} = \sin x \pm \sin y$ | $2\cos\dfrac{x + y}{2}\cos\dfrac{x - y}{2} = \cos x + \cos y$

$-2\sin\dfrac{x + y}{2}\sin\dfrac{x - y}{2} = \cos x - \cos y$ | $\sin u \cos v = \dfrac{1}{2}[\sin(u + v) + \sin(u - v)]$ | $\cos u \sin v = \dfrac{1}{2}[\sin(u + v) - \sin(u - v)]$

$\cos u \cos v = \dfrac{1}{2}[\cos(u + v) + \cos(u - v)]$ | $\sin u \sin v = \dfrac{1}{2}[\cos(u + v) - \cos(u - v)]$

Row-Echelon Form
$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 1 & -2 \end{bmatrix}$

Reduced Row-Echelon Form
$\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$

Using matrix inverses ($AX = B \Rightarrow X = A^{-1}B$)
$\begin{bmatrix} 2 & -5 \\ 3 & -6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ 36 \end{bmatrix} \Longrightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 & \frac{5}{3} \\ -1 & \frac{2}{3} \end{bmatrix}\begin{bmatrix} 15 \\ 36 \end{bmatrix} = \begin{bmatrix} 30 \\ 9 \end{bmatrix}$

Matrix Multiplication!

$\begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 5 & 2 \\ 0 & 4 & 7 \end{bmatrix} = \begin{bmatrix} 1 \cdot (-1) + 3 \cdot 0 & 1 \cdot 5 + 3 \cdot 4 & 1 \cdot 2 + 3 \cdot 7 \\ (-1) \cdot (-1) + 0 \cdot 0 & (-1) \cdot 5 + 0 \cdot 4 & (-1) \cdot 2 + 0 \cdot 7 \end{bmatrix} = \begin{bmatrix} -1 & 17 & 23 \\ 1 & -5 & -2 \end{bmatrix}$

2×2 Matrix Inverse
If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $A^{-1} = \dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$n \times n$ Matrix Inverse
$\left[\begin{array}{ccc|ccc} 1 & -2 & -4 & 1 & 0 & 0 \\ 2 & -3 & -6 & 0 & 1 & 0 \\ -3 & 6 & 15 & 0 & 0 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 2 & 0 \\ 0 & 1 & 0 & -4 & 1 & -\frac{2}{3} \\ 0 & 0 & 1 & 1 & 0 & \frac{1}{2} \end{array}\right]$

2×2 Matrix Determinant
$\det(A) = |A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

Minor M_{ij}: Take the matrix and delete the ith row and the jth column. Find the determinant

Cofactor A_{ij}
$(-1)^{i+j}M_{ij}$

$n \times n$ Matrix Determinant (can move along any row/column)
$\det(A) = |A| = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{vmatrix} = a_{11}A_{11} + a_{12}A_{12} + \cdots + a_{1n}A_{1n}$

Common Sums
$$\sum_{k=1}^{n} c = nc$$
$$\sum_{k=1}^{n} k = \dfrac{n(n+1)}{2}$$
$$\sum_{k=1}^{n} k^2 = \dfrac{n(n+1)(2n+1)}{6}$$
$$\sum_{k=1}^{n} k^3 = \dfrac{n^2(n+1)^2}{4}$$

Algebra of Functions

Let f and g be functions with domains A and B.

$(f + g)(x) = f(x) + g(x)$ Domain $A \cap B$

$(f - g)(x) = f(x) - g(x)$ Domain $A \cap B$

$(fg)(x) = f(x)g(x)$ Domain $A \cap B$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ Domain $\{x \in A \cap B \mid g(x) \neq 0\}$

$(f \circ g)(x) = f(g(x))$ Domain $\{x \in B \mid g(x) \in A\}$

Polynomial Synthetic Division

$(x^3 + x^2 - 1) \div (x + 2)$

$$
\begin{array}{r|rrrr}
-2 & 1 & 1 & 0 & -1 \\
   &   & -2 & 2 & -4 \\
\hline
   & 1 & -1 & 2 & -5
\end{array}
$$

Result is $x^2 - x + 2 - \frac{5}{x+2}$

Polynomial Long Division

$$
\begin{array}{r}
x^2 \quad - x + 2 \\
x+2\overline{\smash{\big)}\,x^3 + x^2 \quad\quad - 1} \\
\underline{-x^3 - 2x^2} \\
-x^2 \\
\underline{x^2 + 2x} \\
2x - 1
\end{array}
$$

Rational Roots Theorem

$2x^3 + 2x^2 - 3x - 6$

$\pm 1, \pm 2$ $\pm 1, \pm 2, \pm 3, \pm 6$

Possible rational roots:

$\pm 1, \pm\frac{1}{2}, \pm 2, \pm 3, \pm\frac{3}{2}, \pm 6$

Decartes' Rule of Signs

Count num. of sign changes

$P(x) = 3x^6 + 4x^5 + 3x^3 - x - 3$

1 positive real root

$P(-x) = 3x^6 - 4x^5 - 3x^3 + x - 3$

1 or 3 negative real roots

Logarithm Formulas

$\log(m \cdot n) = \log m + \log n$

$\log\left(\frac{m}{n}\right) = \log m - \log n$

$\log(m^n) = n \cdot \log m$

$\log_b b^x = x = b^{\log_b x}$

Trigonometric Reciprocals

$\cot = \dfrac{1}{\tan}$

$\csc = \dfrac{1}{\sin}$

$\sec = \dfrac{1}{\cos}$

Horizontal Asymptotes

$y = \dfrac{2x^2 - 4x + 5}{x^2 - 2x + 1}$ Original Equation

$= \dfrac{2x^2}{x^2}$ $x \to \infty$, other terms $\to$ tiny

$= 2$ Cancel, horizontal asymptote

Slant Asymptotes

$y = \dfrac{x^2 - 4x - 5}{x - 3}$ Original Equation

$= x - 1 - \dfrac{8}{x - 3}$ Divide

$= x - 1$ $x \to \infty$, other terms $\to$ tiny

Vertical Asymptotes

$y = \dfrac{2x^2 - 4x + 5}{x^2 - 2x + 1}$ Original Equation

$= \dfrac{2x^2 - 4 + 5}{(2x - 1)(x + 2)}$ Factor demoniator

$x = \dfrac{1}{2}$ or $x = -2$ Impossible

End Behavior

$y = x^n$, n is even

$y = -x^n$, n is even

$y = x^n$, n is odd

$y = -x^n$, n is odd

$m \times n$ matrix

$$
\begin{bmatrix}
a_{11} & a_{12} & \cdots & a_{1n} \\
a_{21} & a_{22} & \cdots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
a_{m1} & a_{m2} & \cdots & a_{mn}
\end{bmatrix}
$$

$y = \sin x$ in red; $y = \cos x$ in blue

sin/cos Graph Properties

If in form:

$$y = a\sin k(x - b)$$

amplitude $|a|$, period $2\pi/k$, phase shift b

Allowed row operations

1. Add a multiple of one row to another
2. Multiply a row by a nonzero constant
3. Interchange two rows

If $\begin{cases} ax + by = r \\ cx + dy = s \end{cases}$, then $x = \dfrac{\begin{vmatrix} r & b \\ s & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$ and $y = \dfrac{\begin{vmatrix} a & r \\ c & s \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$

Vertical Parabola

$x^2 = 4py$

$V(0, 0)$, $F(0, p)$, directrix $y = -p$

Ellipse

$\dfrac{x^2}{(a \text{ or } b)^2} + \dfrac{y^2}{(a \text{ or } b)^2} = 1$

$c^2 = a^2 - b^2$

Eccentricity

$e = \dfrac{c}{a},$

Hyperbola

$\dfrac{x \text{ or } y^2}{a^2} - \dfrac{x \text{ or } y^2}{b^2} = 1$

$c^2 = a^2 + b^2$

Shifted Conic

$V(h, k)$, x to $(x - h)$, y to $(y - k)$

Polar Conics

$r = \dfrac{ed}{1 \pm e(\cos \text{ or } \sin)\theta}$

Derivative Formula

$f^{-1}(a) = \lim\limits_{h \to 0} \dfrac{f(a + h) - f(a)}{h}$

Area

$A = \lim\limits_{n \to \infty} \sum\limits_{k=1}^{n} f(x_k)\Delta x$

$\Delta x = \dfrac{b - a}{n}$

$x_k = a + k\Delta x$

Horizontal Parabola

$y^2 = 4px$

$V(0, 0)$, $F(p, 0)$, directrix $x = -p$

Parabolas

Latus rectum is $|4p|$

Ellipses

$a^2 > b^2$

x^2 first of terms means more horizontal, major axis length is $2a$, minor axis length is $2b$, latus rectum is $\frac{2b^2}{a}$, foci on major axis $F(\pm c, 0)$ or $F(0, \pm c)$

Hyperbolas

a^2 forms positive term with x or y, horizontal when x^2 is first of terms, $V(\pm a, 0)$ or $V(0, \pm a)$, $B(0, \pm b)$ or $B(\pm b, 0)$, transverse axis length is $2a$, conjugate axis length is $2b$, asymptote slopes $\pm\frac{b}{a}$ or $\pm\frac{a}{b}$, foci on transverse axis $F(\pm c, 0)$ or $F(0, \pm c)$, latus rectum is $\frac{2b^2}{a}$