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Arc
                                                                                                                 Difference of Cubes
     All Students Take Calculus
                                                                     Law of Cosines
                                                                                                                                                               Length
                                                               a^{2} = b^{2} + c^{2} - 2bc \cdot \cos A \Big| a^{3} \pm b^{3} = (a \pm b)(a^{2} \mp ab + b^{2})\Big|
 I–All pos. II–sin III–tan IV–cos
                                                                                                                                                                s = r\theta
                                                         Change of Base

\begin{array}{c}
\text{Law of Sines} \\
= \frac{\sin B}{B} - \frac{\sin B}{B}
\end{array}

           Heron's Formula
                                                                                          C(x,y) = \begin{pmatrix} x \\ y \end{pmatrix} = \frac{x!}{y!(x-y)!}
                                                                          \frac{\log m}{\log b}
                                                         \log_b m =
 A = \sqrt{s(s-a)(s-b)(s-c)}
                                                                  Area of \Delta
Area = ab \cdot \frac{1}{2} \sin C
Polar to (x, y)
r^2 = x^2 + y^2
\tan \theta = \frac{y}{2}
 Degrees to Radians
                                        Sector Area
                                          A = \frac{1}{2}r^2\theta
          A \cdot \pi
                                                                                                                                     (\log_a b)(\log_c d) = (\log_a d)(\log_c b)
           180
                                                                                                                  z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2) \\ \frac{z_1}{z_2} = \frac{r_1}{r_1} \operatorname{cis}(\theta_1 - \theta_2) \\ |\vec{v}| = \sqrt{a^2 + b^2}
     z = a + bi
                                     z = r \operatorname{cis} \theta
                                                                                                                                                                                                     |c\vec{u}| = |c||\vec{u}|
 |z| = \sqrt{a^2 + b^2}
                                 z^n = r^n \operatorname{cis}(n\theta)
                                      Dot Product
                                                                     \theta \text{ between } \vec{u} \& \vec{v}
\cos \theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{v}||\vec{v}|}
                                                                                                         \vec{u} and \vec{v} are
                                                                                                                                     Component
    Dot Product
                                                                                                                                                                                                            Work
                                                                                                                                     of \vec{u} along \vec{v}
                                          Theorem
                                                                                                       prependicular
 \vec{u} \cdot \vec{v} = a_1 a_2 + b_1 b_2
                                                                                                                                                                                                        W = \vec{F} \cdot \vec{D}
                                   \vec{u} \cdot \vec{v} = |\vec{u}| |\vec{v}| \cos \theta
                                                                                                                                       (\vec{u}\cdot\vec{v})/|\vec{v}|
                                                                                                            \vec{u} \cdot \vec{v} = 0
                                                                                                Trig Identities
  \sin^2 + \cos^2 = 1 \left| \tan^2 + 1 = \sec^2 \right| \left| 1 + \cot^2 = \csc^2 \right| \left| 2 \sin u \cos u = \sin(2u) \right| \left| \cos^2 u - \sin^2 u = \cos(2u) \right| \left| \frac{2 \tan u}{1 - \tan^2 u} = \tan(2u) \right|
                                                                                                                                               \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v} = \tan(u \pm v)
           \sin u \cos v \pm \cos u \sin v = \sin(u \pm v) || \cos u \cos v \mp \sin u \sin v = \cos(u \pm v) ||
                                                                                                                                                                                               \cot = \frac{1}{\tan}
               \csc = \frac{1}{\sin \left| \left| \sec = \frac{1}{\cos \left| \left| \sin \left( \frac{\pi}{2} - u \right) = \cos u \right| \right| \tan \left( \frac{\pi}{2} - u \right) = \cot u \right| \left| \sec \left( \frac{\pi}{2} - u \right) = \csc u \right| \left| \cos \left( \frac{\pi}{2} - u \right) = \sin u \right|
    \cot\left(\frac{\pi}{2}-u\right) = \tan u \left| \csc\left(\frac{\pi}{2}-u\right) = \sec u \right| \left| \frac{1-\cos 2x}{2} = \sin^2 x \right| \left| \frac{1+\cos 2x}{2} = \cos^2 x \right| \left| \frac{1-\cos 2x}{1+\cos 2x} = \tan^2 x \right| \left| \pm \sqrt{\frac{1-\cos u}{2}} = \sin \frac{u}{2} \right|
       \pm\sqrt{\frac{1+\cos u}{2}} = \cos\frac{u}{2}\left[\frac{1-\cos u}{\sin u} = \frac{\sin u}{1+\cos u} = \tan\frac{u}{2}\right]\left[2\sin\frac{x\pm y}{2}\cos\frac{x\mp y}{2} = \sin x \pm \sin y\right]\left[2\cos\frac{x+y}{2}\cos\frac{x-y}{2} = \cos x + \cos y\right]
        -2\sin\frac{x+y}{2}\sin\frac{x-y}{2} = \cos x - \cos y \left| \sin u \cos v \right| = \frac{1}{2}[\sin(u+v) + \sin(u-v)] \left| \cos u \sin v \right| = \frac{1}{2}[\sin(u+v) - \sin(u-v)]
                                       \cos u \cos v = \frac{1}{2} [\cos(u+v) + \cos(u-v)] |\sin u \sin v = \frac{1}{2} [\cos(u+v) - \cos(u-v)]
                                                  Reduced
 Row-Echelon Form
                                                                                                    Using matrix inverses (AX = B \Rightarrow X = A^{-1}B)
                                         Row-Echelon Form
    1 \ 2 \ -1
                                             1 \quad 0 \quad 0 \quad -3
                          -7
    0 1
                                             0 \ 1 \ 0 \ 1
                                              0 \ 0 \ 1 \ -2
                                                                          Matrix Multiplication!
                                                       \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ then } A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}
      2\times 2 Matrix Determinant
                                                                Minor M_{ij}: Take the matrix and | \lceil
                                                                                                                               Cofactor A_{ij}
                                                                  delete the ith row and the jth
                                           =ad-bc
\det(A) = |A| =
                                                                                                                                (-1)^{i+j}M_{ij}
                                                                  column. Find the determinant
         n \times n Matrix Determinant (can move along any row/column)
                                                                        = a_{11}A_{11} + a_{12}A_{12} + \dots + a_{1n}A_{1n} 
 = \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} 
                                          a_{22}
\det(A) = |A| =
                              a_{m1} a_{m2} ···
                                                              a_{mn}
                                                                                                                                          \sum_{n=1}^{\infty} k^3 = \frac{n^2(n+1)^2}{4}
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Algebra of Functions

Let f and g be functions with domains A and B.

$$(f+g)(x) = f(x) + g(x)$$
 Domain $A \cap B$

$$(f-g)(x) = f(x) - g(x)$$
 Domain $A \cap B$

$$(fg)(x) = f(x)g(x)$$
 Domain $A \cap B$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$
 Domain $\{x \in A \cap B \mid g(x) \neq 0\}$

Polynomial Synthetic Division
$$(x^3 + x^2 - 1) \div (x + 2)$$

$$-2 \begin{vmatrix} 1 & 1 & 0 & -1 \\ & -2 & 2 & -4 \\ & 1 & -1 & 2 & -5 \end{vmatrix}$$

Polynomial Long Division
$$\begin{array}{r}
x^2 - x + 2 \\
x + 2) \overline{\smash{\big)}\ x^3 + x^2 - 1} \\
\underline{-x^3 - 2x^2} \\
-x^2 \\
\underline{-x^2 + 2x} \\
2x - 1
\end{array}$$

Rational Roots Theorem
$$2x^3 + 2x^2 - 3x - 6$$

 $\pm 1, \pm 2$ $\pm 1, \pm 2, \pm 3, \pm 6$
Possible rational roots:

 $\pm 1, \pm \frac{1}{2}, \pm 2, \pm 3, \pm \frac{3}{2}, \pm 6$

Decartes' Rule of Signs
Count num. of sign changes
$$P(x) = 3x^6 + 4x^5 + 3x^3 - x - 3$$
1 positive real root
$$P(-x) = 3x^6 - 4x^5 - 3x^3 + x - 3$$
1 or 3 negative real roots

Logarithm Formulas
$$\log(m \cdot n) = \log m + \log n$$

$$\log\left(\frac{m}{n}\right) = \log m - \log n$$

$$\log(m^n) = n \cdot \log m$$

$$\log_b b^x = x = b^{\log_b x}$$

Other trig stuff
$$\cot = \frac{1}{\tan}$$

$$\csc = \frac{1}{\sin}$$

$$\sec = \frac{1}{\cos}$$

Horizontal Asymptotes

$$y = \frac{2x^2 - 4x + 5}{x^2 - 2x + 1}$$
 Original Equation
 $= \frac{2x^2}{x^2}$ $x \to \infty$, other terms \to tiny
 $= 2$ Cancel, horizontal asymptote

$$y = \frac{x^2 - 4x - 5}{x - 3}$$
 Original Equation

$$= x - 1 - \frac{8}{x - 3}$$
 Divide

$$= x - 1$$
 $x \to \infty$, other terms \to tiny

Slant Asymptotes

$$y = \frac{2x^2 - 4x + 5}{x^2 - 2x + 1}$$
 Original Equation

$$= \frac{2x^2 - 4 + 5}{(2x - 1)(x + 2)}$$
 Factor demoniator

$$x = \frac{1}{2}$$
 or $x = -2$ Impossible

End Behavior
$$y = x^{n} \xrightarrow{f} y = -x^{n} \xrightarrow{f} y$$

$$n \text{ is even} \xrightarrow{f} y = -x^{n} \xrightarrow{f} x$$

$$n \text{ is odd} \xrightarrow{f} n \text{ is odd} \xrightarrow{f} x$$

$$\begin{bmatrix} m \times n \text{ matrix} \\ a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

$y = \sin x$ in red; $y = \cos x$ in blue 2π -1

sin/cos Graph Properties If in form:

 $y = a \sin k(x - b)$ amplitude |a|, period $2\pi/k$, phase shift b

Allowed row operations

- 1. Add a multiple of one row to another
- 2. Multiply a row by a nonzero constant
- 3. Interchange two rows

If
$$\begin{cases} ax + by = r \\ cx + dy = s \end{cases}$$
 then $x = \begin{vmatrix} r & b \\ s & d \\ a & b \\ c & d \end{vmatrix}$ and $y = \begin{vmatrix} a & r \\ c & s \\ a & b \\ c & d \end{vmatrix}$

Parabola

$$x^2 = 4py$$

 $V(0,0), F(0,p),$
directrix $y = -p$

Ellipse
$$\frac{x^2}{(a \text{ or } b)^2} + \frac{y^2}{(a \text{ or } b)^2} = 1$$

$$c^2 = a^2 - b^2$$

Eccentricity

Hyperbola
$$\frac{x^2}{ab^2} - \frac{y^2}{b^2} = 1$$

$$c^2 = a^2 + b^2$$

General Conic
$$\frac{d(P, F)}{d(P, l)} = e$$

$$r = \frac{\text{Polar Conics}}{1 \pm e(\cos \text{ or } \sin)\theta}$$

Derivative Formula
$$f^{-1}(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Area
$$A = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k) \Delta x$$

$$\Delta x = \frac{b-a}{n}$$

$$x_k = a + k\Delta x$$