`github.com/cg505/formulaPage` rev 201412171751

All Students Take Calculus
I–All pos. II–sin III–tan IV–cos

Law of Cosines
$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

Difference of Cubes
$$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$$

Arc Length
$$s = r\theta$$

vHeron's Formula
$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

Change of Base
$$\log_b m = \frac{\log m}{\log b}$$

Choose Formula
$$C(x,y) = \binom{x}{y} = \frac{x!}{y!(x-y)!}$$

Law of Sines
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Degrees to Radians
$$\frac{A \cdot \pi}{180} = \theta$$

Sector Area
$$A = \frac{1}{2} r^2 \theta$$

Area of Δ
$$Area = ab \cdot \frac{1}{2} \sin C$$

Polar to (x,y)
$$r^2 = x^2 + y^2$$
$$\tan\theta = \frac{y}{x}$$

$$(\log_a b)(\log_c d) = (\log_a d)(\log_c b)$$

$$z = a + bi$$
$$|z| = \sqrt{a^2 + b^2}$$

$$z = r \operatorname{cis}\theta$$
$$z^n = r^n \operatorname{cis}(n\theta)$$

nth roots of $z = r \operatorname{cis}\theta$
$$w_k = r^{1/n} \operatorname{cis}\left(\frac{\theta + 2k\pi}{n}\right)$$

$$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$$
$$\frac{z_1}{z_2} = \frac{r_1}{r_1} \operatorname{cis}(\theta_1 - \theta_2)$$

$$\vec{v} = \langle a, b \rangle = a\hat{\imath} + b\hat{\jmath}$$
$$|\vec{v}| = \sqrt{a^2 + b^2}$$

$$|c\vec{u}| = |c||\vec{u}|$$

Dot Product
$$\vec{u} \cdot \vec{v} = a_1 a_2 + b_1 b_2$$

Dot Product Theorem
$$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos\theta$$

θ between $\vec{u}$ & $\vec{v}$
$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}$$

$\vec{u}$ and $\vec{v}$ are prependicular
$$\vec{u} \cdot \vec{v} = 0$$

Component of $\vec{u}$ along $\vec{v}$
$$(\vec{u} \cdot \vec{v})/|\vec{v}|$$

$$\operatorname{proj}_{\vec{v}} \vec{u} = \left(\frac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2}\right) \vec{v}$$

Work
$$W = \vec{F} \cdot \vec{D}$$

Trig Identities

$$\sin^2 + \cos^2 = 1$$
$$\tan^2 + 1 = \sec^2$$
$$1 + \cot^2 = \csc^2$$
$$2 \sin u \cos u = \sin(2u)$$
$$\cos^2 u - \sin^2 u = \cos(2u)$$
$$\frac{2\tan u}{1 - \tan^2 u} = \tan(2u)$$
$$\sin u \cos v \pm \cos u \sin v = \sin(u \pm v)$$
$$\cos u \cos v \mp \sin u \sin v = \cos(u \pm v)$$
$$\frac{\tan u \pm \tan v}{1 \mp \tan u \tan v} = \tan(u \pm v)$$
$$\cot = \frac{1}{\tan}$$
$$\csc = \frac{1}{\sin}$$
$$\sec = \frac{1}{\cos}$$
$$\sin\left(\frac{\pi}{2} - u\right) = \cos u$$
$$\tan\left(\frac{\pi}{2} - u\right) = \cot u$$
$$\sec\left(\frac{\pi}{2} - u\right) = \csc u$$
$$\cos\left(\frac{\pi}{2} - u\right) = \sin u$$
$$\cot\left(\frac{\pi}{2} - u\right) = \tan u$$
$$\csc\left(\frac{\pi}{2} - u\right) = \sec u$$
$$\frac{1 - \cos 2x}{2} = \sin^2 x$$
$$\frac{1 + \cos 2x}{2} = \cos^2 x$$
$$\frac{1 - \cos 2x}{1 + \cos 2x} = \tan^2 x$$
$$\pm\sqrt{\frac{1 - \cos u}{2}} = \sin\frac{u}{2}$$
$$\pm\sqrt{\frac{1 + \cos u}{2}} = \cos\frac{u}{2}$$
$$\frac{1 - \cos u}{\sin u} = \frac{\sin u}{1 + \cos u} = \tan\frac{u}{2}$$
$$2 \sin\frac{x \pm y}{2} \cos\frac{x \mp y}{2} = \sin x \pm \sin y$$
$$2 \cos\frac{x + y}{2} \cos\frac{x - y}{2} = \cos x + \cos y$$
$$-2 \sin\frac{x + y}{2} \sin\frac{x - y}{2} = \cos x - \cos y$$
$$\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$$
$$\cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$$
$$\cos u \cos v = \frac{1}{2}[\cos(u + v) + \cos(u - v)]$$
$$\sin u \sin v = \frac{1}{2}[\cos(u + v) - \cos(u - v)]$$

Row-Echelon Form
$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

Reduced Row-Echelon Form
$$\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

Using matrix inverses ($AX = B \Rightarrow X = A^{-1}B$)
$$\begin{bmatrix} 2 & -5 \\ 3 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ 36 \end{bmatrix} \Longrightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 & \frac{5}{3} \\ -1 & \frac{2}{3} \end{bmatrix} \begin{bmatrix} 15 \\ 36 \end{bmatrix} = \begin{bmatrix} 30 \\ 9 \end{bmatrix}$$

Matrix Multiplication!
$$\begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 5 & 2 \\ 0 & 4 & 7 \end{bmatrix} = \begin{bmatrix} 1 \cdot (-1) + 3 \cdot 0 & 1 \cdot 5 + 3 \cdot 4 & 1 \cdot 2 + 3 \cdot 7 \\ (-1) \cdot (-1) + 0 \cdot 0 & (-1) \cdot 5 + 0 \cdot 4 & (-1) \cdot 2 + 0 \cdot 7 \end{bmatrix} = \begin{bmatrix} -1 & 17 & 23 \\ 1 & -5 & -2 \end{bmatrix}$$

2×2 Matrix Inverse
$$\text{If } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ then } A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$n \times n$ Matrix Inverse
$$\left[\begin{array}{ccc|ccc} 1 & -2 & -4 & 1 & 0 & 0 \\ 2 & -3 & -6 & 0 & 1 & 0 \\ -3 & 6 & 15 & 0 & 0 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 2 & 0 \\ 0 & 1 & 0 & -4 & 1 & -\frac{2}{3} \\ 0 & 0 & 1 & 1 & 0 & \frac{1}{2} \end{array}\right]$$

2×2 Matrix Determinant
$$\det(A) = |A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Minor M_{ij}: Take the matrix and delete the ith row and the jth column. Find the determinant

Cofactor A_{ij}
$$(-1)^{i+j} M_{ij}$$

$n \times n$ Matrix Determinant (can move along any row/column)
$$\det(A) = |A| = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{vmatrix} = a_{11}A_{11} + a_{12}A_{12} + \cdots + a_{1n}A_{1n}$$

Common Sums
$$\sum_{k=1}^{n} c = nc$$
$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$
$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$
$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

Algebra of Functions

Let f and g be functions with domains A and B.

$(f+g)(x) = f(x) + g(x)$	Domain $A \cap B$
$(f-g)(x) = f(x) - g(x)$	Domain $A \cap B$
$(fg)(x) = f(x)g(x)$	Domain $A \cap B$
$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$	Domain $\{x \in A \cap B \mid g(x) \neq 0\}$
$(f \circ g)(x) = f(g(x))$	Domain $\{x \in B \mid g(x) \in A\}$

Polynomial Synthetic Division

$$(x^3 + x^2 - 1) \div (x+2)$$

$$\begin{array}{r|rrrr} -2 & 1 & 1 & 0 & -1 \\ & & -2 & 2 & -4 \\ \hline & 1 & -1 & 2 & -5 \end{array}$$

Polynomial Long Division

$$\begin{array}{rrrrr} & x^2 & -x & +2 & \\ \hline x+2) & x^3 & +x^2 & & -1 \\ & -x^3 & -2x^2 & & \\ \hline & & -x^2 & & \\ & & x^2 & +2x & \\ \hline & & & 2x & -1 \end{array}$$

Rational Roots Theorem

$$2x^3 + 2x^2 - 3x - 6$$

$\pm1, \pm2 \qquad \pm1, \pm2, \pm3, \pm6$

Possible rational roots:

$$\pm1, \pm\tfrac{1}{2}, \pm2, \pm3, \pm\tfrac{3}{2}, \pm6$$

Decartes' Rule of Signs

Count num. of sign changes

$P(x) = 3x^6 + 4x^5 + 3x^3 - x - 3$

1 positive real root

$P(-x) = 3x^6 - 4x^5 - 3x^3 + x - 3$

1 or 3 negative real roots

Logarithm Formulas

$$\log(m \cdot n) = \log m + \log n$$

$$\log\left(\frac{m}{n}\right) = \log m - \log n$$

$$\log(m^n) = n \cdot \log m$$

$$\log_b b^x = x = b^{\log_b x}$$

Other trig stuff

$$\cot = \frac{1}{\tan}$$

$$\csc = \frac{1}{\sin}$$

$$\sec = \frac{1}{\cos}$$

Horizontal Asymptotes

$y = \dfrac{2x^2 - 4x + 5}{x^2 - 2x + 1}$	Original Equation
$= \dfrac{2x^2}{x^2}$	$x \to \infty$, other terms $\to$ tiny
$= 2$	Cancel, horizontal asymptote

Slant Asymptotes

$y = \dfrac{x^2 - 4x - 5}{x - 3}$	Original Equation
$= x - 1 - \dfrac{8}{x-3}$	Divide
$= x - 1$	$x \to \infty$, other terms $\to$ tiny

Vertical Asymptotes

$y = \dfrac{2x^2 - 4x + 5}{x^2 - 2x + 1}$	Original Equation
$= \dfrac{2x^2 - 4 + 5}{(2x-1)(x+2)}$	Factor demoniator
$x = \dfrac{1}{2}$ or $x = -2$	Impossible

End Behavior

$y = x^n$, n is even

$y = -x^n$, n is even

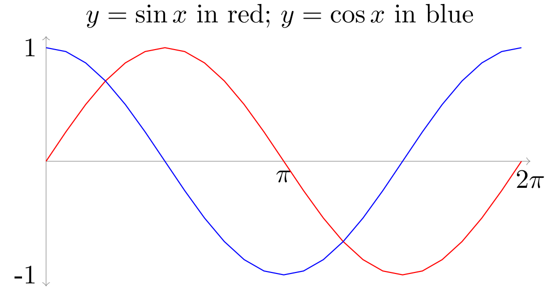
$y = x^n$, n is odd

$y = -x^n$, n is odd

$m \times n$ matrix

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

$y = \sin x$ in red; $y = \cos x$ in blue



sin/cos Graph Properties

If in form:

$$y = a \sin k(x - b)$$

amplitude $|a|$, period $2\pi/k$, phase shift b

Allowed row operations

1. Add a multiple of one row to another
2. Multiply a row by a nonzero constant
3. Interchange two rows

If $\begin{cases} ax + by = r \\ cx + dy = s \end{cases}$ then $x = \dfrac{\begin{vmatrix} r & b \\ s & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$ and $y = \dfrac{\begin{vmatrix} a & r \\ c & s \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$

Parabola

$$x^2 = 4py$$

$V(0,0)$, $F(0,p)$, directrix $y = -p$

Ellipse

$$\frac{x^2}{(a \text{ or } b)^2} + \frac{y^2}{(a \text{ or } b)^2} = 1$$

$$c^2 = a^2 - b^2$$

Eccentricity

$$e = \frac{c}{a},$$

Hyperbola

$$\frac{x^2}{ab^2} - \frac{y^2}{b^2} = 1$$

$$c^2 = a^2 + b^2$$

General Conic

$$\frac{d(P,F)}{d(P,l)} = e$$

Polar Conics

$$r = \frac{ed}{1 \pm e(\cos \text{ or } \sin)\theta}$$

Derivative Formula

$$f^{-1}(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Area

$$A = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k)\Delta x$$

$$\Delta x = \frac{b-a}{n}$$

$$x_k = a + k\Delta x$$