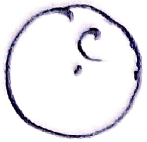
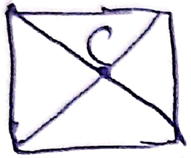
Centroid of some common figure/area

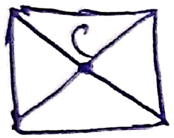
(1) Circle



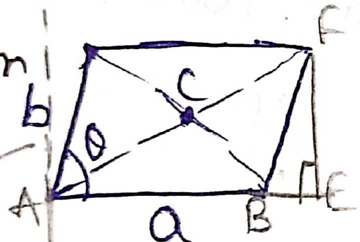
(2) Rectangle



(3) ~~Circle~~ square



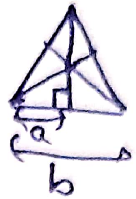
(4) parallelogram



$x_c = \frac{a + b\cos\theta}{2}$ EF = b sinθ

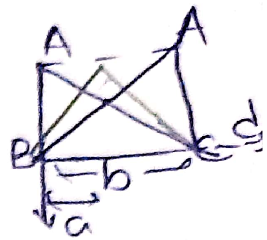
$y_c = \frac{b\sin\theta}{2}$

(5) Triangle:-



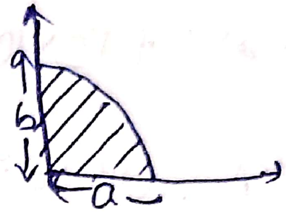
$y_c = \frac{y}{3}$

$x_c = \frac{1}{3}(b+a)$



$a = \frac{1}{2}b$

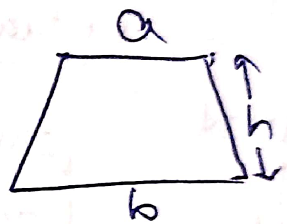
(6) Quarter ellipse



$x = \bar{x} = \frac{4a}{3\pi}$

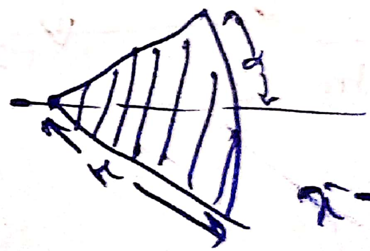
$y_c = \frac{4b}{3\pi}$

(7) Centroid of Trapezium



$y = \frac{h}{3}\frac{(b+2a)}{(b+a)}$

(8) Circular section



$\bar{x} = \frac{2r\sin\alpha}{3\alpha}$

work done $= f \times s \times \cos\theta$

force displacement angle

Virtual work $= f \times \delta s$

virtual displacement

for static eqm

$\Sigma f_x = 0$, $\Sigma f_y = 0$, $\Sigma M = 0$

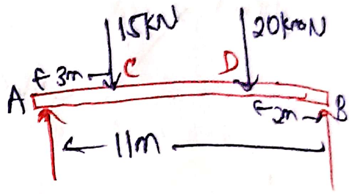
using this we can find tension. Action and reaction of the component on the system.

If Σ virtual work $= 0 \Rightarrow$ system is in eqm

Numerical 1.

Using method of virtual work determine the reaction at the support 'A' and 'B' of transversely loaded beam. find reacn at A & B

using method of moments



$20 \times 10^3 \times 9 + 15 \times 10^3 \times 3 - R_B \times 11 = 0$

$R_B = \frac{15 \times 3 \times 10^3}{11} + \frac{20 \times 10^3 \times 9}{11} = \frac{45}{11} \times 10^3 + \frac{20 \times 9}{11} \times 10^3$

$4.09 \times 10^3 + \frac{180}{11} \times 10^3$

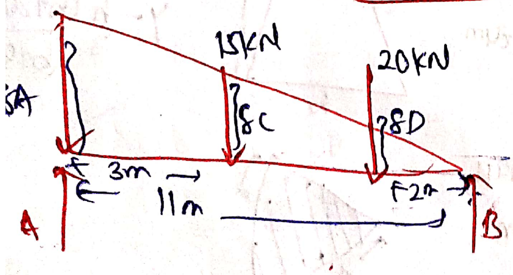
$= \frac{225}{11} = 20.45 = 20.45\,kN$

$- R_A \times 11 + 15 \times 10^3 \times 8 + 20 \times 10^3 \times 2 = 0$

$R_A = \frac{(15 \times 8 + 20 \times 2)\,kN}{11} = \frac{160}{11}\,kN = 14.55$

$= 14.55\,kN$

virtual work :-



Hold the beam at point B' & push 'A' δA upw

$\frac{\delta A}{11} = \frac{\delta C}{8} = \frac{\delta D}{2}$

Now, Applying virtual work on the configuration

$R_A(\delta A) + 15(-\delta C) + 20(-\delta D) = 0$

$R_A(\delta A) = 15 - \left(\frac{\delta A}{11}\right) \cdot 8 + -20\left(\frac{2\delta A}{11}\right) = 0$

$P_A \cdot \delta A = \frac{120(\delta A)}{11} + \frac{40\delta A}{11}$

$P_A = \frac{160}{11} = 14.55$ KN

for P_B

Hold the beam at point A & δB upward

$\frac{\delta B}{11} = \frac{\delta D}{9} = \frac{\delta C}{3}$

$P_B(\delta B) = 15(\delta C) + 20(\delta D)$

$P_B(\delta B) = 15 \times \frac{3\delta B}{11} + 20 \times \frac{9\delta B}{11}$

$P_B = \left(\frac{45+180}{11}\right) \times 10^3 N$

$= \frac{225}{11}$ KN $= 20.45$ KN

$T \sin 65° \times 3.2 \cos\theta = 250 \times 2.2 \cos\theta$

$T = \frac{250 \times 44}{32}$

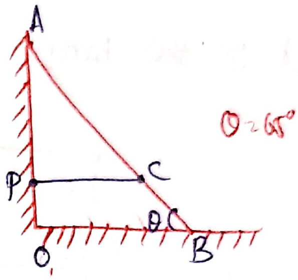
$250 \times \frac{2}{3} \times \cos\theta$

Numerical-2 :-

ladder $AB = 4.4$ mtr

weight of ladder $= 250$ N

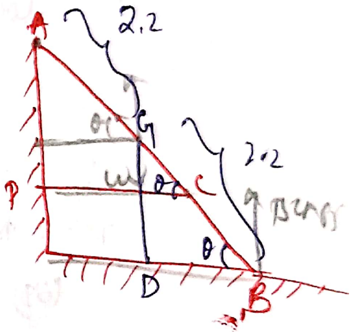
A rope 'PC' is tied with ladder to slip on ground using principle of virtual work. Find the tension on rope PC. $CB = 1.2$



$\frac{CB}{AB} = \frac{PO}{AO} \Rightarrow \frac{1.2}{4.4} = \frac{OP}{OA}$

$OA \cdot 1.2 = OP \cdot 4.4$

$\frac{OA}{4.4} = \frac{OP}{1.2}$



$T \sin\theta = W$

$T = W \operatorname{cosec}\theta$

$= 250 \times \operatorname{cosec} 65°$

$= 250 \times 1.10$

$= 275$

$4.4 \cos\theta$

$DG = 2.2 \sin\theta$

In ΔACP, $PC = (4.4 - 1.2) \cdot \cos\theta = 3.2 \cos\theta$

$\cos\theta = \frac{PC}{AC} \Rightarrow PC = AC \cos\theta = 3.2 \cos\theta$

Then, applying differential displacement

$\frac{\delta DG = 2.2 \cos\theta}{\delta PC \quad 3.2 \sin\theta}$

$\frac{DG = 2.2 \sin\theta}{PC \quad 3.2 \cos\theta} \Rightarrow \frac{DG}{2.2 \sin\theta} = \frac{PC}{3.2 \cos\theta}$

$\delta PC = \frac{\delta DG \times 3.2 \sin\theta}{2.2 \cos\theta}$

by differentiating $\quad \delta(DG) = 2.2 \cos\theta \quad$ and $\delta(PC) = 3.2 \sin\theta$

$= 3.2$

$250 \times (D/n) + 8 \times (8P/3) = 0$

$250 \times (8P/n) = 5 \times 8P/n \times \frac{3.2}{32} \tan\theta$

$250 \times \frac{22}{32} \times \frac{1}{\tan\theta} = 5$

$250 \times \cot\theta \times \frac{22}{32}$

$= 171.875 \times 0.46$

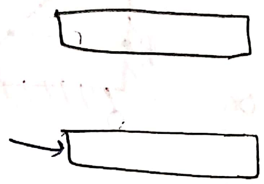
$250 \times 2.2 \cot\theta = 75 \sin 65° \times 3.2 \ldots$

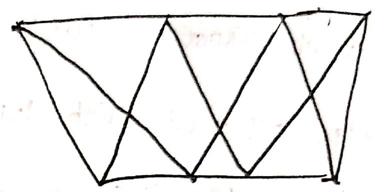
$T = 250 \times \frac{22}{32} \times \cot\theta$

$=$

Truss

A truss is an assemblage of slender bars fastened together at their ends (by balls or root). These slender bars may be either in tension or compression., No transverse load act on the bars.

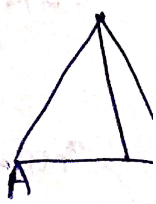




Truss
- Plane truss (2D)
  - Perfect truss: $m = 2j - 3$ (just stable)
  - Deficient truss: $m < (2j-3)$ (unstable)
  - Redendant truss: $m > (2j-3)$ (more than stable)
- Space truss (3D)

m = no. of bars or no. of members

j = no. of joints



plane truss :-

plane truss method can be solved by following methods

(i) method of joints

(ii) method of section

(iii) method of members

method of joints :-

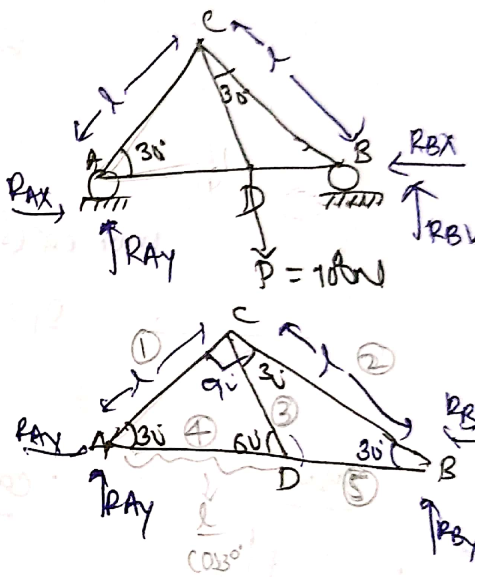
steps :-

(1) checking perfectly rigid or not ($m = 2j - 3$)

(2) finding out unknown reaction by applying eqn of equilibrium $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M = 0$

(3) drawing the FBD of joints and solving the forces on the members calling eqn of equilibrium (more than 2 unknown forces can be avoided)

Numerical :-



$\Sigma M_A = 0$ AD $= l \cos 30°$ BD $=$

$$P \times \frac{l}{\cos 30°} = R_{BY} \times (2l \cos 30°)$$

$$R_{BY} = P \times \frac{1}{2 \times \frac{3}{4}} = \frac{2P}{3}$$

$$R_{BY} = \frac{2P}{3} = \frac{2 \times 1000}{3} = \frac{2000}{3} = 666.6$$

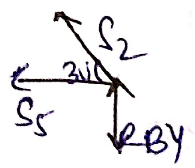
$\Sigma M_B = 0$

or $R_{AY} + R_{BY} = 1000$ N

$R_{AY} = 1000 - 666.6 = 333.3$

$R_{AY} = 333.3$

3) At point 'B' :-



Now $\Sigma F_x = 0$ & $\Sigma F_y = 0$

$\Sigma F_y = 0$ $S_2 \sin 30° + R_{BY} = 0$

$R_{BY} = -\frac{S_2}{2} \Rightarrow S_2 = -2R_{BY}$ $S_2 = -2 \times 666.6 = 1333.2$

$\Sigma F_x = 0$ $S_2 \cos 30° + S_5 = 0$

$S_2 \times \frac{\sqrt{3}}{2} = -S_5 \Rightarrow S_2 = -\frac{2}{\sqrt{3}} S_5$

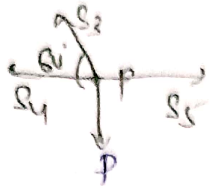
on comparing with the eqn

$-\cancel{R}R_{BY} = \frac{-\cancel{2}}{\sqrt{3}} S_5 \Rightarrow S_5 = \sqrt{3} R_{BY}$

$= \sqrt{3} \times 666.6$

$= 1154.6$

At point p:



$\Sigma f_x = 0$

$S_4 + S_3 \cos 60 - S_5 = 0 \Rightarrow S_4 = (S_5 - \frac{S_3}{2})$

$= (1154.6 - 577)$

$= 577.3$

$\Sigma f_y = 0$

$S_3 \sin 60 = P$

$S_3 = 1000 \times \frac{2}{\sqrt{3}}$

$= 1154.6$

At point '1'

$\Sigma f_y = 0$

$S_2 \sin 30 + S_1 \sin 30 + S_3 \sin 60 = 0$

$= S_2 + S_1 + S_3\sqrt{3} = 0 \rightarrow (5) \Rightarrow S_3 = -\frac{(S_2 + S_1)}{\sqrt{3}}$

$\Sigma f_x = 0$

$S_2 \cos 30 + S_1 \cos 30 + S_3 \cos 60 = 0$

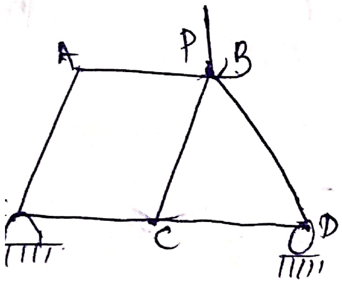
$S_2\sqrt{3} + S_1\sqrt{3} + S_3 = 0$

from eqn (5)

$S_1 = -(S_2 + S_3\sqrt{3})$

$= -($

Case-I



$m = 6$

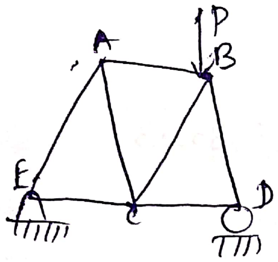
$j = 5$

$m < (2j-3)$

→ unstable

→ under-rigid

Case-II



$m = 7$

$j = 5$

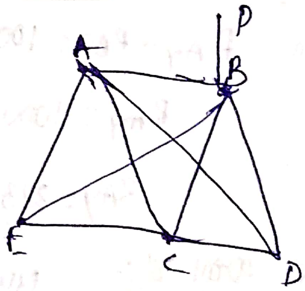
$m = (2j-3)$

→ stable

→ just rigid

→ statistically determinate

Case-III



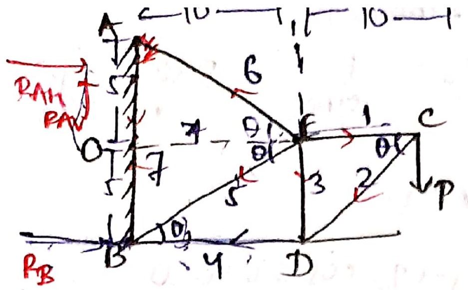
$m = 9$

$j = 5$

$m > (2j-3)$

→ over rigid

→ statically indeterm



In $\triangle COE$

$\tan\theta = \frac{5}{10} = \frac{1}{2}$

$\boxed{\theta = 26.16}$

At 'B'

$\Sigma F_y = 0 \quad T_5 \sin\theta + T_7 = 0$

$T_5 \sin\theta = -T_7$

$\Sigma F_x = 0$

$\cancel{R_B} = \cancel{T_4} + T_5 \cos\theta$

$2P = 2P + T_5 \sin\theta$

$\Sigma M_A = 0$

$R_B \times 1\cancel{0} = P \times 2\cancel{0}$

$R_B = 2P$

from $\Sigma F_x = 0$ at B

since $R_B \simeq T_4$

$T_5 \cos\theta = 0$

$\boxed{T_5 = 0}$

$\Sigma F_y = 0$

$T_5 \sin\theta = -T_7$

$\boxed{T_7 = 0}$

In $\triangle AOE$

$\tan\theta = \frac{5}{10} = \frac{1}{2} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{2}\right) = 26.16$

At 'C'

$\Sigma F_y = 0$

$+T_2 \sin 26.16 = P$

$\boxed{T_2 = \frac{+P}{0.44} = +2.22P}$

$\Sigma F_x = 0$

$T_2 \cos 26.16 + T_1 = 0$

$T_1 = -T_2 \cos 26.16$

$= 2.22P \times 0.9$

$\boxed{\simeq 2P}$

$\Sigma M_c = 0$

$-R_{An} \times 10 + R_B \times 20 + R_{AV} \times 20 = 0$

At 'D'

$\Sigma F_x = 0$

$T_4 = T_2 \cos\theta = T_2 \cos 26.16$

$= 2.22P \times 0.89$

$\boxed{\simeq 2P}$

$\Sigma F_y = 0$

$T_3 = T_2 \sin 26.16$

$= 2.22P \times 0.44$

$\boxed{= 0.98P}$

At (6)

$\Sigma F_x = 0$ $\quad F_1 = F_6 \cos\theta + \cancel{F_5 \cos\theta}$

$$\frac{F_1}{\cos\theta} = F_6$$

$$\boxed{\Rightarrow F_6 = 2.24P}$$

$\Sigma F_y = 0$

$$\cancel{F_5 \sin\theta} + F_3 = F_6 \sin\theta$$

$$F_3 = 2.24P \times \sin 26.6$$

$$= 2.24P \times 0.44$$

$$\boxed{= 0.98P}$$