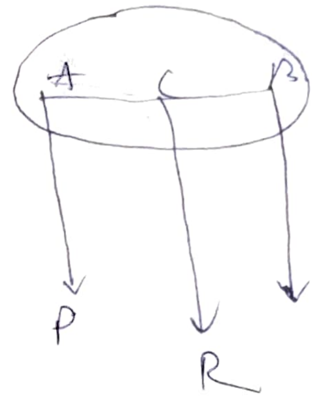
Parallel forces in a plane

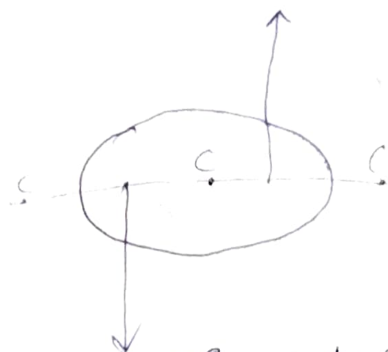
$$\frac{AC}{BC} = \frac{Q}{P}$$



Magnitude of resultant

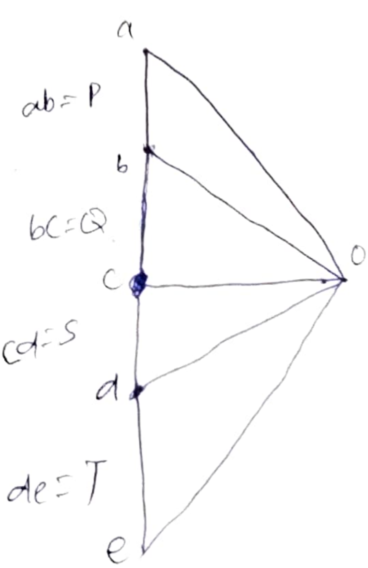
$R = P + Q$

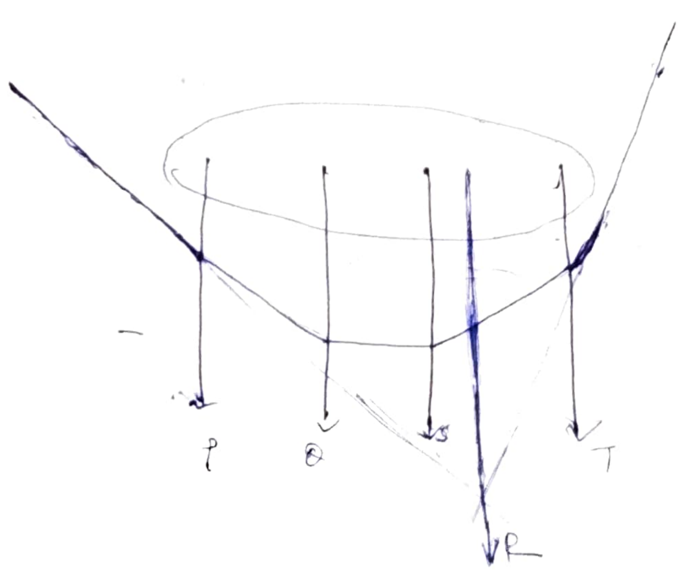
$$\frac{AC}{BC} = \frac{Q'}{P}$$



C can be present

Graphical method





Centre of gravity — Centre of mass.

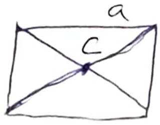
Central point : Centroid, CG, CM

Central point
- Centroid (For length, area & volume)
- CM, CG (For mass & weight)

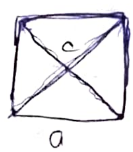
Centroid

① Circle

② Rectangle



③ square

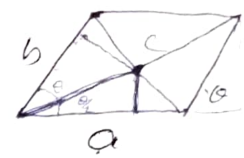


$\cos\frac{\theta}{2} = \frac{a}{b/2}$

$\Rightarrow \frac{b}{2} = \frac{a}{\cos}$

$a = \frac{b}{2}\cos\frac{\theta}{2}$

④ Parallelogram



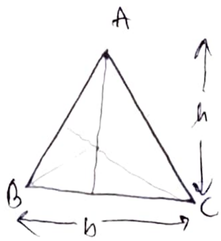
$X_c = \frac{a + b\cos\theta}{2}$

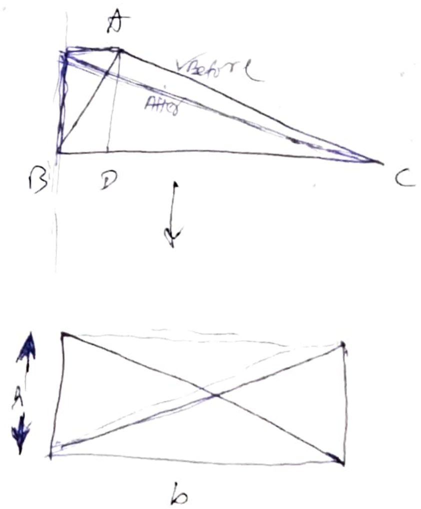
$Y_c = \frac{b\sin\theta}{2}$

⑤ Triangle.

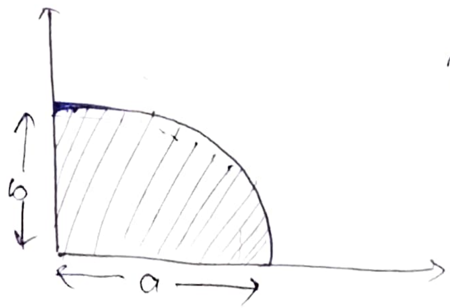
$\bar{y} = y_c = \frac{h}{3}$

$\bar{x} = \frac{1}{3}(b+a)$





⑥ Quarter ellipse



$X_c = \bar{x} = \frac{4a}{3\pi}$

$Y_c = \frac{4b}{3\pi}$

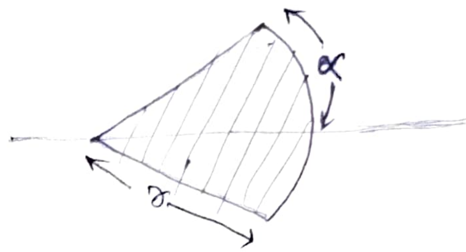
Derivation is required.

(7) Trapezium

$$\bar{y} = \frac{h}{3} \frac{(b+2a)}{(b+a)}$$

(8) Circular section

$$\bar{x} = \frac{2r\sin\alpha}{3\alpha}$$ → Derivation is required.



Virtual work

Workdone $= f\,s\cos\theta$

[θ = Angle between direc^n of the force and displacement vector]

Virtual work $= F \times \delta s$

Virtual work → Imaginary work with very small displacement

If Σ Virtual work $= 0$
then ⇒ System is in eq^m.

(δs = Virtual displacement)

$\Sigma f_x = 0$
$\Sigma f_y = 0$
$\Sigma M = 0$.
we use the above formula for finding tension, x & y for static eq^m.

C.G.

For continuous element, the centroid

$$x_c = \bar{x} = \frac{\int x\,dL}{\int dL}, \qquad y_c = \bar{y} = \frac{\int y\,dL}{\int dL} \quad \Big\}\text{ For 1 D element}$$

$$x_c = \bar{x} = \frac{\int x\,dA}{\int dA}, \qquad y_c = \bar{y} = \frac{\int y\,dA}{\int dA} \quad \Big\}\text{ for 2-D element}$$

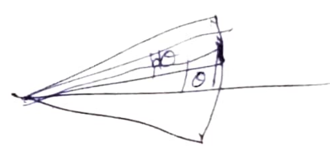
where,

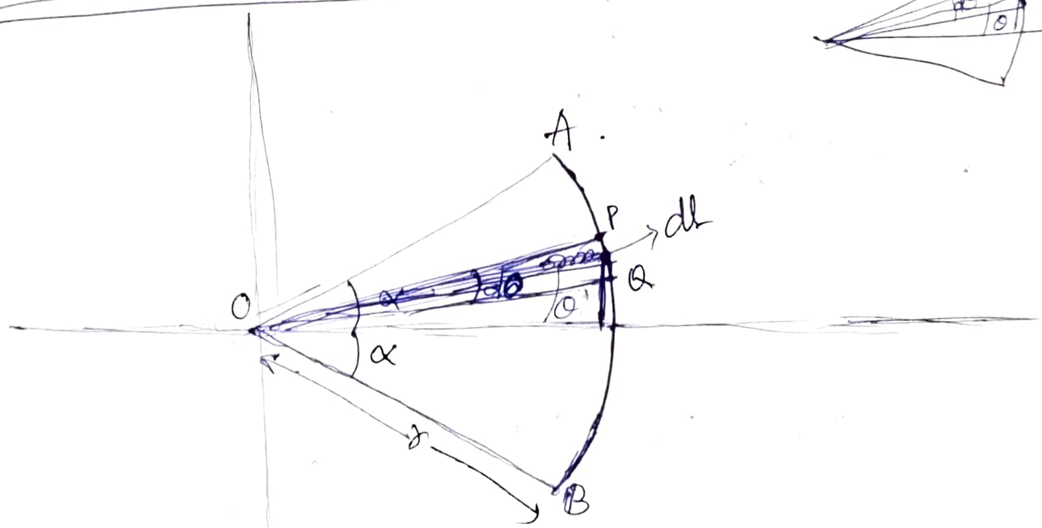
dL = Length of very small elementary section, considered for finding out the centroid.

dA = Area " " " " " " " "

y, x = distance of centroid of such elementary section from reference axis.

Centroid of circular arc





$$dL = r.d\theta$$

$$x = r\cos\theta$$

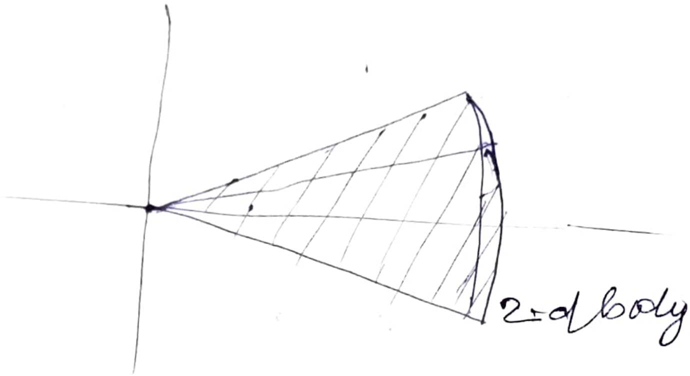
$$\bar{x} = \frac{\int_{-\alpha}^{\alpha} r\cos\theta . r\,d\theta}{\int_{-\alpha}^{\alpha} r\,d\theta} = \frac{\int_{-\alpha}^{\alpha} r^2\cos\theta\,d\theta}{\int_{-\alpha}^{\alpha} r\,d\theta} = \frac{2r^2\sin\alpha}{2r\alpha}$$

$$= \frac{r\sin\alpha}{\alpha}$$

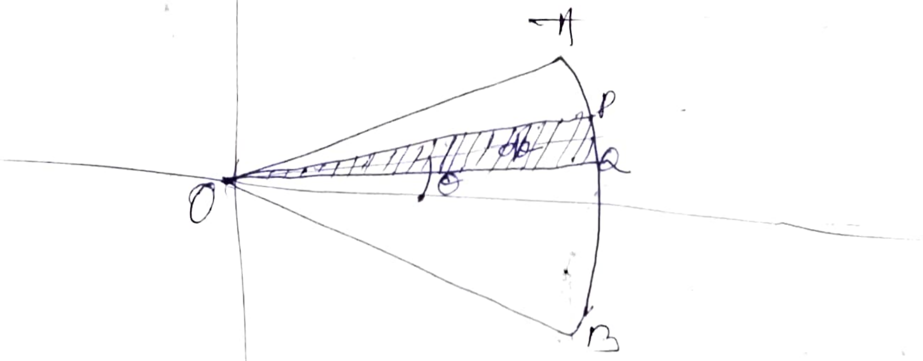
For semicircular arc

$$\bar{x} = \frac{r \sin \frac{\pi}{2}}{\pi/2} = \frac{2r}{\pi}$$

Centroid of circular section.



πr^2



$$dA = \frac{1}{2} \times r \, d\theta \times r$$

$$x = \frac{2}{3} r \cos\theta$$

Distance from centroid of Δ shape to vertical line

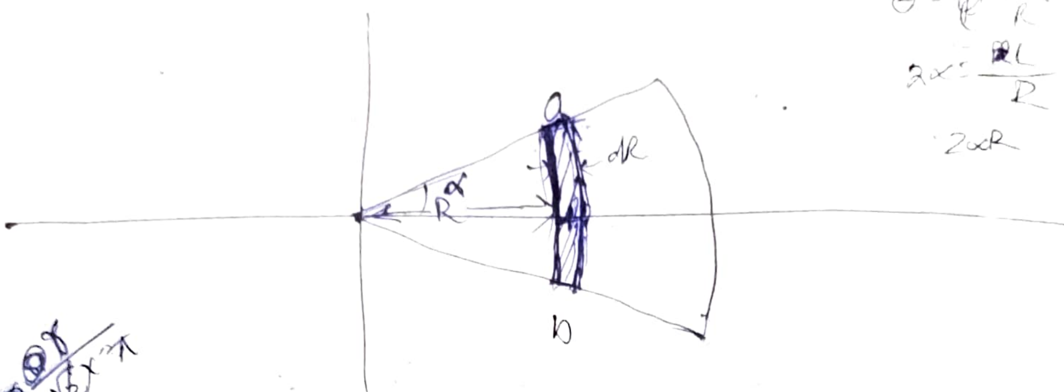
$$\bar{x} = \frac{\int_{-\alpha}^{\alpha} \frac{2}{3} r \cos\theta \, \frac{1}{2} \times r^2 \, d\theta}{\int_{-\alpha}^{\alpha} \frac{1}{2} r^2 \, d\theta} = \frac{\frac{1}{3}\int_{-\alpha}^{\alpha} r^3 \cos\theta \, d\theta}{\frac{1}{2}\int_{-\alpha}^{\alpha} r^2 \, d\theta}$$

$$= \frac{\frac{2}{3} r^3 \sin\alpha}{\frac{2}{2} r^2 \alpha} = \frac{\frac{2}{3} r \sin\alpha}{\alpha}$$

For a semicircle

$$\bar{x} = \frac{2}{3} r \frac{\sin \pi/2}{\pi/2}$$

$$= \frac{4r}{3\pi}$$



$2\alpha = \frac{L}{R}$

$2\alpha R$

$\frac{4\sqrt{2}}{3} \times \frac{r}{\sqrt{2} \times \pi}$

$$dA = 2\alpha R dR$$

$$x = \frac{R \sin\alpha}{\alpha}$$

$$\bar{x} = \frac{\int x \, dA}{\int dA} = \frac{\int \frac{R\sin\alpha}{\alpha} 2\alpha R dR}{\int 2\alpha R dR}$$

$$= \frac{\frac{2}{3} R^3 \sin\alpha}{\alpha R^2} = \frac{2R\sin\alpha}{3\alpha}$$

HW ~~IMP~~ (EXAM)

Find the centroid of a quarter ellipse.

$\frac{2r}{3}$

$\frac{4r}{3\pi}$

For a semicircle

$$\bar{x} = \frac{2}{3} r \frac{\sin \pi/2}{\pi/2}$$

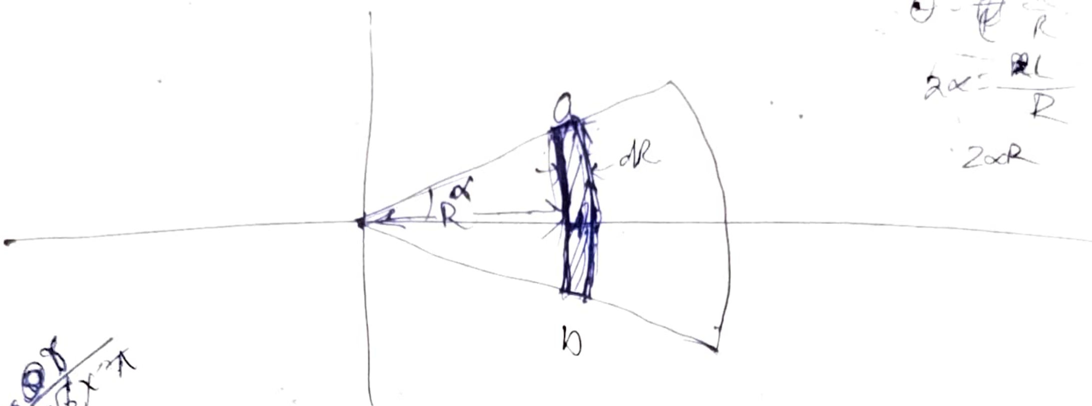
$$= \frac{4r}{3\pi}$$

$\theta = \frac{2\pi r}{r}$

$\theta = \frac{l}{R}$

$2\alpha = \frac{l}{R}$

$2\alpha R$



$\frac{4\sqrt{2}}{3} \times \frac{r}{\sqrt{2} \times \pi}$

$$dA = 2\alpha R dR$$

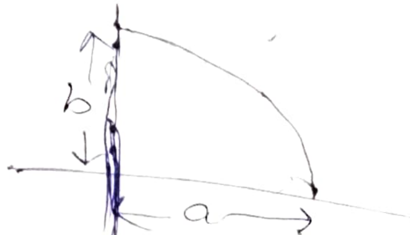
$$x = \frac{R \sin\alpha}{\alpha}$$

$$\bar{x} = \frac{\int x\, dA}{\int dA} = \frac{\int \frac{R\sin\alpha}{\alpha} 2\alpha R dR}{\int 2\alpha R dR}$$

$$= \frac{\frac{2}{3} R^3 \sin\alpha}{\alpha R^2} = \frac{2R\sin\alpha}{3\alpha}$$

IMP (Exam)

HW Find the centroid of a quarter ellipse.



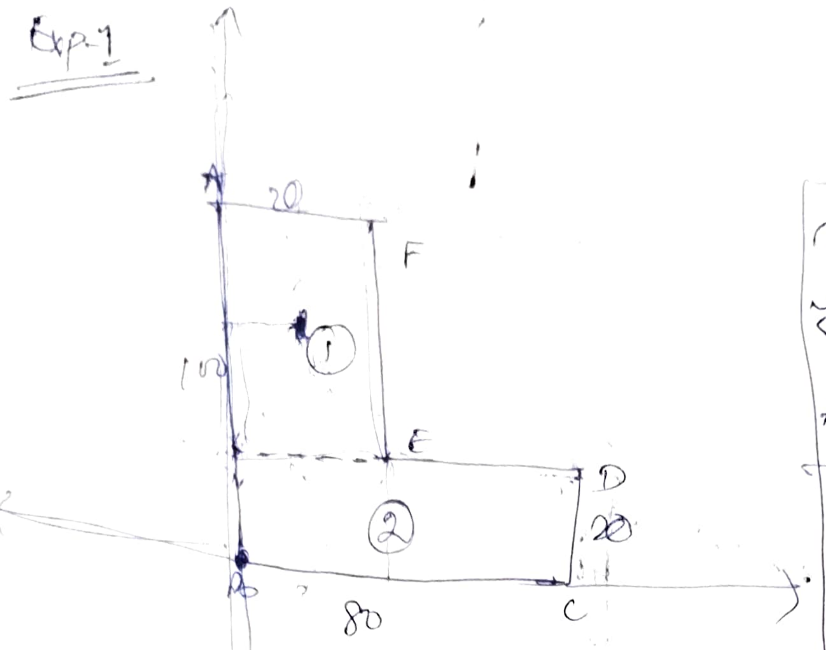
$\frac{2r}{3}$

$\frac{4r}{3\pi}$

Centroid of composite figure

Dabuly

Exp-1



$x_1 = \frac{20}{2} = 10$

$y_1 = 20 + \left(\frac{80}{2}\right) = 60$

$A_1 = 20 \times 80 = 1600$

$x_2 = 40$

$y_2 = 10$

$A_2 = 1600$

Steps

(1) Whether it is symmetry or not. (About X/Y-axis.)

(2) Splitting the whole section into number of known figures.

(3) Reference axis selection (Prefer left most & buttom most axis)

(4) x_1, y_1, A_1

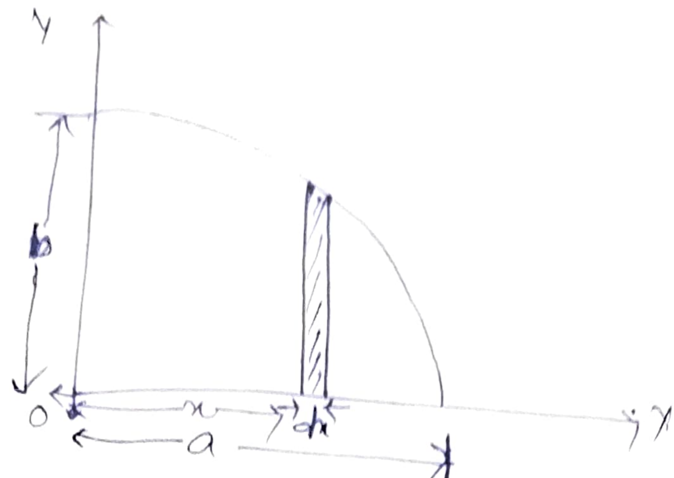
$$\bar{x} = x_c = \frac{\Sigma A_i x_i}{\Sigma A_i} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2} = \frac{16000 + 64000}{3200}$$

$$= \frac{\cancel{8000}}{\cancel{3200}} = 25$$

$$\bar{y} = y_c = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{96000 + 16000}{3200} = \frac{\cancel{112000}}{\cancel{3200}}$$

$$= \frac{\cancel{1120}}{\cancel{32}} = 35$$

Q. Find the centroid of a quarter ellipse



We have eqn of ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\Rightarrow b^2x^2+a^2y^2=a^2b^2$$

$$\Rightarrow a^2y^2=a^2b^2-b^2x^2$$

$$\Rightarrow y^2=\frac{b^2}{a^2}(a^2-x^2)$$

$$\Rightarrow y=\frac{b}{a}\sqrt{a^2-x^2}$$

$$dA=y\,dx$$

$$=\frac{b}{a}\sqrt{a^2-x^2}\,dx$$

$$A=\frac{1}{4}\pi ab$$

$$\bar{x}=\frac{\int_0^a x\cdot\frac{b}{a}\sqrt{a^2-x^2}\,dx}{A}$$

$$\Rightarrow A\bar{x}=\frac{b}{a}\int_0^a x\sqrt{a^2-x^2}\,dx$$

Let

$-x^2=t$

$\Rightarrow -2x\,dx=dt$

$\Rightarrow x\,dx=-\frac{1}{2}dt$

$$\Rightarrow \frac{1}{4}\pi ab\bar{x}=\frac{-b}{2a}\int_0^a (a^2-x^2)^{1/2}\,-2x\,dx$$

$$\Rightarrow \frac{1}{4}\pi ab\bar{x}=\frac{-b}{2a}\left[\frac{(a^2-x^2)^{3/2}}{3/2}\right]_0^a$$

$$\Rightarrow \frac{1}{4}\pi ab\bar{x} = \frac{-b}{3a}\left[(a^2-a^2)^{3/2} - (a^2-0^2)^{3/2}\right]$$

$$\Rightarrow \frac{1}{4}\pi ab\bar{x} = \frac{-b}{3a} \times (-a^3)$$

$$\Rightarrow \bar{x} = \frac{+b}{3a} \times \frac{4a^2}{\pi ab}$$

$$\Rightarrow \bar{x} = \frac{4a}{3\pi}$$

$$\bar{y} = \frac{\int_0^a \frac{1}{2}y \cdot y\,dx}{\frac{1}{4}\pi ab}$$

$$= \frac{\frac{1}{2}\int_0^a y^2\,dx}{\frac{1}{4}\pi ab}$$

$$= \frac{\frac{1}{2}\int_0^a \frac{b^2}{a^2}(a^2-x^2)\,dx}{\frac{1}{4}\pi ab}$$

$$= \frac{\frac{b^2}{2a^2}\int_0^a (a^2-x^2)\,dx}{\frac{1}{4}\pi ab}$$

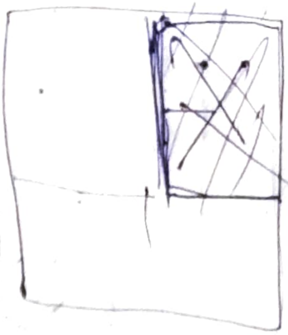
$$= \frac{\frac{b^2}{2a^2}\left[a^2x - \frac{1}{3}x^3\right]_0^a}{\frac{1}{4}\pi ab}$$

$$= \frac{\frac{b^2}{2a^2}\left[\left(a^3-\frac{1}{3}a^3\right) - \left(0^3 - \frac{1}{3}\cdot 0^3\right)\right]}{\frac{1}{4}\pi ab}$$

$$= \frac{\frac{b^2}{2a^2} \times \frac{2}{3}a^3}{\frac{1}{4}\pi ab}$$

$$\Rightarrow \bar{y} = \frac{4b}{3\pi} \quad \underline{\text{Ans}}$$

or



$$\bar{x} = \frac{A_1 x_1 - A_2 x_2}{A_1 - A_2}$$

$$= \frac{(8000 \times 40) - (4800 \times 50)}{3200}$$

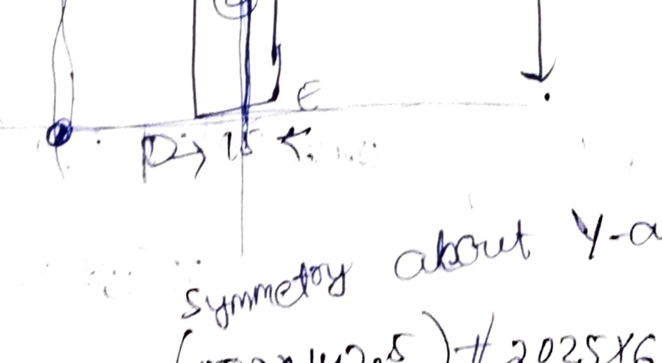
$$= \frac{320000 - 240000}{3200}$$

$$= \frac{80000}{3200} = 25$$

$$\bar{y} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2} = 35$$

Ex

Find the centroid of T section.

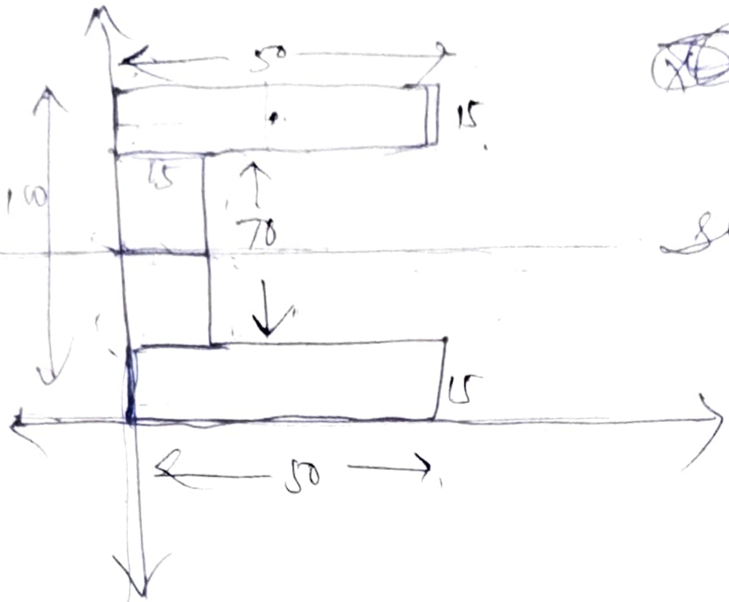


Symmetry about Y-axis.

1500 × 142.5

$$\bar{y} = \frac{(1500 \times 142.5) + (2025 \times 67.5)}{1500 + 2025}$$

$= 99.41$



Symmetry about x-axis

750
$+525$

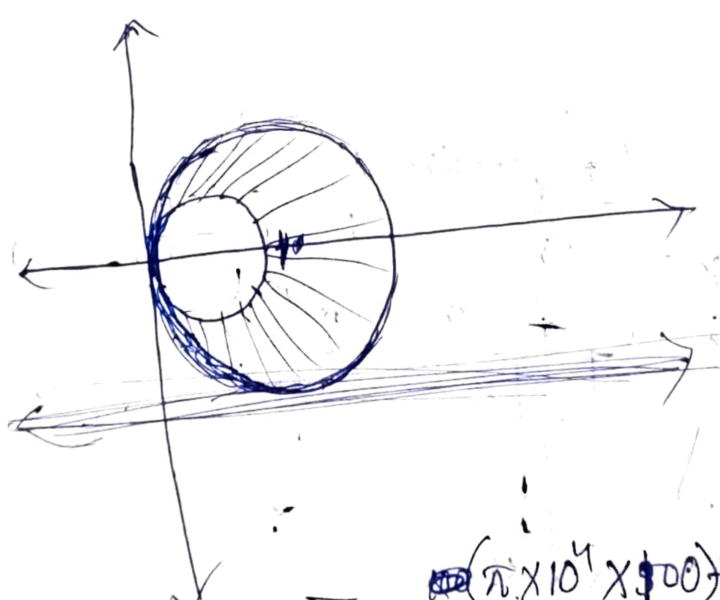
35×15

1275

$$\bar{x} = \frac{(750\times25)+(7.5\times1050)+(750\times25)}{750+1050+750}$$

$= 17.79$

Ex

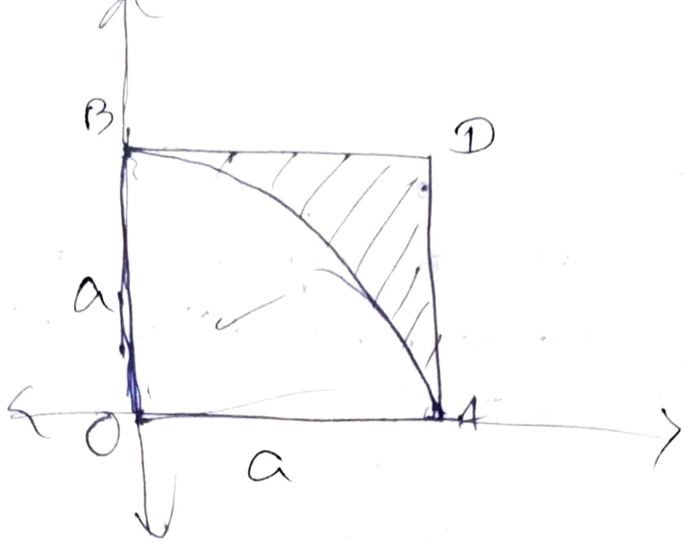


$$\bar{x} = \frac{(\pi\times10^4\times100)-(\pi\times1600\times20)}{\pi(10000-1600)}$$

$$= \frac{\cancel{\pi}\times(500000-32000)}{\cancel{\pi}(10000-1600)}$$

$= 111.428$

Ex



$$A_2 = a^2 - \frac{\pi a^2}{4}$$

$$\cancel{\bar{x} = \frac{\frac{a^3}{2} - a^2\left(1-\frac{\pi}{4}\right)\frac{4a^3}{3\pi}}{a^2 - a^2 + \frac{\pi a^2}{4}} = a^3\left(\frac{1}{2}\right)}$$

$$\bar{X} = \frac{a^3}{2} - \frac{\pi a^2}{4} \times \frac{4a}{3\pi}$$

$$= \frac{\frac{a^3}{2} - \frac{a^3}{3}}{a^2 - \frac{\pi a^2}{4}}$$

$$= \frac{a^3\left(\frac{1}{2}-\frac{1}{3}\right)}{a^2\left(1-\frac{\pi}{4}\right)}$$

$$= 0.776a$$

$$\bar{y} = 0.776a$$

$$\frac{r\sin\alpha}{\alpha}$$

$$= \frac{r\sin\frac{\pi}{2}}{}$$

Pappus theorem

① The area of surface generated by rotating any plane curve about non-intersecting axis in it's plane is equal to the product of length of curve (L) & distance travelled by its centroid.

② The volume of a solid, generated by rotating any plane area/figure about non-intersecting axis in it's plane is equal to the product of area of figure and distance travelled by its centroid.

$$\left(\frac{\pi R^2}{2}\right) \times \left(2 \times \pi \times \frac{4R}{3\pi}\right) \qquad R_x = \frac{4R}{3\pi}$$

$$= \frac{4}{3}\pi R^3$$

$$\pi R \times \left(2\pi \times \frac{2R}{\pi}\right)$$

$$4\pi R^2$$

Moment of Inertia (MI)

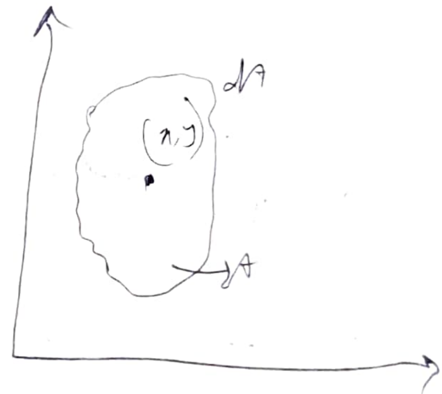
The concept, which gives the quantitative estimate of relative distribution of area or mass of a body w.r.t a reference axis, is termed as MI of the body.

$F = m \cdot a$

$T = I \cdot \alpha$
↳ Mass moment of inertia $= MK^2$ — K = Radius of gyration.

Area moment of Inertia $= Ax^2$

MI of an area of plane figure w.r.t a reference axis.



Area of small figure considered $= dA$

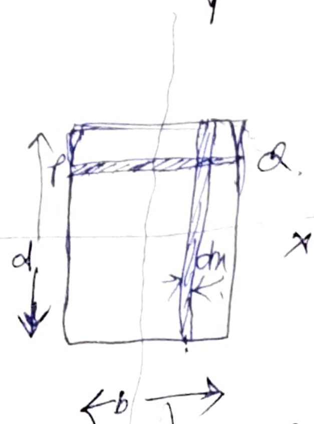
Moment of " " " " " $= dA \cdot x$
w.r.t (y-axis)

M.I. of such area $= dA \cdot x \times x$
(Second moment) $= dA \cdot x^2$

M.I of whole area $= \int dA \cdot x^2$

M.I. of whole area w.r.t. X axis $= \int dA \, y^2$ (I_{XX})

MI of Rectangular section



$dA = b\,dy$

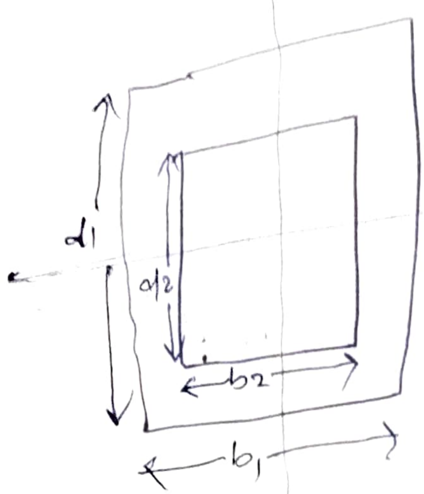
MI about x-axis $= \int dA \, y^2$

$= \int b y^2 \, dy$

$I_{XX} = \frac{b}{3}\left[y^3\right]_{-d/2}^{d/2} = \left[\frac{by^3}{3}\right]_{-d/2}^{d/2} = \frac{b}{3}\left(\frac{d^3}{8} + \frac{d^3}{8}\right) = \frac{bd^3}{12}$

$$I_{yy} = \frac{db^3}{12}$$

MI of hollow rectangular sol^n



$$I_{xx} = \frac{b_1 d_1^3 - b_2 d_2^3}{12}$$

$$I_{yy} = \frac{d_1 b_1^3 - d_2 b_2^3}{12}$$

Polar MI

$I_{xx}, I_{yy} \rightarrow$ Rectangular MI

I_{zz} : Polar MI
or J

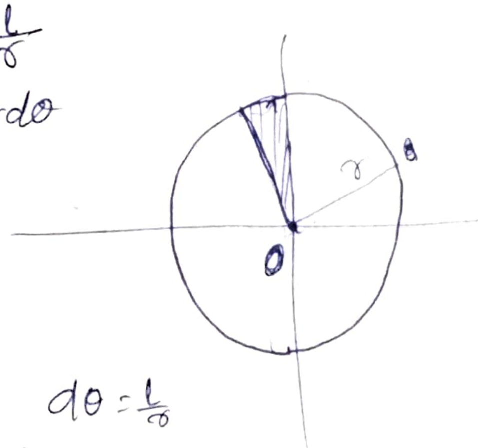
$$J = I_{zz} = \int r^2 dA = \int (x^2 + y^2)\, dA = \int x^2 dA + \int y^2 . dA$$

$$= I_{xx} + I_{yy}$$

MI of circular section

$d\theta = \frac{l}{r}$

$l = r d\theta$



$$dA = \frac{1}{2} r^2 d\theta$$

$$I_{xx} = \int dA \; r^2$$

$$= \int \frac{1}{2} r^2 d\theta . \times r^2$$

$$= \int \frac{1}{2} r^4 d\theta$$

$d\theta = \frac{l}{r}$

MI of circular section

$dA = 2\pi k\,dk$

$I_{zz} = \int r^2 dA$

$= \int_0^r 2\pi k\,dk \cdot k^2$

$= \frac{\pi}{2} r^4 = \frac{\pi}{32} d^4$

I_{zz}

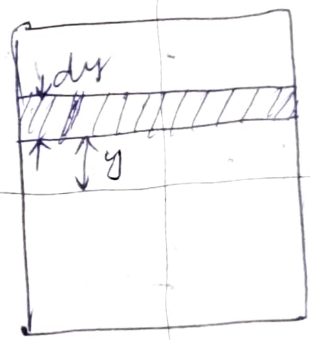
$I_{zz} = I_{xx} + I_{yy} = 2I_{xx} = 2I_{yy}$ $\quad I_{xx} = I_{yy} = \frac{\pi r^4}{4}$

For hollow circular section

$\frac{1}{64}(d_1^4 - d_2^4)$

Parallel axis theorem

H.W, derivn



$I_{AB} = I_G + A \cdot h^2$

where,

I_{AB} = MI of area

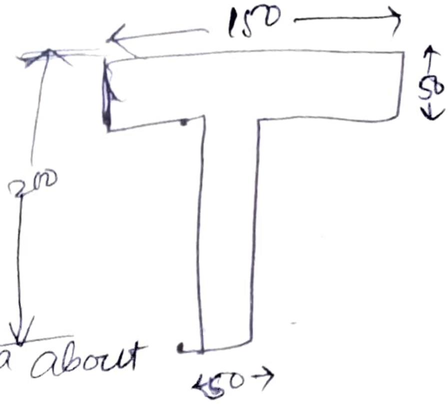
I_G = " " " its centroid $= I_{xx} = \frac{bd^3}{12}$

h =

#

M.I of T-section about it's centroid axis

Steps



① Splitting the whole section into familiar figures

③ Find out its centroid.

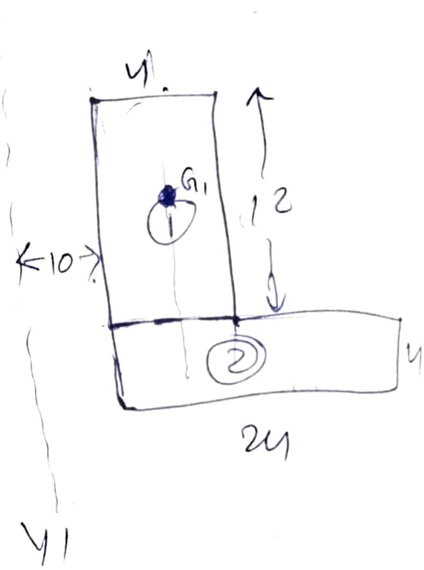
② Find out the MI of those area about their centroid.

④ Transform M.I. of the splitted section about the required axis (using parallel axis theorem).

⑤ Algebraic summation of these MI gives MI of whole section

Ex

Find the centroid of whole section about Y-Y' axis



Rectangle 1

$$I_{GY1} = \frac{12\times4^3}{12} = 64\ cm^4$$

$$I_{AB} = I_G + Ah^2$$

$$I_{YY1} = 64\ cm^4 + 48\times(12)^2$$
$$= 6976\ cm^4$$

For rectangle-2

$$I_{GY2} = \frac{4\times24^3}{12} = \frac{4\times24\times24^2}{12}$$
$$= 4608\ cm^4$$

$$I_{YY2} = 4608 + (96)\times22^2$$

~~6144 cm4~~ $= 51072\ cm^4$

$MI = I_{yy1} + I_{yy2}$

$= 6976 + 51072$

$= 58048$

Find centroid of whole section about base

$I_{yy1} = 64 + 48 \times 10^2 = 4864 \ cm^4$

$I_{yy2} = 4608 + 96 \times 24$

$= 4992 \ cm^4$

$I_{AB} =$

$I_{GX1} = \frac{4 \times 12 \times 12 \times 12}{12} = 576 \ cm^4$

$I_{yy1} = 576 + 100 \times 48$

$= 5376$

$I_{GX2} = \frac{24 \times 4 \times 4 \times 4}{12} = 128 \ cm^4$

$I_{yy2} = 128 + 96 \times 4 = 512 \ cm^4.$

$I_{AB} = I_{yy1} + I_{yy2} = 5376 + 512$

$= 5888 \ cm^4$

$MI = I_{yy1} + I_{yy2}$

$= 6976 + 51072$

$= 58048$

Find centroid of whole section about base

$I_{yy1} = 64 + 48 \times 10^2 = 4864 \text{ cm}^4$

$I_{yy2} = 4608 + 96 \times 4$

$= 4992 \text{ cm}^4$

$I_{AB} =$

$I_{Gx_1} = \frac{4 \times \cancel{12} \times 12 \times 12}{\cancel{12}} = 576 \text{ cm}^4$

$I_{yy1} = 576 + 100 \times 48$

$= 5376$

$I_{Gx2} = \frac{\cancel{24}^{2} \times 4 \times 4 \times 4}{\cancel{12}} = 128 \text{ cm}^4$

$I_{yy2} = 128 + 96 \times 4 = 512 \text{ cm}^4.$

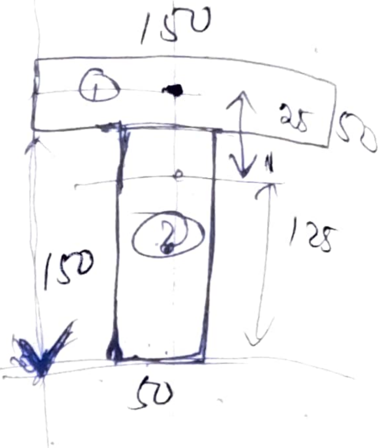
$I_{AB} = I_{yy1} + I_{yy2} = 5376 + 512$

$= 5888 \text{ cm}^4$

$$\bar{x} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2}$$

Q

MI of T.



$$I_{GY1} = \frac{50 \times 150^3}{12}$$

$$= 14062500 \text{ cm}^4$$

$$I_{GY2} = \frac{150 \times 50^3}{12}$$

$$= 1562500 \text{ cm}^4$$

$$\bar{Y} = \frac{(150 \times 50 \times 175) + (150 \times 50 \times 75)}{15000}$$

$$= 125$$

$$\bar{X} = \frac{(7500 \times 75) + (7500 \times 75)}{15000}$$

$$= 75$$

About y-axis

$$I_{yy1} = I_{GY1} + Ah^2$$

$$= 14062500 + (7500 \times 50^2) = 32812500$$

$$= \cancel{14062500} \quad \cancel{1144.37500}$$

$$I_{yy2} = 1562500 + (7500 \times 50^2)$$

$$= 20312500$$

About AB: $I_{AB} = 32812500 + 20312500$

$$= 53125000 \text{ cm}^4$$

14062500
1562500

10 m/sec

3600 → 3600 × 10

= 36000 / 1000

63.64

$\bar{y} = 125$ form base

$$I_{x1} = \frac{150 \times 50^3}{12} = 1.5625 \times 10^6 \text{ mm}^4$$

$$I_{G1} = I_{x1} + A_1 h_1^2$$

$$=$$

$$I_{x2} = \frac{50 \times 150^3}{12}$$

$$I_{G2} = I_{x2} + A_2 h_2^2$$

$$= 14062500$$

$$I_{xx} = I_{G1} + I_{G1}$$

$$= 53.125 \times 10^6 \text{ mm}^4$$

$$I_{yy} = \frac{50 \times 150^3}{12}$$

$$I_{G1} = $$