DATA STRUCTURES

LECTURE-3

ALGORITHM ANALYSIS

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Rate of Growth

The rate at which the running time increases as a function of input is called rate of growth.

Example: $n^4 + 2n^2 + 100n + 500$

In the case below, n^4, $2n^2$, $100n$ and 500 are the individual costs of some function and approximate to n^4 since n^4 is the highest rate of growth.

i.e. $n^4 + 2n^2 + 100n + 500 \approx n^4$

Commonly Used Rates of Growth

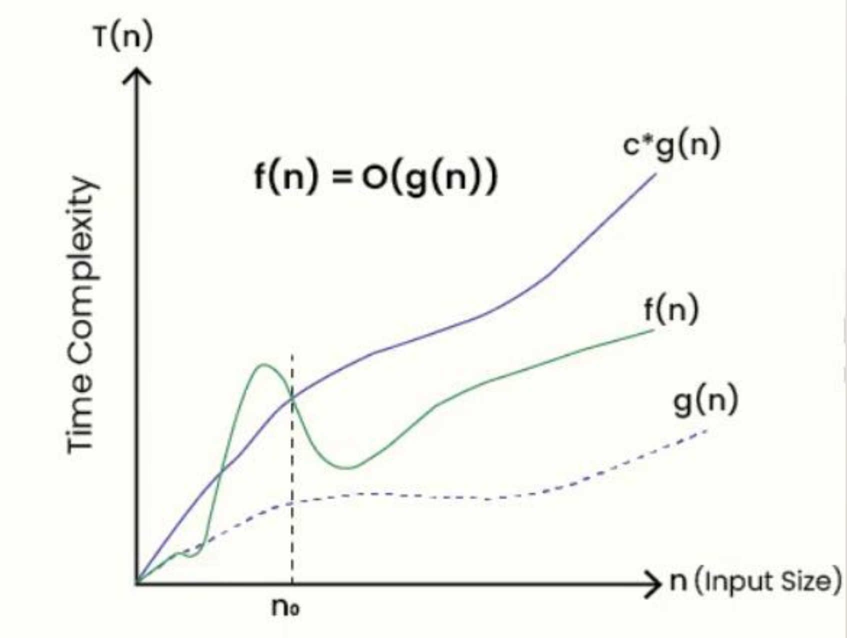
Time Complexity	Name	Example
1	Constant	Adding an element to the front of a linked list
$logn$	Logarithmic	Finding an element in a sorted array
n	Linear	Finding an element in an unsorted array
$nlogn$	Linear Logarithmic	Sorting n items by 'divide-and-conquer' - Mergesort
n^2	Quadratic	Shortest path between two nodes in a graph
n^3	Cubic	Matrix Multiplication
2^n	Exponential	The Towers of Hanoi problem

Big-O Visualization:

f(n) is **O(g(n))** if there exist constants **c > 0** and $\mathbf{n_0}$ >= 0 such that

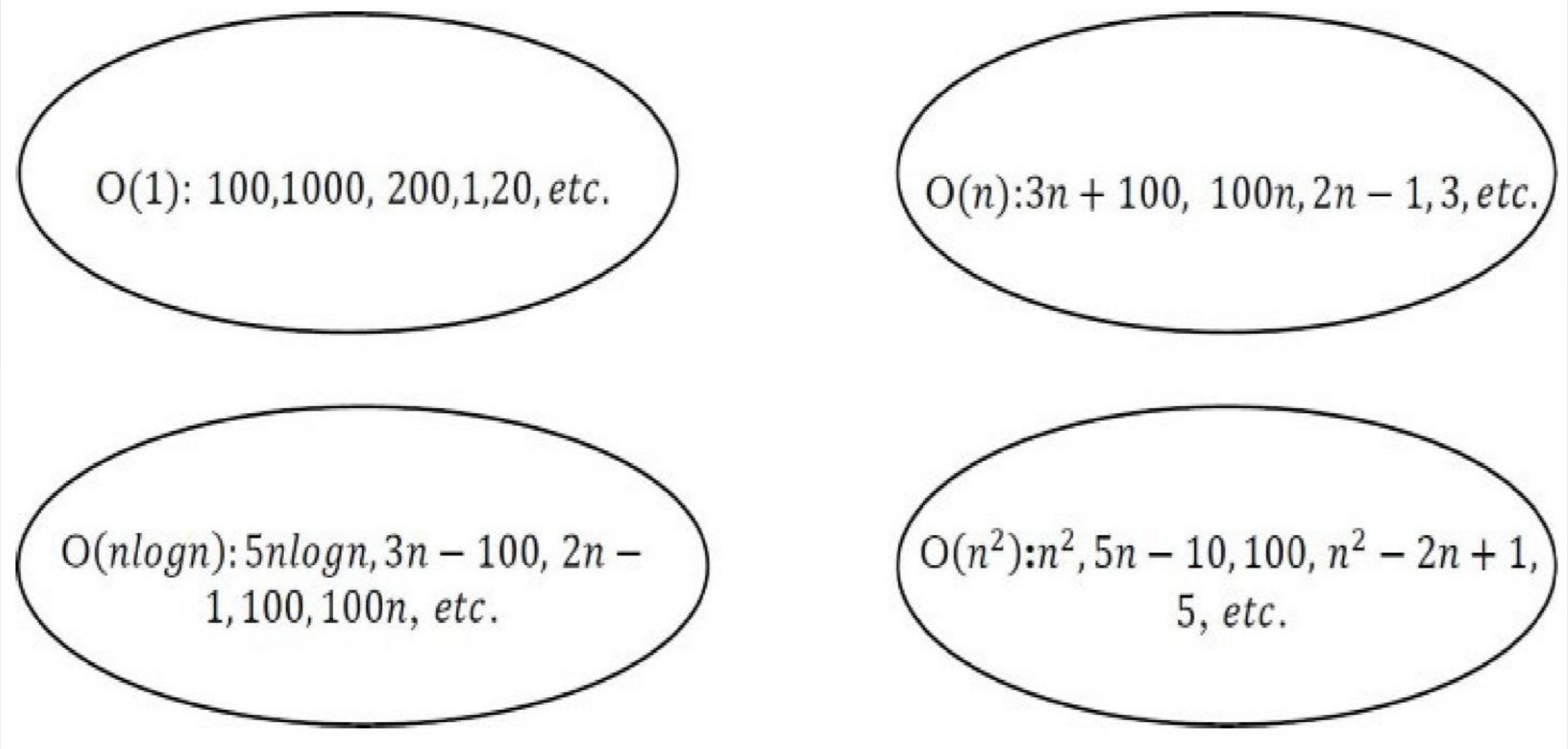
$$\mathbf{f(n) <= c*g(n)} \text{ for all } \mathbf{n >= n_0}.$$

- g(n) is an asymptotic upper bound for f(n).
- Objective is to give the smallest rate of growth g(n) which is greater than or equal to the given algorithms' rate of growth f(n).



In the figure, n0 is the point from which we need to consider the rate of growth for a given algorithm. Below n0, the rate of growth could be different. n0 is called threshold for the given function.

$O(g(n))$ is the set of functions with smaller or the same order of growth as $g(n)$. For example; $O(n^2)$ includes $O(1)$, $O(n)$, $O(nlogn)$, etc.



Find upper bound for $f(n) = 3n + 8$?

There is no unique set of values for n0 and c in proving the asymptotic bounds.

Let us consider, $100n + 5 = O(n)$.
For this function there are multiple $n0$ and c values possible.

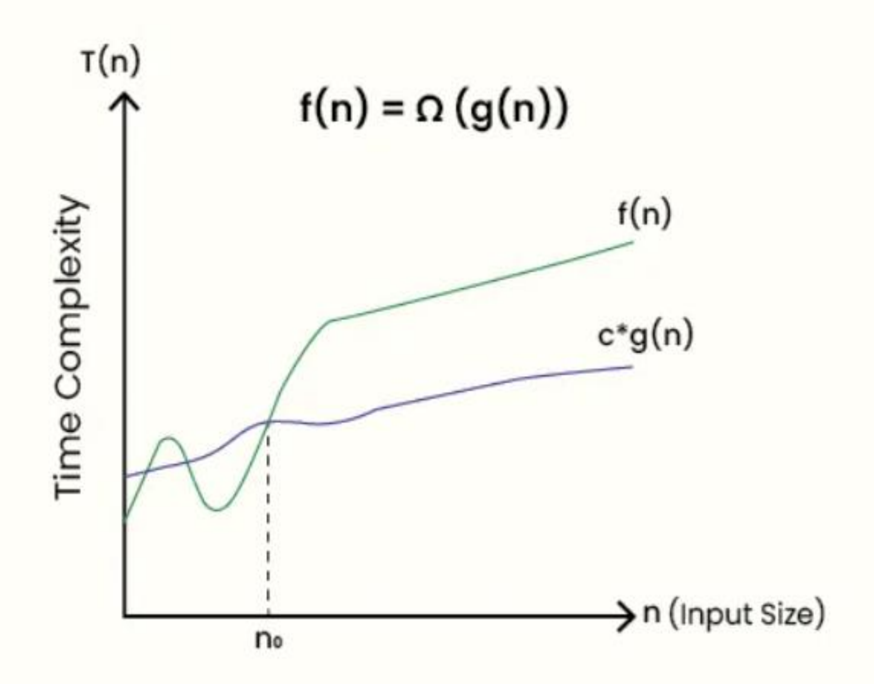
Solution1: $100n + 5 \le 100n + n = 101n \le 101n$, for all $n \ge 5$, $n0 = 5$ and $c = 101$ is a solution.

Solution2: $100n + 5 \le 100n + 5n = 105n \le 105n$, for all $n > 1$, $n0 = 1$ and $c = 105$ is also a solution.

Big-Omega Ω Notation: Lower Bounding Function

$f(n) = \Omega(g(n))$, if there exists constants **$c > 0$** and **n_0** **>=** 0 such that

$$f(n) >= c*g(n) \text{ for all } n >= n_0.$$



if $f(n) = 100n^2 + 10n + 50$, $g(n)$ is $\Omega(n^2)$.

Find lower bound for $f(n) = 5n^2$.