**Positive Trigonometric Functions**
I–All pos. II–sin III–tan IV–cos

**Law of Cosines**
$a^2 = b^2 + c^2 - 2bc \cdot \cos A$

**Difference of Cubes**
$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$

**Arc Length**
$s = r\theta$

**Heron's Formula**
$A = \sqrt{s(s-a)(s-b)(s-c)}$

**Change of Base**
$\log_b m = \dfrac{\log m}{\log b}$

**Choose Formula**
$C(x, y) = \dbinom{x}{y} = \dfrac{x!}{y!(x-y)!}$

**Law of Sines**
$\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

**Degrees to Radians**
$\dfrac{A \cdot \pi}{180} = \theta$

**Sector Area**
$A = \dfrac{1}{2} r^2 \theta$

**Area of $\Delta$**
$Area = ab \cdot \dfrac{1}{2} \sin C$

**Polar to $(x, y)$**
$r^2 = x^2 + y^2$
$\tan\theta = \dfrac{y}{x}$

$(\log_a b)(\log_c d) = (\log_a d)(\log_c b)$

$z = a + bi$
$|z| = \sqrt{a^2 + b^2}$

$z = r \operatorname{cis}\theta$
$z^n = r^n \operatorname{cis}(n\theta)$

**$n$th roots of $z = r \operatorname{cis}\theta$**
$w_k = r^{1/n} \operatorname{cis}\left(\dfrac{\theta + 2k\pi}{n}\right)$

$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$
$\dfrac{z_1}{z_2} = \dfrac{r_1}{r_1} \operatorname{cis}(\theta_1 - \theta_2)$

$\vec{v} = \langle a, b\rangle = a\hat{\imath} + b\hat{\jmath}$
$|\vec{v}| = \sqrt{a^2 + b^2}$

$|c\vec{u}| = |c||\vec{u}|$

**Dot Product**
$\vec{u} \cdot \vec{v} = a_1 a_2 + b_1 b_2$

**Dot Product Theorem**
$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos\theta$

**$\theta$ between $\vec{u}$ & $\vec{v}$**
$\cos\theta = \dfrac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}$

**$\vec{u}$ and $\vec{v}$ are prependicular**
$\vec{u} \cdot \vec{v} = 0$

**Component of $\vec{u}$ along $\vec{v}$**
$(\vec{u} \cdot \vec{v}) / |\vec{v}|$

$\operatorname{proj}_{\vec{v}} \vec{u} = \left(\dfrac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2}\right) \vec{v}$

**Work**
$W = \vec{F} \cdot \vec{D}$

## Trig Identities

$\sin^2 + \cos^2 = 1$

$\tan^2 + 1 = \sec^2$

$1 + \cot^2 = \csc^2$

$2 \sin u \cos u = \sin(2u)$

$\cos^2 u - \sin^2 u = \cos(2u)$

$\frac{2 \tan u}{1 - \tan^2 u} = \tan(2u)$

$\sin u \cos v \pm \cos u \sin v = \sin(u \pm v)$

$\cos u \cos v \mp \sin u \sin v = \cos(u \pm v)$

$\frac{\tan u \pm \tan v}{1 \mp \tan u \tan v} = \tan(u \pm v)$

$\cot = \frac{1}{\tan}$

$\csc = \frac{1}{\sin}$

$\sec = \frac{1}{\cos}$

$\sin\left(\frac{\pi}{2} - u\right) = \cos u$

$\tan\left(\frac{\pi}{2} - u\right) = \cot u$

$\sec\left(\frac{\pi}{2} - u\right) = \csc u$

$\cos\left(\frac{\pi}{2} - u\right) = \sin u$

$\cot\left(\frac{\pi}{2} - u\right) = \tan u$

$\csc\left(\frac{\pi}{2} - u\right) = \sec u$

$\frac{1 - \cos 2x}{2} = \sin^2 x$

$\frac{1 + \cos 2x}{2} = \cos^2 x$

$\frac{1 - \cos 2x}{1 + \cos 2x} = \tan^2 x$

$\pm\sqrt{\frac{1 - \cos u}{2}} = \sin\frac{u}{2}$

$\pm\sqrt{\frac{1 + \cos u}{2}} = \cos\frac{u}{2}$

$\frac{1 - \cos u}{\sin u} = \frac{\sin u}{1 + \cos u} = \tan\frac{u}{2}$

$2 \sin\frac{x \pm y}{2} \cos\frac{x \mp y}{2} = \sin x \pm \sin y$

$2 \cos\frac{x + y}{2} \cos\frac{x - y}{2} = \cos x + \cos y$

$-2 \sin\frac{x + y}{2} \sin\frac{x - y}{2} = \cos x - \cos y$

$\sin u \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$

$\cos u \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$

$\cos u \cos v = \frac{1}{2}[\cos(u + v) + \cos(u - v)]$

$\sin u \sin v = \frac{1}{2}[\cos(u + v) - \cos(u - v)]$

**Row-Echelon Form**
$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

**Reduced Row-Echelon Form**
$$\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

**Using matrix inverses ($AX = B \Rightarrow X = A^{-1}B$)**
$$\begin{bmatrix} 2 & -5 \\ 3 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ 36 \end{bmatrix} \Longrightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 & \frac{5}{3} \\ -1 & \frac{2}{3} \end{bmatrix} \begin{bmatrix} 15 \\ 36 \end{bmatrix} = \begin{bmatrix} 30 \\ 9 \end{bmatrix}$$

**Matrix Multiplication (columns of first = rows of second)**
$$\begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 5 & 2 \\ 0 & 4 & 7 \end{bmatrix} = \begin{bmatrix} 1 \cdot (-1) + 3 \cdot 0 & 1 \cdot 5 + 3 \cdot 4 & 1 \cdot 2 + 3 \cdot 7 \\ (-1) \cdot (-1) + 0 \cdot 0 & (-1) \cdot 5 + 0 \cdot 4 & (-1) \cdot 2 + 0 \cdot 7 \end{bmatrix} = \begin{bmatrix} -1 & 17 & 23 \\ 1 & -5 & -2 \end{bmatrix}$$

**$2 \times 2$ Matrix Inverse**
If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $A^{-1} = \dfrac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

**$n \times n$ Matrix Inverse**
$$\left[\begin{array}{ccc|ccc} 1 & -2 & -4 & 1 & 0 & 0 \\ 2 & -3 & -6 & 0 & 1 & 0 \\ -3 & 6 & 15 & 0 & 0 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -3 & 2 & 0 \\ 0 & 1 & 0 & -4 & 1 & -\frac{2}{3} \\ 0 & 0 & 1 & 1 & 0 & \frac{1}{2} \end{array}\right]$$

**$2 \times 2$ Matrix Determinant**
$\det(A) = |A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

**Minor $M_{ij}$**: Take the matrix and delete the $i$th row and the $j$th column. Find the determinant

**Cofactor $A_{ij}$**
$(-1)^{i+j} M_{ij}$

**Common Sums**
$$\sum_{k=1}^{n} c = nc$$
$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$
$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$
$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

**$n \times n$ Matrix Determinant (can move along any row/column)**
$$\det(A) = |A| = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{vmatrix} = a_{11} A_{11} + a_{12} A_{12} + \cdots + a_{1n} A_{1n}$$

### Algebra of Functions

Let $f$ and $g$ be functions with domains $A$ and $B$.

| | |
|---|---|
| $(f+g)(x) = f(x) + g(x)$ | Domain $A \cap B$ |
| $(f-g)(x) = f(x) - g(x)$ | Domain $A \cap B$ |
| $(fg)(x) = f(x)g(x)$ | Domain $A \cap B$ |
| $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ | Domain $\{x \in A \cap B \mid g(x) \neq 0\}$ |
| $(f \circ g)(x) = f(g(x))$ | Domain $\{x \in B \mid g(x) \in A\}$ |

### Polynomial Synthetic Division

$(x^3 + x^2 - 1) \div (x+2)$

$$\begin{array}{r|rrrr} -2 & 1 & 1 & 0 & -1 \\ & & -2 & 2 & -4 \\ \hline & 1 & -1 & 2 & -5 \end{array}$$

Result is $x^2 - x + 2 - \frac{5}{x+2}$

### Polynomial Long Division

$$\begin{array}{rl} & x^2 \quad - x + 2 \\ x+2) & x^3 + x^2 \qquad\quad - 1 \\ & -x^3 - 2x^2 \\ \hline & \qquad -x^2 \\ & \qquad x^2 + 2x \\ \hline & \qquad\qquad 2x - 1 \end{array}$$

### Rational Roots Theorem

$2x^3 + 2x^2 - 3x - 6$

$\pm 1, \pm 2 \qquad \pm 1, \pm 2, \pm 3, \pm 6$

Possible rational roots:

$\pm 1, \pm \frac{1}{2}, \pm 2, \pm 3, \pm \frac{3}{2}, \pm 6$

### Decartes' Rule of Signs

Count num. of sign changes

$P(x) = 3x^6 + 4x^5 + 3x^3 - x - 3$

1 positive real root

$P(-x) = 3x^6 - 4x^5 - 3x^3 + x - 3$

1 or 3 negative real roots

### Logarithm Formulas

$\log(m \cdot n) = \log m + \log n$

$\log\left(\frac{m}{n}\right) = \log m - \log n$

$\log(m^n) = n \cdot \log m$

$\log_b b^x = x = b^{\log_b x}$

### Trigonometric Reciprocals

$\cot = \frac{1}{\tan}$

$\csc = \frac{1}{\sin}$

$\sec = \frac{1}{\cos}$

### Horizontal Asymptotes

| | |
|---|---|
| $y = \frac{2x^2 - 4x + 5}{x^2 - 2x + 1}$ | Original Equation |
| $= \frac{2x^2}{x^2}$ | $x \to \infty$, other terms $\to$ tiny |
| $= 2$ | Cancel, horizontal asymptote |

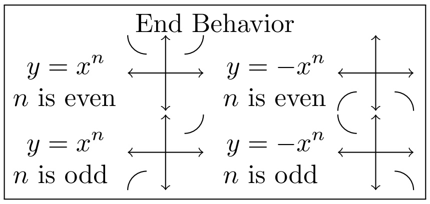
### Slant Asymptotes

| | |
|---|---|
| $y = \frac{x^2 - 4x - 5}{x - 3}$ | Original Equation |
| $= x - 1 - \frac{8}{x-3}$ | Divide |
| $= x - 1$ | $x \to \infty$, other terms $\to$ tiny |

### Vertical Asymptotes

| | |
|---|---|
| $y = \frac{2x^2 - 4x + 5}{x^2 - 2x + 1}$ | Original Equation |
| $= \frac{2x^2 - 4 + 5}{(2x-1)(x+2)}$ | Factor demoniator |
| $x = \frac{1}{2}$ or $x = -2$ | Impossible |

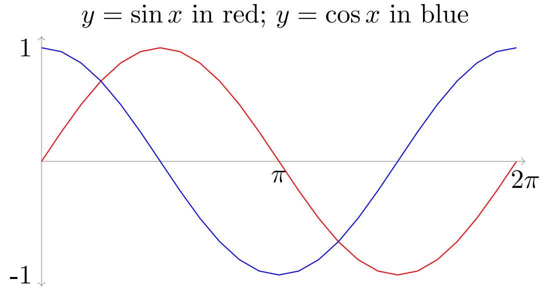
### End Behavior




### $m \times n$ matrix

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

$y = \sin x$ in red; $y = \cos x$ in blue




### sin/cos Graph Properties

If in form:

$y = a \sin k(x - b)$

amplitude $|a|$, period $2\pi/k$, phase shift $b$

$\sin^{-1}, \tan^{-1}$: $R[-\pi/2, \pi/2]$,

$\cos^{-1}$: $R[0, \pi]$,

$D[-1, 1]$ except $\tan^{-1}$: $D[-\infty, \infty]$

### Allowed row operations

1. Add a multiple of one row to another
2. Multiply a row by a nonzero constant
3. Interchange two rows

If $\begin{cases} ax + by = r \\ cx + dy = s \end{cases}$, then $x = \dfrac{\begin{vmatrix} r & b \\ s & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$ and $y = \dfrac{\begin{vmatrix} a & r \\ c & s \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$

### Vertical Parabola

$x^2 = 4py$

$V(0,0)$, $F(0,p)$,

directrix $y = -p$

### Ellipse

$\frac{x^2}{(a \text{ or } b)^2} + \frac{y^2}{(a \text{ or } b)^2} = 1$

$c^2 = a^2 - b^2$

### Eccentricity

$e = \frac{c}{a}$,

### Hyperbola

$\frac{x \text{ or } y^2}{a^2} - \frac{x \text{ or } y^2}{b^2} = 1$

$c^2 = a^2 + b^2$

### Shifted Conic

$V(h,k)$, $x$ to $(x-h)$, $y$ to $(y-k)$

### Polar Conics

$r = \frac{ed}{1 \pm e(\cos \text{ or } \sin)\theta}$

### Derivative Formula

$f^{-1}(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

### Area

$A = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k)\Delta x$

$\Delta x = \frac{b-a}{n}$

$x_k = a + k\Delta x$

### Horizontal Parabola

$y^2 = 4px$

$V(0,0)$, $F(p,0)$,

directrix $x = -p$

### Parabolas

Latus rectum is $|4p|$

### Ellipses

$a^2 > b^2$

$x^2$ first of terms means more horizontal, major axis length is $2a$, minor axis length is $2b$, latus rectum is $\frac{2b^2}{a}$, foci on major axis $F(\pm c, 0)$ or $F(0, \pm c)$

**Hyperbolas** $a^2$ forms positive term with $x$ or $y$, horizontal when $x^2$ is first of terms, $V(\pm a, 0)$ or $V(0, \pm a)$, $B(0, \pm b)$ or $B(\pm b, 0)$, transverse axis length is $2a$, conjugate axis length is $2b$, asymtote slopes $\pm \frac{b}{a}$ or $\pm \frac{a}{b}$, foci on transverse axis $F(\pm c, 0)$ or $F(0, \pm c)$, latus rectum is $\frac{2b^2}{a}$