MODULE-04

Rectilinear motion

(1) Distance (s)
(2) Displacement
(3) speed
(4) Velocity $= \frac{ds}{dt}$
(5) Acceleration $= \frac{dv}{dt}$

Uniform velocity

$a = 0$
$v = $ constant

Unifor acceleration

$a =$ const
v is not constant
$\nabla v =$ const

Variable acc^n

a is not const.

Motion under uniform acceleration

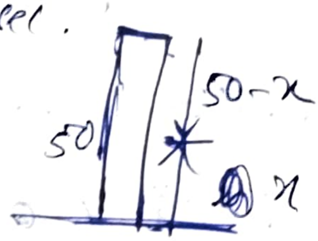
$u =$ initial speed / velocity
$v =$ final "
$a =$ acc^n
$t =$ time duration
$s =$ Distance travelled

① $v = u + at$
② $s = ut + \frac{1}{2}at^2$
③ $v^2 = u^2 + 2as$

$S_n = u + \frac{a}{2}(2n-1)$

Ex

A stone is dropped from the top of the tower (50 m)
At the same time, another stone is thrown upward from the foot of tower with velocity 25 m/sec.
At what height.



$50 - x = +\frac{1}{2}\times 9.8 \times t^2$

$\Rightarrow 50 - x = +4.9t^2$

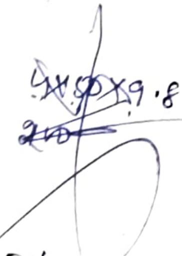
$s = ut + \frac{1}{2}at^2$

$x = 25t - 4.9t^2$

$\Rightarrow 50 - 25t + \cancel{4.9t^2} = \cancel{4.9t^2}$

$\cancel{\Rightarrow 9.8t^2 - 25t + 50 = 0}$

$\Rightarrow \boxed{t = 2.}$



$t = 25 \pm 625$ ...

$x = 25\times 2 - 4.9\times 4$

$= 50 - 19.6$

$= 30.4$

Curvilinear motion

Projectile

Terminology

- Trajectory
- Velocity of projecⁿ
- Angle of projecⁿ
- Time of flight
- Range (R)

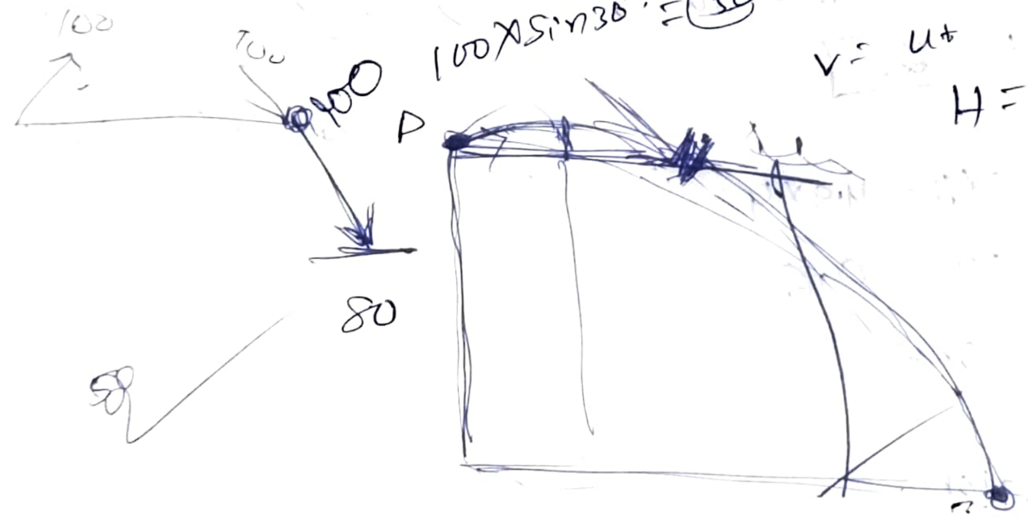
$$t = \frac{2u\sin\alpha}{g}$$

$$\text{Max}^{m}\ \text{height}\ (H) = \frac{u^2\sin^2\alpha}{2g}$$

$$R = \frac{u^2\sin 2\alpha}{g}$$

Ex A bullet is fired at an angle of 30° to horizontal at point 'P' on a hill and strikes the target, which is 80 m lower than 'P'. The initial velocity of bullet is 100 m/sec. ~~Calculate~~ Calculate the actual velocity of the bullet ~~strikes~~ will strike the target.

$100 \times \sin 30^\circ = 50$

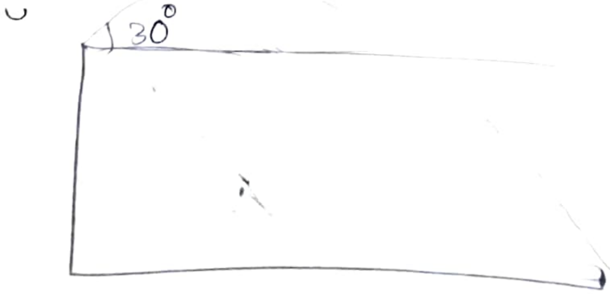


$v = u +$

$$H = \frac{\cancel{100}^{25} \times 100 \times \frac{1}{4}}{2 \times 9.8}$$

127.55

207.55



$u = 100\ m/s$

$$R = \frac{u^2 \sin 60^\circ}{9.8} = \frac{50 \times \sqrt{3} \times 100}{9.8} = 883.7$$

Q ~~$v = u + at$~~

$$v = \sqrt{u^2 + 2as}$$

$$= \sqrt{(100 \times 100) + 2 \times 9.8 \times 80}$$

$$= 107.554$$

Work, Power, Energy

Work = $F \times S \times \cos\theta$

Power : Rate of W

Energy : Capacity to do work

Mechanical energy → PE = mgh
→ KE = $\frac{1}{2}mv^2$

Workdone on/by spring

$W = \frac{1}{2}kx^2$

↓

strain energy of the spring

$W = \frac{1}{2} \times k \times (2.5)^2$

$W + mgh = \frac{1}{2}$

$\frac{1}{2}k \times (2.5)^2 + 0.3 \times mg$

$\boxed{k = \frac{F}{l}}$

$0.3 \times mg$

$mgx = \frac{1}{2}k(0.25)^2$

$mg(0.3 + l) = \frac{1}{2} \times kx^2$

$\Rightarrow \sqrt{\frac{l}{0.3 + l}} = \left(\frac{0.25}{x}\right)^2$

$mg \times h = \frac{1}{2}kx^2$

$0.3mg + \frac{1}{2}k \times (0.25)^2 = \frac{1}{2}kx^2$

$mg(0.3$

$W = kx$

Ex

When a ball of weight 'w' rest on the spring, it produces static deflection of 2.5 mm. How much it will deflect if the same ball is dropped from 0.3 mtr above the spring.

$l = \frac{k \times 0.25^2}{2mg}$

$mgl = \frac{1}{2}k \times (0.25)^2$

$\Rightarrow l = \frac{\frac{1}{2} \times k \times (0.25)^2}{mg}$

$a = 2u \sin\alpha$

$0.3 \times mg +$

$mgl = \frac{1}{2} k \times (6.25)^2$

$\Rightarrow mg = \frac{1}{2}$

$mgh = \frac{1}{2} kx^2$

$\Rightarrow h = \dfrac{\frac{k}{2}(6.25)}{mg}$

$\dfrac{\frac{k}{2}(6.25) + \frac{1}{2}kx^2}{h}$

$mgl = \left(\frac{1}{2}k\right) \times (6.25)$

$h + \frac{1}{2}$

$\dfrac{mgl}{6.25}$

$mg(h+l) = \frac{1}{2}kx^2$

$\Rightarrow mg($

$mg \times 0.3 = \dfrac{mgl}{6.25} \times x^2$

6.25

$mgl = \frac{k}{2} \times (6.25)$ | $mg(0.3 + l) = \frac{k}{2} x^2$

$\dfrac{l}{0.3+l} = \dfrac{6.25}{x^2}$

$\dfrac{0.3+l}{l} = \dfrac{x^2}{6.25}$

0

$mg(h+\delta) = \frac{1}{2}k\delta^2$

$k = \dfrac{W}{\delta_0 = 2.5}$

$\Rightarrow W(h+\delta) = \frac{1}{2}k\delta^2$

$\Rightarrow W = 2.5k$

$\Rightarrow \cancel{2.5}^{5} k(0.3+\delta) = 0.5k\delta^2$

$\Rightarrow 1.5 + 5\delta = \delta^2$

$\Rightarrow \delta^2 - 5\delta - 1.5 = 0$

$\dfrac{5 \pm \sqrt{25+6}}{2}$

$= \dfrac{5 \pm \sqrt{31}}{2} = \dfrac{5+}{2}$

$\Rightarrow mg(h+\delta) = \frac{1}{2}k\delta^2$

$\Rightarrow \frac{2.54 \times 9.8}{5}(0.3 + \delta) = 0.5 k \delta^2$

$\Rightarrow 1.5 + 5\delta = \delta^2$

Ex

An arrow weighing 0.15 N is shot from a bow of force of 155 N at full draw 400 mm. Find the velocity of the arrow when it leaves.

$k = \frac{F}{x}$

$F = 155\ N$

$\frac{1}{2}kx^2$

$x = 400\ m$

$k = \frac{F}{x} = \frac{155}{400}$

$155 \times 400 = \frac{1}{2} \times \frac{0.15}{9.8} \times v^2$

$\frac{1}{2} \times \frac{.155}{400} \times .400 \times 400$

$\frac{1}{2} \times \frac{155}{\cancel{0.4}} \times \cancel{0.16}$ $\quad$ 0.2 $\quad$ ~~0.4~~

$= \frac{1}{2} \times \frac{0.15}{9.8} \times v^2$

$\frac{2480}{3}$

$\Rightarrow v = 69.644$

$t = 2u\sin\alpha$

Momentum & Impulse

Momentum = Mass X Velocity = $m \times v$

Impulse ÷ When a large force act for very short period of time

$$Impulse = F \times (\Delta t)$$

$$= ma \times \Delta t$$

$$= m \times \frac{v_2 - v_1}{\Delta t} \times \Delta t$$

$$= mv_2 - mv_1 = m\Delta v$$

Conservation of momentum & impulse

$$\Sigma Mv_2 = \Sigma Mv_1 + F\Delta t$$

$$\Rightarrow \Sigma mv_2 = \Sigma mv_1$$

Conservation of momentum

When there is no impulse

$$\Sigma mv_2 = \Sigma mv_1$$

Ex:-

Recoil of gun, when bullet is fired from gun.

$$M_1 V_1 = M_2 V_2$$

m_1 = mass of bullet

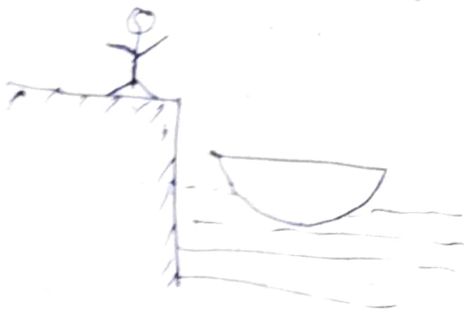
v_1 = velocity " "

M_2 = mass of gun

V_2 = velocity " "

Ex.7

motion of boat when man jumps into it.



$mv_1 = (m+M)v_2$

Q

A man weighing 700N, stand in a boat so that he is 4.5 m from platform, on the shore. He walks 2.4 towards the platform & then stop. Now the man is how far from the platform? (weight of boat 900 N)

$\underline{mg}$

$\frac{700 \times}{9.8} \frac{2.4}{t} = \frac{(700+900) \times}{9.8} \frac{S}{t}$

$\Rightarrow S = \frac{700 \times 2.4}{1600}$

$= 1.87$

$\frac{}{4.5}$

2.4
-1.87

0.53

4.5

1.05

$= 3.15$.

$\frac{700}{9.8} \times \frac{2.4}{t} = \frac{(700+900) \times S}{9.8 \times t}$

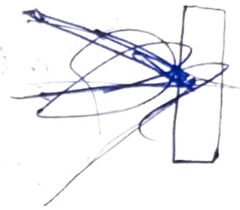
$\Rightarrow S = 3.15$

$t = 2u\sin\alpha$

Q: A ball (of mass 100 gm) with velocity 25 m/sec is thrown to batsman. After ball is hit by bat it has the velocity of 40 m/sec.

~~towards the bowler at (40°)~~.

in the direction, which is 40° to bowler's direction. The ball & bat are contact for 0.015 sec.

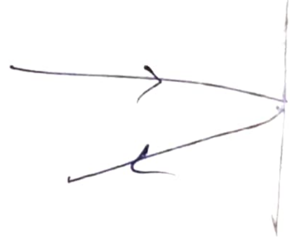


Determine the

$$100 \times (40\cos 40^\circ) - 100 \times (25\cos 0) = F \times 0.015$$

$$\Rightarrow F = \frac{100 \times (40\cos 40^\circ - 25\cos 40^\circ)}{0.015}$$

V_2



$$0.1\,(40\cos 40^\circ) - (0.1 \times 25) = F \times 0.015$$

$$\Rightarrow F = 37.61$$

$F_{net} =$

$$f_x \times (\Delta t) = mV_2 - mV_1 = (0.1)(40\cos 40) - (0.1 \cdot 25)$$

$$= 370.9$$

$$f_y \times 0.015 = mV_{y_2} - mV_{y_1}$$

$$= (0.1)(40 \times \sin 40^\circ) - 0 = 171.

Impact

Collision of 2 bodies, whether we have active and reactive forces acting for very short time interval is termed as ~~Impact~~.

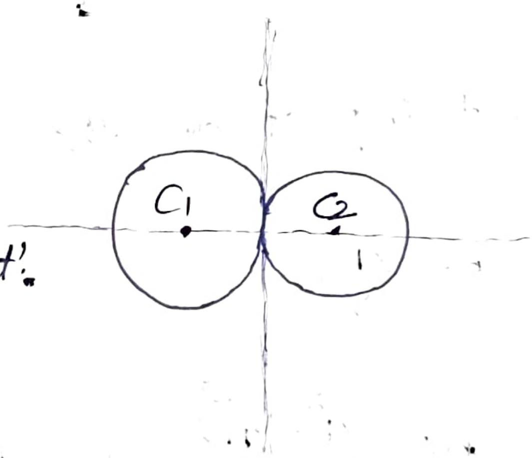
Impact

Magnitude of impact depends on

(1) Velocity
(2) Masses
(3) Elastic properties.

Line of impact

The common normal to the surface of 2 bodies in contact during impact is termed as 'Line of impact'.



Types of ~~contact~~ impact

(1) Central & non central
(2) Direct & oblique impact.

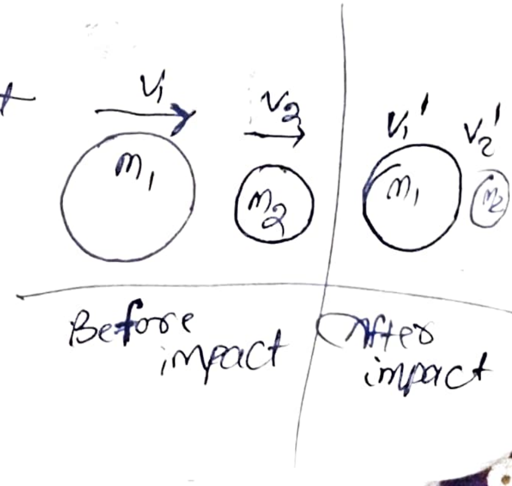
If the centre of masses colliding (i.e. C_1 & C_2) lie on the line of impact then it is said as central impact.

Lowe of conservation of momentum

Since there is no external force act

$$m_1V_1 + m_2V_2 = m_1V_1' + m_2V_2'$$

$V_1 - V_2$ = Velocity of approach
$V_2' - V_1'$ = velocity of separation.



Newton's law of collision

$$(v_2' - v_1') \propto (v_1 - v_2)$$

$$\Rightarrow \frac{v_2' - v_1'}{v_1 - v_2} = K = e \quad \text{[Coefficient of restitution]}$$

(i) For elastic impact
$e = 1$

(ii) For plastic impact
$e = 0$

$u = 0 \quad v =$

$v = u + at$

$\Rightarrow 2 = 0 + ax$

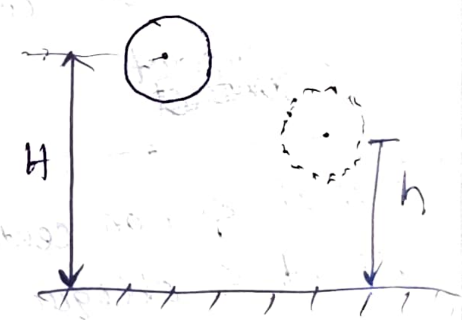
$s = ut + \frac{1}{2}at^2$

$\Rightarrow s = 0$

$a = \frac{1}{2}$

Direct impact of the body with fixed plane

$$e = \frac{v_1'}{v_1}$$



$\cancel{h = \frac{v_1^2}{2g}}$

$v_1 = \sqrt{2gH}$

$v_1' = \sqrt{2gh}$

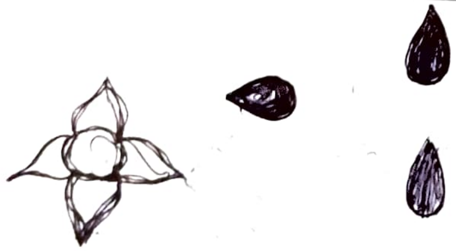
$$\boxed{e = \sqrt{\frac{2gh}{2gH}} = \sqrt{\frac{h}{H}}}$$

$v^2 - u^2 = 2as$

$\Rightarrow a = \frac{v^2 - u^2}{2s}$

$= \frac{4 - 0}{2 \times 2}$

$\boxed{a = 1}$



D- Alembert's principle

↳ Equation of dynamic equilibrium:

The kinetic body is in equilibrium under the resultant forces acting on it along with the inertia forces.

$$\Sigma F = ma$$

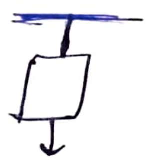
$$\Rightarrow \underbrace{\Sigma F}_{\text{Resultant force}} - \underbrace{ma}_{\text{Inertial force}} = 0$$

Ex

A lift of total weight 5KN, starts to move upward with a constant velocity (2m/sec) after travelling 2m distance.

① Find the tension of lift's cable during accn.

$a = 1$ $\qquad\qquad a = \frac{v^2 - u^2}{2s}$



$T - (5\times10^3) - 5\times10^3 \times \times 1) = 0$

$\Rightarrow T = 9.8 \times 5 \times 10^3 \times 2$

$= 9.8 \times 10^4$

$T - 5g = 0$

$T = 5\times10^3 + \frac{5\times10^3}{9.8}$

T-

$T - 5\times10^3 - \frac{5\times10^3}{9.8} \times 1$ $\qquad \Rightarrow T = 5-$

$\Rightarrow T = 5.510$ KN

The above lift while moving up with a velocity 2m/sec uniformly retard to stop in 2 sec.

(ii) Find the tension in the cable during retard^n.

(iii) " " force of the floor under the feet of man (weight 600 N)

$v = u + at$

$\Rightarrow 0 = 2 + a \times 2$

$\Rightarrow \boxed{a = -1 \text{ m/sec}^2}$

$T - 5 \times 10^3 + \frac{5}{9.8} \times 10^3 \times 1 = 0$

$\Rightarrow T = \left(5 - \frac{5}{9.8}\right) 10^3$

$= 4.5$ KN

(iii) $f = 600 \times \frac{a}{g}$

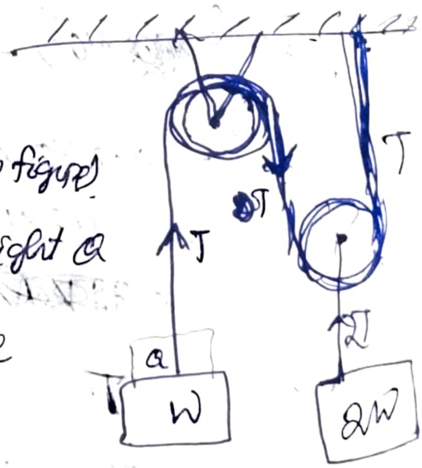
$600 + 5000 = N +$

(iii) $R = W\left(1 - \frac{a}{g}\right)$

$= 538.775$

Ex

Weight, 'W' and '2W' are supported by string and pulled (as shown in figure). Find the magnitude of additional weight Q is added to load 'W', which will give downward acc^n of $a = 0.1g$.



$(W + Q) - T = \frac{W}{9.8} \times 0.1$

$T = W$

$\Rightarrow$

$(W + Q) - W$

$\Rightarrow Q = \frac{W \times 0.1}{9.8} = 0.01W$

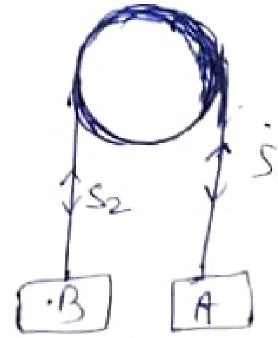
Kinematic rotation of rigid bodies

Rigid body rotation, like pulleys, shaft, flywheel etc have motion of rotation (i.e. angular momentum) about it's own axis.

	Linear motion	Angular motion
(1) Initial velocity	u	ω_0
(2) Final "	$V = u + at$	$W = \omega_0 + \alpha t$
(3) Angular acc^n	a	α
(4) Displacement	$S = ut + \frac{1}{2}at^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
(5) :	$F = ma$	$T = I\alpha$
	$KE = \frac{1}{2}mV^2$	$E = \frac{1}{2}IW^2$
	$WD = F \times S$	$WD = T.\theta$

Ex

Two bodies A & B are attached to the strong (as shown in the fig.) The pulley has the mass of 100 kg & 800 mm dia & $k = 400$ mm, Find the torque, required to raise the mass A with acc^n 1 m/sec².



B = 600 kg | A = 800 kg

$$S_1 - mg + ma = 0$$

$$\Rightarrow S_1 = M(g+a)$$

$$= 800 \times (1 + 9.8)$$

$$= 800 \times 10.8$$

$$= 8640\ N$$

$$S_2 = M_b(g-a) = 600 \times (g-a) = 600 \times 8.8 = 5280$$

$S_1 - S_2 = 8640 - 5280 = 3360\ N$

$T_1 = (S_1 - S_2) \times r = 3360 \times 0.4$

$= 1344\ Nm$

Torque required to rotate the pully.

$\therefore T_2 = I \cdot \alpha$

$= m_p k^2 \times \frac{a}{r}$

$= 100 \times 0.16 \times \frac{1}{0.4}$

$= \frac{16 \times 10}{4} = 40\ Nm$

Total torque required $= T = T_1 + T_2$

$= 1344 + 40$

$= 1384\ Nm$