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MODULE: I PROPERTIES OF MATTER

Idea of elastic constants(Y, K, η and σ), relation between elastic constants, torsion pendulum, determination of η, Cantilever at one end.

MODULE: II OSCILLATION AND WAVES

Review of Simple Harmonic Oscillation and application to compound pendulum, Damped harmonic oscillation, Forced oscillation, Resonance, (Amplitude Resonance, Velocity resonance, Sharpness of resonance).

MODULE: III OPTICS

Concept of wave and wave equation. Superposition of many harmonic waves. Interference: Concept of coherent sources (Division of wavefront and division of amplitude) Interference in thin parallel film, Newton's ring (Theory, Application: Determination of wave length of light, Refractive index of liquid).
Concept of diffraction (Huygen's principle), types of diffraction, Fraunhoffer diffraction due to single slit and diffraction grating. Determination of wavelength, Dispersive power and resolving power of plane diffraction grating, Polarisation: Double refraction, Half wave plate and quarter wave plate.

MODULE: IV ELECTROMAGNETISM

Vector Calculus: Gradient, Divergence, Curl (Mathematical concept), Gauss divergence theorem and Stoke's theorem (statements only),

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Derivation of Maxwell's electromagnetic equation in differential form and in integral form. Electromagnetic wave equations for E and B in vacuum and conducting medium, transverse nature of EM waves.

MODULE : V QUANTUM PHYSICS & PHOTONICS

Wave particle duality. Matter waves (de Broglie Hypothesis), Wave functions, Observable as Operators. Eigen functions and Eigen values, Normalization, Expectation values, Schrodinger equation (Time dependent and time independent), Particle in a box.

Lasers: Introduction, Characteristics of Lasers, Einstein's coefficients & Relation between them, Lasing action (Population Inversion, Three and Four level pumping schemes), Different types of Lasers (Ruby Laser, He-Ne Laser).

OSCILLATIONS

Date: 20/08/19

Periodic Motion → Repeated motion in a particular (equal) int. of time.

Oscillation means one type of periodic motion that can occur when a body is subjected to a force that varies with time.

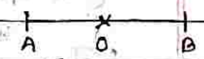
Oscillation → oscillatory motion

The body or object or particle which is under oscillatory motion is known as oscillator.

The to & fro motion about a mean position or equilibrium pos^n is known as oscillatory motion.

All oscillatory motions are periodic but all periodic motions may not be oscillatory.

Characteristics of Oscillation



Mean Position → at which no net force acts on the body.

→ It is also known as eqb^m pos^n

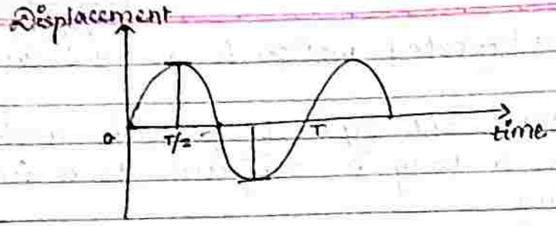
Displacement (x) is the distance of an oscillator at any instant of time from mean position.

Amplitude (A) → The max^m displ. on either side of mean pos^n is called amplitude.

Time Period (T) → The time taken to complete one oscillation.

Frequency → The no. of complete oscillations made by the oscillating body in 1 sec is called the frequency.

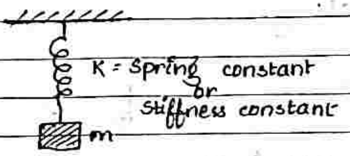
$\nu = \frac{1}{T}$



Simple Harmonic Motion (SHM)

The oscillatory motion in which the force and acceleration is directly proportional to the displacement and is oppositely directed is called the SHM.

→ The SHM is periodic & oscillatory in nature.



Let us consider a spring-mass system (oscillator) consist of a spring of spring constant K, mass m performing a SHM with displacement x at any instant of time t & the restoring force acting on it is F. Acc to the defⁿ of SHM

$$F \propto x$$

$$\Rightarrow \boxed{F = -Kx} \quad —①$$

Here $K \to$ spring constant / Force constant / stiffness const.
The -ve sign indicates that the restoring force and disp are opposite.
But from Newton's law, we know

$$F = ma \quad \text{and} \quad a = \frac{d^2x}{dt^2}$$

$$F = m\frac{d^2x}{dt^2} \quad —(2)$$

Equating the RHS of eqⁿs ① & ② we have

$$m\frac{d^2x}{dt^2} = -Kx$$

$$\frac{d^2x}{dt^2} = -\frac{K}{m}x$$

Let $\frac{K}{m} = \omega^2$ —(3) where ω = angular frequency

Using eqⁿ (3) in above eqⁿ

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

$$\text{or} \quad \boxed{\frac{d^2x}{dt^2} + \omega^2 x = 0} \quad —(4)$$

Differential Equⁿ of SHM

Since $\omega = \sqrt{\frac{K}{m}}$, but $\omega = 2\pi\nu = \frac{2\pi}{T}$

$$\frac{2\pi}{T} = \sqrt{\frac{K}{m}}$$

$$\therefore \boxed{T = 2\pi\sqrt{\frac{m}{K}}} \quad —(5)$$

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Solution of eqⁿ ④ :- $x = Ae^{i\omega t} + Be^{-i\omega t}$ A & B → arbitrary const.

or $x = A\sin(\omega t + \delta)$
or $x = A\cos(\omega t + \delta)$ } A → amp

phase, initial phase angle / epoch

x = Displacement
ωt = phase at any instant

The phase of an oscillator ~~at ant~~ is defined as the posⁿ & dirⁿ of that oscillator at any instant of time during the oscillatory motion.

Displacement: $x = A\sin(\omega t + \theta)$

Velocity: $v = \frac{dx}{dt} = \frac{d}{dt}[A\sin(\omega t + \theta)]$

$= \omega A \cos(\omega t + \theta)$

since $x = A\sin(\omega t+\theta) \Rightarrow \sin(\omega t + \theta) = \frac{x}{A}$

so $\cos(\omega t + \theta) = \sqrt{1 - \frac{x^2}{A^2}} = \frac{\sqrt{A^2 - x^2}}{A}$

$v = \omega \cancel{A} \times \frac{\sqrt{A^2-x^2}}{\cancel{A}} = \omega\sqrt{A^2-x^2}$

$v = \omega\sqrt{A^2 - x^2}$

At $x = 0 \Rightarrow v = \omega A = v_{max}$

At $x = A \Rightarrow v = 0 = v_{min}$

Acceleration: $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

$a = \frac{d}{dt}[\omega A\cos(\omega t + \theta)]$

$= -\omega^2 A \sin(\omega t + \theta) = -\omega^2 x$

At $x = 0 \Rightarrow a = 0 = a_{min}$

At $x = A \Rightarrow a = -\omega^2 A = a_{max}$

Energy of a particle executing SHM

Total energy, $E = K.E + P.E$

$KE = \frac{1}{2}mv^2 = \frac{1}{2}m[\omega\sqrt{A^2-x^2}]^2 = \frac{1}{2}m\omega^2(A^2-x^2)$

$PE = \int_0^x F dx = \int_0^x (m\omega^2 x)\, dx$ $\quad f = ma = m\omega^2 x$

$= m\omega^2 \int_0^x x\,dx = m\omega^2 \frac{x^2}{2} = \frac{1}{2} m\omega^2 x^2$

Now, $E = \frac{1}{2}m\omega^2(A^2-x^2) + \frac{1}{2}m\omega^2 x^2$

$= \frac{1}{2}m\omega^2 A^2$

$\boxed{E = \frac{1}{2}m\omega^2 A^2}$

$\boxed{KE = \frac{1}{2}m\omega^2(A^2-x^2)}$

$\boxed{PE = \frac{1}{2}m\omega^2 x^2}$

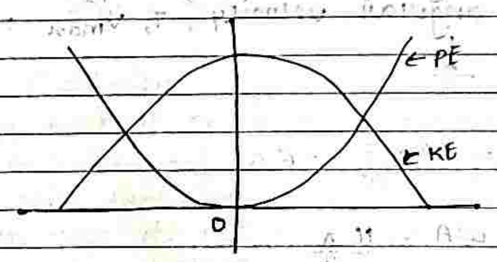
When $PE = KE$

or $\frac{1}{2}\cancel{m\omega^2}x^2 = \frac{1}{2}\cancel{m\omega^2}(A^2-x^2)$

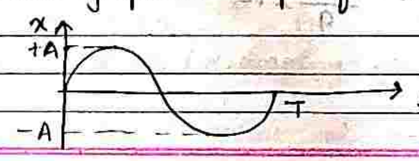
or $x^2 = A^2 - x^2$

or $2x^2 = A^2$

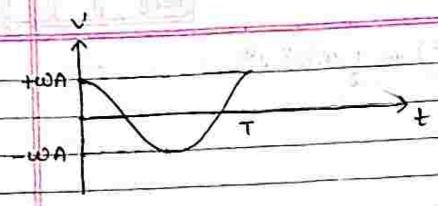
or $\boxed{x = \frac{A}{\sqrt{2}}}$



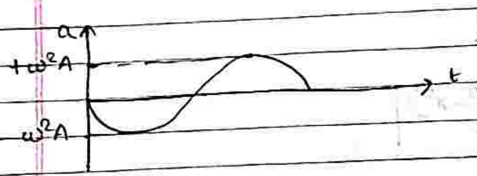
Give the graphical rep. of displ., v & a in SHM



$x = A\sin\omega t$



$v = A\omega \cos \omega t$



$a = -\omega^2 A \sin \omega t$

<u>Conditions for the occurence of SHM</u>

1. There must be a pos^n of stable eq^{bm}
2. There should not be any dissipation/loss of energy during SHM
3. The acceleration should be proportional to displacement & opposite in direction.

Q. The displ. of a particle of mass 0.2 kg executing SHM is indicated by $y = 10 \sin\left(\frac{\pi}{3}t - \frac{\pi}{12}\right)$ m. Calculate the amp., the angular velocity, T, v_{max}, a_{max}, the epoch, total energy

amp. $\to 10$, angular velocity $\to \frac{\pi}{3}$, epoch $\left(\frac{\pi}{12}\right)$

$T = \frac{2\pi}{\omega} = 2\cancel{\pi} \times \frac{3}{\cancel{\pi}} = 6$ s.

$v_{max} = \omega A = \frac{10\pi}{3}$

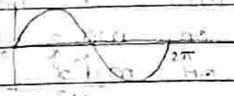
$a_{max} = -\omega^2 A = \frac{-\pi^2}{9} \times 10$

total energy $= \frac{1}{2} m\omega^2 A^2$

$= \frac{1}{\cancel{2}} \times \frac{2}{10} \times \frac{\pi^2}{9} \times 10 \times 10$

$= \frac{100}{9}$

Q. A point is executing SHM with time period 2π sec. At its mean position, its speed is 0.05 m sec^{-1}. What is its speed when it is at a distance of 0.04 m from the mean position?



$T = 2\pi$ sec $= \frac{2\pi}{\omega} \Rightarrow \omega = 1$ rad/s

$x = 0$, $v_{max} = 0.05$ m $sec^{-1} = A\omega$

$x' = 0.04$ m, $v' = ?$ $\Rightarrow A = 0.05$ m

$v = \omega\sqrt{A^2 - x^2}$

$v = 1\sqrt{\left(\frac{5}{100}\right)^2 - \left(\frac{4}{100}\right)^2} = \sqrt{\frac{25-16}{(100)^2}} = \frac{3}{100} = 0.03$ m/s.

Q. The differential eq^n of a SHM is given by $2\frac{d^2x}{dt^2} + 18\pi^2 x = 0$. Calculate the natural freq. of the body.

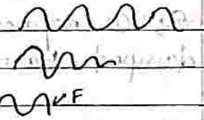
$\frac{d^2x}{dt^2} + (9\pi^2)x = 0$ $\to \omega^2$ $\Rightarrow \omega = 3\pi$

$T = \frac{2\pi}{\omega}$

$\Rightarrow \omega = 2\pi\nu$

$\Rightarrow \nu = \frac{3\pi}{2\pi} = 1.5$ Hz

Oscillation
- Free undamped oscillation
- Free damped oscillation
- Forced oscillation



27/8/19 <u>Damped Oscillation</u>

When any external or internal resistive or frictional force acts on an oscillating body the amplitude of oscillation decreases with time, and the motion of the body ultimately stops. Such type of osillation with decaying amplitude is called damped oscillation. There are two types of forces acting on the damped oscillating body. They are

i) Restoring force. $F_{res} = -kx$

ii) Resistive force / Damping force. $F_{damp} \propto v$ (vel. of oscillating body)

$F_{damp} = -bv$

where $b \to$ constant of proportionality

= damping ~~coeffecient~~ constant = damping force per unit vel.

and v = velocity

$= \frac{dx}{dt}$

Hence, $F = F_{res} + F_{damp}$

or $ma = -kx - bv$

or $m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}$

or $\frac{d^2x}{dt^2} = \frac{-k}{m}x - \frac{b}{m}\frac{dx}{dt}$ (on dividing m throughout the eq^n)

Taking, $\frac{k}{m} = \omega^2$ and $\frac{b}{m} = 2\beta$

where, β = damping ~~constant~~ coefficient

We get, $\frac{d^2x}{dt^2} = -\omega^2 x - 2\beta\frac{dx}{dt}$

or $\boxed{\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega^2 x = 0}$ —— (1)

Eq^n (1) represents the second order differential eq^n of damped harmonic motion.

Solution: Let $x = Ae^{\alpha t}$ —— (2a)

then $\frac{dx}{dt} = \alpha Ae^{\alpha t}$ —— (2b) and $\frac{d^2x}{dt^2} = \alpha^2 Ae^{\alpha t}$ —— (2c)

Using eq^{ns} (2a), (2b), (2c) in eq^n (1)

$\alpha^2 Ae^{\alpha t} + 2\beta\alpha Ae^{\alpha t} + \omega^2 Ae^{\alpha t} = 0$

or $Ae^{\alpha t}[\alpha^2 + 2\beta\alpha + \omega^2] = 0$ —— (3)

In eq^n (3), since $Ae^{\alpha t} \neq 0$,

so $\alpha^2 + 2\beta\alpha + \omega^2 = 0$ —— (4)

Roots: $\alpha = \frac{-2\beta \pm \sqrt{4\beta^2 - 4\omega^2}}{2 \cdot 1}$

$= -\beta \pm \sqrt{\beta^2 - \omega^2}$

Hence, the roots are

$\alpha_1 = -\beta + \sqrt{\beta^2 - \omega^2}$ —— (5a)

$\alpha_2 = -\beta - \sqrt{\beta^2 - \omega^2}$ —— (5b)

Now the sol^n of second order differential eq^n (1)

$x = A_1 e^{\alpha_1 t} + A_2 e^{\alpha_2 t}$ —— (6)

Putting the values of α_1 and α_2 from eq^{ns} (5a) & (5b) in eq^n (6) we get

$x = A_1 e^{[-\beta + \sqrt{\beta^2 - \omega^2}]t} + A_2 e^{[-\beta - \sqrt{\beta^2 - \omega^2}]t}$

$\boxed{x = e^{-\beta t}\left[A_1 e^{(\sqrt{\beta^2 - \omega^2})t} + A_2 e^{(-\sqrt{\beta^2 - \omega^2})t}\right]}$ —— (7)

Eq^n (7) represents the sol^n of a damped oscillatory motion. Here A_1 and A_2 are the arbitrary constants whose values can be determined on putting the boundary conditions.

Conditions:

Based on the relationship between β and ω there are 3 conditions in a damped oscillation.

1) Underdamping condition, $\beta^2 < \omega^2 \rightarrow$ oscillatory in nature
2) Critical damping, $\beta^2 = \omega^2 \rightarrow$ Non-oscillatory
3) Overdamping, $\beta^2 > \omega^2 \rightarrow$ Non-oscillatory

Condition 1 : Underdamping $(\beta^2 < \omega^2)$

In the underdamping condition the damping force is weak.

So $\beta^2 < \omega^2$

and $\beta^2 - \omega^2$ is negative

Let $\beta^2 - \omega^2 = -\omega_1^2$

Further, $\beta^2 - \omega^2 = i^2 \omega_1^2$

and $\sqrt{\beta^2 - \omega^2} = i\omega_1$ —— (8)

Using eq^n (8) in eq^n (7) we get the displacement

$$x = e^{-\beta t}\left[A_1 e^{i\omega_1 t} + A_2 e^{-i\omega_1 t}\right]$$

Now on expanding we get

$$x = e^{-\beta t}\left[A_1(\cos\omega_1 t + i\sin\omega_1 t) + A_2(\cos\omega_1 t - i\sin\omega_1 t)\right]$$

$$= e^{-\beta t}\left[\cos\omega_1 t\,(A_1 + A_2) + i\sin\omega_1 t\,(A_1 - A_2)\right]$$

Substituting, $A_1 + A_2 = A_0 \cos\phi$

and $i(A_1 - A_2) = A_0 \sin\phi$

$$x = x(t)$$
$$= e^{-\beta t}\left[\cos\omega_1 t \cdot A_0\cos\phi + \sin\omega_1 t \cdot A_0 \sin\phi\right]$$
$$= A_0 e^{-\beta t}\left[\cos\omega_1 t \cdot \cos\phi + \sin\omega_1 t \cdot \sin\phi\right]$$
$$= A_0 e^{-\beta t} \cdot \cos(\omega_1 t - \phi)$$

Hence the sol^n of underdamped oscillation is given by

$$x(t) = A_0 e^{-\beta t} \cos(\omega_1 t - \phi)$$

$$\boxed{x(t) = A_0 e^{-\beta t} \cos\left[\sqrt{(\omega^2 - \beta^2)}\, t - \phi\right]} \quad (9)$$

Eq^n (9) represents the sol^n of a damped harmonic oscillator with amplitude $A_0 e^{-\beta t}$ and time period

$$T = \frac{2\pi}{\omega_1} = \frac{2\pi}{\sqrt{\omega^2 - \beta^2}}$$

Condition 3 : Over damping $(\beta^2 > \omega^2)$

So $\beta^2 - \omega^2 = +ve$ and real qty.

The quantities $(-\beta + \sqrt{\beta^2 - \omega^2})$ and $(-\beta - \sqrt{\beta^2 - \omega^2})$ are negative quantities. Hence as t increases the displacement of the particle decreases and finally becomes zero. This type of motion is called overdamped or dead beat.

28/8/19 Condition II : Critical Damping

In critical damp condition $\omega^2 = \beta^2$

$$x(t) = e^{-\beta t}[A_1 + A_2] \quad \text{when } \sqrt{\beta^2 - \omega^2} = 0$$

If we take, $\omega^2 \approx \beta^2 \Rightarrow \sqrt{\beta^2 - \omega^2} \to 0$ or $\sqrt{\beta^2 - \omega^2} \approx h$

h = negligibly small qty.

Now the eq^n for displacement can be expressed as

$$x = e^{-\beta t}\left[A_1 e^{ht} + A_2 e^{-ht}\right]$$

Now expanding the exponential terms of the above eq^n we have

$$x = e^{-\beta t}\left[A_1(1 + ht + \dots) + A_2(1 - ht + \dots)\right]$$

$$= e^{-\beta t}\left[(A_1 + A_2) + ht(A_1 - A_2) + \dots\right]$$

Since h is very small so we can neglect the higher order terms and thus we have

$$x = e^{-\beta t}\left[(A_1 + A_2) + h(A_1 - A_2)t\right]$$

Putting $A_1 + A_2 = B$,

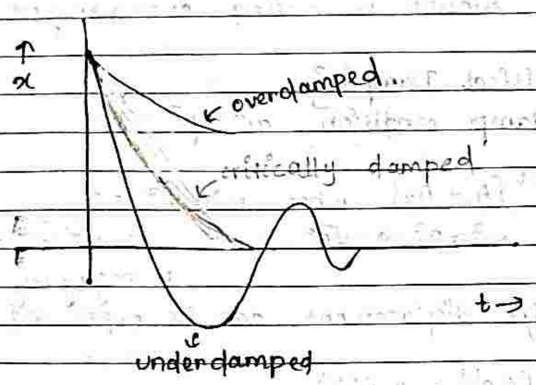
$h(A_1 - A_2) = C$

we have $\boxed{x = e^{-\beta t}[B+Ct]}$

This is the eq^n for displacement under critical damp condition.

In this case, when t increases the second term, $B+Ct$ increase whereas at the same time $e^{-\beta t}$ decreases and thus it lead to the critical damping condition.

So whenever we plot a graph between x and t, we have



Attenuation Parameters in damped Oscillation.

There are different attenuation parameters associated with underdamped oscillatory motion. They are

1) Logarithmic decrement (λ)
2) Quality factor (Q)
3) Relaxation time (τ)

Logarithmic decrement

In underdamped oscillation, $x = A_0 e^{-\beta t}\cos\left[\sqrt{(\omega^2-\beta^2)}\,t - \phi\right]$

The amplitude $= A_0 e^{-\beta t}$

Logarithmic decrement measures the rate at which the amplitude of damped oscillation dies away or decays.

At $t=0$, amplitude $= A_0$

At $t=T$, $A_1 = A_0 e^{-\beta T}$

At $t=2T$, $A_2 = A_0 e^{-2\beta T}$

⋮

where T = Period of oscillation

Now, on taking the ratio between successive amplitudes we have

$$\frac{A_0}{A_1} = \frac{A_0}{A_0 e^{-\beta T}} = e^{\beta T}$$

$$\frac{A_1}{A_2} = \frac{A_0 e^{-\beta T}}{A_0 e^{-2\beta T}} = e^{\beta T}$$

$$\frac{A_2}{A_3} = e^{\beta T}$$

$$\frac{A_{n-1}}{A_n} = e^{\beta T}$$

$$\therefore \frac{A_0}{A_1} = \frac{A_1}{A_2} = \frac{A_2}{A_3} = \ldots = e^{\beta T} = e^{\lambda} \text{ (say)}$$

where $\lambda = \beta T$

Taking the natural logarithm of the above eq^n we get

$$\boxed{\lambda = \log_e \frac{A_0}{A_1} = \log_e \frac{A_1}{A_2} = \ldots}$$

This is the expression for logarithmic decrement.

Hence logarithmic decrement is defined as the logarithm of the ratio between two successive amplitudes of oscillation which are separated by one time period.

Quality Factor

The quality factor of an oscillator measures the rate at which the energy decays. Numerically it is expressed as

$$Q = 2\pi \frac{\text{Energy stored}}{\text{Energy lost}}$$

The mechanical energy of a damped harmonic oscillator is given as

$$E = \frac{1}{2} m\omega^2 A^2$$

but $A_{\text{under damped}} = A_0 e^{-\beta t}$

$$E_{\text{under damped}} = \frac{1}{2} m\omega^2 A_0^2 e^{-2\beta t} = E(t)$$

Putting, $K = m\omega^2 \Rightarrow E_{\text{under damped}} = \frac{1}{2} K A_0^2 e^{-2\beta t} = E_0 e^{-2\beta t}$

Hence, amplitude of energy, $E_0 = \frac{1}{2} K A_0^2$

The average power dissipated/lost during one cycle is expressed as

$$\langle P(t) \rangle = \frac{d}{dt} \langle E(t) \rangle = \frac{d}{dt}\left(E_0 e^{-2\beta t}\right)$$

$$= -2\beta E_0 e^{-2\beta t} = -2\beta \langle E(t) \rangle = -2\beta E$$

Now, the quality factor

$$Q = 2\pi \frac{E}{P \cdot T} = 2\pi \frac{E}{(2\beta E)(T)}$$

$$\boxed{Q = \frac{\pi}{\beta T}}$$

As $T = \frac{2\pi}{\omega}$

$$Q = \frac{\pi}{\beta \cdot \frac{2\pi}{\omega}} = \frac{\omega}{2\beta}$$

3) Relaxation Time (τ)

It is defined as the time required for the amplitude to fall by a factor of $\frac{1}{e}$.

In other words, when $A = \frac{A_0}{e}$,

then $t = \tau$

As we know, $A = A_0 e^{-\beta t}$ the ampl. of under damped osc.

At $t = \tau$, $A = \frac{A_0}{e}$

$$\frac{A_0}{e} = A_0 e^{-\beta \tau}$$

$$e^{-1} = e^{-\beta \tau}$$

$$\Rightarrow \beta\tau = 1 \Rightarrow \boxed{\tau = \frac{1}{\beta}}$$

Express the relaxation time in terms of the quality factor.

$$Q = \frac{\omega}{2\beta} \Rightarrow \beta = \frac{\omega}{2Q}$$

$$\tau = \frac{1}{\beta} = \frac{2Q}{\omega}$$

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Forced Oscillation

It is defined as the oscillatory motion in which the body oscillates with a freq. other than its natural freq. under the action of an external periodic force.

When an external periodic force (driving force) is applied to a body (driven oscillator), then in the body which was vibrating with its own frequency would be driven to vibrate with the frequency of the driving force. To maintain the oscillation in presence of damping external energy related to the periodic force must be applied from an external source.

In forced oscillation the forces acting on a body (the oscillator) :

1) Restoring force

$$F_{res} = -kx \quad \text{— (1a)}$$

2) Damping Force

$$F_{damp} = -bv = -b\frac{dx}{dt} \longrightarrow (1b)$$

3) External Periodic Force

$$F_{ext} = F_0 \sin pt \longrightarrow (1c)$$

where, k = force constant

b = damping coefficient

F_0 = amplitude of externally applied periodic force

p = angular freq. of the applied ext. force

Let m = mass of the oscillating body

The total force in this case

$$F = F_{res} + F_{damp} + F_{ext}$$

or $$ma = -kx - bv + F_0 \sin pt$$

but $a = \frac{d^2x}{dt^2}$

x = displ.

$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt} + F_0 \sin pt$$

or $$\frac{d^2x}{dt^2} = \frac{-k}{m}x - \frac{b}{m}\frac{dx}{dt} + \frac{F_0}{m}\sin pt \quad \text{— (2)}$$

Putting, $\frac{k}{m} = \omega^2$ — (3a)

$\frac{b}{m} = 2\beta$ — (3b)

$\frac{F_0}{m} = f$ — (3c)

Using equations (3a), (3b) and (3c) in equation (2), we get

$$\frac{d^2x}{dt^2} = -\omega^2 x - 2\beta\frac{dx}{dt} + f\sin pt$$

or $$\boxed{\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega^2 x = f\sin pt} \quad \text{— (4)}$$

Eq^n (4) represents the second order differential equation of forced oscillation.

Sol^n of (4) :-

Let $x = A\sin(pt-\theta)$

$$\frac{dx}{dt} = pA\cos(pt-\theta)$$

$$\frac{d^2x}{dt^2} = -p^2 A\sin(pt-\theta)$$

Putting these values in eq^n (4) we get

$$-p^2 A\sin(pt-\theta) + 2\beta A p\cos(pt-\theta) + \omega^2 A\sin(pt-\theta) = f\sin pt$$
$$= f\sin(pt-\theta+\theta)$$

$$= f[\sin\{(pt-\theta)+\theta\}]$$
$$= f[\sin\theta\cos(pt-\theta) + \cos\theta\sin(pt-\theta)]$$

$$A(\omega^2-p^2)\sin(pt-\theta) + 2\beta A p\cos(pt-\theta) = f[\sin(pt-\theta).\cos\theta + \cos(pt-\theta).\sin\theta]$$
$$= f\cos\theta.\sin(pt-\theta) + f\sin\theta.\cos(pt-\theta) \quad \text{—(5)}$$

Comparing the coefficients from both sides of eq^n (5), we get

$$f\cos\theta = A(\omega^2-p^2) \quad \text{—(6a)}$$
$$f\sin\theta = 2\beta A P \quad \text{—(6b)}$$

Now, $f = \sqrt{f^2.1}$
$$= \sqrt{f^2(\sin^2\theta+\cos^2\theta)}$$
$$f = \sqrt{f^2\cos^2\theta + f^2\sin^2\theta}$$
$$= \sqrt{[A(\omega^2-p^2)]^2 + (2\beta AP)^2}$$
$$\boxed{f = A\sqrt{(\omega^2-p^2)^2+4\beta^2p^2}} \quad \text{—(7)}$$

Now, v.v.i
$$\boxed{A = \frac{f}{\sqrt{(\omega^2-p^2)^2+4\beta^2p^2}}} \quad \text{—(8)}$$

Expression for amplitude of a body performing forced oscillatory motion.

$$\tan\theta = \frac{f\sin\theta}{f\cos\theta} = \frac{2\beta \cancel{A} P}{\cancel{A}(\omega^2-p^2)} = \frac{2\beta P}{(\omega^2-p^2)}$$

Phase, $\boxed{\theta = \tan^{-1}\frac{2\beta P}{(\omega^2-p^2)}}$ —(9)

eq^n (9) represents the phase of forced oscillation.

Now the sol^n of forced oscillation can also be expressed as

$$x = A\sin(pt-\theta)$$
$$\boxed{x = \frac{f}{\sqrt{(\omega^2-p^2)^2+4\beta^2p^2}}\sin(pt-\theta)} \quad \text{—(10)}$$

10/9/19 Phase Difference between driven Oscillator and driving force

Suppose the angular frequency p of the applied periodic force is gradually increased from 0 to ∞, then

i) When $p=0$, $\tan\theta = 0 \Rightarrow \theta = 0$
Thus there is no difference of phase between the driven oscillator and driving force.

ii) When $p=\omega$,
$\tan\theta = \infty \Rightarrow \theta = \frac{\pi}{2}$ (Resonance cond^n)

At resonance cond^n, the driven oscillator lags behind the driving force by an angle of $\frac{\pi}{2}$.

iii) When $p<\omega$, $\tan\theta = +ve$
This implies that the difference of phase has a value in between 0 & $\frac{\pi}{2}$.

iv) When $p>\omega$, $\tan\theta = -ve$
which indicates θ lies between $\frac{\pi}{2}$ and π.

$$\frac{\pi}{2} < \theta < \pi$$

Thus for all values of frequencies (p) of the applied periodic force θ lies in between 0 and π and at resonance $\theta = \frac{\pi}{2}$.

Resonance

Resonance is the condition in forced oscillation at which the driven oscillator oscillates with the same frequency as that of the driving force. At resonance condition the amplitude, velocity and the corresponding energy attain maximum value.

1) Amplitude Resonance

In forced oscillatory motion, amplitude A is given by

$$A = \frac{f}{\sqrt{(\omega^2-p^2)^2+4\beta^2p^2}}$$

When $A \to A_{max}$, then $\frac{d}{dp}\left[(\omega^2-p^2)^2+4\beta^2p^2\right]=0$

or $2(\omega^2-p^2)\times(-2p) + 4\beta^2\times 2p = 0$

or $-4p(\omega^2-p^2) + 8\beta^2 p = 0$

or $\omega^2-p^2 = 2\beta^2 \Rightarrow \boxed{p = \sqrt{\omega^2-2\beta^2}}$

It is the expression for the frequency of externally applied periodic force at resonance condition.

$$A_{max} = \frac{f}{\sqrt{(\omega^2-\omega^2+2\beta^2)^2+4\beta^2p^2}}$$

$$= \frac{f}{\sqrt{4\beta^4+4\beta^2p^2}} = \frac{f}{2\beta\sqrt{\beta^2+p^2}}$$

$$= \frac{f}{2\beta\sqrt{\beta^2+\omega^2-2\beta^2}}$$

$$= \frac{f}{2\beta\sqrt{\omega^2-\beta^2}}$$

We can take, $A_{max} \to A_{reso}$

$$\boxed{A_{reso} = \frac{f}{2\beta\sqrt{\omega^2-\beta^2}}}$$

Spl. Case :-

When the damping force is very small then β is also very small. At this condition

$$A_{reso} \approx \frac{f}{2\beta\sqrt{\omega^2}} \approx \frac{f}{2\beta\omega}$$

2) Velocity Resonance

When the amplitude of velocity of the forced oscillation under the action of impressed periodic force becomes max. then the phenomenon is known as velocity resonance. In a forced oscillatory motion the displ. of oscillating body is given by

$$x = A\sin(pt-\theta)$$

$$\text{Velocity, } v = \frac{dx}{dt} = Ap\cos(pt-\theta)$$

The peak value of velocity amplitude

$$v_0 = A.p$$

$$= \frac{f.p}{\sqrt{(\omega^2-p^2)^2+4\beta^2p^2}} = \frac{f.p}{p\sqrt{\left(\frac{\omega^2-p^2}{p}\right)^2+4\beta^2}}$$

$$v_0 = \frac{f}{\sqrt{\left(\frac{\omega^2}{p}-p\right)^2+4\beta^2}}$$

As p varies the velocity v also changes. Furthere when $\omega = p$ the denominator of the above eqⁿ becomes min. and v_0 attains the max. value $(v_0)_{max}$.

$$\text{At } \omega \approx p, \quad (v_0)_{max} = \frac{f}{\sqrt{4p^2}} = \frac{f}{2\beta}$$

$\therefore$ Velocity resonance occurs when the natural frequency of the body becomes equal to the frequency of applied periodic force.

Sharpness of Resonance

The condⁿ of resonance is given by $p = \sqrt{\omega^2 - 2\beta^2}$.
If the freq. changes from this value the amplitude falls. When the fall of amplitude for a small departure from the resonance condition is very large the resonance is said to be sharp. Hence sharpness of resonance can be defined as the rate of fall of amplitude with the change of frequency of the driving force.

$$A_{reso} = \frac{f}{2\beta\omega}$$

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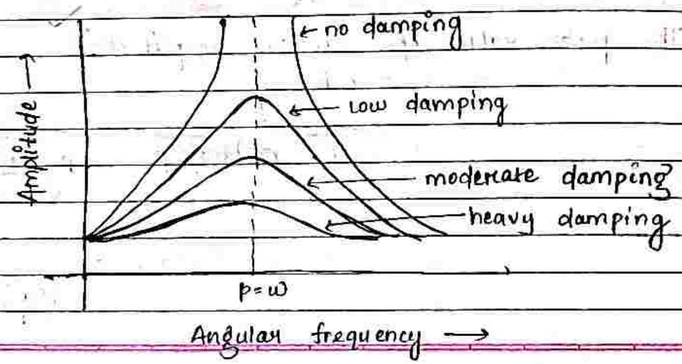
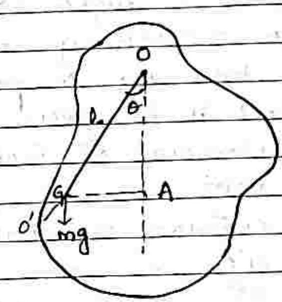


At resonance, $A_{reso} = \frac{f}{2\beta\omega}$

Compound Pendulum

A compound pendulum is a rigid body capable of oscillating freely about a horizontal axis passing through it. A rigid body may be of any shape & any internal structure.

L → length of simp. pen.



compound pendulum.

Let us consider a compound pendulum of mass 'm' vibrating on making an angle θ about a horizontal axis passing through it. Here G is the center of gravity of the pendulum & L is the distance between the points O and G (OG).

Here the point O represents the centre of suspension. It is the point where the axis of vibration meets the vertical plane through the centre of oscillation.

The centre of oscillation is a point at a distance equal to the length of equivalent simple pendulum from the point of suspension.

When the pendulum is displaced from its position of rest so that OG makes an angle θ with OA then the weight of the pendulum 'mg' acting vertically downward direction, produces an equal and opposite reaction at 'O'. Further this is represented by the restoring torque.

∴ So the restoring torque (τ) due to weight 'mg' of the compound pendulum is given by $mgl\sin\theta$ —(1a)

During oscillation θ is very small for which $\sin\theta \approx \theta$. Hence, $\tau = mgl\theta$ —(1b)

To continue the oscillatory motion we observe that the torque brings back the compound pendulum to its original position and it is given by $\tau = -I\alpha$ →(2) where,

I = Moment of inertia $= mK^2$

α = Angular acceleration $= \frac{d^2\theta}{dt^2}$

Comparing eqns (1b) and (2), we get

$$mgl\theta = -I\alpha = -mK^2 \frac{d^2\theta}{dt^2}$$

or $$mK^2 \frac{d^2\theta}{dt^2} + mgl\theta = 0$$

or $$\frac{d^2\theta}{dt^2} + \frac{gl}{K^2}\theta = 0 \quad \text{—(3a)}$$

This eqn is equivalent to the second order differential eqn of SHM

$$\frac{d^2\theta}{dt^2} + \omega^2\theta = 0 \quad \text{—(3b)}$$ θ = Angular displ.

Comparing eqns (3a) & (3b) we can put $\omega^2 = \frac{gl}{K^2}$, but

time period $= T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{gl}{K^2}}} = 2\pi\sqrt{\frac{K^2}{gl}}$

So, $$\boxed{T = 2\pi\sqrt{\frac{K^2}{lg}}} \quad \text{—(4)}$$

Here $K \rightarrow$ radius of gyration

Further, on considering the parallel axis theorem we can take

$$mK^2 = mk^2 + ml^2$$

and $$\boxed{K^2 = k^2 + l^2} \quad \text{—(5)}$$

∵ The radius of gyration 'K' ~~ab~~ of a body about an axis of rotation is the radial distance at which the mass of the body is concentrated without altering the moment of inertia of that body about that axis.

Eqn (4) can be modified as

$$T = 2\pi\sqrt{\frac{k^2 + l^2}{lg}} = 2\pi\sqrt{\frac{\left(\frac{k^2}{l} + l\right)}{g}} = 2\pi\sqrt{\frac{L}{g}}$$

$$\boxed{T = 2\pi\sqrt{\frac{L}{g}}} \quad \text{—(6)}$$ Time period of compound pendulum.

L = Equivalent length of simple pendulum.

g = accn due to gravity.

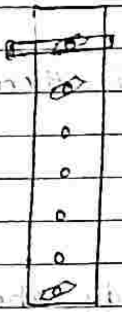
$$\boxed{L = \frac{K^2}{l} + l}$$ length of equivalent simple pendulum.

Different forms of Compound Pendulum

There are diff. eg. of compound pendulum. Among them the widely used compound pendulum are

1) Bar Pendulum

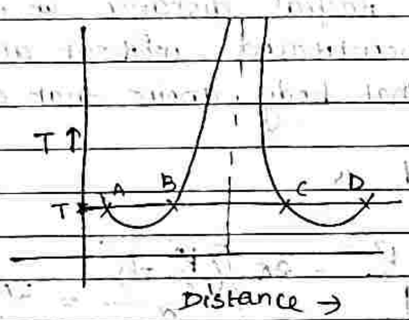
2) Kater's Pendulum



$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$T^2 = 4\pi^2\frac{L}{g}$$

$$\boxed{g = \frac{4\pi^2 L}{T^2}}$$



$$\frac{AC + BD}{2} = L$$

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The Kater's Pendulum is a compound pendulum consisting of a metal (iron, copper or brass) bar having two knife edges K_1 & K_2 facing each other on either side of the centre of gravity of the bar. A heavy weight W is fixed below the knife edge K_2. Two other weights W_1 & W_2 are clamped on the bar and capable of sliding along the bar. W_2 is heavier than W_1.

The lighter weight W_1 is provided with a micrometer screw for final adjustment in either direction. With the help of W_1, W_2 & W the centre of gravity of the system can be fixed and readings are to be taken. The acceleration due to gravity in this system can be calculated on using the following eqⁿ :-

$$g = \frac{8\pi^2}{\left(\frac{T_1^2 + T_2^2}{l_1 + l_2}\right) + \left(\frac{T_1^2 - T_2^2}{l_1 - l_2}\right)},$$

where g = acc. due to gravity

T_1 = Time period about K_1

T_2 = Time period about the knife edge K_2

l_1 = distance of the knife edge K_1 from the CG.

l_2 = distance " " " " K_2 " " "

PROPERTIES OF MATTER

Property → Response of a material w.r.t applied stimulus

Mechanical Property → " " " " " " force/load

Deformation
- Temporary → Elastic Materials → Elasticity
- Permanent → Plastic " → Plasticity

Important terms associated with elasticity

1) Stress = $\frac{\text{Force}}{\text{Area}}$

Types of stress

1) Normal stress 2) Tangential stress
3) Tensile stress 4) Compressive stress
5) Bending " 6) Shear "
7) Twisting "

2) Strain Ratio between change in dimension to the original dimension.

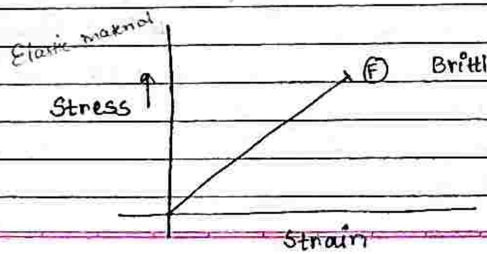
Types of strains

1) Longitudinal or linear strain 2) Volumetric strain
3) Shear strain

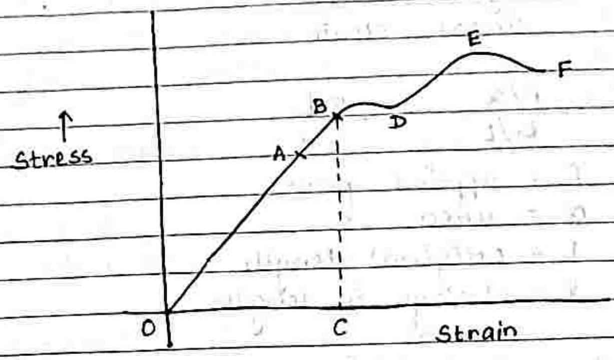
3) Hooke's law

Within elastic limit, stress ∝ strain

and $\frac{\text{stress}}{\text{strain}} = E$ = Modulus of Elasticity = Coefficient of elasticity



4) Stress-Strain Curve



OA = Elastic limit
AB = Partially elastic and partially plastic
BC = Residual strain
B = Yield Point
DE = Plastic region
F = Fracture point = Breaking point

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Modulus of elasticity = $\frac{\text{Stress}}{\text{strain}}$

Corresponding to different types of strains there are 3 types of moduli of elasticity

1. Linear elasticity or Young's modulus of elasticity corresponding to linear strain. (Y)

2. Elasticity of volume or Bulk modulus → Volume strain (K)

3. Elasticity of shape or shear modulus or modulus of rigidity → shear strain (η)

1) Young's Modulus of Elasticity

$$Y = \frac{\text{longitudinal stress}}{\text{linear strain}}$$

$$Y = \frac{F/a}{l/L} = \frac{F.L}{a.l}$$

where F = applied force
a = area
L = original length
l = change in length

2) Bulk Modulus of Elasticity

$$K = \frac{F/a}{v/V}$$

$$K = \frac{F.V}{a.v} = \frac{P.V}{v}$$

$$P = \text{Pressure} = \frac{F}{a}$$

V = Original volume
v = Change in volume

Bulk modulus is also called as incompressibility and its reciprocal is called as compressibility

3) Rigidity Modulus of Elasticity → Torsion modulus

$$\eta = \frac{F/a}{\theta} = \frac{F/a}{l/L} = \frac{FL}{a.l}$$

In this case, there is a change in the shape of the body but no change in its volume. The above relationship is exactly similar to the Young's Modulus of Elasticity with the only difference that here F/a is the tangential stress and the displacement l is at right angle to L whereas in Young's modulus of rigidity F/a is the longitudinal stress and $\frac{l}{L}$ is the longitudinal displ (either tension or compression)

* Spl Note

$$\text{Work done} = \int_0^l F.dl$$

(i) Elongation strain, $Y = \frac{F.L}{a.l} \Rightarrow F = \frac{Yal}{L}$

$$W = \int_0^l \frac{Ya}{L}.l\,dl = \frac{Ya}{L}.\frac{l^2}{2} = \frac{1}{2}\frac{Yal}{L}.l = \frac{1}{2}F.l$$

$$\text{Work done per unit volume} = \frac{\frac{1}{2}F.l}{La} = \frac{1}{2}.\frac{F}{a}.\frac{l}{L}$$

$$= \frac{1}{2} \times \text{stress} \times \text{strain}$$

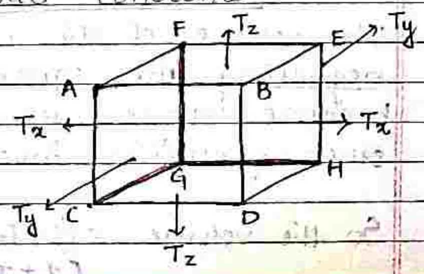
Valid for all types of strains

$$\boxed{\text{Work done per unit volume} = \frac{1}{2}\cdot\text{stress}\cdot\text{strain}}$$

↑

$Y \rightarrow$ long. strain
$K \rightarrow$ vol. "
$\eta \rightarrow$ shear "

Relationship between elastic constants



Let us consider a cube ABCDEFGH of unit length and let the forces T_x, T_y and T_z act perpendicularly to the faces as follows.

$T_x \longrightarrow$ BEHD, AFGC
$T_y \longrightarrow$ ABDC, EFGH
$T_z \rightarrow$ ABEF, CDHG

If α = increase in length per unit tension along the direction of the force and,
β = contraction produced per unit length per unit tension in a direction perpendicular to the applied force.

Thus the elongation produced in the edges AB, BE and BD will be $T_x\alpha$, $T_y\alpha$, $T_z\alpha$ respectively and the contraction produced perpendicular to the above edges will be $T_x\beta$, $T_y\beta$ and $T_z\beta$ respectively.
$T_y\beta, T_z\beta$; $T_y\beta, T_z\beta$ and $T_x\beta$ $T_y\beta$

The lengths of the edges of the cube become:

$AB = 1 + T_x\alpha - T_y\beta - T_z\beta$
$BE = 1 + T_y\alpha - T_x\beta - T_z\beta$
$BD = 1 + T_z\alpha - T_x\beta - T_y\beta$

Hence, the volume of the cube is $= AB \times BE \times BD$
$= (1+T_x\alpha - T_y\beta - T_z\beta)(1+T_y\alpha - T_x\beta - T_z\beta)(1+T_z\alpha - T_x\beta - T_y\beta)$

The volume of the cube is to be determined on neglecting the squares and products of α and β, because those are very small compare to the other quantities involved.

So the volume $= (1+T_x\alpha - T_y\beta - T_z\beta)(1+T_y\alpha - T_x\beta - T_z\beta)(1+T_z\alpha - T_x\beta - T_y\beta)$

$= (1+T_x\alpha - T_y\beta - T_z\beta + T_y\alpha - T_x\beta - T_z\beta)\ (1+T_z\alpha - T_x\beta - T_y\beta)$
+ higher order terms + multipl of α, β

$= 1+T_z\alpha - T_x\beta - T_y\beta + T_x\alpha - T_y\beta - T_z\beta + T_y\alpha - T_x\beta - T_z\beta$
+ higher order terms + multipl of α, β

$= 1 + T_x\alpha + T_y\alpha + T_z\alpha - 2T_x\beta - 2T_y\beta - 2T_z\beta$

$= 1 + \alpha(T_x + T_y + T_z) - 2\beta(T_x + T_y + T_z)$

$= 1 + (\alpha - 2\beta)(T_x + T_y + T_z)$

If we assume $T_x = T_y = T_z = T$ (say) then the above eq^n will be modified as
Volume $= 1 + [(\alpha - 2\beta) \times 3T]$

Thus the increase in volume of the cube
$= \cancel{1} + 3T(\alpha - 2\beta) - \cancel{1} = 3T(\alpha - 2\beta)$

* Spl. Note

Instead of the tension T in the outward direction if we apply a pressure P to compress the cube then the reduction in volume is found to be $3P(\alpha - 2\beta)$,
P = Pressure

19/9/19 ∴ Volume of the unit cube $= \frac{3P(\alpha - 2\beta)}{1} = 3P(\alpha - 2\beta)$

Now, the Bulk modulus, $K = \frac{\text{Stress}}{\text{Volume strain}} = \frac{P}{3P(\alpha - 2\beta)}$

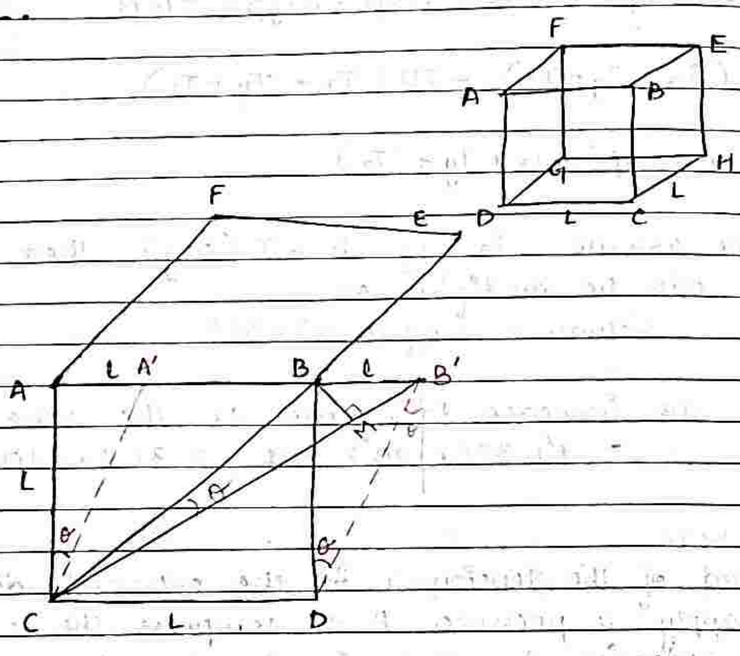
or $K = \frac{1}{3(\alpha - 2\beta)}$

Compressibility → Reciprocal of Bulk Modulus

$= \frac{1}{K} = 3(\alpha - 2\beta)$

Modulus of Rigidity

Let us consider a cube ABCDEFGH with edge length L.



Let us consider the top face ABEF of the cube be sheared by a shearing force F related to the bottom face CDGH such that A takes up the position A' and B takes up the position B' respectively by making an angle θ with the original edges.

$\angle ACA' = \theta = \angle BDB'$

The tensile stress $= \frac{F}{\text{area of the face ABEF}} = \frac{F}{L^2} = T$ (say)

Let the displacement $AA' = BB' = l$

Shear strain $= \frac{l}{L} = \theta$

Rigidity modulus, $\eta = \frac{T}{\theta}$

We can see that a shearing stress along AB is equivalent to an equal linear tensile stress along the diagonal CB and an equal compressive stress along AD at right angles to each other.

The tensile stress T along CB = Compressive stress T along AD

Let α = longitudinal strain per unit stress
β = lateral " " " "

Extension of diagonal CB due to tensile stress
$= CB \cdot T \cdot \alpha$

Extension or change in length of AD due to compressive stress $= CB \cdot T \cdot \beta$

Total extension in length of CB $= CB(T\cdot\alpha + T\cdot\beta)$
$= CB \times T(\alpha + \beta)$
$= L\sqrt{2} \times T(\alpha + \beta)$

Drop a perpendicular BM from CB to CB' so that the increase in length of CB is MB'.

$$MB' = L\sqrt{2} \times T(\alpha+\beta)$$

Here we can find in $BB'M$ $\triangle$,

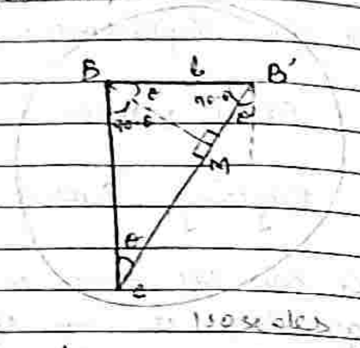
$\cos(90-\theta) = \frac{MB'}{BB'}$

or $\sin\theta = \frac{MB'}{BB'}$

or $MB' = BB' \sin\theta$

$= BB' \sin 45^\circ$

$= \frac{l}{\sqrt{2}}$



$B'M = L\sqrt{2} \times T(\alpha+\beta) = \frac{l}{\sqrt{2}}$

or $\frac{T}{(l/L)} = \frac{1}{2(\alpha+\beta)}$

or $\frac{T}{\theta} = \frac{1}{2(\alpha+\beta)}$

but $\frac{T}{\theta} = \eta$

$$\boxed{\eta = \frac{1}{2(\alpha+\beta)}}$$

Derivation practice karna hi

Young's Modulus $Y = \frac{\text{stress}}{\text{linear strain}}$

On considering a unit stress for the linear strain or extension we have

$$\boxed{Y = \frac{1}{\alpha}}$$

Relations connecting the elastic constants

We have, Bulk modulus, $K = \frac{1}{3(\alpha-2\beta)}$ —— ①

Rigidity modulus, $\eta = \frac{1}{2(\alpha+\beta)}$ —— ②

Young's modulus, $Y = \frac{1}{\alpha}$ —— ③

From eq^n ①, we have

$\alpha - 2\beta = \frac{1}{3K}$ —— (1a)

② → $\alpha+\beta = \frac{1}{2\eta}$ —— (2a)

(2a) − (1a) ⇒ $\alpha+\beta-\alpha+2\beta = \frac{1}{2\eta} - \frac{1}{3K}$

or $3\beta = \frac{3K-2\eta}{6\eta K}$

⇒ $\boxed{\beta = \frac{3K-2\eta}{18\eta K}}$ —— ④

$2\times(2a) + (1a)$ ⇒ $2\alpha + \cancel{2\beta} + \alpha - \cancel{2\beta} = \frac{1}{\eta} + \frac{1}{3K}$

or $3\alpha = \frac{3K+\eta}{3K\eta}$

$\boxed{\alpha = \frac{3K+\eta}{9K\eta}}$ —— ⑤

Since $Y = \frac{1}{\alpha} = \frac{9K\eta}{3K+\eta}$

or $\frac{3K+\eta}{K\eta} = \frac{9}{Y}$

$\boxed{\frac{3}{\eta} + \frac{1}{K} = \frac{9}{Y}}$ —— ⑥ → Rel^n ship between Y, K, η

Relation of Y, K and η in terms of Poisson's ratio (σ)

Poisson's ratio is defined as the ratio between the lateral strain to the longitudinal strain.

$$\sigma = \frac{\beta}{\alpha}$$

σ = P.R.
β = lat. strain
α = long. strain

Since $K = \frac{1}{3(\alpha-2\beta)} \Rightarrow 3K(\alpha-2\beta) = 1$

or $3K\left(1-2\frac{\beta}{\alpha}\right) = \frac{1}{\alpha}$

$\Rightarrow \boxed{3K(1-2\sigma) = Y} \rightarrow (Y, K, \sigma)$

Shear strain $= 2(\alpha+\beta)P$

$$\eta = \frac{\text{Shear stress}}{\text{shear strain}}$$

$$\eta = \frac{1}{2(\alpha+\beta)}$$

or $2\eta(\alpha+\beta) = 1$

$\Rightarrow 2\eta\left(1+\frac{\beta}{\alpha}\right) = \frac{1}{\alpha}$

$\Rightarrow \boxed{2\eta(1+\sigma) = Y} \rightarrow (Y, \eta, \sigma)$

Since, $Y = 3K(1-2\sigma)$, $Y = 2\eta(1+\sigma)$

or $\frac{Y}{3K} = 1-2\sigma$ and $\frac{Y}{\eta} = 2(1+\sigma) = 2+2\sigma$

On adding, $\frac{Y}{3K} + \frac{Y}{\eta} = 1-\cancel{2\sigma}+2+\cancel{2\sigma} = 3$

or $\frac{3Y}{3K} + \frac{3Y}{\eta} = 9$

or $\frac{Y}{K} + \frac{3Y}{\eta} = 9$

$Y\left(\frac{1}{K}+\frac{3}{\eta}\right) = 9$

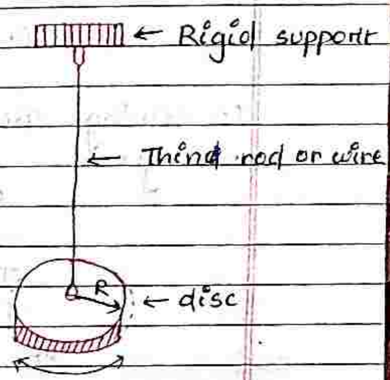
or $\boxed{\frac{1}{K} + \frac{3}{\eta} = \frac{9}{Y}}$ (Y, K, η)

$\boxed{-1 < \sigma < 0.5}$

Q. Show that the limiting values of σ are -1 and 0.5 respectively.

Torsion Pendulum

A torsional pendulum consist of a disc or cylinder suspended from a thin rod or wire. When the mass is twisted about the axis of the wire, the wire exerts a torque on the mass tending to rotate it back to its original position. If twisted and release, the mass will oscillate back and forth executing SHM.



The working principle is based on the theory of simple harmonic motion with the analog of displ. replaced by angular displ. θ,

force $\longrightarrow$ Torque, τ

Spring const $\longrightarrow$ Torsion const, C

For a small angular displacement θ the reaction in terms of τ is given as $-C\theta$

$$\tau = -C\theta$$

The -ve sign indicates the restoring couple.

If a mass with moment of inertia I is attached to the rod or wire then the torque will give an angular acceleration according to the expression

$$\tau = I\frac{d^2\theta}{dt^2}$$

On comparing both the equns we have

$$I\frac{d^2\theta}{dt^2} = -C\theta$$

or $$\frac{d^2\theta}{dt^2} = -\frac{C}{I}\theta$$

On solving the above second order differential eqn, we get

$$\omega = \sqrt{\frac{C}{I}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{C}}$$

or $$C = \frac{4\pi^2 I}{T^2}$$

If we consider the mass of the disc & M

then $$M.I_{\text{about axis through com \& perpendicular to plane}} = \frac{1}{2}MR^2$$

R = radius of the disc

Then the restoring couple exerted by the suspended wire or rod of length l for 1 radian twist is given by ~~for~~ (just an expression, remember)

$$C = \frac{\eta\pi r^4}{2l}$$

where η = rigidity modulus of the material of the wire

r = radius of the wire

l = length of wire

$$C = 4\pi^2\frac{I}{T^2}$$

On comparing, we get

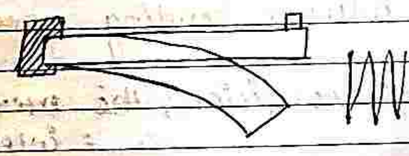
$$\frac{\eta r^4\pi}{2l} = \frac{4\pi^2 I}{T^2}$$

or $$\boxed{\eta = \frac{8\pi I l}{T^2 r^4}}$$ N/m^2 or $dyne/cm^2$

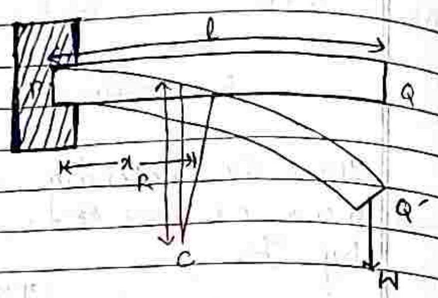
depends on material not the dimension

Cantilever

A cantilever is a beam fixed horizontally at one end and loaded at the other end.



Let us consider a cantilever. Let PQ be a long beam of uniform cross-sectional area clamped horizontally at one end P and loaded with a mass m at the other end Q. Let l is the length of the cantilever.



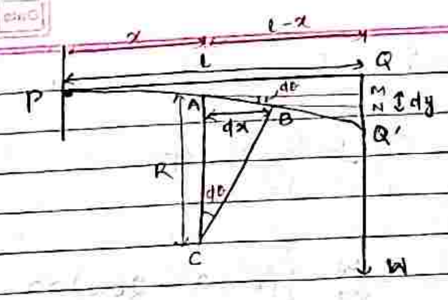
PQ = original pos^n of the cantilever
PQ' = Pos^n of the cantilever when it is loaded.
C = Centre of curvature
R = Radius " "

Due to the applied load at the free end a couple is created between the two forces, i.e.

1. Force applied at the free end towards downward direction and
2. Reaction acting in upward direction at the supporting end.

The external bending couple tends to bend in the cw direction. Since one end of the beam is fixed, ~~at~~ the external bending couple must be balanced by another equal and opposite couple created by the Cantilever due to its elastic nature which is further called as internal bending ~~mo~~moment.

Under eq^{bm} condition, the external bending moment = internal bending moment



Due to the applied load at the free end of the Cantilever the external bending moment $= W(l-x)$

The internal bending moment $= \frac{YI}{R}$

At eq^{bm} condition,

$$\frac{YI}{R} = W(l-x)$$

$$\Rightarrow R = \frac{YI}{W(l-x)} = \text{Radius of curvature}$$

In the bending position the point B is at a distance dx from A so that we have

$$AB = R \cdot d\theta$$
$$\Rightarrow dx = R\,d\theta$$
$$\text{and} \quad R = \frac{dx}{d\theta}$$

$$\text{As} \quad W(l-x) = \frac{YI}{R} = YI \cdot \frac{d\theta}{dx}$$

$$\Rightarrow d\theta = \frac{W(l-x)}{YI} \cdot dx$$

Now the depression of B below A i.e. = MN

and MN = dy

or $dy = (l-x)\,d\theta$

$$= (l-x) \cdot \frac{W(l-x)}{YI}\,dx$$

mid-sem → oscillation, Properties of matter, concept of wave & wave eqn upto thin parallel film
2 hrs 1 – 5Q 2 – LQ 3 4
20 m 2 m 5 m 5m

The total depression is found to be

$$y = \int_0^y dy = \int_0^l \frac{W(l-x)^2}{YI}\,dx$$

$$= \frac{W}{YI}\int_0^l (l^2+x^2-2lx)\,dx$$

$$= \frac{W}{YI}\left[l^2x + \frac{x^3}{3} - 2l\frac{x^2}{2}\right]_0^l$$

$$= \frac{W}{YI}\left[l^3 + \frac{l^3}{3} - l^3\right] = \frac{W}{YI}\left[l^3 - \frac{2l^3}{3}\right]$$

$$\boxed{y = \frac{Wl^3}{3YI}} \rightarrow \text{long thin beams}$$

If we take, $W = mg$

learn $\boxed{y = \frac{mgl^3}{3YI}}$

Spl. Cases (for g.k)

1) If the beam is of Rectangular Cross Section
For a beam of breadth b and thickness d, the moment of inertia is given as

$$I = \frac{bd^3}{12}$$

$$y = \frac{4Wl^3}{Ybd^3}$$

2) Circular Cross Section
For a beam of radius r, the geometric moment of inertia, $I = \frac{\pi r^4}{12} \Rightarrow y = \frac{4Wl^3}{Y\pi r^4}$

WAVES

Date: 20 09 19

Concept of Wave
Waves are the entities spread over space and time. Wave is defined as a form of disturbance which travels from one part of a medium to the other part due to repeated periodic motions of the particles of the medium above their mean position.

Types: longitudinal wave, transverse wave; Progressive wave, Stationary wave

→ Waves are characterised by wavelength, frequency, time period, phase etc.

→ Wave is represented through a wave function (ψ).

→ Wave function, $\psi = \psi(\vec{r}, t)$
$\psi = \psi(x,t)$ or
$\psi = \psi(y,t)$ or
$\psi = \psi(z,t)$

function of space and time

If the periodic variation of the wave function with time and position is sinusoidal, the wave is called a harmonic wave or a sinusoidal wave and it is expressed as $\psi = \psi(\vec{r},t) = A\sin(\vec{k}\cdot\vec{r} - \omega t + \alpha)$

or $\psi(\vec{r},t) = A\cos(\vec{k}\cdot\vec{r} - \omega t + \alpha)$

or

$$\psi(\vec{r},t) = Ae^{i(\vec{k}\cdot\vec{r} - \omega t + \theta)}$$

Here, A = Amplitude of the wave
ω = Angular frequency $= 2\pi\nu$
$\vec{k}$ = wave propagation vector

$$K = |\vec{K}| = \frac{2\pi}{\lambda}$$

λ = wavelength of the wave

In this way, the propagation of wave in a single direction can be represented by

$$\psi(x,t) = A\sin(Kx - \omega t + \alpha) \text{ along } x\text{-axis}$$

If sign of coeff of t & x (ω, K) — same $\Rightarrow$ wave travel along $-x$ dirn; opp $\Rightarrow$ $+x$ dirn

Wave Equation

The dependence of the wave function on time and space is described by a partial differential equation involving time and space derivatives of the wave function.

Let us consider a wave with wave function

$$\psi(x,t) = A\sin(Kx - \omega t + \alpha)$$

$$\frac{\partial \psi}{\partial t} = -\omega A\cos(Kx - \omega t + \alpha)$$

$$\frac{\partial \psi}{\partial x} = KA\cos(Kx - \omega t + \alpha)$$

$$\frac{\partial}{\partial t}\left(\frac{\partial \psi}{\partial t}\right) = \frac{\partial^2 \psi}{\partial t^2} = -\omega^2 A\sin(Kx - \omega t + \alpha)$$

$$= -\omega^2 \psi$$

Similarly, $\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial x}\right) = \frac{\partial^2 \psi}{\partial x^2} = -K^2 A\sin(Kx - \omega t + \alpha)$

$$= -K^2\psi$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\omega^2\psi \quad \text{and} \quad \frac{\partial^2 \psi}{\partial x^2} = -K^2\psi$$

$$\Rightarrow \psi = -\frac{1}{\omega^2}\frac{\partial^2 \psi}{\partial t^2} \quad \text{and} \quad \psi = -\frac{1}{K^2}\frac{\partial^2 \psi}{\partial x^2}$$

On comparing the above equ^ns we have

$$\frac{\cancel{-}1}{\omega^2}\frac{\partial^2 \psi}{\partial t^2} = \frac{\cancel{-}1}{K^2}\frac{\partial^2 \psi}{\partial x^2}$$

or
$$\frac{\partial^2 \psi}{\partial t^2} = \frac{\omega^2}{K^2}\frac{\partial^2 \psi}{\partial x^2}$$

but $\omega = 2\pi\nu$ and $K = \frac{2\pi}{\lambda}$ = Propagation constant

$$\therefore \frac{\omega^2}{K^2} = \frac{4\pi^2\nu^2}{4\pi^2/\lambda^2} = \nu^2\lambda^2 = (\nu\lambda)^2 = v^2$$

where, $v = \nu\lambda$ = velocity of the wave

Hence,
$$\boxed{\frac{\partial^2 \psi}{\partial t^2} = v^2\frac{\partial^2 \psi}{\partial x^2}}$$

Wave Equation →

or

$$\boxed{\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}}$$

} Second order differential equn of wave motion

Superposition of Waves

The phenomenon in which two or more waves moving simultaneously in a medium combine each other is called as superposition of waves.

Principle of Superposition

When more than one wave simultaneously propagate through a region, the resultant wave function is a linear combination of the wave function of the individual waves.

There are two types of superposition

1.) Coherent superposition
2.) Incoherent Superposition

Coherent Superposition

In this case the phase difference of consecutive wave remains constant. The intensity of the resultant wave differs from the sum of the intensities of the individual component waves.

Incoherent Superposition

The superposition betn two waves is incoherent when their phase difference changes randomly. The resultant intensity is equal to the sum of the intensities of the component waves.

Theory of Two Beam Superposition

Let us consider 2 waves travelling in the same dirn with same freq. ω, phase diff δ superpose on each other. The displacements are given by

$$y_1 = a_1 \sin \omega t \quad \text{—(1)}$$

$$y_2 = a_2 \sin(\omega t + \delta) \quad \text{—(2)}$$

Acc. to the principle of superposition the resultant displacement

$$y = y_1 + y_2$$

$$= a_1 \sin \omega t + a_2 \sin(\omega t + \delta)$$

$$= a_1 \sin \omega t + a_2 \sin \omega t \cos \delta + a_2 \cos \omega t \sin \delta$$

$$= \sin \omega t (a_1 + a_2 \cos \delta) + \cos \omega t (a_2 \sin \delta) \quad \text{—(3)}$$

Let $a_1 + a_2 \cos \delta = A \cos \theta$ and —(4a)

$a_2 \sin \delta = A \sin \theta$ —(4b)

Putting the values of $a_1 + a_2 \cos \delta$ & $a_2 \sin \delta$ in eqn(3) we have

Imp. $y = A \sin \omega t \cos \theta + A \cos \omega t \sin \theta$

or $\boxed{y = A \sin(\omega t + \theta)}$ —(5)

Eqn (5) is the eqn for resultant displ. due to superposition of two waves.

$$\text{Now} \quad A^2 = A^2(\sin^2\theta + \cos^2\theta)$$

$$= A^2 \sin^2\theta + A^2 \cos^2\theta$$

$$= (a_2 \sin \delta)^2 + (a_1 + a_2 \cos \delta)^2$$

$$A^2 = a_2^2 \sin^2 \delta + a_1^2 + a_2^2 \cos^2 \delta + 2a_1 a_2 \cos \delta$$

or $\boxed{A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos \delta}$ —(6)

Since $A^2 = I$ = Intensity of wave

If we take $a_1^2 = I_1$

$a_2^2 = I_2$

then $\boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta}$ —(7)

Now, the resultant amplitude, $\boxed{A = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos \delta}}$ due to superposition of two waves.

Here, $2a_1 a_2 \cos \delta$ = Interference term

21/9/19

Superposition
- two beam
- multiple beam

Superposition
- Coherent
- Incoherent

Intensity is defined as the energy crossing per unit area per unit time.

Intensity $\propto A^2$

Intensity $= KA^2$

If we take $K = 1$

$\boxed{\text{Intensity} = A^2}$

V.V. Imp.

Intensity Distribution

$I = A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\delta$

1) Maximum Intensity

$I \longrightarrow I_{max}$, when $\cos\delta = 1$

$\Rightarrow \delta = \pm 2n\pi$

where, $n = 0, 1, 2, 3, \ldots$

$\therefore \delta_{max} = \pm 2n\pi$ = Phase difference

As, Path difference $= \frac{\lambda}{2\pi}$ Phase difference

If we take, Path difference $= \Delta$

then $\Delta_{max} = \frac{\lambda}{2\pi} \times \delta_{max}$

$= \frac{\lambda}{2\pi} \times 2n\pi$

$= n\lambda = (2n)\frac{\lambda}{2}$

The path difference for maximum intensity is an even multiple of $\frac{\lambda}{2}$ and the corresponding phase difference is an even multiple of π.

For I_{max}, $A \longrightarrow A_{max}$ with $\cos\delta = 1$

So $A_{max} = \sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cdot 1}$

$= \sqrt{(a_1+a_2)^2} = a_1 + a_2$

$\therefore I_{max} = A_{max}^2 = (a_1+a_2)^2$

$I_{max} = a_1^2 + a_2^2 + 2a_1a_2$

$$\boxed{I_{max} = I_1 + I_2 + 2\sqrt{I_1I_2}}$$

where, $I_1 = a_1^2$

$I_2 = a_2^2$

$2\sqrt{I_1I_2} = 2a_1a_2$

= superposing form = Interference form

2) Minimum Intensity

$I \longrightarrow I_{min}$, when $\cos\delta = -1$

$\Rightarrow \delta = \pm(2n+1)\pi$

where, $n = 0, 1, 2, 3, \ldots$

$\delta_{min} = (2n+1)\pi$ = Phase difference

As, Path difference $= \frac{\lambda}{2\pi} \times$ Phase difference

If we take, Path diff. $= \Delta$

then $\Delta_{min} = \frac{\lambda}{2\pi} \times \delta_{min}$

$= \frac{\lambda}{2\cancel{\pi}} \times (2n+1)\cancel{\pi}$

$= (2n+1)\frac{\lambda}{2}$

For I_{min} $A \longrightarrow A_{min}$ with $\cos\delta = -1$

So $A_{min} = \sqrt{a_1^2 + a_2^2 - 2a_1a_2 \cdot 1} = \sqrt{(a_1-a_2)^2} = a_1 - a_2$

$\therefore I_{min} = A_{min}^2 = (a_1-a_2)^2$

$I_{min} = a_1^2 + a_2^2 - 2a_1a_2$

$$\boxed{I_{min} = I_1 + I_2 - 2\sqrt{I_1I_2}}$$

where, $I_1 = a_1^2$

$I_2 = a_2^2$

$2\sqrt{I_1I_2} = 2a_1a_2$

= Superposing form

= Interference form

For min. intensity, the path difference is an odd multiple of $\lambda/2$ and the corresponding phase difference is an odd multiple of π.

Path difference = $\frac{\lambda}{2\pi} \times$ Phase difference

Phase difference = $\frac{2\pi}{\lambda} \times$ Path difference

Condition for Coherent Superposition

v.v. Imp

$A_{max} = a_1 + a_2$ $\quad$ $A_{min} = a_1 - a_2$

$I_{max} = (a_1 + a_2)^2$ $\quad$ $I_{min} = (a_1 - a_2)^2$

$\delta_{max} = \pm 2n\pi$ $\quad$ $\delta_{min} = \pm(2n+1)\pi$

$\Delta_{max} = 2n \cdot \frac{\lambda}{2} = n\lambda$ $\quad$ $\Delta_{min} = (2n+1)\frac{\lambda}{2}$

If we assume the coherent superposition of two waves with same amplitude, same frequency and a constant phase difference between them then we can take

$a_1 = a_2 = a$ (say)

So that $\quad I_{max} = (2a)^2 = 4a^2$

$I_{min} = (a-a)^2 = 0$

Completely opp. nature

Intensity Contrast Ratio = $\frac{I_{max}}{I_{min}} \approx \infty$

~~coherent source~~

Condition for Incoherent Superposition

The phase diff. b/w two superimposing waves in incoherent superposition changes randomly with time. So the value of $\cos\delta$ varies between -1 to $+1$ and the time average of the interfering term is zero.

$$\langle 2a_1a_2\cos\delta \rangle = 0$$

In coherent → interferring term +ve
incoherent → " " -ve

Hence, $I = a_1^2 + a_2^2 + \underbrace{2a_1a_2\cos\delta}_{0}$

$I_{incoherent} = a_1^2 + a_2^2 = I_1 + I_2$

$$\boxed{I_{coherent} = a_1^2 + a_2^2 \pm 2a_1a_2}$$ → Interferring term.

Multiple Beam Superposition (Superposition of many waves)

Let us consider the superposition of 'n' no. of waves travelling in same direction with same frequency along with the following parameters.

Let $a_1, a_2, a_3, \ldots a_n$ = amplitudes

$\delta_1, \delta_2, \delta_3, \ldots \delta_n$ = Phase difference betn the consecutive wave

$y_1, y_2, y_3, \ldots y_n$ = displacement associated with each wave

Acc. to the principle of superposition the resultant displacement is sum of the displacements of 'n' no. of waves i.e. $y = y_1 + y_2 + y_3 + \ldots + y_n$ —(i)

$y_1 = a_1\sin(\omega t + \delta_1)$

$y_2 = a_2\sin(\omega t + \delta_2)$

$\vdots$

$y_n = a_n\sin(\omega t + \delta_n)$

$y = a_1\sin(\omega t + \delta_1) + a_2\sin(\omega t + \delta_2) + \ldots + a_n\sin(\omega t + \delta_n)$

or $y = a_1 \sin\omega t \cos\delta_1 + a_1 \cos\omega t \cdot \sin\delta_1 + a_2 \sin\omega t \cdot \cos\delta_2 + a_2 \cos\omega t \cdot \sin\delta_2 + \dots + a_n \sin\omega t \cdot \cos\delta_n + a_n \cos\omega t \cdot \sin\delta_n$

$$= \sin\omega t\,(a_1\cos\delta_1 + a_2\cos\delta_2 + \dots + a_n\cos\delta_n) + \cos\omega t\,(a_1\sin\delta_1 + a_2\sin\delta_2 + \dots + a_n\sin\delta_n) \quad \text{---(2)}$$

Let $a_1\cos\delta_1 + a_2\cos\delta_2 + \dots + a_n\cos\delta_n = A\cos\theta = \sum_{i=1}^{n} a_i\cos\delta_i$ ---(3a)

and $a_1\sin\delta_1 + a_2\sin\delta_2 + \dots + a_n\sin\delta_n = A\sin\theta = \sum_{i=1}^{n} a_i\sin\delta_i$ ---(3b)

Using eqns (3a) & (3b) in eqn (2), we get

$$y = A\cos\theta \cdot \sin\omega t + A\sin\theta \cdot \cos\omega t$$

$$\boxed{y = A\sin(\omega t + \theta)} \quad \text{---(4)}$$

Eqn (4) represents the resultant displacement due to superposition of 'n' no. of waves.

Resultant amplitude, $A = \sqrt{(A\sin\theta)^2 + (A\cos\theta)^2}$

or $A = \sqrt{\left(\sum_{i=1}^{n} a_i\cos\delta_i\right)^2 + \left(\sum_{i=1}^{n} a_i\sin\delta_i\right)^2}$

Imp.

$$\boxed{A = \sqrt{\sum_{i=1}^{n} a_i^2 + 2\sum_{i\neq j} a_i a_j \cos(\delta_j - \delta_i)}} \quad \text{---(5)}$$

(not in syllabus) Multiple beam superposn (in syllabus)

$i = 1, 2$

$(a_1\cos\delta_1 + a_2\cos\delta_2)^2 + (a_1\sin\delta_1 + a_2\sin\delta_2)^2$

$= a_1^2\cos^2\delta_1 + a_2^2\cos^2\delta_2 + 2a_1a_2\cos\delta_1\cdot\cos\delta_2 + a_1^2\sin^2\delta_1 + a_2^2\sin^2\delta_2 + 2a_1a_2\sin\delta_1\cdot\sin\delta_2$

$= a_1^2 + a_2^2 + 2a_1a_2\cos(\delta_2 - \delta_1)$

The 2nd term of eqn (5) is known as the interference term in multiple beam superposition.

In multiple beam superposn < Coherent superposition / Incoherent "

1) Coherent Superposition

When $(\delta_j - \delta_i)$ remains constant during coherent superposn we have the following conditions.

For maximum intensity, $(\delta_j - \delta_i) = \pm 2n\pi$, $n = 0, 1, 2, \dots$

For minimum intensity, $(\delta_j - \delta_i) = \pm(2n+1)\pi$, $n = 0, 1, 2, \dots$

In coherent superposition of multiple beam

$$I_{max} = \sum_{i=1}^{n} a_i^2 + 2\sum_{i\neq j} a_i a_j$$

$$I_{min} = \sum_{i=1}^{n} a_i^2 - 2\sum_{i\neq j} a_i a_j$$

If we consider all the 'n' no. of waves haves same amplitude and a constant phase difference between them, i.e. $a_1 = a_2 = \dots = a_n = a$ (say)

$\cancel{(\delta_2 - \delta_1)}$

Phase difference $= \delta$

$$A^2 = (na\sin\delta)^2 + (na\cos\delta)^2$$

$$= (na)^2 \underbrace{(\sin^2\delta + \cos^2\delta)}_{1}$$

$$= n^2a^2$$

$A^2_{coherent} = n^2a^2$

$\boxed{I_{coherent} = n^2a^2}$

If $a^2 = I_0$

$\boxed{I_{coherent} = n^2 I_0}$

2) Incoherent superposition

In incoherent superposition, the value of $(\delta_j - \delta_i)$ changes randomly with time i.e. $\cos(\delta_j - \delta_i)$ changes from -1 to +1 respectively. Hence the time average

$$\langle \cos(\delta_j - \delta_i) \rangle = 0$$

Intensity of incoherent superposition

$$= \sum_{i=1}^{n} a_i^2 + 2a_ia_j \underbrace{\sum_{i \neq j} \cos(\delta_j - \delta_i)}_{0}$$

$$= \sum_{i=1}^{n} a_i^2$$

$$= a_1^2 + a_2^2 + a_3^2 + \dots + a_n^2$$

On considering the incoherent superposition of 'n' no. of incoherent waves of same amplitude $a_1 = a_2 = a_3 = \dots = a_n = a$ (say)

$I_{incoherent} = na^2$

If we take, $a^2 = I_0$

$\Rightarrow \boxed{I_{incoherent} = nI_0}$

but $\boxed{I_{coherent} = n^2 I_0}$

$I_{coherent} = n \cdot n I_0$

$= n \cdot I_{incoherent}$

$\boxed{I_{coherent} = n \cdot I_{incoherent}}$

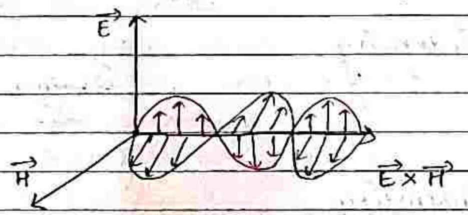
23/9/19

Chapter - Interference

Light → Optics

Study of propagation, properties and applications of light

Light → Transverse electro-magnetic wave



Dual nature → Particle, Wave

Wave → Interference, Diffraction, Polarisation

Interference → Long. wave, Trans. "

Polarisation → only by light wave (T.W)

Interference :

Interference is defined as
→ Superposition of two or more no. of light waves.
→ Distribution of energy

$$\text{Intensity} = \frac{\text{energy}}{\text{Area} \cdot \text{time}}$$

In interference, intensity distribution takes place at the points of maximum and minimum intensities.

→ Superposition
- Constructive → Maximum Int. (Points correspond to max. int. are called Bright Fringes)
- Destructive → Minimum Int. (Dark Fringes)

Fringe :
The alternate arrangement of maximum and minimum intensed points are known as fringes.

Formation of fringes depends on two parameters
→ Path difference
→ Phase difference

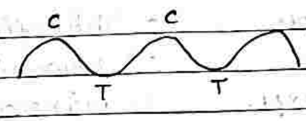
For bright intensity,

$$\Delta_{\substack{bright \\ or \\ max}} = n\lambda \quad , \quad \delta_{\substack{bright \\ or \\ max}} = 2n\pi$$

For dark intensity,

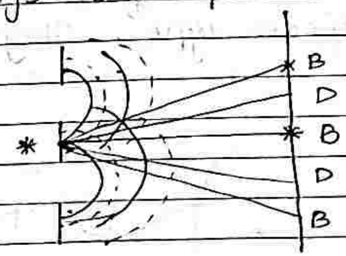
$$\Delta_{\substack{dark \\ or \\ min}} = (2n+1)\frac{\lambda}{2} \quad , \quad \delta_{\substack{dark \\ or \\ min}} = (2n+1)\pi$$

Why do you observe an alternate arrangement of bright and dark fringes in an interference pattern?
Transverse waves are characterised by crest and troughs.



C-C , T-T → Max^m
C-T , T-C → Min^m

→ Young's D-S expt.



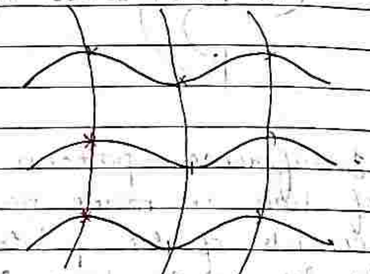
Interference pattern is obtained due to superposition of two or more no. of coherent waves and it consists of the points of maxima and minima (bright and dark fringes)

Imp. What are the conditions to obtain a well observable sustained interference pattern?
For clear observation :
(a) The separation d between the coherent sources must be as small as possible, and the distance D between the source and screen should be as large as possible. This results in larger value of fringe width so that the fringes can be observed clearly.
(b) The background of observation should be dark in order to observe the fringe system clearly.

Methods to produce Coherent Waves
or
Types of Interference Phenomenon

– Division of Wavefronts	– Division of amplitude
→ Young's double slit expt.	→ Thin film interference
→ Fresnel's biprism expt.	→ Newton's ring expt.
→ Lloyd's single mirror expt.	→ Michelson's interferometer expt.

Wavefront
The continuous locus of all the points vibrating in the same phase is known as wavefront.
Wavefront is of different types. They are
- Spherical Wavefront for point source
- Cyclindrical " for linear source
- Plane " for distant source (S at ∞)



Thin Film Interference
This is the phenomenon based on the principle of division of amplitudes.

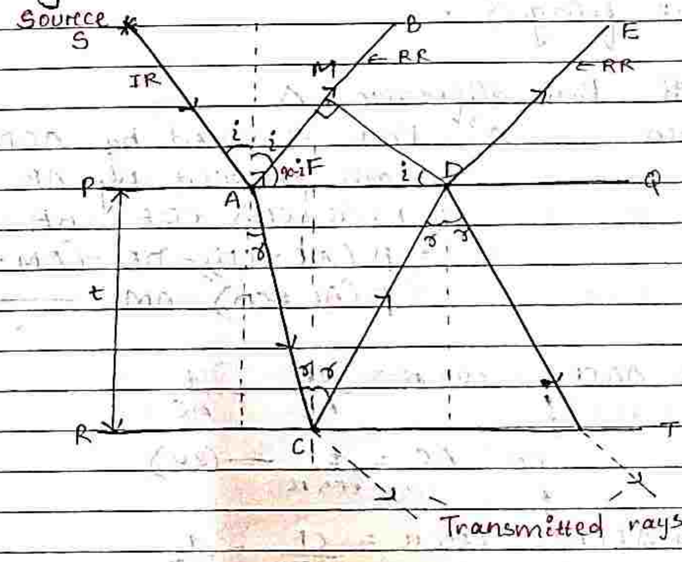
Amplitude of Incident radiation → Reflection, Transmission

Thin film interference pattern is obtained due to
→ Reflected light int.
→ Transmitted " "

Mention different examples of thin film interference
Soap bubble, oil films on water.
Thin film interference is also the mechanism behind the action of antireflection coatings used on glasses and camera lenses.

The fringe pattern obtained due to reflected light int. and due transmitted light interference are complementary to each other.

Theory



~~In AFC tr., $\tan r = \frac{AE}{CF} = \frac{AF}{t}$~~

~~or $AF = t \tan r$~~

~~In CFD tr.~~

Let us consider a thin film of uniform thickness t.

S = Monochromatic source of light

SA = Incident ray

AB, DE = Reflected ray (coherent rays)

μ = Refractive index of the thin film

i = Angle of incidence

r = " " refraction

Let us drop a perpendicular DM on AB.

Imp. If we consider the interfering waves are AB and DE then the extra path travelled by one ray w.r.t the other ray is known as the path difference.

Based on the condition satisfied by the path difference we observe the points of maxima or minima (bright or dark fringes).

If Path difference $= \Delta$

then Δ = Path travelled by ACDE − Path travelled by AB

$= \mu(AC+CD) + DE - AB$

$= \mu(AC+CD) + DE - (AM + MB)$

$= \mu(AC+CD) - AM$ —— (1)

In ΔACF, $\cos r = \frac{CF}{AC} = \frac{t}{AC}$

or $AC = \frac{t}{\cos r}$ —— (2a)

In ΔFCD, $\cos r = \frac{CF}{CD} = \frac{t}{CD}$

or $CD = \frac{t}{\cos r}$ —— (2b)

In AMB triangle, $\sin i = \frac{AM}{AD}$

$AM = AD \sin i$ —— (2c)

In AFC tr., $\tan r = \frac{AF}{CF} = \frac{AF}{t}$

or $AF = t \tan r$ —— (3a)

In CFD tr., $\tan r = \frac{FD}{CF} = \frac{FD}{t}$

or $FD = t \tan r$ —— (3b)

Now, $AD = AF + FD = 2t \tan r$ —— (3c)

Using (3c) in (2c), we get

$AM = 2t \tan r . \sin i$ —— (3d)

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Acc. to Snell's law

$\frac{\sin i}{\sin r} = \mu$

$\Rightarrow \sin i = \mu \sin r$

Using Snell's law in eq^n (3d), we get

$AM = 2t \tan r . \mu \sin r$

$= 2\mu t \frac{\sin^2 r}{\cos r}$ —— (3e)

Using eq^{n}s (2a), (2b) and (3e) in (1), we get

$\Delta = \mu.\left(\frac{t}{\cos r} + \frac{t}{\cos r}\right) - 2\mu t \frac{\sin^2 r}{\cos r}$

$= \frac{2\mu t}{\cos r} - 2\mu t \frac{\sin^2 r}{\cos r}$

$\Delta = \frac{2\mu t}{\cos r}(1 - \sin^2 r)$

$$\Delta = \frac{2\mu t}{\cos r} \times \cos^2 r$$

$$\boxed{\Delta = 2\mu t \cos r} \quad \text{— (4)}$$

$$\left(\text{Path diff.}\right)_{\text{thin film}} = 2\mu t \cos r$$

Thin film interference pattern will be obtained due to superposⁿ of reflected rays
superposⁿ of transmitted rays

applicable only for reflected rays

Principle of Reversibility (stoke's theorem)

Acc. to the Principle of Reversibility, when a ray of light is reflected from an optically denser medium then an extra path difference of $\frac{\lambda}{2}$ or a phase difference of π is introduced. Hence for reflected light interference the path difference given in eqⁿ (4) is modified as

$$\left(\Delta_{path}\right)_{\text{reflected interference}} = 2\mu t \cos r + \frac{\lambda}{2} \quad \text{— (5)}$$

For transmitted rays, the path diff.

$$\left(\Delta_{path}\right)_{\text{transmitted interference}} = 2\mu t \cos r \quad \text{— (6)}$$

Intensity Distribution

Case I : For reflected light interference.

(a) Maximum Intensity

$$\Delta_{max} = n\lambda$$

From (5) → $2\mu t \cos r + \frac{\lambda}{2} = n\lambda$

or $2\mu t \cos r = n\lambda - \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2}$

$$\boxed{2\mu t \cos r = (2n-1)\frac{\lambda}{2}} \quad \text{— (7a)}$$
→ condⁿ for maxᵐ int.

(b) Minimum Intensity

$$\Delta_{min} = (2n+1)\frac{\lambda}{2}$$

From (5) → $2\mu t \cos r + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$

or $2\mu t \cos r + \cancel{\frac{\lambda}{2}} = n\lambda + \cancel{\frac{\lambda}{2}}$

or $\boxed{2\mu t \cos r = n\lambda}$ — (7b)
→ condⁿ for minᵐ int.

Case II : For transmitted light interference

(a) Maximum Intensity

$$\Delta_{path} = 2\mu t \cos r$$

$$\Delta_{max} = n\lambda$$

from (9) → $\boxed{2\mu t \cos r = n\lambda}$ → (8a)
→ condⁿ for maximum int.

(b) Minimum Intensity

$$\Delta_{min} = (2n+1)\frac{\lambda}{2}$$

$$\boxed{2\mu t \cos r = (2n+1)\frac{\lambda}{2}} \quad \text{— (8b)}$$

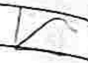
Condⁿ for minᵐ int.

Conclusion :-

From eq^n (7a), (7b) and (8a), (8b), we observe that the conditions of max. & min. intensities in reflected & transmitted light interference are complementary to each other.

Important Questions for Mid-Sem

PROPERTIES OF MATTER

1. Define elasticity & mention different types of moduli of elasticity.
2. Distinguish b/w stress & strain. Discuss different types of stress & the corresponding strain.
3. What do you mean by Poisson's ratio?
4. Draw the stress - strain graph of a material. 
5. Define Hooke's law.
6. Derive the relationship among Y, K, σ, η.
7. How to find out the rigidity modulus of a material by using torsional pendulum?
8. Derive the expression for elongation produced in a Cantilever fixed at one end.

OSCILLATION

1. Derive the second order differential eq^n for the following
   i) free undamped $oscill^n$ (SHM)
   ii) free damped $oscill^n$
   iii) forced oscillation
2. Find out the sol^{ns} of eq^{ns} of motion for free undamped, free damped & forced oscillation.
3. Discuss different $cond^{ns}$ of damped oscillation.
4. Define the terms logarithmic decrement, quality factor & relaxation time in a damped harmonic motion.
5. What do you mean by resonance? Obtain the $cond^n$ of reso. in forced $oscill^n$.
6. Define ampl. reso., vel. reso. & sharpness of reso.
7. Give some examples of reso.
8. How to determine the acc^n due to gravity by a compound pendulum?

WAVES

1. What is a wave? Mention different characteristics of a wave.
2. Write the characteristic features of a wave $func^n$.
3. Derive the wave eq^n.

4. Find out the expression for resultant amplitude due to superposition of waves.

5. Mention the condns for intensity distribution for ~~two wave~~ superposition of two waves.

6. What is relationship betn $\Delta\phi$ & Δx ?

7. What do you mean by coherence ? What are the methods to produce coherent waves ?

8. Mention the condns for a well-observable interference pattern.

9. Derive the relationship betn intensities of coherent and incoherent waves.

10. Discuss on thin film interference

17/10/19 Techniques to produce interference → Coherent sources

Production of coh. sor.

division of wavefronts | div. of amplitudes.

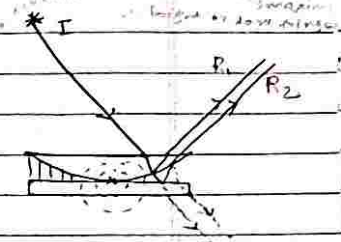
Newton's Rings

Rings → Interference fringes ⟨ dark ring → minima, bright ring → maxima

Principle based on Theory of thin film interference

Newton's ring apparatus

why rings ? ~~In a circular path~~ : since the thickness of air film remain const. in a circular path



$$\Delta_{\text{path diff}}\Big|_{\text{reflected ray}} = 2\mu t\cos r + \frac{\lambda}{2}$$

In Newton's rings, $\Delta_{refl} = 2\mu t\cos r + \frac{\lambda}{2}$

$\mu_{air} = 1$

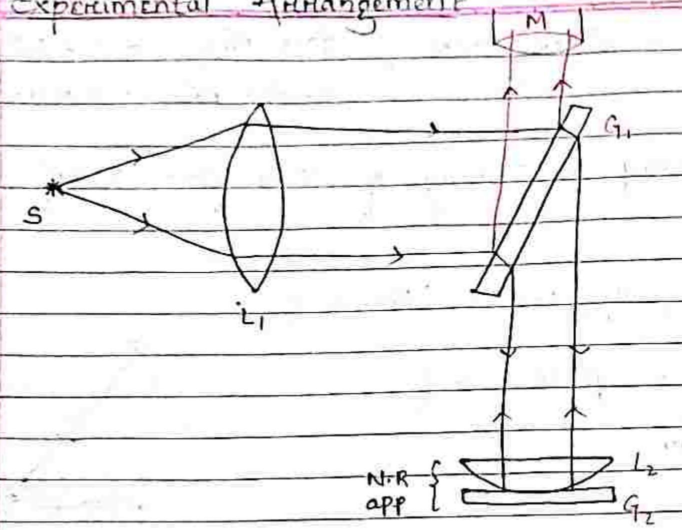
For normal incidence, $r = 0$

$$\Delta_{refl} = 2\cdot1\cdot t\cdot 1 + \frac{\lambda}{2} = 2t + \frac{\lambda}{2}$$

$$\Delta = 2t + \frac{\lambda}{2}$$

In a Newton's ring expt., the points lying in a circular path will either satisfy the condition of maximum int. or the condn of minm intensity & hence the continuous locus of those points will finally give either a bright ring or a dark ring.

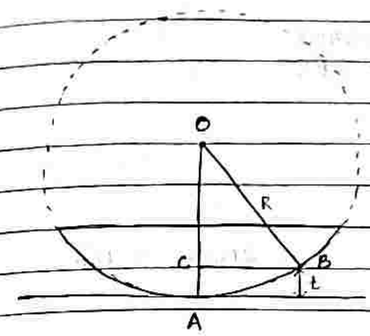
Experimental Arrangement



S = Monochromatic source of light
L_1, L_2 = Lenses
G_1, G_2 = Plane glass plates
M = Microscope

Theory

For air film, $\mu = 1$
For normal incidence, $r = 0$
Let t = thickness of air film
D = Diameter of ring
r = Radius of ring
R = Radius of curvature of the plano convex lens



$OA = OB = R$
$AC = t$
$OC = OA - AC = R - t$

In OCB triangle,
$OB^2 = OC^2 + CB^2$

Let, $CB = r_n$ = Radius of n^{th} ring
Now, $R^2 = (R-t)^2 + r_n^2$
or $R^2 = R^2 + t^2 - 2Rt + r_n^2$
or $t^2 - 2Rt + r_n^2 = 0$

Since, a thin air film is enclosed in between the two surfaces (plano convex & plane surface) so thickness t is very small as compared to the radius of curvature R. ~~sinc~~ Hence t^2 can be neglected.

$t^2 \to 0$
So $r_n^2 - 2Rt = 0$
$\therefore \boxed{r_n^2 = 2Rt}$

$D_n = 2r_n$
or $r_n = \frac{D_n}{2}$
$\therefore \frac{D_n^2}{4} = 2Rt$
or $\boxed{D_n^2 = 8Rt}$

From these two eqns we find

$\boxed{2t = \frac{r_n^2}{R}}$ or $\boxed{2t = \frac{D_n^2}{4R}}$

Date: 18/10/19

Newton's Rings

Reflected light sup. — Transmitted light sup.

Intensity Distribution

1) Bright rings

$\Delta \rightarrow \Delta_{max}$, when $\Delta = 2n\frac{\lambda}{2} = n\lambda$

$\therefore \; 2t + \frac{\lambda}{2} = n\lambda$

or $2t = \left(n - \frac{1}{2}\right)\lambda = (2n-1)\frac{\lambda}{2}$

but $2t = \frac{D_n^2}{4R}$

$\therefore \frac{D_n^2}{4R} = (2n-1)\frac{\lambda}{2}$

or $D_n^2 = 2(2n-1)\lambda R$

or $D_n = \sqrt{2(2n-1)\lambda R} = \sqrt{2\lambda R}\,\sqrt{2n-1} \Rightarrow D \propto \sqrt{2n-1}$

$= D_{bright\ ring}$ where $n = 1, 2, \ldots$ (Bright rings)

2) Dark rings

$\Delta \rightarrow \Delta_{min}$, $\Delta = (2n+1)\frac{\lambda}{2}$

$2t + \cancel{\frac{\lambda}{2}} = 2n\cdot\frac{\lambda}{2} + \cancel{\frac{\lambda}{2}}$

or $2t = n\lambda$

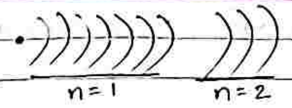
but $2t = \frac{D_n^2}{4R}$

So $\frac{D_n^2}{4R} = n\lambda$

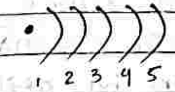
$D_n^2 = 4nR\lambda$

$\therefore D_n = \sqrt{4nR\lambda} = D_{dark\ ring}$

$= \sqrt{2R\lambda}\,\sqrt{2n} \Rightarrow D \propto \sqrt{n}$ where $n = 0, 1, 2, \ldots$



Polychromatic
n is less



Monochromatic
n is more

Q. What are the concluding remarks in Newton's ring expt ?

Newton's rings are example of fringes of equal thickness. These fringes are localised in the film and hence a microscope is necessary for viewing the fringe pattern.

The law of conservation of energy holds good in the Newton's ring experiment.

The points at which light energy appears to be absent (i.e. dark rings) as seen by the reflected waves, at the same points light energy is present (i.e. bright rings) as seen by the transmitted waves.

The regions at which bright Newton's rings are formed by the transmitted wave, at the same regions dark Newton's rings are formed by the reflected wave.

The rings observed by the reflected waves are exactly complementary to those observed by the transmitted light.

1. The central fringe appears dark when observed by the reflected light wave whereas it appears bright when observed by the transmitted light wave.
2. The diameter of nth order bright fringe as seen by the reflected light wave is $D_n = \sqrt{2(2n-1)\lambda R}$. The dia. of n^{th} order dark fringe as seen by the transmitted

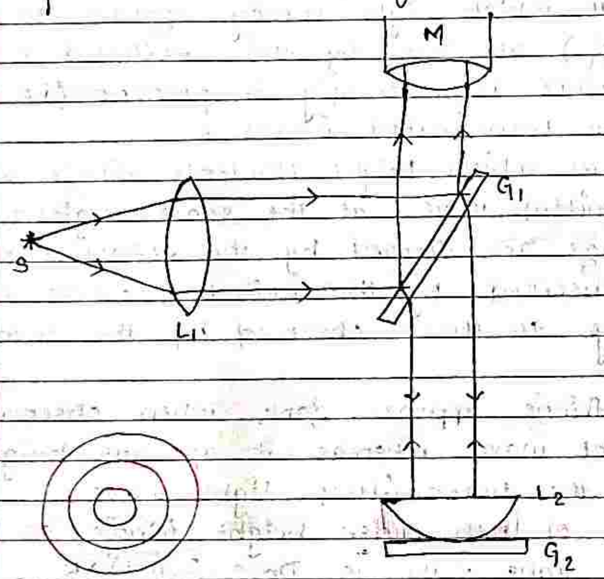
light wave is $D_n = \sqrt{2(2n-1)\lambda R}$. They are located at the same position.

3. The diameter of nth order bright fringe as seen by the transmitted light wave is $D_n = \sqrt{4n\lambda R}$. The dia of nth order dark fringe as seen by reflected light wave is $D_n = \sqrt{4n\lambda R}$. They are located at the same position.

22/10/19 **Applications of Newton's Ring**

LQ
1) Determination of wavelength of monochromatic light
2) Determination of refractive index (μ) of a medium
3) Determination of radius of curvature of a lens

Experimental set-up for all the applications



Theory

1) We know,

$$D_n\big|_{dark} = \sqrt{4n\lambda R}$$

So $D_n^2 = 4n\lambda R$

Similarly, $D_{m+n}^2 = 4(m+n)\lambda R$

So $D_{m+n}^2 - D_n^2 = 4(m+n)\lambda R - 4n\lambda R$

or $D_{m+n}^2 - D_n^2 = 4m\lambda R$

$$\boxed{\lambda = \frac{D_{m+n}^2 - D_n^2}{4mR}}$$

Q. Find out the expression for λ on taking bright rings of different orders.

We know,

$$D_n\big|_{bright} = \sqrt{2(2n-1)\lambda R}$$

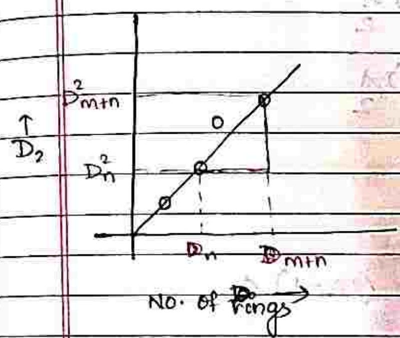
So $D_n^2 = 2(2n-1)\lambda R$

Similarly, $D_{m+n}^2 = 2[2(m+n)-1]\lambda R$

So $D_{m+n}^2 - D_n^2 = 2[2m+2n-1]\lambda R - (4n-2)\lambda R$

or $D_{m+n}^2 - D_n^2 = 4m\lambda R$

$$\boxed{\lambda = \frac{D_{m+n}^2 - D_n^2}{4mR}}$$



$$\text{slope} = \frac{D_{m+n}^2 - D_n^2}{m}$$

Determination of refractive index of a liquid by using Newton's Ring Experiment

Let us consider the refractive index of a liquid is μ. When a drop of liquid is put on the glass plate, then a thin film of liquid will be formed in between the plano convex lens and plane glass plate. The path difference for reflected light

$$\Delta_R = 2\mu t \cos r + \frac{\lambda}{2}$$

Here, $r = 0$, $\mu_{liq} \neq 1$

So $\mu_{liq} = \mu$

$$\therefore \quad \Delta_R = 2\mu t + \frac{\lambda}{2}$$

For bright ring

$$\Delta = n\lambda$$

or $2\mu t + \frac{\lambda}{2} = n\lambda$

$$\Rightarrow \quad 2\mu t = (2n-1)\frac{\lambda}{2}$$

For dark ring

$$\Delta = (2n+1)\frac{\lambda}{2}$$

$$2\mu t + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

or $2\mu t = n\lambda$

but, $2t = \frac{D_n^2}{4R}$

For bright ring; $\mu \cdot \frac{D_n^2}{4R} = (2n-1)\frac{\lambda}{2}$

or $(D_n^2)_l = \frac{2(2n-1)\lambda R}{\mu}$

or $\mu = \frac{2(2n-1)\lambda R}{D_n^2}$

For dark ring; $\mu \cdot \frac{D_n^2}{4R} = n\lambda$

or $(D_n^2)_l = \frac{4n\lambda R}{\mu}$

or $\mu = \frac{4n\lambda R}{D_n^2}$

Let us take dark rings of different orders so that:

$(D_n^2)_l = \frac{4n\lambda R}{\mu}$ and

$(D_{m+n}^2)_l = \frac{4(m+n)\lambda R}{\mu}$

$$(D_{m+n}^2 - D_n^2)_l = \frac{4(m+n)\lambda R - 4n\lambda R}{\mu}$$

$$(D_{m+n}^2 - D_n^2)_{liq} = \frac{4m\lambda R}{\mu}$$

$$= \frac{(D_{m+n}^2 - D_n^2)_{air}}{\mu}$$

Hence, $\boxed{\mu = \frac{(D_{m+n}^2 - D_n^2)_{air}}{(D_{m+n}^2 - D_n^2)_{liq}}}$

3) Determination of radius of curvature of a lens by using Newton's Ring Expt.

If λ = given

$$R = \frac{D_{m+n}^2 - D_n^2}{4\lambda m}$$

Date 23/10/19

Assignment

Q1. Define Interference of light. Derive the expressions for intensity distribution in an interference pattern.

Q2. Prove that conservation ~~combination~~ of light energy takes place during interference. What are different methods to obtain coherent sources?

Q3. Write down the conditions for a well-observable sustained interference pattern.

Q4. Give the theory of thin film interference & write down some examples based on thin film interference.

Q5. Show that the conditions for max. & min. intensity in thin film interference due to reflected light & transmitted light superposition are complementary to each other.

Q6. Derive the relationship between resultant intensity of multiple beams due to coherent and incoherent sup.

Q7. What is the principle of N.R's expt.?

Q8. Why N.Rs are circular in nature?

Q9. Why do we get a dark spot at the centre of N.R pattern due to reflected light superposition?

Q10. Discuss the experimental arrangement & theory of N.R expt with suitable diagram.

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Q11. How to determine the λ of a monochromatic light in N.R expt.?

Q12. How to determine the μ of a liquid by N.R expt.?

Q13. What will happen to the ring pattern of NR expt. when monochr. light is replaced by white light?

Problems

Q. In a N.R expt. the light used is of wavelength 5.9×10^{-5} cm. The dia. of 10th dark ring is 0.5 cm. Find the radius of curvature of the lens and thickness of air film.

$$\lambda = 5.9 \times 10^{-5}\ cm$$

$$(D_{10})_{dark} = 0.5\ cm$$

$$D_n = \sqrt{4n\lambda R} \Rightarrow D_n^2 = 4n\lambda R$$

$$or\quad R = \frac{D_n^2}{4n\lambda}$$

$$but,\ n = 10 \Rightarrow R = \frac{D_{10}^2}{4 \times 10 \times 5.9 \times 10^{-5}} = 106\ cm$$

$$2t = n\lambda \Rightarrow t = \frac{10 \times 5.9 \times 10^{-5}}{2} = 29.5 \times 10^{-5}\ cm$$

Q. In a N.R expt. the dia. of 15th ring is 0.59 cm and dia of 5th ring is 0.336 cm. If the radius of plano convex lens is 100 cm then calculate the wavelength of light used.

$$\lambda = \frac{D_{15}^2 - D_5^2}{4(15-5)R}$$

$$= \frac{0.3481 - 0.112896}{4 \times 10 \times 100}$$

$$= \frac{0.235204}{4000}$$

$$= 5.8801 \times 10^{-5}\ cm$$

Q. In a N.R expt the dia of 10^{th} ring changes from 1.40 cm to 1.27 cm when a drop of liquid is introduced. Calculate the refractive index of the liquid.

$$\mu = \frac{(D^2)_{air}}{(D^2)_{liq}} = \frac{(1.40)^2}{(1.27)^2} = \frac{1.96}{1.6129} = 1.2152$$

Q. The dia of 10^{th} ring is 0.15 cm. The dia. of 5^{th} ring is 0.122 cm then calculate the dia. of 15^{th} ring.

$D_{10} = 0.150$ cm

$D_5 = 0.122$ cm

$D_{15} = ?$

$$\lambda = \frac{D_{10}^2 - D_5^2}{4\times(10-5)\times R}$$

$$4\lambda R = \frac{(0.15)^2 - (0.122)^2}{5}$$

$$D_{15} = \sqrt{4n\lambda R}$$

$$= \sqrt{4\times 15\times \lambda R}$$

Q. The ratio of intensities of maxima & minima of interference fringes is 25 : 9. Determine the ratio of amplitudes & intensities of the two interference beams.

$$\frac{I_{max}}{I_{min}} = \frac{25}{9}$$

a_1, a_2

$$\frac{(a_1+a_2)^2}{(a_1-a_2)^2} = \frac{25}{9}$$

$$\frac{a_1+a_2}{a_1-a_2} = \frac{5}{3}$$

$a_1 = 4$

$a_2 = 1$

$$\frac{a_1}{a_2} = \frac{4}{1}, \quad \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2}$$

encroachment → a gradual advance beyond usual or acceptable limits

Q. In YDSE the screen is placed at a distance 1 m from the slits. The slits are 2 mm apart. They are illuminated by a λ of 589 nm. Calculate the fringe width.

$D = 1$ m

$d = 2$ mm

$\lambda = 589$ nm

$$\beta = \frac{\lambda D}{d} = 294.6 \ \mu m$$

CHAPTER Diffraction

Interference, Diff — Two prop of light — These 2 prop are exhibited by both transverse & longitudinal wave

→ Physical properties of light

→ Wave phenomenon of light nature

Diffraction is defined as bending of light around the corner of an object in which encroachment of light energy takes place into the geometrical shadow.

Imp. Diffraction pattern is observable only when the size of the object is comparable to the wavelength of light.

Diff. pattern consists of fringes.

Bright — Dark – Minima

Bright: Principal or primary maxima, Secondary maxima

Diff. pattern is obtained due to superposition of wavelets.

Q. Define the terms wave, wavefront and wavelet.

Wave is defined as a form of disturbance which travels from one part of a medium to the other part due to ~~reflected~~ repeated periodic motion of the particles of the medium above their mean position.

Wavefront is the continuous locus of all the points vibrating in the same phase.

Every point on the wave front of a propagating light wave can be regarded as a new source of disturbance. The new disturbance spreads out in all directions with the speed of light, and is called secondary wavelet.

Date 24 | 10 | 19

Similarity between Interference and Diffraction

1) Wave property of light.
2) Interference pattern and diffrⁿ pattern are obtained due to the superposition.

Interference fringes — Superposⁿ of waves or wavefronts

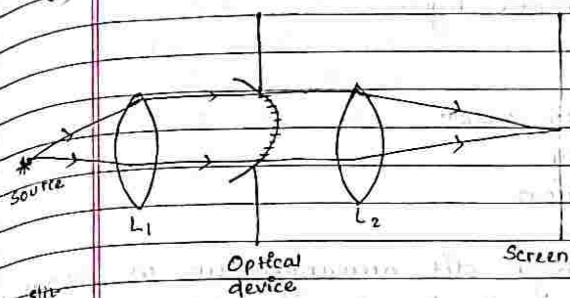
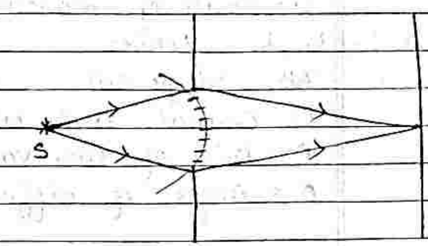
Diffraction fringes — Superposⁿ of wavelets

3) We observe bright and dark fringes in both the patterns.
4) For interference and diffrⁿ expts → Source of light, Optical Instrument, Screen
5) The max. and minimum intensity points are observed through the conditions of path difference or phase difference.

Difference between Interference and Diffraction

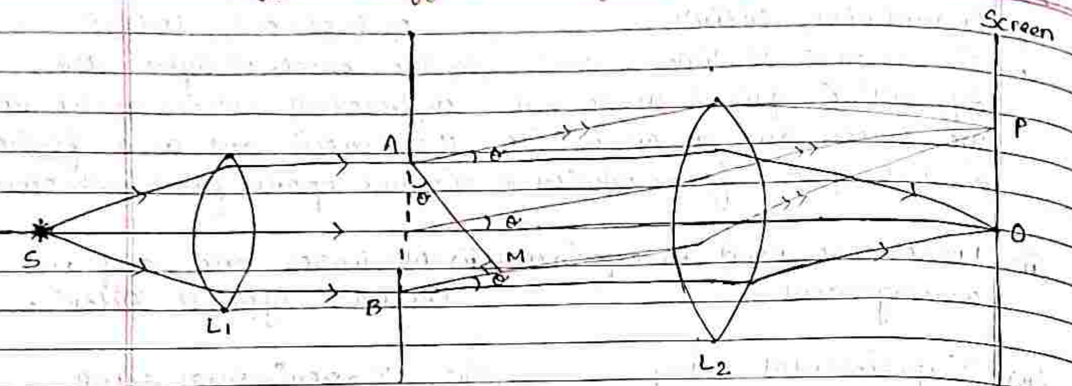
Interference	Diffraction
1) Interference defines the superposⁿ of light waves.	1) Bending nature of light.
2) Int pattern is obtained due to the superposⁿ of wavefronts.	2) Diffrⁿ pattern is obtained due to the superposⁿ of wavelets.
3) The intensity of all the bright fringes remain same throughout the interference pattern.	3) The intensity of bright fringes gradually decreases on either side of the central zero order maxima or bright fringe.
4) The thickness of fringes almost remain same in most of the interference pattern.	4) The thickness of fringes change w.r.t the change in order.

Types of Diffraction

1) Fraunhofer Diffrⁿ	2) Fresnel's Diffrⁿ
(i) The source of light, the diffracting optical device and the screen are at an infinite distance apart from each other.	(i) The source of light, the diffracting optical device and the screen are at a finite distance apart from each other.
(ii) Lenses are used in experimental arrangement.	(ii) No lenses are used in Fresnel's type of Diffrⁿ.
(iii) Experimental setup	(iii) Experimental setup
	
(iv) Plane wavefront is used.	(iv) Spherical and cylindrical wavefronts are used.
(v) Practically observable.	(v) Theoretical concept

Theory of Single Slit

Fraunhoffer Diffraction of a Single Slit



S = Source of monochromatic light
L_1, L_2 = Lenses
AB = Single slit
O = Central zero order max^m
P = Point of observation
θ = Angle of diffraction

Let us consider a single slit arrangement as shown in the figure with an angle of diffraction θ. We can observe the diffr^n pattern due to superpos^n of 'n' no. of wavelets.

Experimental arrangement
In the fig., AB is the section of a narrow slit of width d. A plane wavefront W, of monochromatic light of wavelength λ is incident on the slit. Light is diffracted at the slit and diffracted light is focussed by a convex lens L_2 on the screen placed at its focal plane. The diffraction pattern produced here belongs to the Frauhoffer type.

Each point on the wavefront in the plane of the slit emits secondary wavelets. The wavelets leaving the slit plane normally are focussed at O, that is, a point on the normal to AB drawn at the mid-point of the slit. Since these wavelets travel equal distance, they reach O in the same phase. Hence O is a central bright image. But wavelets travelling at an angle θ are brought to focus in the focal plane at P. This point may be a point of maximum intensity (bright) or a point of minimum intensity (dark) depending upon the phase difference between the wavelets reaching from A and B, that is, the two extreme points on the slit.

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Theory of Single Slit Expt.
To determine the exact intensity at the point P on the Σ, we need to calculate the path diff. or the phase diff. at the point P on the screen.
The path diff. b/w BP and AP = BM
In ABM triangle, $\sin\theta = \frac{BM}{AB}$

Let, $AB = d$ = width of single slit

or $\sin\theta = \frac{BM}{d} \Rightarrow BM = d\sin\theta$ = Path difference betw^n BP & AP

$\therefore \boxed{\Delta = d\sin\theta}$ —①

Phase difference $= \frac{2\pi}{\lambda} \times \Delta = \frac{2\pi}{\lambda} d\sin\theta$ —②

Let us divide the width of single slit into 'n' no. of parts i.e. 'n' no. of wavelets with

a = amplitude of each wavelet

δ = Phase difference between any two consecutive wavelets.

Hence, eq^n ② can be expressed as

$$\frac{2\pi}{\lambda} d\sin\theta = n\delta \quad \text{—(3)}$$

From the principle of superposition theory for 'n' no. of wavelets superposition, the resultant amplitude is given by

$$R = \frac{a \sin\frac{n\delta}{2}}{\sin\frac{\delta}{2}} \quad \text{—(4)}$$

Let us put, $\frac{n\delta}{2} = \alpha$ (say)

$$\Rightarrow \frac{\delta}{2} = \frac{\alpha}{n}$$

$$\therefore R = \frac{a\sin\alpha}{\sin\frac{\alpha}{n}} \quad \text{—(5)}$$

$\because$ n is very large, so $\sin\frac{\alpha}{n} \approx \frac{\alpha}{n}$

$$R = \frac{a\sin\alpha}{\left(\frac{\alpha}{n}\right)}$$

or $R = na\frac{\sin\alpha}{\alpha} = A\frac{\sin\alpha}{\alpha}$, where $\boxed{A = na}$ —(6)

Eq^n (6) represents the resultant amplitude due to superpos^n of 'n' no. of wavelets in a single slit diffr^n.

The resultant intensity, $I = R^2$

$$\boxed{I = A^2 \frac{\sin^2\alpha}{\alpha^2}} \quad \text{—(7)}$$

Intensity Distribution in Single Slit Pattern of Diffraction

$$R = A\frac{\sin\alpha}{\alpha} \quad \text{and} \quad I = A^2\frac{\sin^2\alpha}{\alpha^2}$$

Fringe Pattern
- Maxima
  - Principal
  - Secondary
- Minima

1) Primary or Principal Maxima

$R \to R_{max}$, $R = \frac{A}{\alpha}(\sin\alpha)$

On expanding the sine series & neglecting the higher order terms we get

$$R_{PM} = \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} + \cdots\right] = A\left[1 - \frac{\alpha^2}{3!} + \cdots\right]$$

or $\boxed{R_{PM} = A}$ —(8a)

So $\boxed{I_{PM} = A^2}$ —(8b)

For primary maximum, $\alpha \to 0$

but $\alpha = \frac{n\delta}{2}$

and $n\delta = \frac{2\pi}{\lambda} d\sin\theta$

So $\alpha = \frac{\pi d \sin\theta}{\lambda}$

For $\alpha = 0 \Rightarrow \boxed{\theta = 0}$ —(8c)

At $\theta = 0$, we get the P.M. point.

2) Minima or Dark Fringes

For dark fringes, $R \rightarrow R_{min}$

i.e. $R_{min} = 0$

$A\frac{\sin\alpha}{\alpha} = 0 \Rightarrow \sin\alpha = 0$

then $\alpha = \pm m\pi$, $m = 1, 2, 3, \ldots$

As $\alpha = \frac{\pi d \sin\theta}{\lambda}$

So $\frac{\pi d \sin\theta}{\lambda} = \pm m\pi$

$\Rightarrow \boxed{d\sin\theta = \pm m\lambda}$ —(9a) condⁿ for minᵐ int.

$\boxed{R_{min} = 0, \; I_{min} = 0}$ —(9b)

3) Condition for Secondary Maxima

We know, $I = A^2 \frac{\sin^2\alpha}{\alpha^2}$

From differential calculus it is known that at maxᵐ point $\frac{dI}{d\alpha} = 0$

So $\frac{d}{d\alpha}\left[\frac{A^2 \sin^2\alpha}{\alpha^2}\right] = 0$

or $A^2 \frac{d}{d\alpha}\left(\frac{\sin^2\alpha}{\alpha^2}\right) = 0$

$\because A^2 \neq 0$, so $\frac{d}{d\alpha}\left(\frac{\sin^2\alpha}{\alpha^2}\right) = 0$

$\frac{1}{\alpha^2}(2\sin\alpha \cdot \cos\alpha) - \frac{2}{\alpha^3}\sin^2\alpha = 0$

or $\frac{\cancel{2}}{\cancel{\alpha^2}} \cancel{\sin\alpha}\cos\alpha = \frac{\cancel{2}}{\alpha^3}\sin^{\cancel{2}}\alpha$

or $\cos\alpha = \frac{\sin\alpha}{\alpha}$

or $\boxed{\alpha = \tan\alpha}$ —(10)

$\frac{\alpha^2 \frac{d}{d\alpha}\sin^2\alpha - \sin^2\alpha \frac{d}{d\alpha}\alpha^2}{(\alpha^2)^2}$

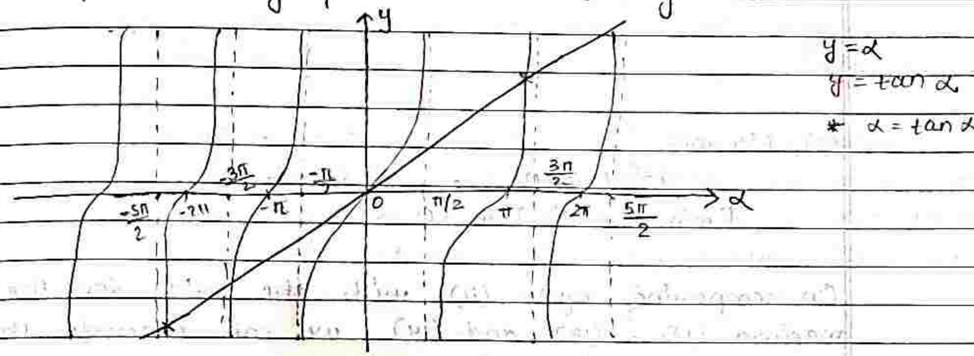
$\frac{\alpha^2 \times 2\sin\alpha\cos\alpha - \sin^2\alpha \, 2\alpha}{(\alpha^2)(\alpha^2)}$

$\frac{\cancel{\alpha^2}\sin 2\alpha - \sin^2\alpha \, 2\cancel{\alpha}}{\alpha^2 \, \alpha^{\cancel{2}}}$

$\frac{2\sin\alpha\,(2\alpha\cos\alpha - \sin\alpha)}{\alpha^2}$

Eqⁿ (10) represents the condⁿ for secondary maximum in single slit diffʳⁿ.

To determine the values of α which should satisfy the eqn (10), let us take $y = \tan\alpha$ & $y = \alpha$ & plot the graph in betⁿ α & y.



From this graph we can observe the secondary maxima occurs at $\alpha = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, \ldots\ldots$

or $\alpha = \pm (2n+1)\frac{\pi}{2}$, $n = 1, 2, 3, \ldots$

29/10/19 Conclusions

1) Intensity distribution

(a) Primary Maximum

$\theta = 0°$, $R_{PM} = A$, $I_{PM} = A^2 = I_0$ —(11)

(b) Secondary Maxima

$\alpha = \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2}, \pm \frac{7\pi}{2}, \ldots$

$I_{SM} = A^2 \frac{\sin^2\alpha}{\alpha^2}$

For $\alpha = \pm \frac{3\pi}{2}$, $I_{SM_1} = A^2 \frac{\sin^2 \frac{3\pi}{2}}{\left(\frac{3\pi}{2}\right)^2} \approx \frac{A^2}{22} = \frac{I_0}{22}$ —(12)

For $\alpha = \pm \frac{5\pi}{2}$, $I_{SM_2} = \frac{I_0}{62}$ —(13)

For $\alpha = \pm \frac{7\pi}{2}$, $I_{SM_3} = \frac{I_0}{128}$ —(14)

(c) Minima

$d\sin\theta = m\lambda$, $m = 1, 2, 3, \ldots$

$R_{min} = 0 \Rightarrow I_{min} = 0$

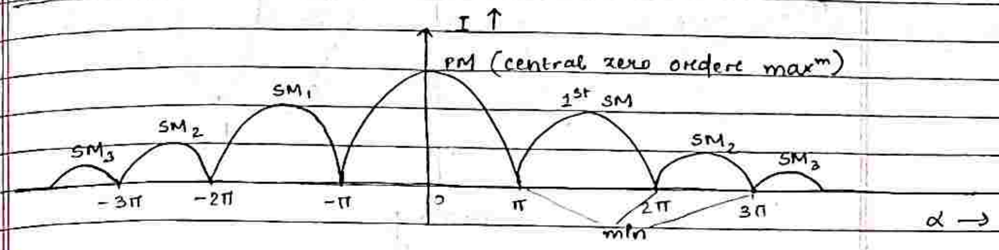
On comparing equ^ns (11) with the eq^ns for the secondary maxima (12), (13) and (14) we can conclude that

2) $I_0 > \frac{I_0}{22} > \frac{I_0}{62} > \frac{I_0}{128} \ldots$

$\Rightarrow I_{PM} > I_{SM_1} > I_{SM_2} > I_{SM_3} > \ldots$

Hence it is observed that the intensity of bright fringes decreases on either side of the central zero order maximum in the diffraction pattern.

3) Plot betw^n I vs α



Diffraction Grating

It is an optical device consist of parallel equidistant slits. There are two types of diffraction grating.

Reflection type of D.G

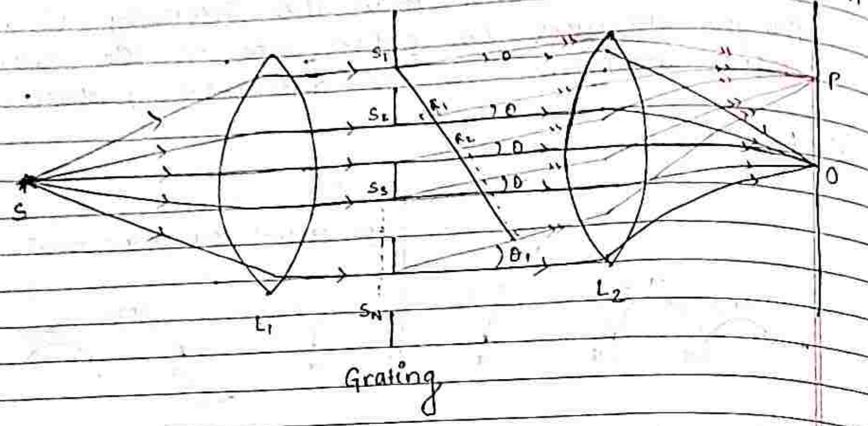
Transmission " " "

Diffraction grating has a combination of large no. of parallel slits. Hence it is known as multiple slits or n-slits.

→ In a Fraunhoffer diffraction of n-slits the grating is placed in bet^n the source & screen and the diffraction pattern is obtained on the screen which is due to the effect of diffraction at each slit and interference due to n no. of slits.

N-slits → Grating

Transmission grating



S = Source of monochromatic light

$S_1, S_2, \ldots, S_N$ = slits

θ = Angle of diffr^n

L_1, L_2 = Lenses

O = Central zero order max^m

P = Point under observation

Theory.

30/10/19 Let us consider a diffraction grating with 'N' slits having equal width and separated by equal distance throughout.

Let d = width of each slit

b = Distance of separation bet^n consecutive slits

and $b+d$ = Grating element $= S_1S_2 = S_2S_3 = \ldots$

In a grating expt. the diffr^n pattern is obtained due to the combined effect of

1. Superposition of wavelets at individual slits (diffr^n effect at single slit).

2. Superposition of wavefronts due to the presence of n-slits (interference effect).

To obtain the conditions for maxima or minima at the point P we have to find out the path difference and phase difference. With reference to fig. the path diff bet^n S_2P and $S_1P = S_2R_1$

In $S_1S_2R_1$ triangle, $\sin\theta = \frac{S_2R_1}{S_1S_2}$

but $S_1S_2 = b+d$

or $\sin\theta = \frac{S_2R_1}{(b+d)}$

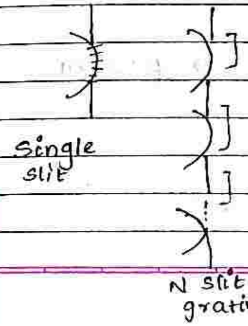
$\Rightarrow S_2R_1 = (b+d)\sin\theta$ —(1)

Similarly we can find the path diff. bet^n two rays coming from consecutive slits as

$S_3R_2 = (b+d)\sin\theta$

$S_4R_3 = (b+d)\sin\theta$

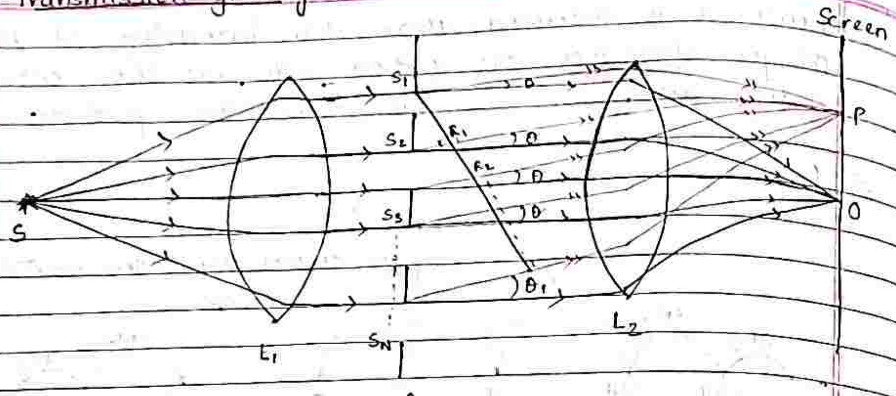
The corresponding phase diff bet^n consecutive slits is given by $\frac{2\pi}{\lambda} \times (b+d)\sin\theta = 2\beta$ (say) —(2)



single slit	N-slits
a	R
δ	2β
n	N
R	R'

N-slits → Grating

Transmission grating



Grating

S = Source of monochromatic light

$S_1, S_2, \dots, S_N$ = Slits

θ = Angle of diffrⁿ

L_1, L_2 = Lenses

O = Central zero order max^m

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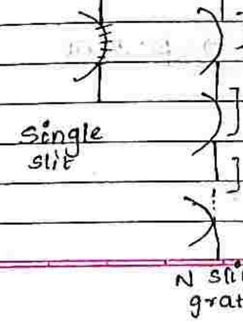


Single slit	N-slits
a	R
δ	2β
n	N
R	R'

On considering this analogy we can obtain the resultant amplitude of superposⁿ of N no. of wavefronts in a grating expt. The resultant amplitude

$$R' = \frac{R\sin\frac{N(2\beta)}{2}}{\sin\frac{2\beta}{2}} = \frac{R\sin N\beta}{\sin\beta} \quad \text{—(3)}$$

The resultant intensity $= R'^2 = R^2\dfrac{\sin^2 N\beta}{\sin^2\beta}$ —(4a)

As, $R = A\dfrac{\sin\alpha}{\alpha}$

$$\Rightarrow I_{grating} = \underbrace{\left(A\frac{\sin\alpha}{\alpha}\right)^2}_{\text{Diffraction term}} \underbrace{\left(\frac{\sin^2 N\beta}{\sin^2\beta}\right)}_{\text{Interference term}} \quad \text{—(4b)}$$

Intensity Distribution

Diffⁿ Pattern:
- Maxima
  - Principal or Primary
  - Secondary
- Minima

$R' = R\dfrac{\sin N\beta}{\sin\beta}$, $I' = R^2\dfrac{\sin^2 N\beta}{\sin^2\beta}$

1) Principal Maxima

$R' \to R'_{PM}$, when $\sin\beta \approx 0 \Rightarrow \beta = \pm n\pi$

As $\cancel{2}\beta = \dfrac{\cancel{2}\pi}{\lambda}\times(b+d)\sin\theta$

$\Rightarrow \beta = \dfrac{\pi(b+d)\sin\theta}{\lambda}$

For PM, $\dfrac{\cancel{\pi}(b+d)\sin\theta}{\lambda} = \pm n\cancel{\pi}$

$\Rightarrow \boxed{(b+d)\sin\theta = \pm n\lambda}$ —(5) Grating Eqⁿ

where, $b+d \to$ grating element, $\theta \to$ angle of diffrⁿ, $n \to$ order of grating fringes

when $\sin\beta \to 0$, $R' = R\dfrac{\sin N\beta}{\sin\beta} \approx \dfrac{0}{0}$

On applying L' Hospital rule,

$$R'_{PM} = \lim_{\beta\to\pm n\pi} R\frac{\frac{d}{d\beta}(\sin N\beta)}{\frac{d}{d\beta}(\sin\beta)}$$

$$= R\lim_{\beta\to n\pi}\frac{N\cos N\beta}{\cos\beta} = RN\left(\lim_{\beta\to n\pi}\frac{\cos N\beta}{\cos\beta}\right)$$

$\boxed{R'_{PM} = RN}$ —(6)

$I_{PM} = R'^2_{PM} = R^2N^2$ —(7)

(2) Minima

$R' \to R'_{min}$, when $\sin N\beta \approx 0$

$\Rightarrow N\beta = \pm m\pi$

but $m \neq 0, N, 2N, 3N, \dots, nN$, why?

on putting the values $0, N, 2N, 3N, \dots nN$ for m, we get the condⁿ for PM

$N\times\dfrac{\pi(b+d)\sin\theta}{\lambda} = m\pi$

$\Rightarrow \boxed{N(b+d)\sin\theta = \pm m\lambda}$ —(8) Eqⁿ for minima

$\therefore R'_{min} = 0$, $I'_{min} = 0$

3) Secondary Maxima.

$$I \rightarrow I_{SM} = R^2 \frac{\sin^2 N\beta}{\sin^2 \beta}$$

$$R \left\{ \sin^2\beta \frac{d}{d\beta} \sin^2 N\beta \right. - \sin^2 N\beta \frac{d}{d\beta} \sin^2 \beta \left. \right\}$$

$$\frac{dI_{SM}}{d\beta} = 0$$

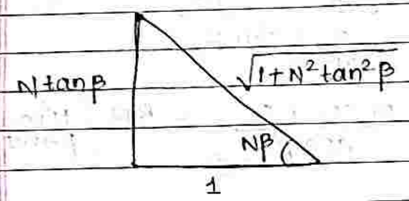
$$R^2 \left\{ \sin^2\beta \times 2 \sin N\beta \cos N\beta \times N - \sin^2 N\beta \times 2 \sin\beta \cos\beta \right\}$$

$$\frac{d}{d\beta}\left[R^2 \frac{\sin^2 N\beta}{\sin^2 \beta}\right] = 0$$

or $R^2 \left[\frac{1}{\sin^2\beta} \cdot 2 \sin N\beta \cdot\right.$

$$N \sin\beta \cos N\beta = \sin N\beta \cos\beta$$

$$\boxed{\tan N\beta = N \tan\beta} \quad \text{--- (9)}$$



$$R'_{SM} = R \frac{\sin N\beta}{\sin\beta}$$

$$I_{SM} = R^2 \frac{\sin^2 N\beta}{\sin^2\beta}$$

$$\sin N\beta = \frac{N \tan\beta}{\sqrt{1+N^2\tan^2\beta}}$$

$$I_{SM} = \frac{R^2 N^2}{1+(N^2-1)\sin^2\beta} \quad \text{--- (10)} \rightarrow \text{how ??}$$

Conclusion

1) In diffr^n grating expt; more than one number of fringes for primary maximum will be obtained. Since the grating eq^n is given by

$$(b+d)\sin\theta = \pm n\lambda, \quad n = 0, 1, 2, \ldots$$

i.e. a no. of different orders of P.M. points are situated symmetrically on either side of central zero order maximum.

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2) Diffraction pattern of grating < Diffraction, Interference.

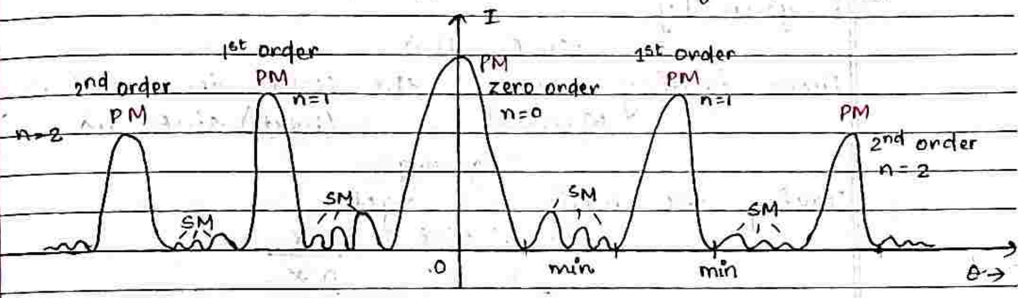
3) $I_{PM} = R^2 N^2$, $\quad I_{SM} = \frac{R^2 N^2}{1+(N^2-1)\sin^2\beta}$

$$\frac{I_{SM}}{I_{PM}} = \frac{1}{1+(N^2-1)\sin^2\beta}$$

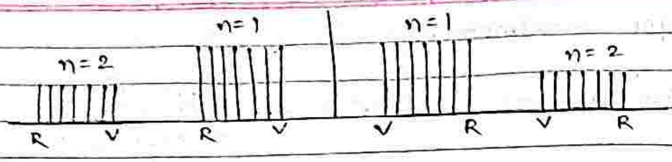
Since, N is very large, $\quad I_{SM} <<< I_{PM}$ Imp

$$I_{SM} = \frac{I_{PM}}{\text{very large value}}$$

Hence it is concluded that the fringe pattern obtained in a grating expt. is due to the primary maximum only, because the intensity of secondary maxima points are very small as compared to the primary maxima. Thus the secondary maxima intensities ~~will be~~ are suppressed in the diffr^n pattern of grating.



If the monochromatic source of light is replaced by a white light source then fringes of different colours with different orders from violet to red will be observed on either side of the central zero order maximum.



Short Q. 5) Maximum Order

The grating eqn is given by

$$(b+d)\sin\theta = n\lambda$$

$n \to n_{max}$ when $\theta \to 90°$

$$n_{max} = \frac{(b+d)\sin\theta_{max}}{\lambda}$$

$$= \frac{(b+d)\sin 90°}{\lambda}$$

$\therefore$ $\boxed{n_{max} = \frac{b+d}{\lambda}}$ Imp. formula for numerical

Imp. 6) Missing Order or Absent Spectra

For single slit diffrn theory the condn for minimum is given by

$$d\sin\theta = m\lambda$$

From grating theory, the condn for maxm is given by

$$\text{Maxima}_{\text{grating}} = (b+d)\sin\theta = n\lambda$$

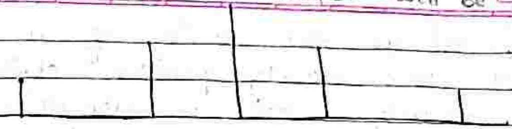
Taking the ratio we get

$$\frac{(b+d)\cancel{\sin\theta}}{d\cancel{\sin\theta}} = \frac{n\cancel{\lambda}}{m\cancel{\lambda}}$$

$$\Rightarrow \boxed{\frac{b+d}{d} = \frac{n}{m}}$$

This condn is required to determine the absent spectra in a diffrn grating profile.

When the condition for the principal maximum due to N-slits and the condition for minimum for the single slit are satisfied simultaneously for a given value of θ, then the principal maximum of that order will be absent or missing.



For ex:

(i) If $b = d \Rightarrow \frac{2\cancel{d}}{\cancel{d}} = \frac{n}{m}$

or $\frac{n}{m} = 2 \Rightarrow n = 2m$

If $m = 1, 2, 3, \ldots \Rightarrow n = 2, 4, 6, \ldots$

(ii) If $b = 2d \Rightarrow \frac{n}{m} = 3$

$\therefore$ so $n = 3m$

when $m = 1, 2, 3, \ldots$

then $n = 3, 6, 9, \ldots$ missing orders

7) Dispersive Power of Grating

Dispersive power is the rate of change of angle of diffrn w.r.t change in wavelength & it is expressed as $\frac{d\theta}{d\lambda}$.

As the grating eqn is given by

$$(b+d)\sin\theta = n\lambda$$

On differentiating both sides of this eqn, we get

$$\text{or } \frac{d\theta}{} \cos\theta(b+d) = n\,d\lambda$$

$$\text{or } \boxed{\frac{d\theta}{d\lambda} = \frac{n}{(b+d)\cos\theta}}$$

8) **Resolving Power of Grating**

It is defined as the ability of an instrument to distinguish two closely placed objects or spectral lines of nearly same wavelength.

According to Rayleigh's criterion for resolution two spectral lines of nearly equal wavelengths are said to be just resolved, when the primary maximum of first spectral line exactly coincides with the first minimum of the second spectral line and vice-versa.

Primary max^m (1st spect. line) = First min^m (2nd spect. line)

Using this cond^n the resolving power of grating can be obtained as

R.P of grating $\boxed{\frac{\lambda}{d\lambda} = nN}$

n = no. of orders

N = no. of lines present in a grating

Characteristics of Grating Spectra

1. Grating spectra are straight fringes of max^m & min^m intensity.
2. A no. of P.M. fringes are observed on either side of the central zero order max^m.
3. The intensity of P.M fringes gradually decreases with increase in order on either side of the central zero order max^m.
4. For white light source coloured spectral lines of different orders are observed.

Applications of Grating

1) Determination of wavelength of monochromatic light.
2) " " grating element of a diffr^n grating.
3) " " angle of diffr^n in a diffr^n grating ~~element~~ expt.

Q1. Define diffraction & hence discuss the Huygen's Principle of diffraction.

Q2. Distinguish bet^n interference and diffr^n.

Q3. ~~What~~ Discuss different types of diffr^n.

Q4. Give the theory of Fraunhoffer diffr^n at a single slit.

(OR)

What are the cond^{n}s of intensity distribution in a single slit diffr^n?

(OR)

Why the intensity of higher order fringes decreases in a single slit diffr^n?

Q5. Derive the grating eq^n.

Q6. Discuss the theory of diffr^n grating with suitable ray dia.

Q7. How to determine the wavelength of a monochromatic light by using grating?

Use Rayleigh's criterion for resolution

Q8. Derive the expression for R.P of grating.

Q9. What are the characteristics of grating spectra.

Numericals

Q. A ||el beam of monochr. light falls normally on a plane diffrn grating having 5000 lines per cm. A second order spectral line is diffracted through an angle of 30°. What would be the wavelength of light?

$n = 2 \qquad \lambda = ?$

$\theta = 30°$

$b+d = \frac{1}{5000}$ cm

$\frac{(b+d)\sin\theta}{n} = \lambda$

Q. Light of wavelength 5000 Å is incident on a plane transmission grating of width 3 cm and 15000 lines. Find the angle of diffrn in 1st order.

$\lambda = 5000$ Å $=$ ______ cm

$b+d = \frac{3}{15000}$ cm

$n = 1 \qquad \theta = ?$

$\theta = \sin^{-1}\left(\frac{n\lambda}{b+d}\right)$

Q. What is the highest order spectrum which may be seen with light of wavelength 6000 Å by means of a grating with 5000 lines per cm?

2.12 → 2
3.33 → 3
3.5 → 3
3.7 → 3

$(b+d)\sin\theta = n_{max}\lambda$

$n_{max} = \frac{b+d}{\lambda}$

POLARISATION

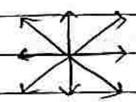
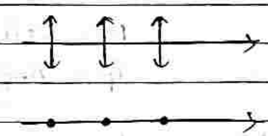
→ Wave property

→ Wave < Longitudinal, Transverse } → Interference, Diffraction

→ Polarisation is exhibited by the transverse wave only.

→ Light is a transverse e-m wave.

Polarisation is the one sidedness, asymmetrical property of light in which the vibrations will be restricted in one direction.

	Unpolarised light	Polarised light
1)	The vibration is found in all direction in case of unpolarised light.	Vibration is restricted in one direction.
2)		
3)	Sun, Bulb, Tube light etc.	Laser

Plane polarised light

Methods to produce Plane Polarised Light

Types
1) Plane polarised light or linearly pol. light
2) Circularly " "
3) Elliptically " "
4) Partially " "

Superposⁿ of plane polarised with with a phase difference of $\frac{\pi}{2}$ having amplitudes a and b

If $a = b$ $\quad (x^2+y^2=a^2) \Rightarrow$ CPL

If $a \neq b$ $\quad \left(\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\right) \Rightarrow$ EPL

Methods to produce PPL

1) By Reflection
2) " Refraction
3) " Double Refraction
4) " Selective Absorption
5) " Scattering

Brewster's Law

$$\boxed{\mu = \tan i_p}$$

μ = Refractive index
i_p = Angle of Polarisation

Malus Law

$I \propto \cos^2\theta$

$$\boxed{I = I_0\cos^2\theta}$$

Instrumentation

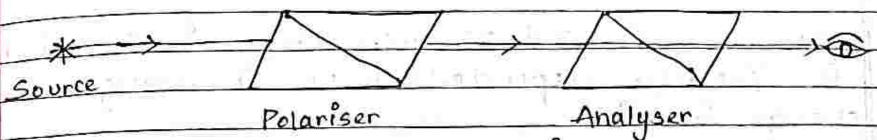
Source $\Rightarrow$ Ordinary unpolarised light.

Optical Device < Polariser / Analyser

Screen

Polariser is the instrument used to obtain polarised light from ordinary unpolarised light.

Analyser is the instrument used to analyse the polarised light.



Acc. to Malus law, $I = I_0\cos^2\theta$

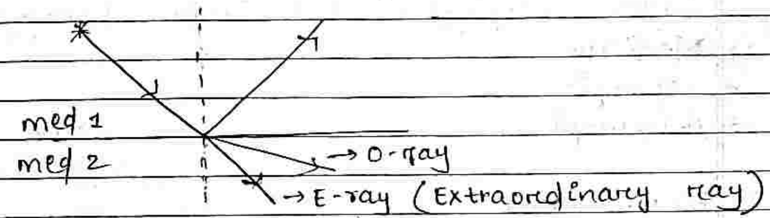
Nicol prism < Polariser / Analyser

when $\theta = 0 \Rightarrow I = I_0 \rightarrow max^m$

$\theta = \frac{\pi}{2} \Rightarrow I = 0 \rightarrow min^m$

Hence polarisation can also be defined as the process, by which an unpolarised light will be converted into polarised light by various experimental methods.

Double Refraction Birefringence



There are certain special type of substances known as double refracting substances in which we observe two refracted rays. so the splitting of ~~refrac~~ incident ray by two refracted rays is known as double refraction.

One of the doubly refracted ray obeys Snell's law i.e. O-ray & the other ray does not obeys Snell's law is known as E-ray (Extraordinary ray)

→ The vel. of O-Ray is const in all dirⁿs and also the refractive index of O-Ray is constant in all dirⁿs.

→ The vel. & as well as refractive index of E-Ray is direction dependent i.e. it changes w.r.t change in dirⁿ.

58/11/19 Plane of Polarisation
The plane containing the dirⁿ of propagation of light & ⊥ar to the dirⁿ of vibration is known as plane of polarisation.

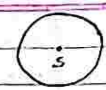
Plane of Vibration
The plane containing the dirⁿ of propagation of light & the dirⁿ of vibration of light is called as the plane of vibration.

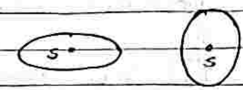
Terms associated with Double Refraction
1) Medium
→ Isotropic dirⁿ independent property
→ Anisotropic dirⁿ dependent property → DR crystals

2) Birefringence another name of double refrⁿ

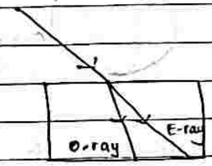
3) DR rays < O-Ray, E-Ray

4) Huygen's Principle of DR
Acc. to Huygen's theory each point on a wavefront acts as a source of secondary wavelets and spreads its disturbances in all directions with a speed of propaga. of the wave.

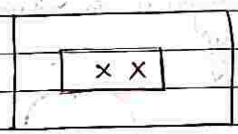
Acc. to this theory, O-ray is represented by 


E-ray

5) Double Refracting Crystals

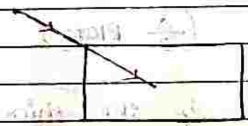


Calcite
Quartz
Tourmaline
Topaz etc.



Imp. 6) Optic Axis
It is the direction inside a double refracting crystal along which the E-ray and O-ray will have the same velocity.

DR → Anisotropic medium



7) Uniaxial and Biaxial Crystals
Uniaxial crystals are those DR crystals which possess 1 optical axis. Ex- calcite, Quartz
Biaxial crystals → Two optical axes Eg- Copper sulphate, Aragonite

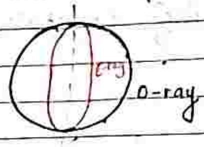
8) Positive and Negative DR crystals

Depending on the velocities of O-Ray and E-ray inside the DR crystals they are classified into positive and negative crystals.

$\mu_E > \mu_O$

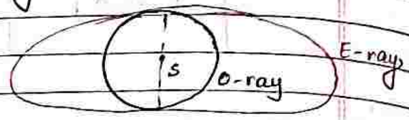
Positive Crystal :- If the vel. of O-ray > vel. of E-ray then the type of crystal is known as +ve crystal & it is representy by

$\mu = \frac{c}{v}$



$\mu_E < \mu_O$

Negative Crystal :- If the vel of E-ray > vel. of O-ray inside a DR crystal, the crystal is known as a -ve crystal & it is represented by



9) Phase Retardation Plates

The optical device which makes a path difference betn O-ray & E-ray by retarding the motion of one of the rays is called a retardation plate. There are two types of retardation plate :-

Quarter Wave Plate ($\frac{\lambda}{4}$-Plate)

Half Wave Plate ($\frac{\lambda}{2}$-Plate)

Quarter Wave Plate :- If the thickness of a DR crystal is adjusted in such a way that it produces a path diff. of $\frac{\lambda}{4}$ or phase difference of $\frac{\pi}{2}$ betn the O-ray & E-ray then the plate is called a quarter wave plate.

DR < Uniaxial, Biaxial

Negative, Positive

$$\boxed{t = \frac{\lambda}{4(\mu_O \sim \mu_E)}}$$ for a quarter wave plate

For a -ve crystal, $t = \frac{\lambda}{4(\mu_O - \mu_E)}$

For a +ve crystal, $t = \frac{\lambda}{4(\mu_E - \mu_O)}$

Quarter wave plate is used to produce & analyse circularly & elliptically polarised light.

Half Wave Plate :- If the thickness of a DR crystal is adjusted in such a way that it produces a path difference of $\frac{\lambda}{2}$ or a phase difference of π betn O-ray and E-ray then the plate is called a half wave plate.

For a half wave plate, the thickness is given by

$$\boxed{t = \frac{\lambda}{2(\mu_O \sim \mu_E)}}$$ for a half wave plate

For a -ve crystal, $t = \frac{\lambda}{2(\mu_O - \mu_E)}$

For a +ve crystal, $t = \frac{\lambda}{2(\mu_E - \mu_O)}$

ELECTROMAGNETISM

Date 6 | 11 | 19

→ Vector Calculus

→ Fundamental theories of electricity and magnetism

→ Maxwell's equ^ns < Differential form / Integral form

→ E-M wave equ^ns

→ Transverse nature of e-m wave.

Field: It is the region or space surrounding a physical quantity where the influence of the physical quantity is observed.

Field < Scalar field $\Rightarrow \phi$ / Vector field $\Rightarrow \vec{V} = V_x\hat{i} + V_y\hat{j} + V_z\hat{k}$

V_x, V_y, V_z are the components of the vector field $\vec{V}$ along x, y and z axis.

$\hat{i}, \hat{j}, \hat{k}$ are unit vectors along x, y and z dir^n.

$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1 = \hat{i}\times\hat{j} = \hat{j}\times\hat{k} = \hat{k}\times\hat{i}$

$\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$

Vector Calculus

Del operator or Differential operator

$$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

Laplacean Operator

$$\nabla^2 = \vec{\nabla}\cdot\vec{\nabla}$$

$$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)$$

$$= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Operations between $\vec{\nabla}$ and Field

Page No. | Date

1) **Gradient** (grad)

grad (scalar field) = $\vec{\nabla}\phi$

where ϕ is the scalar field which have only the mag.

$$\text{grad}\ \phi = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\phi$$

or $$\vec{\nabla}\phi = \hat{i}\frac{\partial\phi}{\partial x} + \hat{j}\frac{\partial\phi}{\partial y} + \hat{k}\frac{\partial\phi}{\partial z}$$

$\Rightarrow$ Vector qty.

2) **Divergence** (div)

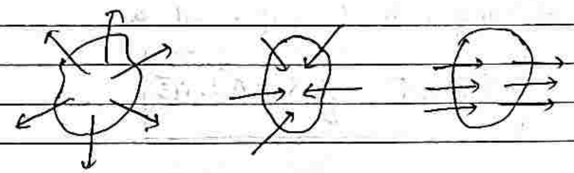
$\text{div}\ \vec{A} = \vec{\nabla}\cdot\vec{A}$

$$\vec{\nabla}\cdot\bar{A} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\left(\hat{i}A_x + \hat{j}A_y + \hat{k}A_z\right)$$

$$= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

→ Scalar qty.

Divergence < +ve / -ve / zero



+ve -ve zero

Solenoid Vector

If $\vec{A}$ is said to be a solenoidal vector, then $\vec{\nabla}\cdot\bar{A} = 0$

3) **Curl**

$$\text{Curl}\ \vec{A} = \vec{\nabla}\times\vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{j}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{k}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$\Rightarrow$ vector qty.

* When curl $\bar{A} = 0 \Rightarrow \bar{A}$ = Irrotational field

Gauss Divergence Theorem

It states that the surface integral of a vector taken over a closed surface enclosing a volume is equal to the volume integral of the divergence of a vector over that volume. It is expressed as

$$\boxed{\oint \vec{A} \cdot d\vec{s} = \oint_V (\vec{\nabla} \vec{A})\, dv}$$

Stoke's Theorem

It states that the line integral of a vector A around a closed curve C is equal to the surface integral of the curl of that vector A taken over a surface S bounded by the curve C.

Mathematically, it is expressed as

$$\boxed{\oint_C \vec{A} \cdot d\vec{l} = \oint_S (\vec{\nabla} \times \vec{A}) \cdot d\vec{s}}$$

Q. Find the divergence of the position vector in xyz plane.

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{\nabla} \cdot \vec{r} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \cdot (\hat{i}x + \hat{j}y + \hat{k}z)$$

$$= \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z} = 1+1+1 = 3$$

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Date
Page

Q. Evaluate $\vec{\nabla} q$ where $q = ax^2 - 2by + c^2z^2$ where a, b, c are constants at 1, -2, 3.

$$\vec{\nabla} q = \text{grad } q$$
$$= \hat{i}\frac{\partial q}{\partial x} + \hat{j}\frac{\partial q}{\partial y} + \hat{k}\frac{\partial q}{\partial z}$$

$$= \hat{i}\frac{\partial}{\partial x}(ax^2 - 2by + c^2z^2) + \hat{j}\frac{\partial}{\partial y}(ax^2 - 2by + c^2z^2) + \hat{k}\frac{\partial}{\partial z}(ax^2 - 2by + c^2z^2)$$

$$= \hat{i}[2ax] + \hat{j}[-2b] + \hat{k}(c^2 2z)$$

$\vec{\nabla} q|_{(1,-2,3)}$ $= \hat{i}(2)(1)x + \hat{j}(-2)\times(-2) + \hat{k}(9\times2\times3) = 2x\hat{i} + 4\hat{j} + 18z\hat{k}$

Q. Evaluate curl $\vec{A}$ where $\vec{A} = xy\hat{i} + yz\hat{j} + zx\hat{k}$
$$= A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\text{curl } \vec{A} = \vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & yz & zx \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial}{\partial y}zx - \frac{\partial}{\partial z}yz\right) - \hat{j}\left(\frac{\partial}{\partial x}zx - \frac{\partial}{\partial z}xy\right) + \hat{k}\left(\frac{\partial}{\partial x}yz - \frac{\partial}{\partial y}xy\right)$$

$$= (-y)\hat{i} - x\hat{j} + (-x)\hat{k}$$

8/11/19

Maxwell's Electromagnetic Equations

1) Gauss law of electrostatics
2) " " " magnetostatics
3) Faraday's law of e-m induction
4) Ampere's (modified form) circuital law

Maxwell's 1st Eqn

Gauss law of electrostatics

Acc. to gauss law of electrostatics the electric flux from a closed surface is equal to $\frac{1}{\varepsilon_0}$ times the net charge enclosed by the surface. Mathematically, it is expressed as

$\oint \vec{E}\cdot d\vec{s} = \frac{1}{\varepsilon_0} q$ — Integral form of Maxwell's 1st Eqn

$\downarrow$ electric flux

— ① $\Rightarrow \oint \vec{D}\cdot d\vec{s} = q$

If ρ = charge density

then $q = \int \rho\, dv$ —(2)

① $\Rightarrow \oint \vec{E}\cdot d\vec{s} = \frac{1}{\varepsilon_0}\int \rho\, dv$ —(3) Integral form

or $\oint \varepsilon_0 \vec{E}\, d\vec{s} = \int \rho\, dv$

but $\varepsilon_0 \vec{E} = \vec{D}$ = Displacement vector

or $\oint \vec{D}\cdot d\vec{s} = \int \rho\, dv$ —(4) Integral form

Acc. to Gauss divergence theorem we can express the LHS of eqn (4) as

$$\oint \vec{D}\cdot d\vec{s} = \oint (\vec{\nabla}\cdot\vec{D})\, dv \quad —(5)$$

$\Downarrow$ Gauss dvg thm

On comparing eqns (4) & (5)

$\boxed{\vec{\nabla}\cdot\vec{D} = \rho}$ 1st $\Rightarrow$ Differential form

$D = \varepsilon_0 E$

$D = \varepsilon E$

Displacement Current

Acc. to Maxwell, it is not only the current that produces a magnetic field but a changing electric field in vacuum or in a dielectric medium also produce magnetic field. This implies that a change in electric field is equivalent to a current which flows till the electric field is changing. This equivalent current is known as the displacement current.

Q. Distinguish betn displ. current and conduction current.

Maxwell's 2nd Eqn

Gauss law of Magnetostatics

Acc. to Gauss law of Magnetostatics the magnetic flux enclosed in a surface is equal to zero. The magnetic flux,

$$\phi_B = \int \vec{B}\cdot d\vec{s}$$

Acc. to this law, $\oint \vec{B}\cdot d\vec{s} = 0$ —(1) $\Rightarrow$ Integral form

On applying Gauss divergence theorem

$$\oint \vec{B}\cdot d\vec{s} = \oint (\vec{\nabla}\cdot\vec{B})\, dv$$

but $\oint (\vec{\nabla}\cdot\vec{B})\, dv = 0$

$\Rightarrow \boxed{\vec{\nabla}\cdot\vec{B} = 0}$ —(2) $\Rightarrow$ Differential form

Magnetic monopole does not exist.

Maxwell's 3rd eqn

Faraday's Law of Electromagnetic Induction

Acc. to Faraday's law of electromagnetic induction the induced emf in a circuit is equal to the rate of change of magnetic flux associated with it.

If ε = Induced emf

ϕ_B = Magnetic flux $= \int B\cdot ds$

$\varepsilon = -\frac{\partial \phi_B}{\partial t}$ —(1)

Q. What is the significance of a -ve sign in eqn (1)?

The cause of induced emf opposes it

LHS ① $\Rightarrow$ $\varepsilon = \int \vec{E}\cdot dl$ —(2a)

$\vec{E}$ = Electric field intensity

RHS ① $\Rightarrow$ $\frac{-\partial \phi_B}{\partial t} = \frac{-\partial}{\partial t}\int B\cdot ds$ —(2b)

Using eqns (2a) & (2b) in eqn ①, we get

$$\int E\cdot dl = -\frac{\partial}{\partial t}\int B\cdot ds \quad —(3) \Rightarrow \text{Integral form}$$

$$\int E\, dl = -\frac{\partial \phi_B}{\partial t}$$

On applying Stoke's theorem to LHS of eq^n (3)

LHS (3) $\Rightarrow \int \vec{E}.d\vec{l} = \int (\vec{\nabla}\times\vec{E}).ds$ —(4)

Comparing eq^{n}s (3) & (4), we find

$$\vec{\nabla}\times\vec{E} = -\frac{\partial B}{\partial t}$$

$$\Rightarrow \vec{\nabla}\times\vec{E} + \frac{\partial B}{\partial t} = 0 \quad —(5)$$ Differential form of Maxwell's 3rd eqn

Maxwell's 4th Eq^n

Ampere's Circuital Law

Acc. to Ampere's circuital law the line integral of the magnetic field is equal to μ_0 times the current flowing through a closed loop.

Mathematically, it is expressed as

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I \quad —(1)$$

μ_0 = Permeability in free space

I = Current flowing through the surface

Applying Stoke's th^m on LHS of eq^n (1), we find

LHS (1) $\Rightarrow \oint \vec{B}d\vec{l} = \int (\vec{\nabla}\times\vec{B}).ds$ —(2)

If we take J as the current density.

$$I = \int J.ds$$

The modified form of Ampere's law introduces the presence of displacement current.

Let us consider a capacitor with capacitance $C = \frac{q}{V} = \frac{\varepsilon_0 A}{d}$

d = distance of separation

A = Area

$$\text{or} \quad \frac{q}{V} = \frac{\varepsilon_0 A}{d}$$

$$\Rightarrow q = \varepsilon_0 A \frac{V}{d} = \varepsilon_0 A \vec{E}$$

$$q = \varepsilon_0 \vec{E}.A = A.\vec{D}$$

but $I = \frac{\partial q}{dt} = A\frac{\partial D}{\partial t}$ — Displacement current

Hence the displ. current density, $J = \frac{I}{A} = \frac{\partial D}{\partial t}$ —(3)

On introducing the displacement current the Ampere's law in modified form can be expressed as

$$\int \vec{B}\,ds = \mu_0(I + I_D)$$

$$= \mu_0 \int (J + J_D).ds \quad —(4) \text{ Integral form}$$

$$\int \vec{B}.dl = \mu_0 \int \left(J + \frac{\partial D}{\partial t}\right)\partial s \quad —(4a) — \text{Integral form}$$

As we know by using the Stoke's th^m on LHS of eq^n (4a) the line Integral can be converted into surface integral & can be expressed as

$$\oint \vec{B}.d\vec{l} = \oint(\vec{\nabla}\times\vec{B}).ds \quad —(4b) \rightarrow \text{Stoke's th}^m$$

$$(4a) \rightarrow \vec{\nabla}\times B = \mu_0\left(J + \frac{\partial D}{\partial t}\right) \quad —(5a)$$

On comparing eq^{n}s (4a) & (4b) we get (5a).

$$\left.\begin{aligned} \text{or} \quad \frac{\vec{\nabla}\times\vec{B}}{\mu_0} &= \vec{J} + \frac{\partial\vec{D}}{\partial t} \\ \text{or} \quad \vec{\nabla}\times\vec{H} &= J + \frac{\partial D}{\partial t} \\ \text{or} \quad \vec{\nabla}\times\vec{H} &= J + \varepsilon_0\frac{\partial E}{\partial t} \end{aligned}\right\} —(6) \text{ Differential form}$$

since $D = \varepsilon_0 E$

Summary of Maxwell's e-m Eq^{n}s

	Differential form	Integral form
1)	$\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$ or $\vec{\nabla}.\vec{D} = \rho$	1) $\oint \vec{E}.ds = \frac{q}{\varepsilon_0}$ or $\int D.ds = \int \rho\,dv$
2)	$\vec{\nabla}.\vec{B} = 0$	2) $\int \vec{B}.ds = 0$
3)	$\vec{\nabla}\times\vec{E} = -\frac{\partial B}{\partial t}$	3) $\oint E.dl = -\frac{\partial}{\partial t}\int \vec{B}.ds$
4)	$\vec{\nabla}\times\vec{B} = \mu_0\left(J + \frac{\partial D}{\partial t}\right)$ or $\vec{\nabla}\times\vec{H} = J + \frac{\partial D}{\partial t}$	4) $\oint \vec{H}.dl = \int \left(J + \frac{\partial D}{\partial t}\right)ds$ or $\oint B.dl = \mu_0 \int \left(J + \frac{\partial D}{\partial t}\right)ds$

$\vec{E}$ = Electric field intensity
$\vec{D}$ = Displacement vector
ρ = charge density
q = charge
$\vec{B}$ = Mag. field
J = Current density
$\vec{H}$ = Mag. field intensity
ε_0 = Permittivity in free space
μ_0 = Permeability " "

Significance of Maxwell's Equns

1) The changing electric field induces changing magnetic field and vice-versa.

2) The electric & mag. fields are ⊥ar to each other in an e-m wave and also perpendicular to the direction of propagation of wave.

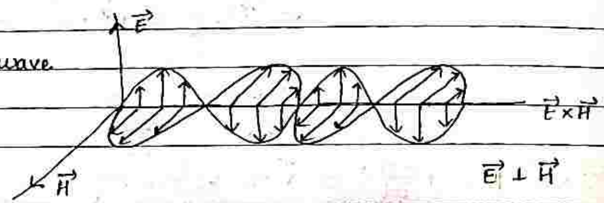
3) The speed of e-m wave is given by $C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

4) In electromagnetic waves the electric and magnetic field oscillates in phase with each other.

5) Maxwell's electromagnetic eqns are the fundamental equns of electromagnetic theory.

6) EM waves are transverse in nature.

Transverse e-m wave



$\vec{E} \perp \vec{H}$

Electromagnetic Wave Equns < In vacuum / In a conducting medium

EM wave is a transverse wave in which the electric component is ⊥ar to the mag. component & both of them are ⊥ar to the dirn of propagation of waves.

Electromagnetic wave eqn in free space

In free space, $\rho = 0$ and $J = 0$

so the Maxwell's e-m equns in free space are

$$\vec{\nabla}\cdot\vec{D} = 0 \quad \text{—(1)} \quad , \quad D = \varepsilon_0 \vec{E}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \quad \text{—(2)}$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{—(3)}$$

$$\vec{\nabla}\times\vec{B} = \mu_0 \frac{\partial \vec{D}}{\partial t} \quad \text{—(4)}$$

(4) $\Rightarrow \vec{\nabla}\times\vec{B} = \mu_0 \frac{\partial \vec{D}}{\partial t} = \mu_0\varepsilon_0 \frac{\partial \vec{E}}{\partial t}$ —(4a)

(3) $\Rightarrow \vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$

Taking curl on both sides

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B}) \quad \text{—(5)}$$

On applying the theorems of vector calculus on LHS of eqn(5), we get

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = \vec{\nabla}(\underbrace{\cancel{\vec{\nabla}\cdot\vec{E}}}_{0}) - (\vec{\nabla}\cdot\vec{\nabla})\vec{E}$$

Here, $\vec{\nabla}\cdot\vec{D} = 0 \Rightarrow \vec{\nabla}\cdot\vec{E} = 0$

$\vec{\nabla}\cdot\vec{\nabla} = \nabla^2$

$$\Rightarrow \vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = -\nabla^2\vec{E} \quad \text{—(6a)}$$

RHS (5) $\Rightarrow -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B}) = -\frac{\partial}{\partial t}\left[\mu_0\varepsilon_0 \frac{\partial E}{\partial t}\right]$ using (4)

$$= -\mu_0\varepsilon_0 \frac{\partial^2 E}{\partial t^2} \quad \text{—(6b)}$$

Using eqns (6a) & (6b) in eqn (5), we get

$$\cancel{-}\nabla^2 E = \cancel{-}\mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$$

or $$\boxed{\nabla^2 E = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}}$$ — (7) e-m wave eqn for electric component

Similarly we can find the e-m wave eqn for mag. component as

$$\boxed{\nabla^2 B = \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2}}$$ → (8) e-m wave eqn for magnetic component

Q. Derive eqn (8)

$\nabla \times (\nabla \times E)$

$\nabla \times (\nabla \times B) = \nabla\underbrace{(\nabla \cdot B)}_{0} - (\nabla \cdot \nabla) B$

* Special Case :

It is known that the general wave eqn is given by

$$\nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$$

ψ = Wave function $= \psi(\vec{r}, t)$

v = Velocity of wave

We obtained, $\nabla^2 E = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$ and $\nabla^2 B = \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2}$

On comparing these eqns, $\frac{1}{v^2} = \mu_0 \epsilon_0 \Rightarrow \boxed{v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}}$

Electromagnetic Wave Eqn in a Charge Free Conducting Medium

Charge free $\rho = 0$ conducting medium $\therefore \epsilon_0 \to \epsilon$, $\mu_0 \to \mu$

If σ = conductivity

$J = \sigma E$

The Maxwell's equns for this case are

$\vec{\nabla} \cdot \vec{D} = 0$ — (1) $\Rightarrow \vec{\nabla} \cdot \vec{E} = 0$

$\vec{\nabla} \cdot \vec{B} = 0$ — (2)

$\vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t}$ — (3)

$\vec{\nabla} \times \vec{B} = \mu \left(J + \frac{\partial D}{\partial t} \right)$

$= \mu \left(\sigma E + \frac{\partial D}{\partial t} \right)$ — (4)

$= \mu \left(\sigma E + \epsilon \frac{\partial E}{\partial t} \right)$ $D = \epsilon E$

(3) $\vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t}$ — (3)

Taking curl on both sides

$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\frac{\partial}{\partial t} (\vec{\nabla} \times \vec{B})$

but $\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla} \underbrace{(\vec{\nabla} \cdot \vec{E})}_{0} - \underbrace{(\vec{\nabla} \cdot \vec{\nabla})}_{\nabla^2} \vec{E}$

$= -\nabla^2 E$ — (5a)

RHS $\Rightarrow -\frac{\partial}{\partial t} (\vec{\nabla} \times \vec{B}) = -\frac{\partial}{\partial t} \left[\mu \left(\sigma E + \epsilon \frac{\partial E}{\partial t} \right) \right]$

$= -\frac{\partial}{\partial t} \left(\mu \sigma E + \mu \epsilon \frac{\partial E}{\partial t} \right)$

$= -\mu \sigma \frac{\partial E}{\partial t} - \mu \epsilon \frac{\partial^2 E}{\partial t^2}$ — (5b)

On using equns (5a) & (5b) in equn (3), we get

$$\cancel{-}\nabla^2 E = \cancel{-}\left[\mu \sigma \frac{\partial E}{\partial t} + \mu \epsilon \frac{\partial^2 E}{\partial t^2} \right]$$

or $$\boxed{\nabla^2 E = \mu \sigma \frac{\partial E}{\partial t} + \mu \epsilon \frac{\partial^2 E}{\partial t^2}}$$ (c) for electric component

Similarly for magnetic component the e-m eqn for a charge free conducting medium is given by

$$\boxed{\nabla^2 B = \mu\sigma \frac{\partial B}{\partial t} + \mu\epsilon \frac{\partial^2 B}{\partial t^2}} \quad \text{—— (7) for magnetic comp.}$$

Q. Derive eqn (7).

5/11/19 **Transverse Nature of Electro-magnetic Wave**

An e-m wave consist of both electric & magnetic field components.

The plane wave soln of electric field intensity vector $\vec{E}$ & mag. field intensity vector $\vec{H}$ can be written as

$$\vec{E} = \vec{E}(\vec{r},t) = E_0 e^{i(\vec{K}\cdot\vec{r}-\omega t)} = \hat{e} E_0 e^{i(\vec{K}\cdot\vec{r}-\omega t)}$$

$$\vec{H} = \vec{H}(\vec{r},t) = H_0 e^{i(\vec{K}\cdot\vec{r}-\omega t)} = \hat{b} H_0 e^{i(\vec{K}\cdot\vec{r}-\omega t)}$$

$\vec{E}$ = Electric field intensity
$\vec{H}$ = Magnetic " "
E_0 = Peak value of $\vec{E}$
H_0 = Peak value of ~~B~~ $\vec{H}$
$\vec{r}$ = Space coordinate
t = time coordinate
ω = angular frequency
$\vec{K}$ = wave propagation vector = $\hat{i}K_x + \hat{j}K_y + \hat{k}K_z$
$i = \sqrt{-1}$
$\hat{e}, \hat{b}$ = unit vectors associated with $\vec{E}$ and $\vec{H}$

For a charge free region, Maxwell's first eqn can be expressed as

$$\vec{\nabla}\cdot\vec{E} = 0$$

$$\vec{\nabla}\cdot\left[\hat{e}E_0 e^{i(\vec{K}\cdot\vec{r}-\omega t)}\right] = 0$$

From vector calculus, it is known that

$$\vec{\nabla}\cdot(\phi\vec{A}) = \vec{\nabla}\phi\cdot\vec{A} + \phi(\vec{\nabla}\cdot\vec{A})$$

Here ϕ = scalar field
$\vec{A}$ = vector field
$\vec{\nabla}\phi$ = grad ϕ
$\vec{\nabla}\cdot\vec{A}$ = div $\vec{A}$

Using this identity in the above eqn we find

$$\vec{\nabla}\left[E_0 e^{i(\vec{K}\cdot\vec{r}-\omega t)}\right]\cdot\hat{e} + E_0 e^{i(\vec{K}\cdot\vec{r}-\omega t)} \underbrace{(\vec{\nabla}\cdot\hat{e})}_{0} = 0$$

Further it is known that the divergence of a constant unit vector is 0, i.e $\vec{\nabla}\cdot\hat{e} = 0$

So $\vec{\nabla}\left[E_0 e^{i(\vec{K}\cdot\vec{r}-\omega t)}\right]\cdot\hat{e} = 0$

Since, $\hat{K} = \hat{i}K_x + \hat{j}K_y + \hat{k}K_z$

Hence $\left[(i\vec{K})e^{i(\vec{K}\cdot\vec{r}-\omega t)}\right]\cdot\hat{e} = 0$

or $i(\vec{K}\cdot\hat{e}) = 0$ since $e^{i(\vec{K}\cdot\vec{r}-\omega t)} \neq 0$

or $(\vec{K}\cdot\hat{e}) = 0$

$\Rightarrow$ Transverse nature of electric field
because $\hat{K} \rightarrow$ wave propagation vector
$\hat{e} \rightarrow$ unit vector of $\vec{E}$

Similarly on considering the Maxwell's 2nd eqn

$$\vec{\nabla}\cdot\vec{B} = 0$$

and following the same procedure as above we can prove that $\vec{K}\cdot\hat{b} = 0 \Rightarrow$ tr. nature of $\vec{B}$ or $\vec{H}$.

Hence from these two observations we can explain that the electric and magnetic components of an e-m wave are $\perp$ar to the dir^n of propagation of e-m wave. Hence e-m wave is transverse in nature.

Q1. Define gradient, divergence and curl and hence explain their significance.

Q2. Derive the Maxwell's electr. eqns of e-m induction in differential form and in integral form.

Q3. Distinguish bet^n displ. current & conduction current.

Q4. Derive the eq^n of continuity.

Q5. Define Faraday's law of electromagnetic induction.

Q6. Prove that magnetic monopoles does not exist.

Q7. Give the interpretation of Maxwell's eqn.

Q8. Derive the e-m wave eq^n in a free space in vacuum

Q9. Derive the e-m wave eqns in a conducting medium.

Q10. Prove that $\vec{J} = \sigma\vec{E}$

Q11. Prove the transverse nature of e-m wave.

Numericals

Q1. ~~Show that~~ Examine the vector $\vec{A} = (x+3y)\hat{i} + (y+az)\hat{j} + (x+az)\hat{k}$ is solenoidal is or not.

Q2. Show that the vector field $\vec{A} = (x^2+xy^2)\hat{i} + (y^2+x^2y)\hat{j}$ is irrotational.

Q3. If $\vec{A} = (x+y+1)\hat{i} + \hat{j} - (x+y)\hat{k}$ then prove that $\vec{A}\cdot(\vec{\nabla}\times\vec{A}) = 0$

Q4. If the potential $V(x,y,z) = (4x^2+2y^2+z^2)^{1/2}$, find $\vec{E}$ at $(1,1,1)$
Hint:- $E = -\nabla V$

Q5. If $\phi = 3x^2y - y^3z^2$, find $\vec{\nabla}\phi$ at $(1,-2,-1)$

QUANTUM PHYSICS

Wave-particle duality
Matter waves (de-Broglie hypothesis)
Wave function
Observables, Operators
Eigen value, eigen functions
Normalization, ~~Epe.~~ Expectation values
Schrodinger's equ^n^s < time dep / time ind.

Particle in box

Wave-Particle Duality

de-Broglie hypothesis explains that a wave is always associated with every moving particle and its corresponding wavelength which is called as de-Broglie's wavelength given by

$$\lambda = \frac{h}{p}$$

where, $\lambda \rightarrow$ wavelength of a wave
$p \rightarrow$ momentum $= mv$
$h \rightarrow$ Planck's constant

There are different forms of de-Broglie equ^n^:

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$K.E = \frac{1}{2}mv^2 = \frac{m^2v^2}{2m} = \frac{p^2}{2m} = E \text{ (say)}$$

or $p = \sqrt{2mE}$

1) $$\lambda = \frac{h}{\sqrt{2mE}}$$

where E represents the K.E of an accelerated particle

2) If the particle is accelerated with the help of an electrostatic energy $E = qV \Rightarrow \lambda = \frac{h}{\sqrt{2mqV}}$

3) When the particle is accelerated by thermal energy the Kinetic energy is given by

$$E = \frac{3}{2}KT$$

$$\lambda = \frac{h}{\sqrt{2m \times \frac{3}{2}KT}} = \frac{h}{\sqrt{3mKT}}$$

4) When the particle is moving with a velocity comparable to that of light, then the mass of the particle no longer remains constant but it will vary according to the eq^n^

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\lambda = \frac{h}{mv}$$

$$= \frac{h}{\frac{m_0}{\frac{\sqrt{c^2-v^2}}{c}} \times v} = \frac{h\sqrt{c^2-v^2}}{m_0 v c}$$

Q. Distinguish between electromagnetic wave and matter wave

Date ____
Page ____

Q. Mention the properties of matter wave.

* The wave particle duality of matter wave was experimentally proved by Davisson - Germer expt.

Wave Function

As per the theory of quantum mechanics every physical system is characterized by a wave function which contains all information about the system.

→ Wave functions are the mathematical representations of particles in motion which gives the probabilistic description of the particle.

→ A wave function is a function of space and time coordinates.

$$\psi = \psi(r, t)$$

In 1-D, $\psi = \psi(x, t)$
or $\psi(y, t)$
or $\psi(z, t)$

$\psi = A e^{i(kx - \omega t)}$
or $\psi = A \sin(kx - \omega t)$
or $\psi = A \cos(kx - \omega t)$
or $\psi = \ldots + \ldots$

ψ = wave function
A = Amplitude
$i = \sqrt{-1}$
k = wave propagation vector $= \frac{2\pi}{\lambda}$

λ = wavelength

x = displacement

$\omega = 2\pi\nu$

t = time at any instant

The probability density is given by $\psi\psi^* = |\psi|^2$

→ The wave f^n ψ must be continuous and single-valued.

→ The wave f^n must satisfy Schrodinger's wave eq^n.

→ Acc. to the principle of superposition the wave function of a system is a linear combination of different possible allowed state which is expressed as

for numerical problems $\psi = c_1\psi_1 + c_2\psi_2 + c_3\psi_3 + \ldots$

where $c_1, c_2, c_3, \ldots$ are the coefficients
$\psi_1, \psi_2, \psi_3, \ldots$ are the wave functions associated with the allowed states.

For such systems the probability $= |C_i|^2$, $i = 1, 2, 3, \ldots$

Normalization

It is expressed as $N = \frac{1}{\sqrt{\int_V |\psi|^2 dV}}$

where N is the Normalization constant

$Cond^n$ for normalization : $\int_V |\psi|^2 dV = 1$

Observables and Operators

Any dynamic quantity like position, co-ordinates, energy, linear momentum, angular momentum etc. which can be observed or measured is known as observables which can further be explained as follows :

Observables	Operators in 3D	Operators in 1D
Total Energy (E)	$i\hbar\frac{\partial}{\partial t}$	$i\hbar\frac{\partial}{\partial t}$
Linear momentum (p)	$-i\hbar\vec{\nabla}$	$-i\hbar\frac{\partial}{\partial x}$
Kinetic energy $\left(\frac{p^2}{2m}\right)$	$\frac{-\hbar^2}{2m}\nabla^2$	$\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$
Position	r	x.
Potential energy	V	V
Hamiltonian	$\frac{-\hbar^2}{2m}\nabla^2 + V$	$\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V$

Eigen functions and Eigen Values

In quantum mechanics a system can be defined by a number of definite states which are called as eigen states and are represented by eigen functions.

→ The actual state of a system is a linear combination of these eigen states with different relative probabilities. So each observable property of a physical system can have a set of definite allowed values called as eigen values.

Expectation Values

If q_1, q_2, q_3 etc. are the eigen values of a physical quantity then the expectation value is given by

$$\langle Q\rangle = \frac{P_1q_1 + P_2q_2 + P_3q_3 + \cdots}{P_1 + P_2 + P_3 + \cdots} = \frac{\sum_n P_nq_n}{\sum_n P_n}$$

where $P_1, P_2, P_3, \ldots$ are different relative probabilities of a system.

ImpQ1. What are the characteristic features of a wave function?

→ A wave function is a function of space and time co-ordinates.

→ The wave function must be continuous and single-valued.

→ The wave function must satisfy Schrodinger's wave eqⁿ.

Schrodinger's Wave Equation

Schrodinger eqⁿ is one of the most fundamental eqⁿ required to explain different quantum mechanical principles & theories. It gives the info. about the physical system.

To derive Schrodinger's Wave eqⁿ the wave particle duality principle is used.

Schrodinger's wave eqⁿ has two forms < Time Dependent / Time Independent

Time Dependent Schrodinger's Eqn

Let us consider a particle of mass 'm' moving with a velocity 'v'. The behaviour of the particle is explained through a wave function given by

$$\Psi = \Psi(x,t) = Ae^{i(Kx-\omega t)} \quad \text{—(1)}$$

where A = Amplitude

$K = \frac{2\pi}{\lambda}$

$i = \sqrt{-1}$

$\omega = 2\pi\nu$

Differentiating eqⁿ (1) w.r.t x

$$\frac{\partial\Psi}{\partial x} = iKAe^{i(Kx-\omega t)} = iK\Psi \quad \text{—(2a)}$$

On further differentiation of equⁿ (2a) w.r.t x, we get

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial x}\right) = -K^2 A e^{i(Kx-\omega t)} = -K^2\psi \quad \text{—(2b)}$$

Differentiating equⁿ (1) w.r.t 't' we get

$$\frac{\partial \psi}{\partial t} = -i\omega A e^{i(Kx-\omega t)} = -i\omega\psi \quad \text{—(2c)}$$

The energy of a particle is given by

$$E = h\nu$$

As $\omega = 2\pi\nu$

$\Rightarrow \nu = \frac{\omega}{2\pi}$

$$= \frac{h\omega}{2\pi} = \frac{h}{2\pi}\cdot\omega = \hbar\omega \quad \text{—(3)}$$

where, $\hbar = \frac{h}{2\pi}$ reduced Planck constant.

From de-Broglie's hypothesis on wave particle duality, we know that

$$\lambda = \frac{h}{p} \Rightarrow p = \frac{h}{\lambda}$$

On dividing the numerator & denominator of the above equⁿ, we have

$$p = \frac{h/2\pi}{\lambda/2\pi} = \frac{h}{2\pi}\times\frac{2\pi}{\lambda} = \hbar k \quad \text{—(4)}$$

Since the kinetic energy is given by

$$E = \frac{p^2}{2m} = \frac{1}{2}mv^2 = \frac{\hbar^2 K^2}{2m} \quad \text{—(5)}$$

~~Multiplying equⁿ (3) by $i\hbar$, we get~~

~~$i\hbar E = i\hbar^2\omega$~~

Multiplying equⁿ (2c) with $i\hbar$, we get

$$i\hbar\frac{\partial \psi}{\partial t} = (i\hbar)(-i\omega\psi)$$

$$= -i^2\hbar\omega\psi = \hbar\omega\psi$$

$$= E\psi \quad (\text{using } ③)$$

Thus we find $i\hbar\frac{\partial \psi}{\partial t} = E\psi$ —(6)

Using equⁿ (5) in equⁿ (6) we get

$$i\hbar\frac{\partial \psi}{\partial t} = \frac{\hbar^2 K^2}{2m}\psi \quad \text{—(7)}$$

As it was found that $\frac{\partial^2 \psi}{\partial x^2} = -K^2\psi$ from (2b)

$$\Rightarrow K^2\psi = -\frac{\partial^2 \psi}{\partial x^2} \quad \text{—(8)}$$

Using equⁿ (8) in the R.H.S of equⁿ (7), we get

$$i\hbar\frac{\partial \psi}{\partial t} = \frac{\hbar^2}{2m}\left(-\frac{\partial^2 \psi}{\partial x^2}\right) = \frac{-\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2}$$

$$\therefore \boxed{i\hbar\frac{\partial \psi}{\partial t} = \frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2}} \quad \text{—(9)}$$

Time Dependent Schrodinger's equⁿ

Equⁿ (9) holds good for a free particle where the potential energy, $V = 0$.

* Special Case

1) For any particle with potential energy, V the time dependent form of Schrodinger equⁿ (9) can be modified as

$$i\hbar\frac{\partial \psi}{\partial t} = \frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi \quad \text{—(10)}$$

2) In 3-D, eqⁿ (9) can be expressed as

$$i\hbar\frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi \quad \text{—(11)}$$

where, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

Time Independent Schrodinger's Equation

The Schrodinger's eq^n in time dependent form is given by
$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi$$

but $i\hbar\frac{\partial\Psi}{\partial t} = E\Psi$ (Refer equ^n (6))

Hence $E\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi$

or $(E-V)\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}$

or $\frac{2m}{\hbar^2}(E-V)\Psi = -\frac{\partial^2\Psi}{\partial x^2}$

or $\boxed{\frac{\partial^2\Psi}{\partial x^2} + \frac{2m}{\hbar^2}(E-V)\Psi = 0}$ —(12) Time independent S.E in 1-D

* In 3-D
$$\nabla^2\Psi + \frac{2m}{\hbar^2}(E-V)\Psi = 0$$

In general equation (12) is taken as the reference equation to derive different app' equ^{n}s for quantum physics.

Free Particle in a Box.

Let us consider a particle of mass 'm' moving freely in a box. For a free particle, $V = 0$

The time ind. S.E in 1-D
$$\frac{\partial^2\Psi}{\partial x^2} + \frac{2m}{\hbar^2}(E-V)\Psi = 0 \quad \text{——(1)}$$

Since, $V=0 \Rightarrow \frac{\partial^2\Psi}{\partial x^2} + \frac{2mE}{\hbar^2}\Psi = 0$ ——(2)

Let $\frac{2mE}{\hbar^2} = K^2$ ——(3)

Using eq^n (3) in eq^n (2) we get
$$\frac{\partial^2\Psi}{\partial x^2} + K^2\Psi = 0 \quad \text{——(4)}$$

Equ^n (4) is a second order differential equ^n.

Taking $\frac{\partial^2}{\partial x^2} = D^2$

(4) $\rightarrow$ $D^2\Psi + K^2\Psi = 0$

Auxiliary equ^n: $D^2 + K^2 = 0$ or $m^2 + K^2 = 0$
or $m^2 = -K^2 = i^2K^2$

Roots $= \pm iK$, $m_1 = iK$, $m_2 = -iK$

Sol^n: $\Psi = Ae^{m_1 x} + Be^{m_2 x}$

So $\Psi = Ae^{iKx} + Be^{-iKx}$ —(5)

Eq^n (5) represents the sol^n of the 2nd order differential eq^n of the free particle moving inside the box.

In eq^n (5), A and B are the arbitrary constants whose values can be found out by putting the boundary conditions.

The eigen functions of eq^n (5) are Ae^{iKx} and Be^{-iKx}

The corresponding eigen values $E = \frac{\hbar^2K^2}{2m}$

Since both the eigen functions have the same eigen value as explained above so it satisfy the condition of degeneration.

LASER

Date 14/11/19

Introduction
Characteristics
Einstein Coefficients
Lasing action – Population Inversion
3-level pumping
4-level "

Ruby laser
He-Ne Laser

LASER → **L**ight **A**mplication by **S**timulated **E**mission of **R**adiation

Characteristics of Laser
1) highly monochromatic in nature
2) " coherent " "
3) " polarised " "
4) " intensed or energetic " "
5) " directional
6) " focussed

LQ Atomic Transition (Einstein's Coefficients and their Relation)
Movement of atoms from one energy level to another energy level is known as atomic transition. Let us consider there are two energy levels E_1 and E_2 in a system where E_1 is the ground state and E_2 is the excited state.

$$E_2 - E_1 = h\nu$$

$h\nu$ amount of energy is to be supplied to the atoms present in the ground state for the transition from E_1 to E_2.

In the ground state, energy is minimum whereas life time is more. In the excited state, energy is max. whereas life time is less.

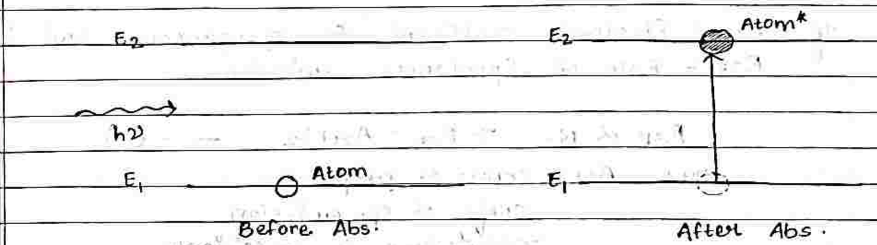
During the transition process, E_1 to E_2 transition takes place by absorption of energy. E_2 to E_1 transition takes place by emission of energy.

$E_1 \rightarrow E_2 \Rightarrow$ Absorption of energy
$E_2 \rightarrow E_1 \Rightarrow$ Emission of energy < Spontaneous / Stimulated

Atomic transition is of 3 types :-
Stimulated Absorption
Spontaneous Emission
Stimulated "
} Interaction between e-m rad^n with matter.

The coefficients of the above transition processes are called the Einstein's coefficients.

1) Stimulated Absorption Process (Einstein's B-coefficients)



Atom + Photon $\longrightarrow$ Atom*

Let N_1 = No. of atoms present at E_1
N_2 = No. of atoms present at E_2
R_{Ab} = Rate of absorption
$U(\nu)$ = Energy density

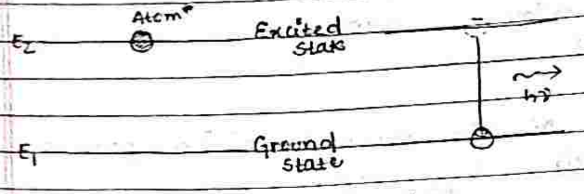
During stimulated absorption

$$\left.\begin{array}{l} R_{Ab} \propto N_1 \\ \propto u(\nu) \end{array}\right\} R_{Ab} = B_{12} N_1 u(\nu) \quad \text{——— (I)}$$

where, B_{12} = Constant of proportionality
= Coefficient of st. Abs.
= Einstein's B coefficient

During the transition from $E_2 \to E_1 \Rightarrow$ Emission < Normal Excited state (10^{-8} sec) → Spontaneous Emission; Metastable Excited state (10^{-3} sec) → Stimulated "

2) Spontaneous Emission Process (Einstein's A-co-efficient)



During spontaneous emission

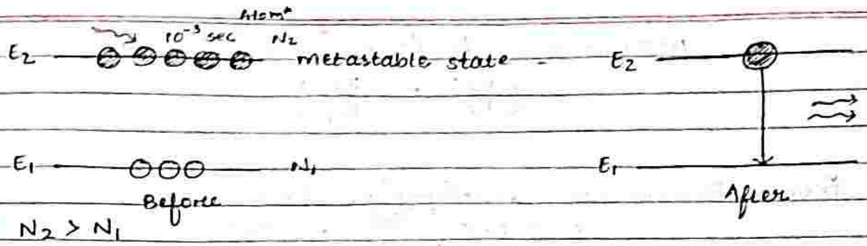
$$Atom^* = Atom + h\nu$$

If A_{21} = Einstein's coefficient for spontaneous emission
R_{sp} = Rate of Spontaneous emission

$$R_{sp} \propto N_2 \Rightarrow R_{sp} = A_{21} N_2 \quad \text{——— (II)}$$

Here A_{21} = Const. of prop.
= Coeff of sp. emission
= Einstein's A-coefficient

3) Stimulated Emission Process (Einstein's B-coefficient)
Stimulated emission takes place when there is metastable excited state. In metastable excited state the life time of atoms is very large, i.e. of the order of 10^{-3} sec.



$N_2 > N_1$

During stimulated emission process

$$Atom^* + h\nu \longrightarrow Atom + 2h\nu$$

If, R_{st} = Rate of stimulated emission

$$\left.\begin{array}{l} R_{st} \propto N_2 \\ \propto u(\nu) \end{array}\right\} R_{st} = B_{21} N_2 u(\nu) \quad \text{——— (3)}$$

where, B_{21} = Const. of prop.
= Coefficient of st. emission
= Einstein's B-coeff for st. em.

Relationship between Einstein's A, B coeff.
From the transition process, we obtained the following eq^ns

$$R_{Ab} = B_{12} N_1 u(\nu)$$
$$R_{sp} = A_{21} N_2$$
$$R_{st} = B_{21} N_2 u(\nu)$$

At thermal equilibrium, rate of absorption = rate of emission

i.e., $R_{Ab} = R_{sp} + R_{st}$

or $B_{12} N_1 u(\nu) = A_{21} N_2 + B_{21} N_2 u(\nu)$

or $u(\nu)\,[B_{12} N_1 - B_{21} N_2] = A_{21} N_2$

or $u(\nu) = \dfrac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2}$

or $u(\nu) = \dfrac{A_{21}}{B_{12} \frac{N_1}{N_2} - B_{21}}$

On dividing B_{12} from the numerator and denominator of the above eq^n we find

$$u(\nu) = \frac{(A_{21}/B_{12})}{\left(\frac{N_1}{N_2}\right) - \left(\frac{B_{21}}{B_{12}}\right)} \quad \text{--- (4)}$$

From Boltzmann distribution law

$$N_1 = N_0 e^{-E_1/K_BT} \quad \text{--- (5a)}$$

$$N_2 = N_0 e^{-E_2/K_BT} \quad \text{--- (5b)}$$

where K_B = Boltzmann constant

T = Absolute temperature

N_0 = Total no. of atoms present in the system before transition

So $\frac{N_1}{N_2} = e^{(E_2-E_1)/K_BT}$

$= e^{h\nu/K_BT}$ --- (6)

At thermal equilibrium, $B_{12} = B_{21} \Rightarrow \frac{B_{21}}{B_{12}} = 1$ --- (7)

Using eqns (6) and (7) in eqn (4), we get

$$u(\nu) = (A_{21}/B_{12}) \boxed{\frac{1}{e^{h\nu/K_BT} - 1}} \quad \text{--- (8)}$$

Further on considering the Planck's hypothesis we can express the energy density as

$$u(\nu) = \frac{8\pi h\nu^3}{c^3} \boxed{\frac{1}{e^{h\nu/K_BT} - 1}} \quad \text{--- (9)}$$

Comparing eqns (8) and (9), we find

$$\boxed{\frac{A_{21}}{B_{12}} = \frac{8\pi h\nu^3}{c^3}} \quad \text{--- (10)}$$

Eqn (10) represents the relationship between Einstein's A, B coefficients.

Lasing Action

The mechanism by which laser light is produced is known as lasing action.

1) Metastable state → Pumping Mechanism
2) Population Inversion
3) Stimulated Emission
4) For amplification → Resonator or Resonant Cavity

Population Inversion

Population Inversion is the condition at which more no. of atoms are present in the higher energy state as compared to the lower energy state.

~~During~~ For population inversion, $N_2 > N_1$

Because of population inversion stimulated emission can take place in laser devices.

Pumping

The mechanism to achieve population inversion.

There are different types of pumping mechanisms:

1) Optical Pumping
2) Thermal "
3) Electric discharge
4) chemical pumping
5) Nuclear "
6) Semiconductor "

15/11/19 Components of Laser Devices

1) Active Medium ⟨ Solid / Liquid / Gas → Active centers + Photons ⇒ Laser

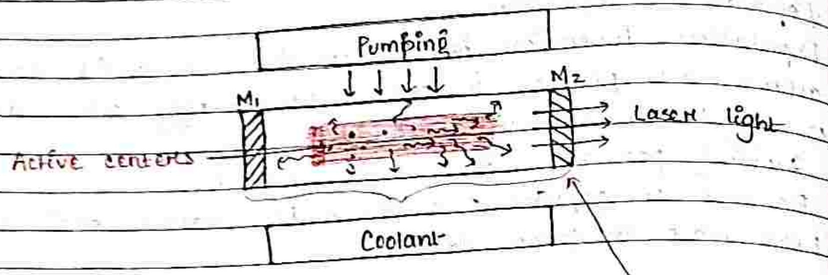
2) Pumping System

3) Resonant Cavity

4) Coolant arrangement

5) Brewster's Window

Block Diagram of Laser Device



$M_1 \rightarrow$ Fully reflective mirror
$M_2 \rightarrow$ Partially reflective mirror

Based on the type of ~~ept~~ active medium used in a laser device the laser devices are classified as

✓ Solid state laser ⇒ Ruby Laser → 3-level laser
✓ Gaseous laser ⇒ He-Ne Laser → 4-level Laser
Liquid state Laser
Semiconductor "

11/11/19 **Ruby Laser**

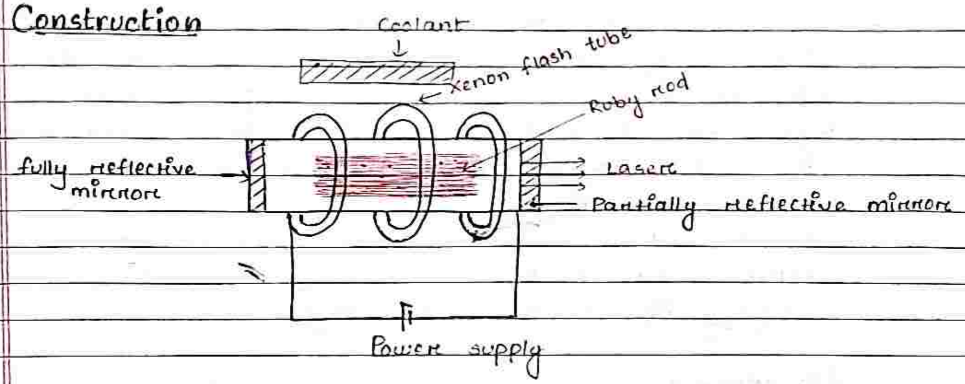
Ruby laser is the first solid state laser invented in 1960.
Ruby belongs to the family of gems.
It is basically Al_2O_3 crystal doped with chromium (Cr^{3+}) ion which gives it characteristic red colour. The colour of ruby rod depends upon the concentration of chromium in doped in it.

In ruby laser chromium ions are active centres which are responsible for the lasing behaviour of ruby laser device. The pumping system consist of a xenon flash tube which produces a pulsed laser light of wavelength 6943 Å when the lasing action is completed.

Keypoints

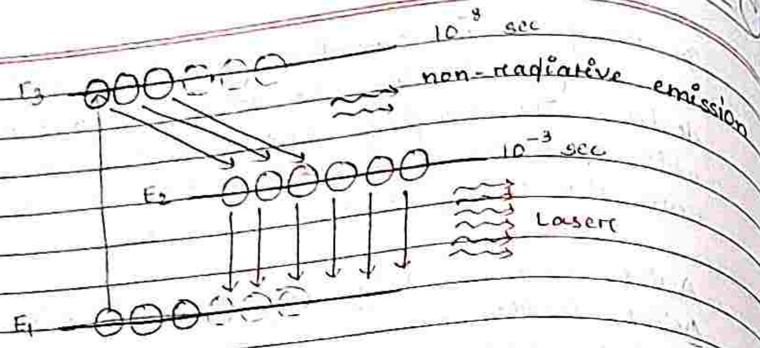
Active medium - Ruby Crystal doped with Cr^{3+}
Active center - Cr^{3+}
Pumping system - Xenon flash tube
3-level Energy system

Construction



Ruby crystal is taken in the form of a symmetrical rod of about 10 cm in length and about 1 cm in diameter. Its ends are polished in such a way that the end faces are exactly parallel to each other. The ruby crystal is enclosed inside a tube with a coil of xenon flash light. The xenon flash tube produces flashes of light and activates the active centres. The system is cooled with the coolant arrangement.

Working



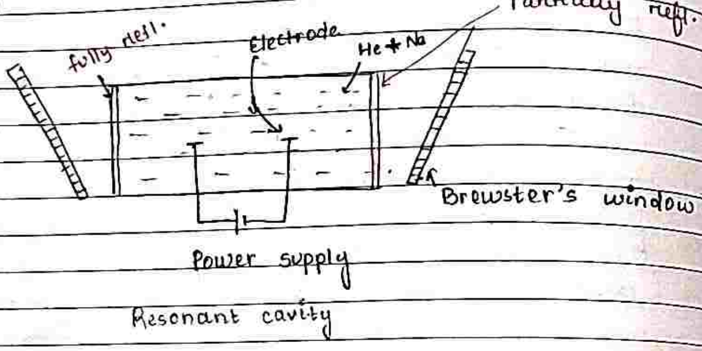
$N_2 > N_1$

He-Ne Laser

It is one of the types of first gas laser invented in 1961. It consists of a mixture of He & Ne with a suitable proportion. Ne gas will act as an active medium with electric discharge as the pumping system. The best known He-Ne laser operates at the wavelength of 6328 Å & produces a continuous laser beam as the output.

Active medium - Mixture of He and Ne gas
Active centre - Ne.
Pumping Mech - Electric discharge

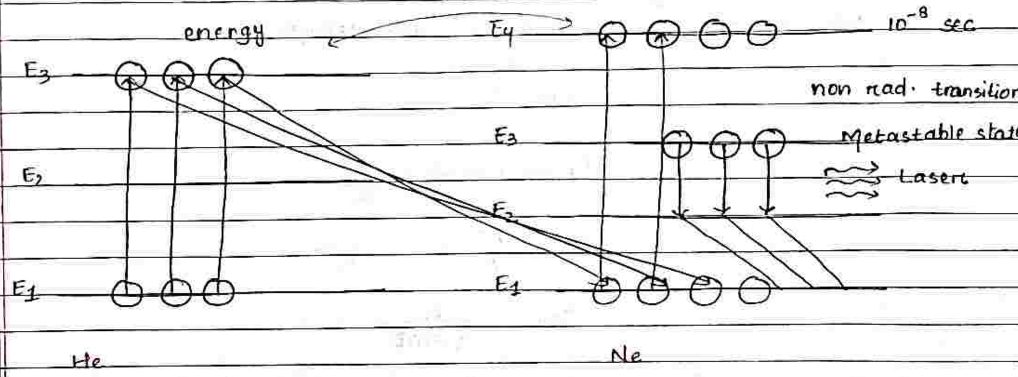
Construction



In He-Ne laser system a mixture of He and Ne gas in the ratio of 10:1 is taken inside a quartz tube at low pressure. The pumping source is provided by an electric discharge of very high voltage through the electrodes. The optical cavity consist of a fully reflective mirror at the input end and partially reflective mirror at the output end. An arrangement of Brewster's window is found in He-Ne laser to obtain a highly polarised laser light.

Working

The working of He-Ne laser is based on 4-energy level system.



QUANTUM MECHANICS

Q. Calculate the de-Broglie wavelength of a particle of mass 10 g moving with a speed of 300 m/s.

$m = 10\ g$

$v = 300\ m\,s^{-1}$

$\lambda = \frac{h}{mv}$

Q. If an electron beam in a television receivers tube is accelerated through a p.d. of 10,000 volts, Calculate the de-Broglie wavelength of the electron.

$\lambda = \frac{h}{\sqrt{2mqV}} = \frac{h}{\sqrt{2meV}}$

$\lambda = \frac{12.26}{\sqrt{V}}\ Å$

Q. Between a photon and an electron of same energy 100 eV, which one has shorter wavelength.

$E = 100\ eV$

$\lambda_{ph} = ?$

$\lambda_{el} = ?$

$E_{photon} = h\nu = \frac{hc}{\lambda}$

$\lambda_{ph} = \frac{hc}{E}$

$\lambda_{el} = \frac{h}{\sqrt{2mE}}$

Q. A proton is moving freely with kinetic energy 43.9 eV. Calculate the de-Broglie wavelength.

$E = 43.9\ eV$

$= \qquad J$

$\lambda = \frac{h}{\sqrt{2m_pE}}$

Q. If an electron has a wavelength of 1 Å, find the energy and momentum.

$\lambda_e = 1\ Å$

$\lambda = \frac{h}{\sqrt{2m_eE}}$

$\lambda^2 = \frac{h^2}{2m_eE}$

$\Rightarrow E_e = \frac{h^2}{2m_e\lambda^2}$

$\lambda = \frac{h}{p}$

$p = \frac{h}{\lambda}$

1. What is wave function? Give the physical interpretation of a wave f^n.
2. State wave-particle duality principle.
3. What do you mean by matter wave & mention the properties
4. State de-Broglie hypothesis & prove it.
5. P.T. the de-Broglie wavelength of an electron is $\frac{12.26}{\sqrt{V}}\ Å$.
6. What is the diff. bet^n matter wave & e.m wave?
7. Mention the characteristics of a wave f^n.
8. Define probability & probability density.
9. Write a note on operators used in quantum physics.

10. "All operators in quantum mechanics have eigen functions and eigen values". Define this.

11. What do you mean by eigen function and eigen values & hence explain the degenerate and non-degenerate states.

12. State the basic postulates of quantum mechanics.

13. Derive the Schrodinger's wave eqⁿ (i) in time dep. form
(ii) " " indep. "

14. Discuss the behaviour of a particle in a box using Schrodinger's wave eqⁿ.

LASER

1. What is laser?
2. Explain the characteristics of laser.
3. Write the difference betⁿ ordinary light & laser light.
4. How atomic transition occurs, when radiation interacts with matter?
5. Distinguish betⁿ spontaneous em. & st. em. of radiation.
6. Why stimulated em. is required for laser?
7. Why do you mean by population inversion? How is it achieved?
8. Explain (i) life-time of an energy level.
   (ii) pumping mechanism
   (iii) metastable state
   (iv) resonant cavity
   (v) negative temp. states
9. What are the functional components of a laser device?
10. Draw the block diagram of a laser device.
11. Explain, why a minimum of three-energy levels required for lasing action.

12. Derive the relationship between Einstein's A, B coefficient.
13. Describe the construction and working of a 3-level ruby laser.
14. What are the advantages of a 4-level laser as compared to a 3-level laser?
15. Explain the principle, construction & working of a 4-level He-Ne laser device.
16. What are the applications of laser?

Q. The He-Ne laser emits laser beam of wavelength 632.8 nm. Calculate the energy difference in eV between the two energy levels of the Ne atom.

$\lambda = 632.8$ nm

$E_g = h\nu = \frac{hc}{\lambda}$

E_2
$h\nu$
E_1

Q. A semiconductor laser emits light of wavelength 1.55 μm. Find its band gap.

$E_g = \frac{hc}{\lambda}$

Q. What is the coherence length of a laser beam of wavelength 7400 Å and coherence time 4×10^{-3} sec?

Coherence length, $L_c = \tau_c \cdot c$

$\lambda = 7400$ Å

coherence time, $\tau_c = 4 \times 10^{-3}$ sec