Exact equation :-

The equation of the form $M(x,y)\,dx + N(x,y)\,dy = 0$ is said to be exact if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

How to get solution?

$$\int M\,dx + \int N\,dy = c$$

(Treating $y$ constant) (Only those term which are free from $x$)

Q! Check whether the equation is exact or not then find the solution

$$(3x^2 + 2e^y)\,dx + (2xe^y + 3y^2)\,dy = 0$$

Soln

$$M(x,y) = 3x^2 + 2e^y$$
$$N(x,y) = 2xe^y + 3y^2$$

$$\frac{\partial M}{\partial y} = 2e^y$$

$$\frac{\partial N}{\partial x} = 2e^y$$

Here, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\Rightarrow$ eqn is exact

$$\int M\,dx = \int (3x^2 + 2e^y)\,dx$$
$$= x^3 + 2xe^y$$

$$\int N\,dy = \int 3y^2\,dy = y^3$$

$$\int (3x^2+2e^y)\,dx + \int 3y^2\,dy = c$$

$$\Rightarrow x^3 + 2xe^y + y^3 = c$$

Q:- What about $e^x(\cos y\,dx \mp \sin y\,dy) = 0$

$y(0) = 0$

Sol:

$$e^x \cos y\,dx - e^x \sin y\,dy = 0$$

$$\frac{\partial M}{\partial y} = -\sin y\, e^x$$

$$\frac{\partial N}{\partial x} = -e^x \sin y$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$\Rightarrow$ eqn is exact

Now

$$\int e^x \cos y\,dx + \int 0\,dy = c$$

$$\Rightarrow e^x \cos y = c \quad \text{——— (1)}$$

Given, $y(0) = 0$

From (1)

$$e^0 \cos 0 = c$$

$$\Rightarrow c = 1$$

Now soln is

$$\boxed{e^x \cos y = 1}$$

$\int (3x^2 + 2e^y)\,dx + \int 3y^2\,dy = c$

$\Rightarrow x^3 + 2xe^y + y^3 = c$

Q:- What about $e^x(\cos y\,dx \mp \sin y\,dy) = 0$

$y(0) = 0$

Sol:

$e^x \cos y\,dx - e^x \sin y\,dy = 0$

$\frac{\partial M}{\partial y} = -\sin y\, e^x$

$\frac{\partial N}{\partial x} = -e^x \sin y$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\Rightarrow$ eqn is exact

Now

$\int e^x \cos y\,dx + \int 0\,dy = c$

$\Rightarrow e^x \cos y = c$ —— (1)

Given, $y(0) = 0$

From (1)

$e^0 \cos 0 = c$

$\Rightarrow c = 1$

Now soln is

$\boxed{e^x \cos y = 1}$

Integrating factor: (I.F) / $\mu$

Integrating factor is a function $f(x,y)$ which is multiplied with a non-exact equation, it will be exact

Ex: $y\,dx - x\,dy = 0$ ——— (1)

the above eqn is not exact

Let $f(x,y) = \frac{1}{y^2}$

Now multiply $f(x,y)$ in eqn (1) we get

$$\frac{y\,dx - x\,dy}{y^2} = 0$$

$$\Rightarrow \underbrace{\frac{1}{y}}_{M}dx - \underbrace{\frac{x}{y^2}}_{N}dy = 0 \quad ——— (2)$$

$$\frac{\partial M}{\partial y} = -\frac{1}{y^2}$$

$$\frac{\partial N}{\partial x} = -\frac{1}{y^2}$$

$$\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$\Rightarrow$ eqn (2) is exact

$$\int \frac{1}{y}\,dx + \int 0\,dy = c$$

$$\Rightarrow \frac{x}{y} = c \quad \text{(solution)}$$

Let $g(x,y) = \dfrac{1}{\frac{1}{2}(y^2-x^2)}$

Now multiply $g(x,y)$ in eqn (1) we get

$$\underbrace{\frac{2y}{y^2-x^2}}_{M} dy - \underbrace{\frac{2x}{y^2-x^2}}_{N} dy = 0 \quad \text{—— (3)}$$

$$\frac{\partial M}{\partial y} = \frac{2(y^2-x^2) - 2y \times 2y}{(y^2-x^2)^2}$$

$$= \frac{2y^2 - 2x^2 - 4y^2}{(y^2-x^2)^2}$$

$$= \frac{-2x^2 - 2y^2}{(y^2-x^2)^2}$$

$$=$$

$$\frac{\partial N}{\partial x} = \frac{-2(y^2-x^2) + 2x \times (-2x)}{(y^2-x^2)^2}$$

$$= \frac{-2y^2 + 2x^2 - 4x^2}{(y^2-x^2)^2}$$

$$= \frac{-2x^2 + 2y^2}{(y^2-x^2)^2}$$

$$\frac{\partial M}{\partial x} = \frac{\partial N}{\partial x}$$

$\Rightarrow$ eqn (3) is exact

How to find integrating factor?

1. If $\dfrac{\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}}{N} = f(x)$ then $I.F = e^{\int f(x)dx}$

2. If $\dfrac{\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}}{M} = g(y)$ then $I.F = e^{\int -g(y)\,dy}$

3. If M & N are homogeneous function of same degree and $M.x + N.y \neq 0$ then $I.F = \dfrac{1}{Mx + Ny}$. But if $Mx + N.y = 0$ then $\dfrac{1}{xy}, \dfrac{1}{x^2}, \dfrac{1}{y^2}$ may be taken as integrating factor.

X

X —— X

Q! Find the solution of the following diff. eqn

$$(5x^3+12x^2+6y^2)dx + 6xy\,dy = 0$$

Soln Given, $\underbrace{(5x^3+12x^2+6y^2)}_{M}dx + \underbrace{6xy}_{N}dy = 0$ —— ①

$$\frac{\partial M}{\partial y} = 12y \quad , \quad \frac{\partial N}{\partial x} = 6y$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \quad \text{(Not exact equation)}$$

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 12y - 6y = 6y$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{6y}{6xy} = \frac{1}{x} = f(x)$$

Now, $I.F = e^{\int f(x)dx} = e^{\int \frac{1}{x}dx} = e^{\ln x}$

$\Rightarrow \mu = x$

Multiply I.F in eqn ①

$$\underbrace{(5x^4 + 12x^3 + 6xy^2)}_{M'}dx + \overbrace{6x^2y}^{N'}dy = 0$$

(Exact equation)

Solution : $\underset{\text{(Treating 'y' as constant)}}{\int M'dx} + \underset{\text{(Taking the term which are free from x)}}{\int N'dy} = c$

$$\Rightarrow \int (5x^4 + 12x^3 + 6xy^2)\,dx + \int 0\,dy = c$$

$$\Rightarrow \boxed{x^5 + 3x^4 + 3x^2y^2 = c}$$

↳ solution

Q:- Find solution for the following differential equation

$$(3x^2y^3e^y + y^3 + y^2)\,dx + (x^3y^3e^y - xy)\,dy = 0$$

Sol:

Given, $$\underbrace{(3x^2y^3e^y + y^3 + y^2)}_{M}\,dx + \underbrace{(x^3y^3e^y - xy)}_{N}\,dy = 0 \quad \text{--- (1)}$$

$$\frac{\partial M}{\partial y} = 9x^2y^2e^y + 3x^2y^3e^y + 3y^2 + 2y$$

$$\frac{\partial N}{\partial x} = 3x^2y^3e^y - y$$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x} \quad \text{(not exact eqn)}$$

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 9x^2y^2e^y + 3y^2 + 3y$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M} \neq \cancel{\frac{9x^2y^2e^y + 3y^2 + 3y}{9x^2y^2e^y + 3x^2y^3e^y}}$$

$$= \frac{9x^2y^2e^y + 3y^2 + 3y}{3x^2y^3e^y + y^3 + y^2} = \frac{3(3x^2y^2e^y + y^2 + y)}{y(3x^2y^2e^y + y^2 + y)}$$

$$= \frac{3}{y} = g(y)$$

Now,

$$I.F = \mu = e^{\int -g(y)\,dy} = e^{\int -\frac{3}{y}\,dy}$$

$$= e^{-3\ln y}$$

$$= \frac{1}{y^3}$$

Now multiplying I.F in eq^n (1)

$$\underbrace{\left(3x^2e^y + 1 + \frac{1}{y}\right)}_{M'} dx + \underbrace{\left(x^2e^y - \frac{x}{y^2}\right)}_{N'} dy = 0$$

(exact eq^n.)

To get solution:

$$\int M'\,dx + \int N'\,dy = C$$

(Taking 'y' as constant) ( Taking the term rest from x )

$$\Rightarrow \int \left(3x^2e^y + 1 + \frac{1}{y}\right) dx + \int 0\,dy = C$$

$$\Rightarrow \boxed{x^3e^y + x + \frac{x}{y} = C}$$

solution

Linear equation :-

↳ In the form of : $\frac{dy}{dx} + P(x)y = Q(x)$

$I.F = \mu = e^{\int P dx}$

To write : $y\mu = \int Q\mu \, dx$ (To find solⁿ of diff. eqⁿ)

e.g. : $y' - y = e^{2x}$

**solⁿ** $I.F = \mu = e^{\int P dx} = e^{\int (-1) dx} = e^{-x}$

$y\mu = \int Q\mu \, dx$

$\Rightarrow \quad ye^{-x} = \int e^{2x} \cdot e^{-x} dx$

$\Rightarrow \quad \boxed{ye^{-x} = e^{x} + c}$

↳ solution of diff. eqⁿ

Q1. Find the solution of the following differential equation

$\frac{dy}{dx} + y\tan x = \sin 2x$ , $y(0) = 1$

Sol$^n$ Given, $\frac{dy}{dx} + \underbrace{y\tan x}_{P(x)} = \overbrace{\sin 2x}^{Q(x)}$ ——— ①

↳ Linear diff. eq$^n$

$I.F = e^{\int P dx} = e^{\int \tan x\, dx} = e^{\ln \sec x} = \sec x = \mu$

~~Multiplying 'μ' in eq ① we get~~

$\cancel{\sec x \frac{dy}{dx} +}$

solution:-
$y\mu = \int Q\mu\, dx$

$\Rightarrow y\sec x = \int \sin 2x \sec x\, dx$

$\Rightarrow y\sec x = 2\int \sin x\, dx$

$\Rightarrow y\sec x = -2\cos x + c$

$\Rightarrow y\sec x + 2\cos x = c$ ——— ②

Given, $y(0) = 1$ (when $x = 0 \Rightarrow y = 1$)

From ②
$1 \times \sec 0 + 2 \times \cos 0 = c$

$\Rightarrow 1 \times 1 + 2 \times 1 = c$

$\Rightarrow c = 3$

Now solution will be

$$\boxed{y\sec x + 2\cos x = 3}$$

Bernoulli's equation :-

$$\frac{dy}{dx} + P(x)y = q(x)y^a \quad a \in R \quad \text{——— (1)}$$

$$u = y^{1-a}$$

$$\frac{du}{dx} = (1-a)y^{-a}\frac{dy}{dx}$$

Dividing $y^a$ in both sides in eqn (1)

$$y^{-a}\frac{dy}{dx} + P(x)y^{1-a} = q(x)$$

$$\Rightarrow \frac{1}{(1-a)}\left(\frac{du}{dx}\right) + uP(x) = q(x)$$

$$\Rightarrow \frac{du}{dx} + (1-a)uP(x) = (1-a)q(x)$$

Apply procedure of linear equation

Q: solve $\quad y' + 4xy + xy^3 = 0$

soln

$$y' + \underbrace{4x}_{P(x)}y = \underbrace{-x}_{q(x)}y^3 \quad \text{——— (1)}$$

Let $\quad u = y^{1-3} = y^{-2}$

$$\frac{du}{dx} = -2y^{-3}y'$$

Dividing $y^3$ in eqn (1) we get

$$y^{-3}y' + 4xy^{-2} = -x$$

$$\Rightarrow -\frac{1}{2}\frac{du}{dx} + 4xu = -x$$

$$\Rightarrow \frac{du}{dx} - 8xu = 2x \quad \text{——— (2)}$$

$$\text{I.F} = \mu = e^{\int -8x\,dx}$$

$$= e^{-4x^2}$$

soln for (2)

$$u \times (\text{I.F}) = \int 2x e^{-4x^2} dx$$

$$\Rightarrow u \cdot e^{-4x^2} = \int 2x e^{-4x^2} dx$$

put $-4x^2 = z$

$\Rightarrow -8x\,dx = dz$

$\Rightarrow 2x\,dx = -\frac{dz}{4}$

$$\Rightarrow u \cdot e^{-4x^2} = \frac{-1}{4}\int e^{z} dz$$

$$\Rightarrow u \cdot e^{-4x^2} = \frac{-1}{4} e^{z} + c$$

$$\Rightarrow u \cdot e^{-4x^2} = -\frac{1}{4} e^{-4x^2} + c$$

$$\Rightarrow u = \frac{-1}{4} + ce^{4x^2}$$

$$\Rightarrow \boxed{y^{-2} = ce^{4x^2} + -\frac{1}{4}} \rightarrow \text{Solution}$$

Application 1st order differential equation :-

① The type rate of change of temperature $\frac{dT}{dt}$ ( $T(t)$ is the temperature) is directly proportional to the difference of temperatures ($T$) and temperature of surrounding ($T_A$).

That means $\frac{dT}{dt} \propto (T - T_A)$

Let the temperature of the room at $t=0$ is $66°F$ at $t=2$, $63°F$. If temperature of surrounding is $32°F$. Then find the temperature of the room after 10 mins

Sol$^n$

$$\frac{dT}{dt} \propto (T - T_A)$$

$$\Rightarrow \frac{dT}{dt} = K(T - T_A)$$

$$\Rightarrow \frac{dT}{T - T_A} = k\,dt$$

$$\Rightarrow \ln(T - T_A) = kt + c$$

$$\Rightarrow T - T_A = e^{kt+c}$$

$$\Rightarrow T = e^{kt+c} + T_A$$

or $$T(t) = e^{kt+c} + T_A$$

at $t=0$, $T = 66$

$$66 = e^{k \times 0 + c} + 32$$

$$\Rightarrow e^{c} = 34$$

$$\Rightarrow c = \ln 34$$

Now,

$$T(t) = e^{kt+c} + T_A$$

at $t=2$, $T=63$

$$63 = e^{K\times 2+\ln 34} + 32$$

$$\Rightarrow \quad 31 = 34\, e^{2K}$$

$$\Rightarrow \quad e^{2K} = \frac{31}{34}$$

$$\Rightarrow \quad K = \frac{1}{2}\ln\frac{31}{34}$$

Now,

$$T(t) = e^{\left(\frac{1}{2}\ln\frac{31}{34}\right)t + \ln 34} + T_A$$

put $t=10$

$$\Rightarrow \quad T(10) = e^{\left(\frac{1}{2}\ln\frac{31}{34}\right)\times 10 + \ln 34} + 32$$

$$= e^{\ln\left(\frac{31}{34}\right)^5 + \ln 34} + 32$$

$$= 34\cdot\left(\frac{31}{34}\right)^5 + 32$$

$$= \frac{(31)^5}{(34)^4} + 32$$

Malthus law of population growth :-

Let $P(t)$ be the population of particular species at time $t$.
The rate of change of population at time '$t$' is proportional to the population present at that time.

$$\frac{dP(t)}{dt} \propto P(t)$$

At $t=0$ population of particular species is 5.3 million. At $t=1$, $P(t) = 4.8$ million. Find population of the species at $t=10$.

Sol$^n$

$$\frac{dP(t)}{dt} \propto P(t)$$

$$\Rightarrow \frac{dP(t)}{dt} = K\,P(t)$$

$$\Rightarrow \frac{dP(t)}{P(t)} = k\,dt$$

$$\Rightarrow \ln(P(t)) = Kt + c$$

$$\Rightarrow P(t) = e^{Kt+c} \qquad \text{—— (1)}$$

at $t=0$, $P(0) = 5.3$

$$5.3 = e^{K \times 0 + c}$$

$$\Rightarrow 5.3 = e^{c}$$

$$\Rightarrow c = \ln 5.3$$

at $t=1$, $P(1) = 4.8$

$$4.8 = e^{K \times 1 + \ln 5.3}$$

$$\Rightarrow 4.8 = 5.3e^{K}$$

$$\Rightarrow K = \ln\left(\frac{4.8}{5.3}\right)$$

Now,

$$P(t) = e^{\left(\ln\left(\frac{4.8}{5.3}\right)\right)t + \ln(5.3)}$$

at $t=10$

$$P(10) = e^{10\ln\left(\frac{4.8}{5.3}\right) + \ln(5.3)}$$

$$\Rightarrow P(10) = \left(\frac{4.8}{5.3}\right)^{10} \cdot (5.3)$$

voltage drop ($E_L$) across an inductor $= L\frac{dI}{dt}$

" " ($E_c$) " a capacitor $= \frac{1}{c} q$

$E_R = RI$

$$L\frac{dI}{dt} + RI = E(t)$$

$$\Rightarrow \frac{dI}{dt} + \left(\frac{R}{L}\right)I = \frac{1}{L}E(t)$$

Q: If $L = 0.1H$, $R = 5\Omega$, $E(t) = 12$ then:- Find ~~the~~ the ~~final eqn for current~~ diff. eqn & current

Soln Diff. eqn will be:-

$$\frac{dI}{dt} + \left(\frac{R}{L}\right)I = \frac{1}{L}E(t)$$

$$\Rightarrow \frac{dI}{dt} + \left(\frac{5}{0.1}\right)I = \frac{1}{0.1} \times 12$$

$$\Rightarrow \boxed{\frac{dI}{dt} + 50I = 120} \rightarrow \text{Linear differential eqn}$$

$$\text{I.F} = \mu = e^{\int 50\,dt} = e^{50t}$$

solution:-

$$I \cdot \mu = \int Q\mu\,dt$$

$$\Rightarrow I \cdot e^{50t} = \int 120e^{50t}\,dt$$

$$\Rightarrow \boxed{I \cdot e^{50t} = \frac{12}{5}e^{50t} + c} \rightarrow \text{solution.}$$

Current, $I = \frac{12}{5} + c \cdot e^{-50t}$

Q: If $E(t) = E_0 \sin\omega t$ then find the current:- ?

Sol^n

D.E will be

$$L\frac{dI}{dt} + IR = E_0 \sin\omega t$$

$$\Rightarrow \frac{dI}{dt} + \left(\frac{R}{L}\right) I = \left(\frac{E_0}{L}\right)\sin\omega t$$

$$I.F = \mu = e^{\int \frac{R}{L} dt} = e^{\frac{R}{L}t}$$

$$I \cdot \mu = \int \frac{E_0}{L} \sin\omega t \cdot \mu \, dt$$

$$\Rightarrow I e^{\frac{R}{L}t} = \frac{E_0}{L} \int \sin\omega t \cdot e^{\frac{R}{L}t} \, dt$$

$$\Rightarrow I e^{\frac{R}{L}t} = \frac{E_0}{L} \frac{e^{\frac{R}{L}t}}{\left(\frac{R}{L}\right)^2 + \omega^2} \times \left(\frac{R}{L}\sin\omega t - \omega \cos\omega t\right) + c$$

$$\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$$

$$\int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$$

$$\Rightarrow I = \frac{E_0 L}{R^2 + \omega^2 L^2}\left(\frac{R}{L}\sin\omega t - \omega\cos\omega t\right) + c e^{-\frac{R}{L}t}$$

Dt- 02/02/2024

# Second order homogenous equation with constant coefficient

## General form:-

$$y'' + ay' + by = 0$$

**Solution**

Step-1: Put $y'' = \lambda^2$, $y' = \lambda$, $y = 1$

Step-2: Characteristic equation became,

$$\lambda^2 + a\lambda + b = 0$$

Step-3: Solve the characteristic equation

(A) If $\lambda = \lambda_1, \lambda_2$

Then gen. solⁿ,

$$y = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x}$$

(B) If $\lambda = \lambda_1 = \lambda_2$

Then general solⁿ

$$y = c_1 e^{\lambda x} + c_2 x e^{\lambda x}$$

(C) Let $\lambda = \alpha \pm i\beta$

The general solⁿ,

$$y = e^{\alpha x}(C_1 \cos\beta x + C_2 \sin\beta x)$$

Q:- $y'' - 3y' + 2y = 0$, $y(0) = 1$, $y'(0) = 1$

Sol$^n$ put $y'' = \lambda^2$, $y' = \lambda$, $y = 1$

$$\lambda^2 - 3\lambda + 2 = 0$$

$$\Rightarrow (\lambda - 2)(\lambda - 1) = 0$$

$$\Rightarrow \lambda = 2, 1$$

general sol$^n$,

$$y = c_1 e^{2x} + c_2 e^{x} \quad \text{——— (1)}$$

$y(0) = 1$

From (1) $\quad 1 = c_1 e^{0} + c_2 e^{0}$

$$\Rightarrow c_1 + c_2 = 1 \quad \text{——— (2)}$$

From (1) $\quad y' = 2c_1 e^{2x} + c_2 e^{x}$

$y'(0) = 1$

$$1 = 2c_1 e^{0} + c_2 e^{0}$$

$$\Rightarrow 2c_1 + c_2 = 1 \quad \text{——— (3)}$$

From (2) & (3)

$$c_1 = 0$$

$$c_2 = 1$$

Now,

$$y = 0 \times e^{2x} + 1 \times e^{x}$$

$$\Rightarrow \boxed{y = e^{x}}$$ solution

Q1: let ~~2nd~~ $y = e^{2x}(c_1 \cos x + c_2 \sin x)$ be the solution of second order ~~diff.~~ ~~eqn~~ linear homogenous eq. Then Find eqn?

Soln

$$y = e^{2x}(c_1 \cos x + c_2 \sin x) \;//$$

$y = e^{\alpha x}(c_1 \cos\beta x + c_2 \sin\beta x)$ when solution of quadratic eqn is

$\alpha = 2, \quad \beta = 1$

$\alpha \pm i\beta = 2 \pm i$ root1 = $2+i$, root2 = $2-i$

$$\cancel{\lambda^2 - (\alpha+\beta)\lambda + \alpha\beta = 0}$$

$$\cancel{\Rightarrow \lambda^2 - 3\lambda + 2 = 0}$$

$$\cancel{\Rightarrow \boxed{y'' - 3y' + 2y = 0}}$$

~~diff. eqn~~

$$\lambda^2 - (\text{root1} + \text{root2})\lambda + (\text{root1} \times \text{root2}) = 0$$

$$\Rightarrow \lambda^2 - 4\lambda + 5 = 0$$

$$\Rightarrow \boxed{y'' - 4y' + 5y = 0}$$

Diff. eqn

# Cauchy Euler equation:

1 $\hookrightarrow x^2y'' + axy' + by = 0$

Characteristic equation: $m^2 + (a-1)m + b = 0$

($m$ is variable)

(1) If $m_1$ & $m_2$ are ~~dis~~ different real roots then general solution,

$$y = C_1 x^{m_1} + C_2 x^{m_2}$$

(2) If $m = m_1 = m_2$

general solution,

$$y = (C_1 + C_2 \ln x) x^m$$

(3) If $m = \mu \pm i\nu$

general solution,

$$y = x^{\mu}\left(A\cos(\nu \ln x) + B\sin(\nu \ln x)\right)$$

Q: $x^2y'' - 2xy' + 2y = 0$, $y(1) = 1.5$, $y'(1) = 1$

Find general solution

Sol$^n$

~~$m^2 + 0$~~

$x^2y'' - 2xy' + 2y = 0$

$(x^2y'' + axy' + by = 0)$

$a = -2$, $b = 2$

Characteristic equation:

$m^2 + (-2-1)m + b = 0$

$\Rightarrow m^2 - 3m + 2 = 0$

$\Rightarrow (m-2)(m-1) = 0$

$\Rightarrow \quad m = 2, 1$

general solution

$$y = C_1 x^2 + C_2 x \quad \text{——— (1)}$$

$y(1) = 1.5$

so, $\quad 1.5 = C_1 \times 1^2 + C_2 \times 1$

$\Rightarrow \quad C_1 + C_2 = 1.5 \quad \text{——— (2)}$

from (1) $\quad y' = 2C_1 x + C_2 \quad \text{——— (3)}$

$y'(1) = 1$

so, $\quad 1 = 2 \times C_1 \times 1 + C_2$

$\Rightarrow \quad 2C_1 + C_2 = 1 \quad \text{——— (4)}$

From (2) & (4)

$C_1 = -0.5$

$C_2 = 2$

Now general solution will be

$$\boxed{y = -0.5x^2 + 2x}$$

Q! $(x^2D^2 - 3xD + 4)y = 0$ Find general solution

solⁿ $(x^2D^2 - 3xD + 4)y = 0$

$\Rightarrow x^2D^2y - 3xDy + 4y = 0$

$\Rightarrow x^2y'' - 3xy' + 4y = 0$

Remember
$D^2y = y''$
$Dy = y'$

$a = -3,\ b = 4$ $(x^2y'' + axy' + by = 0)$

Characteristic equation!

$m^2 + (a-1)m + b = 0$

$\Rightarrow m^2 - 4m + 4 = 0$

$\Rightarrow (m-2)^2 = 0$

$\Rightarrow m = 2, 2$

general solⁿ $y = (c_1 + c_2 \ln x)x^m$

$\Rightarrow \boxed{y = (c_1 + c_2 \ln x)x^2}$

Q!- $x^2y'' + 3xy' + 4 = 0$, find solution.

solⁿ $x^2y'' + 3xy' + 4 = 0$ $(x^2y'' + axy' + by = 0)$

$a = 3,\ b = 4$

Characteristic eqⁿ!

$m^2 + (a-1)m + b = 0$

$\Rightarrow m^2 + 2m + 4 = 0$

$\Rightarrow m = \dfrac{-2 \pm \sqrt{4 - 4 \times 1 \times 4}}{2}$

$$\Rightarrow m = \frac{-2 \pm \sqrt{-12}}{2}$$

$$m = -1 \pm \sqrt{3}\, i$$

$$\mu = -1, \quad \nu = \sqrt{3}$$

general sol$^n$

$$y = x^{\mu}\left[A \cos(\nu \ln x) + B \sin(\nu \ln x)\right]$$

$$\boxed{y = x^{-1}\left[A \cos(\sqrt{3} \ln x) + B \sin(\sqrt{3} \ln x)\right]}$$

Q. $y'' + 4y' + 3y = 0$

8/8

sol$^n$

$$y'' + 4y' + 3y = 0$$

$\Rightarrow$ now,

$$\lambda^2 + 3\lambda + \lambda + 3 = 0$$

$$\Rightarrow (\lambda + 3)(\lambda + 1) = 0$$

$$\Rightarrow \lambda = -3, -1$$

general sol$^n$

$$\boxed{y = C_1 e^{-3x} + C_2 e^{-x}}$$