Extraneous Attribute

X: set of attributes
$A \subseteq X$

Def^n: The functional dependency $X \rightarrow Y$ contains an extraneous attribute A in X if $\{X - A\}^+$ contains Y. (We can remove the attribute without changing the closure of the set of FDs) $\Rightarrow$ Y can be determined from X even after removal of extraneous attribute A from X.

Example

Suppose the set of FDs F is given by the following:

$$F = \{ AB \rightarrow D,\ A \rightarrow C,\ B \rightarrow D,\ CD \rightarrow E \}$$

$AB \rightarrow D$

$$(\{AB\} - \{A\})^+ = B^+ = BD$$

$\Rightarrow$ A is extraneous.

$$\boxed{AB \rightarrow D \quad \text{becomes} \quad B \rightarrow D}$$

$CD \rightarrow E$

$$(\{CD\} - \{C\})^+ = D^+ = D$$

C is not extraneous.

$$(\{CD\} - \{D\})^+ = C^+ = C$$

D is not extraneous.

So, the FD becomes (after removal of extraneous attribute)

$$F = \{ B \rightarrow D,\ A \rightarrow C,\ CD \rightarrow E \}$$

Redundant ~~Attribute~~ FD

The FD $X \to Y$ is redundant in F if $(F - \{X \to Y\})^+ = F^+$ or X^+ contains Y. after removal of $X \to Y$

Example

$F = \{A \to B,\ B \to C,\ A \to C\}$

Test $A \to B$

$(F - \{A \to B\})^* = (\{B \to C,\ A \to C\})^*$

Now, we find, $A^+ = AC$

So, we dont get B in A^+ $(B \notin A^+)$.

$\Rightarrow$ $A \to B$ is not redundant.

Test $B \to C$

$F - \{B \to C\} = \{A \to B,\ A \to C\}$

Now, we find, $B^+ = B$

$\therefore C \notin B^+$

$\Rightarrow$ $B \to C$ is not redundant

Test $A \to C$

$F - \{A \to C\} = \{A \to B,\ B \to C\}$

Now, we find $A^+ = ABC$

$\therefore C \in A^+$

$\Rightarrow$ $A \to C$ is redundent and hence needs to be removed.

$F = \{A \to B,\ B \to C\}$

Finding Minimal Cover

Given a set of FDs F. E is a minimal cover of F if it satisfies the following conditions (steps):

step-1) Each FD has single attribute in R.H.S

step-2) No FD contains extraneous attributes

step-3) Each FD is non-redundant.

Example

Decomposition Rule (IR4)

$F = \{ A \to BC, B \to C, A \to C, AB \to C \}$

Find the minimal cover for the set of FDs F.

Step 1: $F = \{ A \to B, A \to C, B \to C, A \to C, AB \to C \}$ ← Single attribute in R.H.S.

Sol^n: Step 2: Check for extraneous attributes in each FD (if L.H.S is having more than one attribute)

$AB \to C$

$(AB - A)^+ = B^+ = BC$

B^+ contains C.

$\Rightarrow$ A is extraneous.

$\therefore AB \to C$ is replaced by $B \to C$.

$A \to C,$

$\Rightarrow F = \{ A \to B, B \to C, \underbrace{A \to C, B \to C}_{same} \}$

Step 3: Remove the redundant FDs.

$F - \{A \to B\} = \{ A \to C, B \to C \}$

Then, find $A^+ = AC$, $B \notin A^+$

$A \to B$ is non-redundant.

$F - \{B \to C\} = \{ A \to B, A \to C \}$

Then, find $B^+ = B$, $C \notin B^+$ $\therefore B \to C$ is not redundant.

$F - \{A \to C\} = \{A \to B,\ B \to C\}$

$A^+ = ABC$, $C \in A^+$

$\therefore$ $A \to C$ is redundant and needs to be removed.

So, minimal cover of F, i.e. E is given by,

$E = \{A \to B,\ B \to C\}$