

# Principle of Virtual Work

In statics, we are usually concerned with eqs. of rigid bodies.

we are using

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum M = 0$$

& Lami's theorem.

to get

Reaction, Tension etc.  $\gamma$

Method of <sup>Principle</sup> virtual work

is another method to

find out such Reaction, Tension etc of a eqs system of rigid bodies

## Virtual Work

$$\text{Work} = F \times s \times \cos \theta$$

$\nearrow$  distance / displacement

$$\text{Virtual work} = F \times \delta s$$

$\nearrow$  virtual displacement

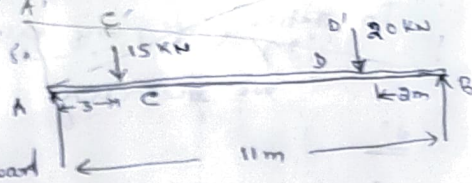
(configuration)

If,  $(\sum \text{Virtual work} = 0)$ , then system is eqs.

Ex-1: Using the method of virtual work, determine the reaction at support A & B of the transversely loaded beam.

sol

Hold the beam at B & push AB at A upward



$$\therefore \delta A = \text{displacement of A} = \Delta A'$$

$$\delta C = \delta C'$$

$$\delta D = \delta D'$$

$$\delta B = 0$$

Applying virtual work in the configuration

$$R_A(\delta A) + 15(\delta C) + 20(\delta D) + R_B \times 0 = 0$$

↓ due to displacement is opposite to Force.

From  $\Delta A'B$

$$\frac{\delta A}{11} = \frac{\delta D}{2} = \frac{\delta C}{8}$$

$$\Rightarrow R_A \times \frac{11(\delta D)}{2} - 15\left(\frac{8}{2}\right)\delta D - 20(\delta D) = 0$$

$$\Rightarrow R_A = 14.55 \text{ kN}$$

(Ans)

Similarly to determine  $R_B = ?$

Keeping Hddy 'A', Push B to B'

Applying virtual work in the configuration

$$R_B(\delta B_2) + 20(\delta D_2) + 15(\delta C_2) = 0$$

$$\left[ \frac{\delta C_2}{3} = \frac{\delta D_2}{11} = \frac{\delta B_2}{9} \right]$$

$$\Rightarrow R_B \times \frac{11}{3}(\delta C_2) - 20 \times 3(\delta C_2) - 15 \times \delta C_2 = 0$$

$$\Rightarrow R_B = 20.45$$

(Ans)

To cross-check use method of moments

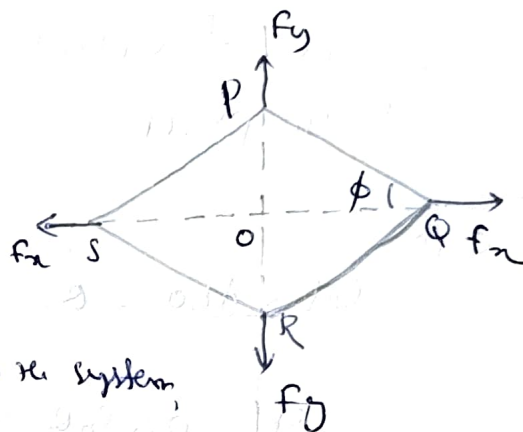
Ex. 2 Four weightless bars of length  $a$  are hinged together at their junctions and form the shape of rhombus, Applying virtual work, Find the relation between  $F_x$  &  $F_y$

Sol<sup>n</sup>

From the geometry,

$$OP = OR = a \sin \phi$$

$$OQ = OS = a \cos \phi$$



Allowing differential, virtual displacement to the system,

$$\delta(OP) = a \cos \phi \cdot \delta \phi$$

$$\delta(OQ) = -a \sin \phi \cdot \delta \phi$$

Applying Principle of virtual work to the system,

$$2F_y \delta(OP) + 2F_x \delta(OQ) = 0$$

$$\Rightarrow 2F_y \times a \cos \phi \cdot (\delta \phi) + 2F_x (-a \sin \phi) \cdot \delta \phi = 0$$

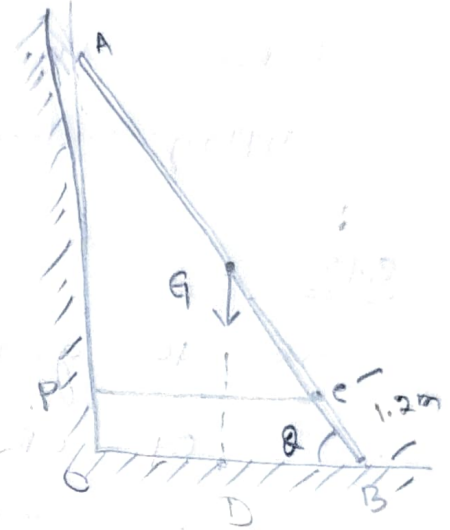
$$\Rightarrow \tan \phi = \frac{F_y}{F_x}$$

### Exp-3

Ladder  $AB = 4.4 \text{ m}$

wt of ladder  $= 250 \text{ N}$

Rope PC is tied with the wall, using  
Principle of virtual work determine the tension  
of rope (PC)



Soln  
Let  $\angle ABO = \theta$

$$DG \therefore y_G = BG \sin \theta = \frac{4.4}{2} \sin \theta = 2.2 \sin \theta$$

$$PC = x_C = AC \cos \theta = 3.2 \cos \theta$$

Applying differential displacements,

$$\delta y_G = 2.2 \cos \theta (\delta \theta)$$

$$\delta x_C = -3.2 \sin \theta (\delta \theta)$$

Now apply Principle of virtual work

$$250 (\delta y_G) + F_{PC} (\delta x_C) = 0$$

$$\Rightarrow 250 (2.2 \cos \theta) - F_{PC} \times 3.2 \sin \theta (\delta \theta) = 0$$

$$\Rightarrow F_{PC} = 171.87 \cot \theta$$

when  $\theta = 65^\circ$

$$F_{PC} = 80.15 \text{ N}$$

Ans