Integration and Laplace Transform Formula Cheat Sheet

Additions:

Multiplication in Laplace Transform:

If a function is multiplied by t^n, the Laplace Transform becomes:

L{t^n f(t)} = (-1)^n (d^n/ds^n) F(s)

Division in Laplace Transform:

If the function is divided by t, the Laplace Transform follows:

L{f(t)/t} = integral from s to infinity of F(u) du

Solution of Differential Equations Using Laplace:

1. Take the Laplace transform of both sides of the differential equation.

2. Solve the resulting algebraic equation in the s-domain.

3. Take the inverse Laplace transform to find the solution f(t).

Example (2nd order):

y'' + ay' + by = f(t)

s^2 Y(s) - sy(0) - y'(0) + a[sY(s) - y(0)] + bY(s) = F(s)

Dirac Delta Function in Laplace Transform:

L{delta(t - a)} = e^(-as)

Integration and Laplace Transform Cheat Sheet:

1. Power Rule for Integration:

integral of x^n dx = (x^(n+1))/(n+1) + C (for n not equal to -1)

2. Logarithmic Rule:

integral of 1/x dx = ln |x| + C

3. Integration of Constant:

integral of a dx = ax + C

4. Exponential Function:

integral of e^x dx = e^x + C

5. Trigonometric Functions:

integral of sin(x) dx = -cos(x) + C

integral of cos(x) dx = sin(x) + C

6. Logarithmic Form for Integrals:

integral of 1/(ax + b) dx = (1/a) ln |ax + b| + C

7. Hyperbolic Functions:

integral of sinh(x) dx = cosh(x) + C

integral of cosh(x) dx = sinh(x) + C

8. Gamma Function:

Gamma(n) = integral from 0 to infinity of x^(n-1) e^(-x) dx = (n-1)!

Laplace Transform Formulas:

1. L{e^(at)} = 1/(s-a)

2. L{sin(at)} = a/(s^2 + a^2)

3. L{cos(at)} = s/(s^2 + a^2)

4. L{sinh(at)} = a/(s^2 - a^2)

5. L{cosh(at)} = s/(s^2 - a^2)

Inverse Laplace Transform Formulas:

1. Inverse Laplace of 1/s = 1

2. Inverse Laplace of n!/s^(n+1) = t^n

3. Inverse Laplace of $1/(s-a) = e^{(at)}$

4. Inverse Laplace of $a/(s^2 + a^2) = \sin(at)$

5. Inverse Laplace of $s/(s^2 + a^2) = \cos(at)$