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Positive Trigonometric
                                                                                                                                                 Arc
                                                                                                    Difference of Cubes
                                                              Law of Cosines
                  Functions
                                                                                                                                              Length
                                                                                               a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)
                                                        a^2 = b^2 + c^2 - 2bc \cdot \cos A
 I-All pos. II-sin III-tan IV-cos
                                                                                                                                               s = r\theta
                                                   Change of Base
                                                                                          Choose Formula
                                                                                                                                           Heron's Formula
                                                   \log_b m = \frac{\log m}{\cdot}
                                                                                                                                   \sin A
                                                                                                                                                              \sin C
                                                                                 C(x,y) =
 A = \sqrt{s(s-a)(s-b)(s-c)}
                                                                   \log b
 Degrees to Radians
                                    Sector Area
                                                                 Area of \Delta
                                      A = \frac{1}{2}r^2\theta
                                                           Area = ab \cdot \frac{1}{2}\sin C
         A \cdot \pi
                                                                                                                       (\log_a b)(\log_c d) = (\log_a d)(\log_c b)
          180
                                                           nth roots of z = r \operatorname{cis} \theta
                                                                                                      z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2) 
 \frac{z_1}{\theta_1} = \frac{r_1}{\theta_2} \operatorname{cis}(\theta_1 - \theta_2)
                                                                                                                                              \vec{v} = \langle a, b \rangle = a\hat{\imath} + b\hat{\jmath}
                                 z = r \operatorname{cis} \theta
     z = a + bi
                                                                                                                                                                               |c\vec{u}| = |c||\vec{u}|
 |z| = \sqrt{a^2 + b^2}
                                                                                                                                                |\vec{v}| = \sqrt{a^2 + b^2}
                             z^n = r^n \operatorname{cis}(n\theta)
                                                              \theta between \vec{u} \cdot \vec{v} \vec{v}
                                  Dot Product
                                                                                                                       Component
                                                                                              \vec{u} and \vec{v} are
    Dot Product
                                                                                                                                                                                      Work
                                     Theorem
                                                                                                                      of \vec{u} along \vec{v}
                                                                                            prependicular
                                                                \cos \theta =
 \vec{u} \cdot \vec{v} = a_1 a_2 + b_1 b_2
                                                                                                                                                                                  W = \vec{F} \cdot \vec{D}
                               \vec{u} \cdot \vec{v} = |\vec{u}| |\vec{v}| \cos \theta
                                                                                                \vec{u}\cdot\vec{v}=0
                                                                                                                        (\vec{u}\cdot\vec{v})/|\vec{v}|
                                                                                      Trig Identities
  \sin^2 + \cos^2 = 1 || \tan^2 + 1 = \sec^2 || 1 + \cot^2 = \csc^2 || 2 \sin u \cos u = \sin(2u) ||
                                                                                                                        \cos^2 u - \sin^2 u = \cos(2u) \frac{2 \tan u}{1 - \tan^2 u} = \tan(2u)
         \sin u \cos v \pm \cos u \sin v = \sin(u \pm v) || \cos u \cos v \mp \sin u \sin v = \cos(u \pm v)|
                                                \left|\sin\left(\frac{\pi}{2}-u\right)=\cos u\right|\left|\tan\left(\frac{\pi}{2}-u\right)=\cot u\right|\left|\sec\left(\frac{\pi}{2}-u\right)=\csc u\right|
                                                                       \frac{1-\cos 2x}{2} = \sin^2 x \left| \frac{1+\cos 2x}{2} = \cos^2 x \right|
    \cot\left(\frac{\pi}{2}-u\right) = \tan u \mid \csc\left(\frac{\pi}{2}-u\right) = \sec u
                                        \frac{1-\cos u}{\sin u} = \frac{\sin u}{1+\cos u} = \tan \frac{u}{2} \left| 2\sin \frac{x\pm y}{2}\cos \frac{x\mp y}{2} = \sin x \pm \sin y \right| 2\cos \frac{x+y}{2}\cos \frac{x-y}{2} = \cos x + \cos y
       \pm \sqrt{\frac{1+\cos u}{2}} = \cos \frac{u}{2}
        -2\sin\frac{x+y}{2}\sin\frac{x-y}{2} = \cos x - \cos y \sin u \cos v = \frac{1}{2}[\sin(u+v) + \sin(u-v)] \cos u \sin v = \frac{1}{2}[\sin(u+v) - \sin(u-v)]
                                  \cos u \cos v = \frac{1}{2} [\cos(u+v) + \cos(u-v)] |\sin u \sin v = \frac{1}{2} [\cos(u+v) - \cos(u-v)]
                                             Reduced
 Row-Echelon Form
                                                                                          Using matrix inverses (AX = B \Rightarrow X = A^{-1}B)
                                     Row-Echelon Form
   1 2
              -1
                         1
                                         1 \quad 0 \quad 0 \quad -3
                                                                           2 -5
   0 1
                 4
                      -7
                                         0 \ 1 \ 0
                                                         1
                                                                           3 -6
                      -2
                                         0 \ 0 \ 1 \ -2
                                                                  Matrix Multiplication!
                                                 \begin{bmatrix} 1 \cdot (-1) + 3 \cdot 0 & 1 \cdot 5 + 3 \cdot 4 & 1 \cdot 2 + 3 \cdot 7 \\ (-1) \cdot (-1) + 0 \cdot 0 & (-1) \cdot 5 + 0 \cdot 4 & (-1) \cdot 2 + 0 \cdot 7 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} 
                                                                                                                      n \times n Matrix Inverse
                                                                                                 -3 -6
                                                                                                                   0 \ 1 \ 0
                                                                                                                                                0 \ 1 \ 0
                                                                                                                                                                  -4 1
                                                                                                    6
                                                                                                                                                0 \ 0 \ 1
                                                                                                                                                                    1 0
     2 \times 2 Matrix Determinant
                                                         Minor M_{ij}: Take the matrix and
                                                                                                                 Cofactor A_{ij}
                                                           delete the ith row and the jth
\det(A) = |A| =
                                                                                                                  (-1)^{i+j}M_{ij}
                                       =ad-bc
                                                           column. Find the determinant
                                                                                                                                  Common Sums
        n \times n Matrix Determinant (can move along any row/column)
                                      a_{12}
                                      a_{22}
                                                                 = a_{11}A_{11} + a_{12}A_{12} + \dots + a_{1n}A_{1n}
\det(A) = |A| =
                                                                                                                        \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}
                                     a_{m2}
                                                       a_{mn}
                                                                                                                            \sum_{n=0}^{n} k^3 = \frac{n^2(n+1)^2}{4}
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Algebra of Functions

Let f and g be functions with domains A and B.

$$(f+g)(x) = f(x) + g(x)$$
 Domain $A \cap B$

$$(f-g)(x) = f(x) - g(x)$$
 Domain $A \cap B$

$$(fg)(x) = f(x)g(x)$$
 Domain $A \cap B$

Domain
$$\{x \in A \cap B \mid g(x) \neq 0\}$$

Domain
$$\{x \in B \mid g(x) \in A\}$$

Polynomial Synthetic Division $(x^3 + x^2 - 1) \div (x + 2)$

Polynomial Long Division
$$\begin{array}{rrr}
x^2 - x + 2 \\
x + 2) \overline{\smash) x^3 + x^2} & -1 \\
\underline{-x^3 - 2x^2} \\
-x^2 \\
\underline{-x^2 + 2x}
\end{array}$$

Rational Roots Theorem $2x^3 + 2x^2 - 3x - 6$

$$2x^{3} + 2x^{2} - 3x - 6$$

 $\pm 1, \pm 2, \pm 3, \pm 6$

Possible rational roots:

$$\pm 1, \pm \tfrac{1}{2}, \pm 2, \pm 3, \pm \tfrac{3}{2}, \pm 6$$

Decartes' Rule of Signs Count num. of sign changes $P(x) = 3x^6 + 4x^5 + 3x^3 - x - 3$ 1 positive real root $P(-x) = 3x^6 - 4x^5 - 3x^3 + x - 3$ 1 or 3 negative real roots

Logarithm Formulas $\log(m \cdot n) = \log m + \log n$

$$\log\left(\frac{m}{n}\right) = \log m - \log n$$

$$\log(m^n) = n \cdot \log m$$

$$\log_b b^x = x = b^{\log_b x}$$

Other trig stuff
$$\cot = \frac{1}{\tan}$$

$$\csc = \frac{1}{\sin}$$

$$\sec = \frac{1}{\cos}$$

Horizontal Asymptotes

$$y = \frac{2x^2 - 4x + 5}{x^2 - 2x + 1}$$

Original Equation

$$=\frac{2x^2}{x^2}$$

 $x \to \infty$, other terms $\to \text{tiny}$

$$= 2$$

Cancel, horizontal asymptote

$$y = \frac{2x^2 - 4x + 5}{x^2 - 2x + 1}$$
 Original Equation

$$=\frac{2x^2-4+5}{\sqrt{2}}$$
 Factor demoniator

$$x = \frac{1}{2}$$
 or $x = -2$ Impossible

Slant Asymptotes

$$y = \frac{x^2 - 4x - 5}{x - 3}$$
 Original Equation

$$= x - 1 - \frac{8}{x - 3} \quad \text{Divide}$$

$$x \to \infty$$
, other terms $\to \text{tiny}$

Vertical Asymptotes

$$y = \frac{2x^2 - 4x + 5}{2}$$

$$=\frac{2x^2-4+5}{(2x-1)(x+2)}$$

$$x = \frac{1}{2}$$
 or $x = -2$ Impossible

End Behavior

$$y = x^n \longleftrightarrow y = -x^n \leftarrow n \text{ is even}$$

$$n \text{ is even} \longleftrightarrow n \text{ is even} \longleftrightarrow n \text{ is even}$$

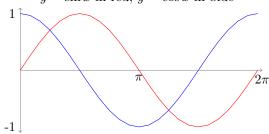
$$\begin{vmatrix} y = x^n & & & \\ n \text{ is odd} & & & \\ & & & \\ \end{vmatrix}$$

$$y = -x^n \leftarrow$$

$$n \text{ is odd}$$

$$a_{21}$$
 a_{22} \cdots a_{2n}

 $y = \sin x$ in red; $y = \cos x$ in blue



sin/cos Graph Properties If in form:

 $y = a \sin k(x - b)$ amplitude |a|, period $2\pi/k$, phase shift b

Allowed row operations

- 1. Add a multiple of one row to another
- 2. Multiply a row by a nonzero constant
- 3. Interchange two rows

If
$$\begin{cases} ax + by = r \\ cx + dy = s \end{cases}$$
 then $x = \begin{vmatrix} r & b \\ s & d \end{vmatrix}$ and $y = \begin{vmatrix} a & r \\ c & s \end{vmatrix}$

Parabola

$$x^2 = 4py$$

 $V(0,0), F(0,p),$
directrix $y = -p$

Ellipse
$$\frac{x^2}{(a \text{ or } b)^2} + \frac{y^2}{(a \text{ or } b)^2} = 1$$

$$c^2 = a^2 - b^2$$

Eccentricity
$$e = \frac{c}{a}$$
,

Hyperbola
$$\frac{x^2}{ab^2} - \frac{y^2}{b^2} = 1$$

$$c^2 - a^2 + b^2$$

General
Conic
$$\frac{d(P, F)}{d(P, I)} = e$$

$$r = \frac{\text{Polar Conics}}{1 \pm e(\cos \text{ or } \sin)\theta}$$

Derivative Formula
$$f^{-1}(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Area
$$A = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k) \Delta x$$

$$\Delta x = \frac{b-a}{n}$$

$$x_k = a + k\Delta x$$