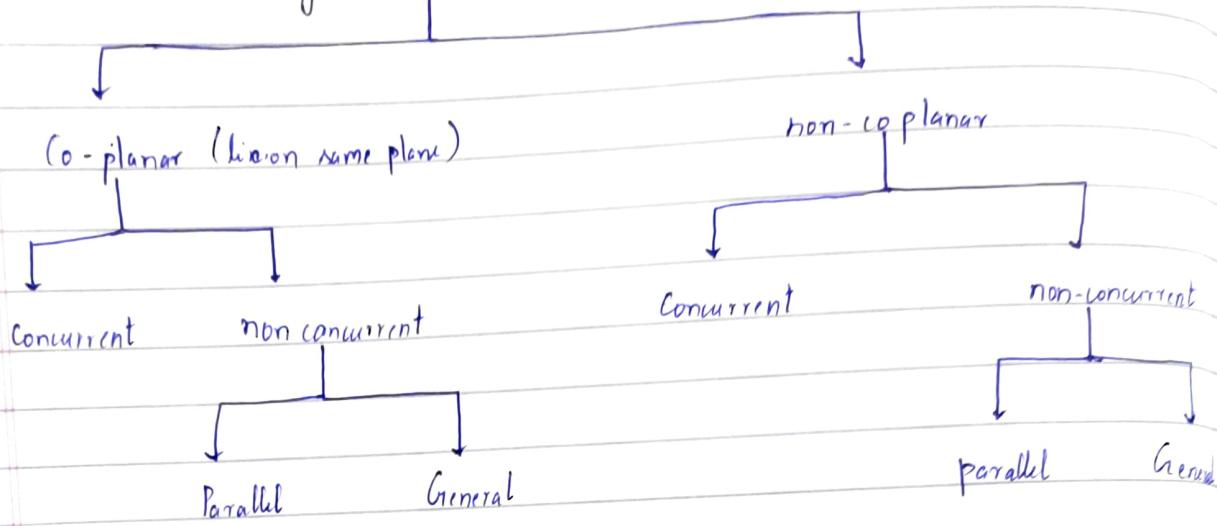
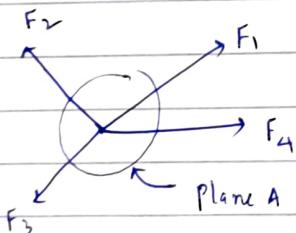


System of forces



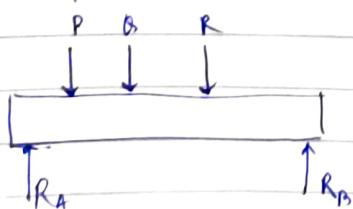
Concurrent force system in a plane :-

In this system lines of action of all forces pass through a single point and forces lie in the same plane



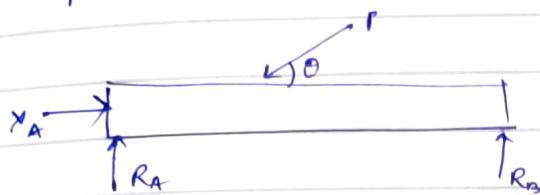
Parallel force system in a plane :-

In this system line of action of all forces lie in the same plane and are parallel to each other



General Force System in a Plane :-

The line of action of all these forces lie in the same plane but they are neither parallel nor concurrent.



- Concurrent force system can act on a particle or a rigid body
- Parallel or General force system can act on a system of particles, a rigid body or a system of rigid bodies.

Statics :

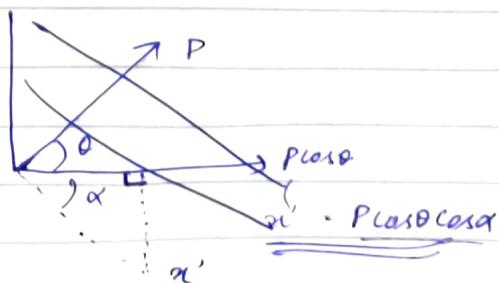
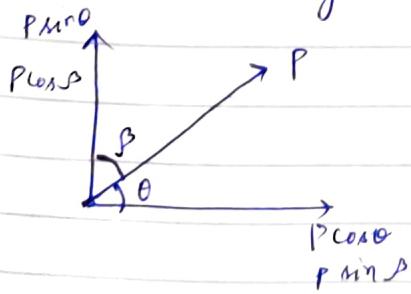
It deals with the equilibrium of the bodies in rest acted upon by force.
Deals with forces in the absence of ~~force~~ motion.

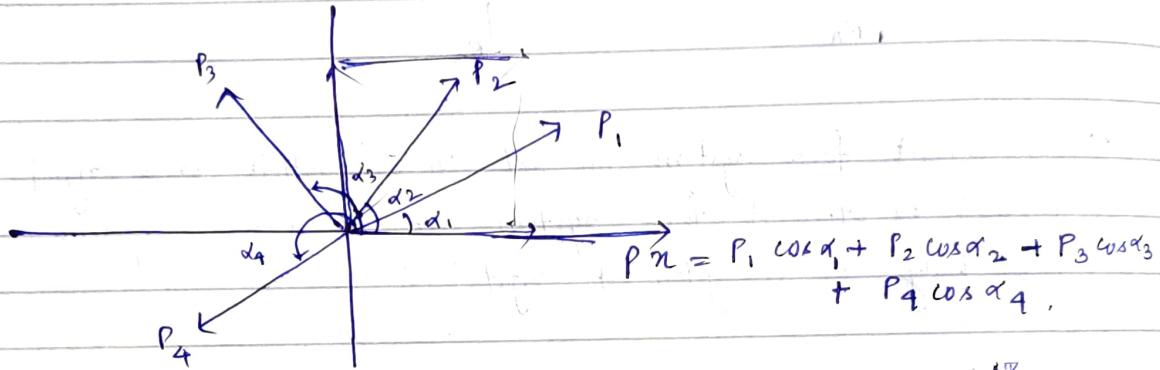
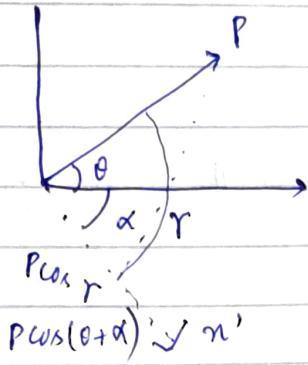
Dynamics :

The branch of mechanics that deals with the effect that forces have on the motion of objects.

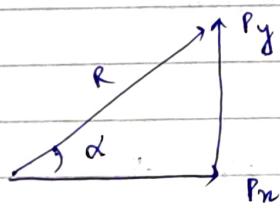
Resolution of forces :-

Replacement of a single force by several components will be equivalent in the action to the given force is called resolution of force.





$$P_y = P_1 \sin \alpha_1 + P_2 \sin \alpha_2 + P_3 \sin \alpha_3 + P_4 \sin \alpha_4.$$



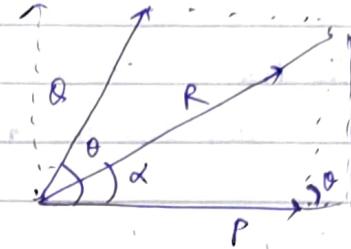
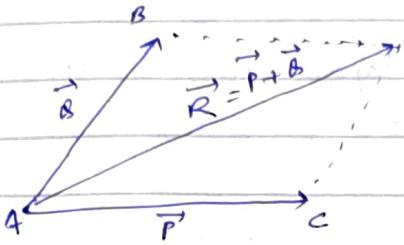
$$R = \sqrt{P_x^2 + P_y^2 + 2 P_x P_y \cos(90^\circ)}$$

$$R = \sqrt{P_x^2 + P_y^2}$$

$$\tan \alpha = \frac{P_y}{P_x}$$

Composition of forces :-

The reduction of given system of forces to the simplest system that will be its equivalent is called problem of composition.



$$X\text{ component} = P + Q \cos \theta + \cancel{P \cos \alpha}$$

$$Y\text{ component} = Q \sin \theta$$

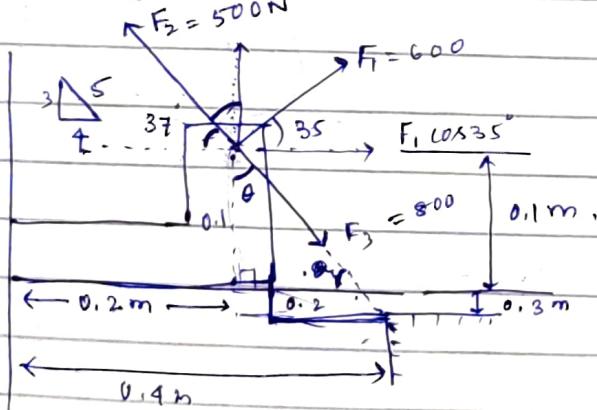
$$R = \sqrt{(P + Q \cos \theta)^2 + (Q \sin \theta)^2}$$

$$= \sqrt{P^2 + Q^2 \cos^2 \theta + 2PQ \cos \theta + Q^2 \sin^2 \theta}$$

$$= \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

$$\tan \alpha = \frac{Q \sin \theta}{P + Q \cos \theta}$$

$$(P + Q \cos \theta)$$



$$\begin{aligned} F_x &=? \\ F_y &=? \end{aligned}$$

$$\begin{aligned} \sqrt{(0.1)^2 + (0.2)^2} \\ = \sqrt{0.01 + 0.04} \end{aligned}$$

$$= \sqrt{0.05}$$

$$= \frac{\sqrt{5}}{10}$$

$$\frac{\sqrt{0.16 + 0.09}}{0.2} = \frac{\sqrt{0.25}}{0.2} = \frac{0.5}{0.2} = 2.5$$

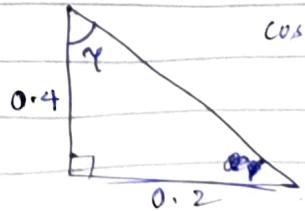
$$\cos \theta = \frac{3}{5}$$

$$\tan \theta = \frac{0.1}{0.2}$$

$$\theta = \tan^{-1}(y_2)$$

$$\cos \theta = \frac{0.2 \times 10}{\sqrt{5}} = \frac{2}{\sqrt{5}}$$

$$\begin{aligned} F_2 \times \frac{3}{5} &= 400 \text{ N} \\ F_2 \times \frac{4}{5} &= 400 \text{ N} \end{aligned}$$

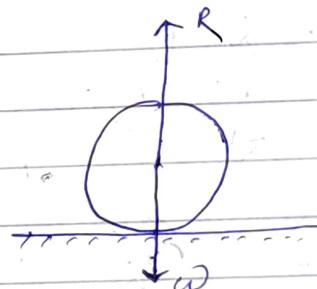
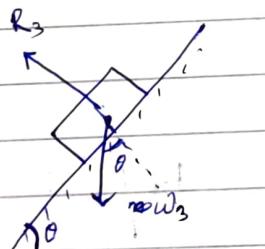
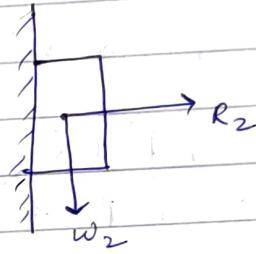
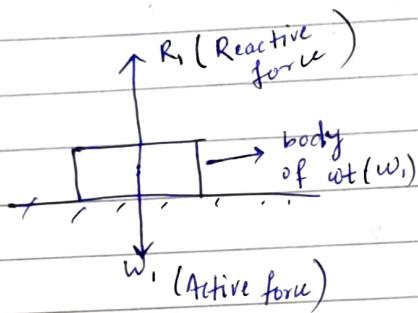


Reaction at planes :

A reaction is always perpendicular to the plane, the horizontal plane having vertical reaction and vertical plane has horizontal reaction.

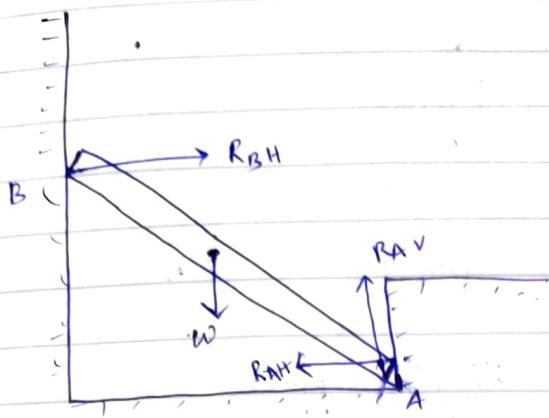
Active Forces : are the forces which are externally applied to the body or forces acting due to gravity. (weight of the body).

Reactive Forces : These are the forces which act as a result of active forces



Reactive force will pass through the centre.

Hinged



Types of Supports

- Simple Support :-

If one end of the beam rests on a fixed support, the support is known as simple support.

- Roller Support :-

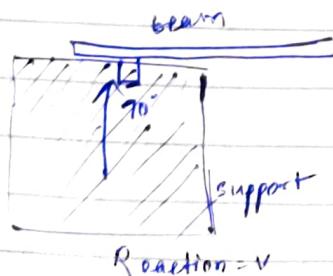
Here one end of the beam is supported on a roller.

- Hinged Support :-

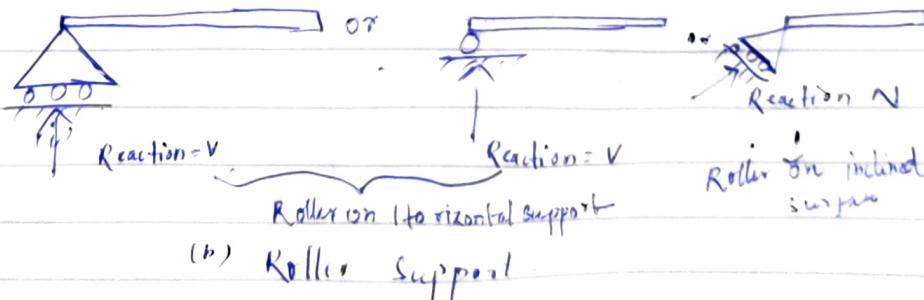
The beam does not move either along or normal to the axis but can rotate.

- Fixed Support :-

The beam is not free to rotate or slide along the length of the beam or in the direction normal to the beam. Therefore three reaction components can be observed for 3-D.

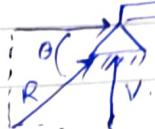
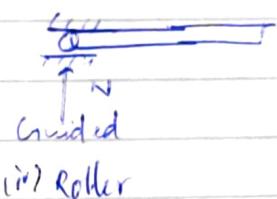


(a) Simple support.



(b) Roller support

Q-



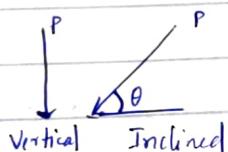
$$\text{Reaction} = R$$

Its components in vertical and horizontal directions

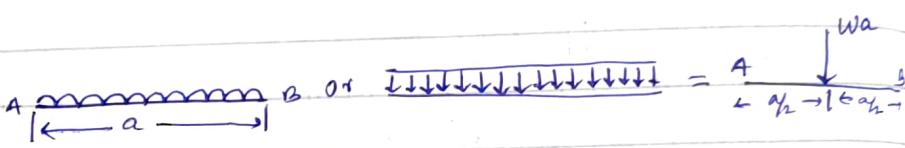
are V & H , respectively.

(i) Hinged support.

Types of Load

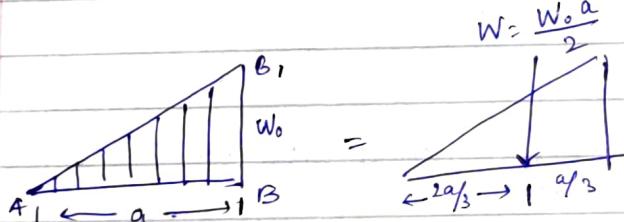


(a) Concentrated or point load

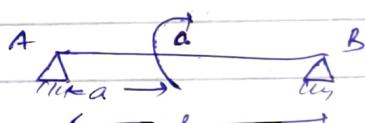


$$\text{Total load} = w.a$$

(b) Uniformly distributed load



(c) Uniformly varying load



(d) pure moment

Necessary Condition for Equilibrium:-

Body is said to be equilibrium under coplanar force, if it satisfies the following condition:

$$\sum F_x = 0$$

$$\sum F_y = 0$$

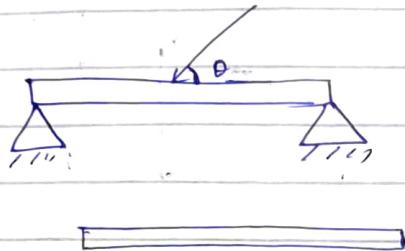
$$\sum M = 0$$

Free Body Diagram:

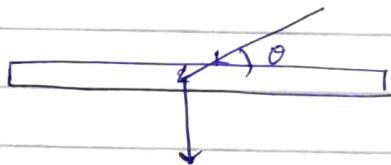
A Free Body diagram is a sketch of isolated body which shows the external forces (applied forces or active forces) on the body and the reaction extended on it by removed element of the surface.

Procedure:

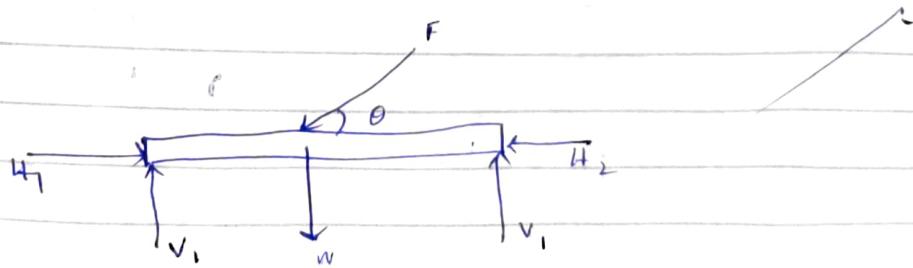
- i) A sketch of body is drawn by removing the surface



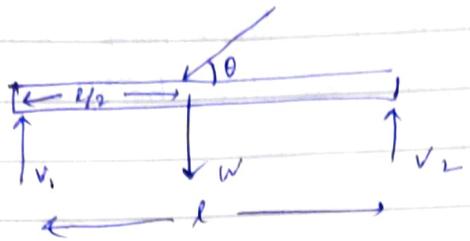
- ii) Indicate the applied or active forces which tend to set the body in motion such as those caused by the weight of the body or applied forces.



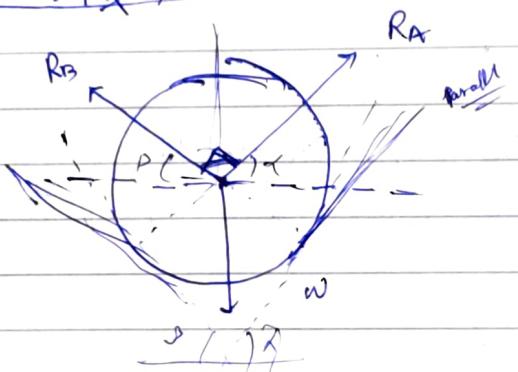
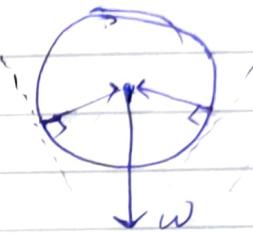
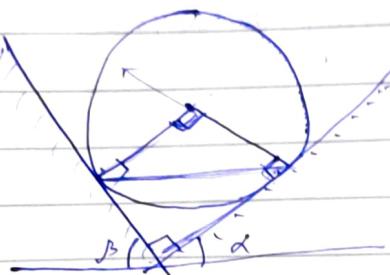
- iii) Also indicate on all the reactive forces such as those are caused by constraints or supports that tend to prevent the motion. (The sense of unknown reactant should be assumed). The correct sense will be determined by the solution of the problem.



iv) All relevant dimensions and angles, reference axis should be shown in the sketch.

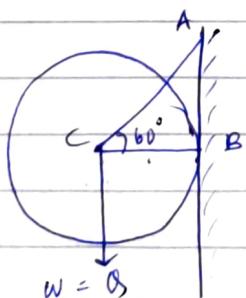


Ex:-



$$\alpha + \beta = 90^\circ$$

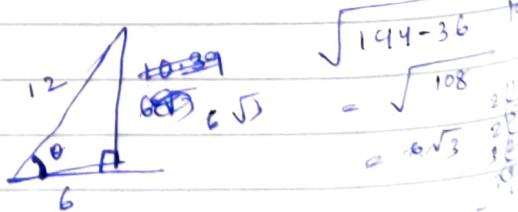
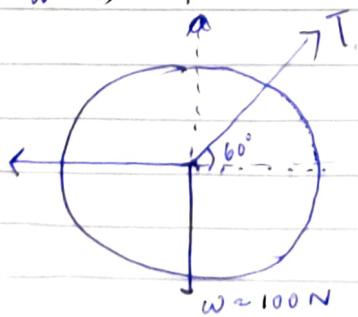
Ex :-



$$AC = 12 \text{ cm}$$

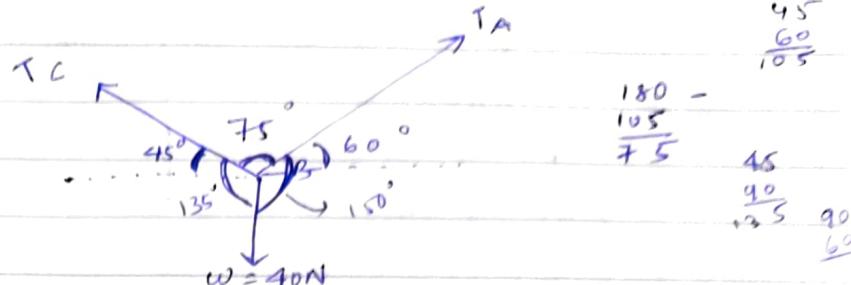
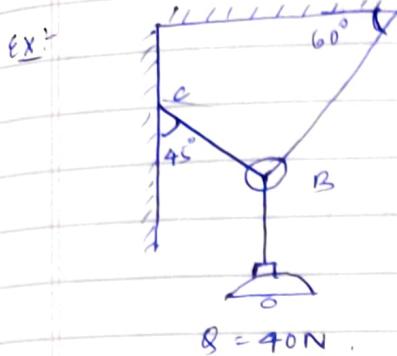
radius of the sphere = 6 cm

wt. of the sphere = 100 N.



$$\cos \theta = \frac{6}{\sqrt{108}} \quad \sin \theta = \frac{12}{\sqrt{108}}$$

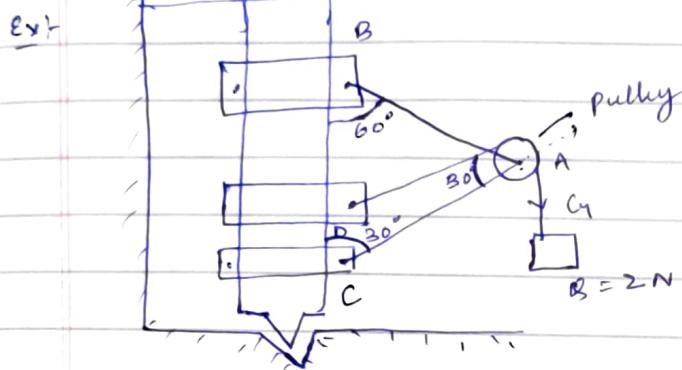
$$\cos \theta = \frac{6}{\sqrt{108}} \quad \theta = 60^\circ$$



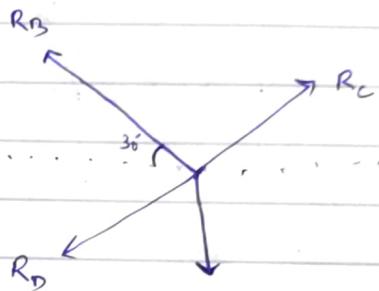
$$\frac{45}{60} - \frac{60}{105}$$

$$\frac{45}{105} - \frac{90}{135}$$

$$90 \\ 60$$

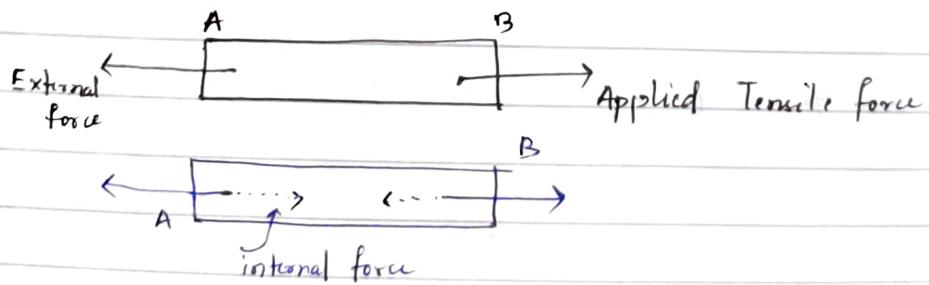


$B_{AB} = AB \& AC$ string of B & C

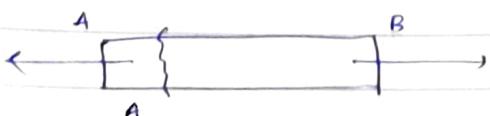


Tension & Compression:

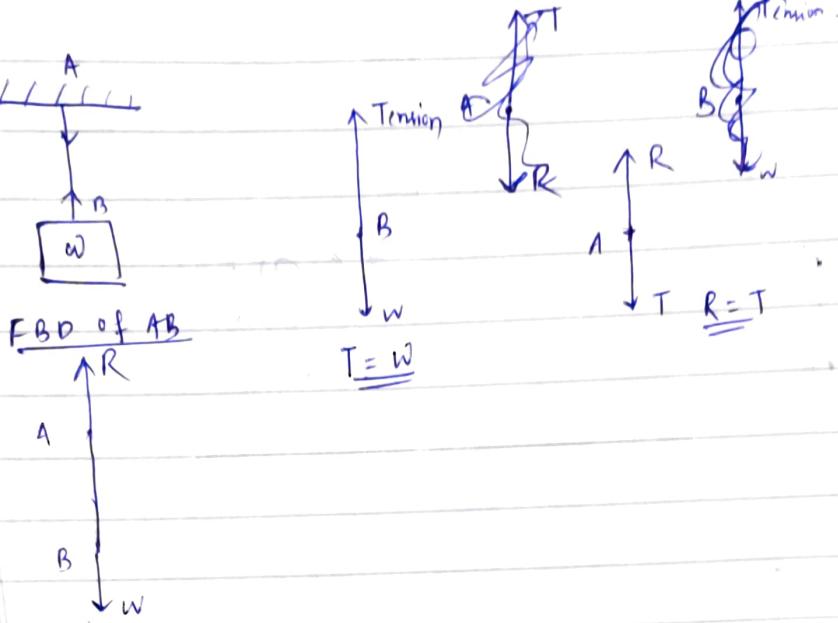
Tension is the force of resistance which opposes the elongation of the
It is always away from the point of action from force and along the



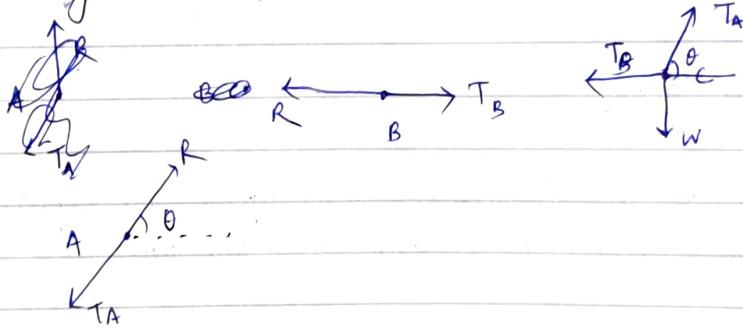
FBD of AB



$EF \leftarrow \boxed{\quad} \rightarrow IF$ which becomes EF



FBD diagram of A, B & C,



Compression is the force of resistance which opposes any contraction of a member under external load.



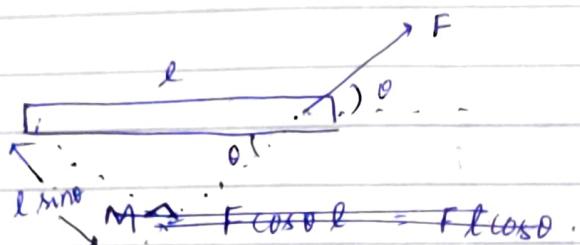
Moment of force:

Moment of the force is defined as product of the force and perpendicular distance of the force ~~from~~ at that point.

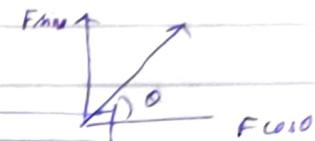
Unit - N/m

$$M = r F$$

↳ r = distance between Force and that point.



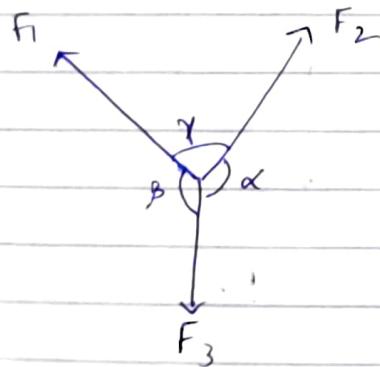
$$M = F l \sin \theta$$



$$M = F l \cos \theta$$

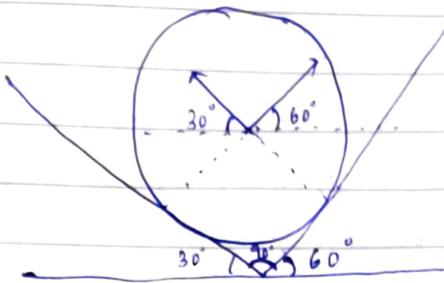
$$= F l \sin \theta$$

Lamis Theorem:

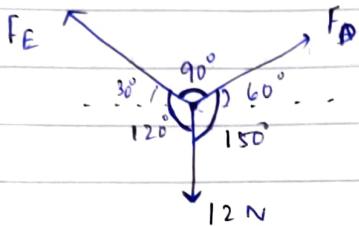


$$\frac{F_1}{\sin \alpha} = \frac{F_2}{\sin \beta} = \frac{F_3}{\sin \gamma}$$

Three forces acting on a point are in equilibrium then each force is proportional to the sine of angle with closing the other two forces.



$$\theta = 12 \text{ N}$$



Determine the forces exerted on the sides of the turf D and E if the surfaces are perfectly smooth.

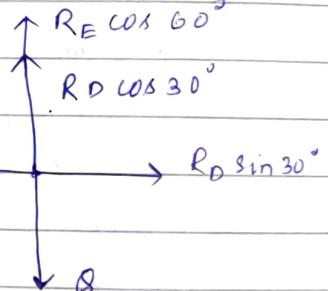
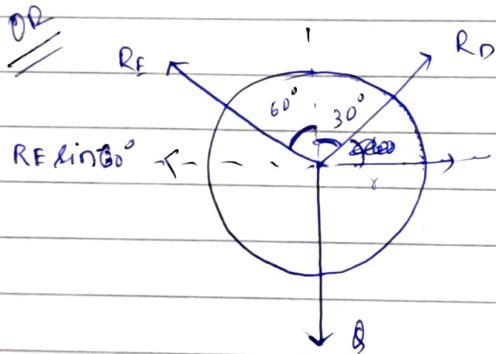
$$\frac{F_E}{0.5} = \frac{12}{1}$$

$$F_E = \frac{12 \times 1}{2} = 6$$

$$\frac{F_E}{\sin(150^\circ)} = \frac{F_D}{\sin(120^\circ)} = \frac{12}{\sin(90^\circ)}$$

$$\frac{F_D}{\sin(90+30^\circ)} = 12$$

$$F_D = 12 \times \frac{\sqrt{3}}{2} = \frac{6\sqrt{3}}{2}$$



$$\theta = \frac{R_E}{2} + \frac{R_D \cdot \sqrt{3}}{2}$$

$$\Rightarrow 24 = R_E + R_D \sqrt{3} R_D$$

$$\frac{R_D}{2} = \frac{\sqrt{3}}{2} R_E$$

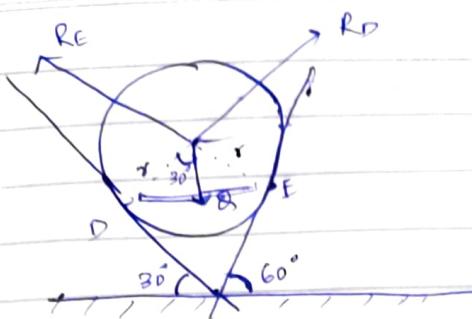
$$R_D = \sqrt{3} R_E$$

$$\Rightarrow 24 = R_E + \sqrt{3} R_E$$

$$\Rightarrow 24 = 4 R_E$$

$$\Rightarrow R_E = 6$$

$$R_D = 6\sqrt{3}$$



Moment about D,

$$\sum M_D = 0$$

$$(R_F \times 0) + R_E r \sin 30^\circ - 0 \cdot r \sin 30^\circ = 0$$

$$\Rightarrow R_E \cancel{x} - 0 \cdot r \sin 30^\circ = 0$$

$$\Rightarrow R_E = 0 \cdot r \sin 30^\circ$$

Moment about E,

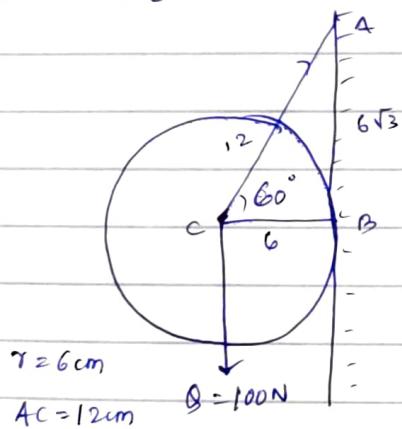
$$(R_E \times 0) - R_D \cancel{x} + 0 \cdot r \sin 60^\circ = 0$$

$$R_D = \frac{0 \cdot r \sqrt{3}}{2}$$

Anticlockwise $\rightarrow +ve$

Clock

Q: Determine 'T' in tie rod and force R_B exerted against the wall B.

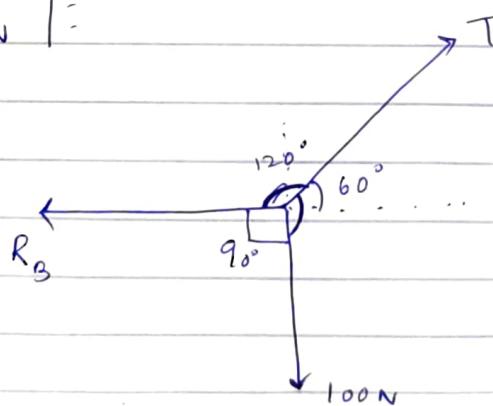


$$\sqrt{144 - 36}$$

$$\frac{144}{36}$$

$$\begin{array}{r} 2 \\ 2 \\ 3 \\ 3 \\ \hline 108 \end{array}$$

$$\cos \theta = \frac{6}{\sqrt{12}} \approx \frac{\sqrt{3}}{2}$$

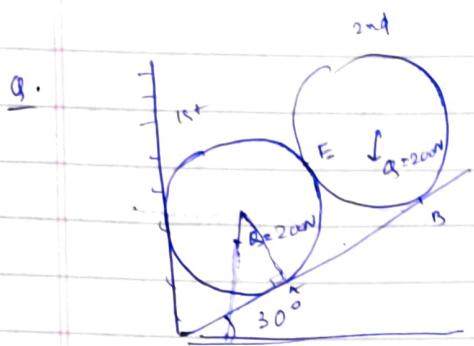


$$\frac{T}{\sin 90^\circ} = \frac{100}{\sin 120^\circ} = \frac{R_B}{\sin(90+60^\circ)}$$

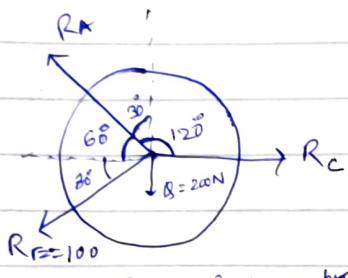
$$\therefore \frac{T}{1} = \frac{100}{\cos 30^\circ} = \frac{R_B}{\cos 60^\circ}$$

$$\therefore T = \frac{100 \times 2}{\sqrt{3}} = \frac{200}{\sqrt{3}}$$

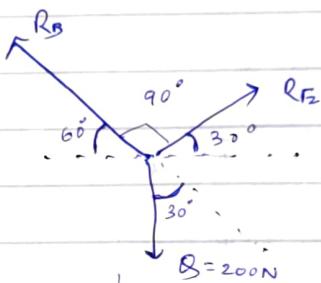
$$R_B = \frac{100 \times \sqrt{3}}{\sqrt{3}} \times \frac{1}{2} = \frac{100}{\sqrt{3}}$$



Find R_A, R_B, R_C



FBD of 1st span



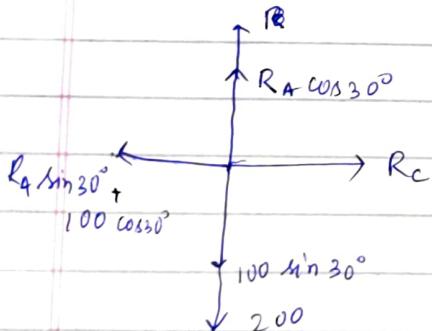
FBD of 2nd span

$$\frac{R_B}{\sin(90+30^\circ)} = \frac{200}{\sin 90^\circ} = \frac{R_E}{\sin(90^\circ+60^\circ)}$$

$$\Rightarrow \frac{R_B}{\cos 30^\circ} = \frac{200}{1} = \frac{R_E}{\sin 60^\circ}$$

$$\therefore \boxed{R_B = \frac{200 \times \sqrt{3}}{2}}$$

$$\boxed{R_E = 100}$$



$$R_C = \frac{R_A}{2} + \frac{100\sqrt{3}}{2}$$

$$\frac{R_A\sqrt{3}}{2} = 200 + 50$$

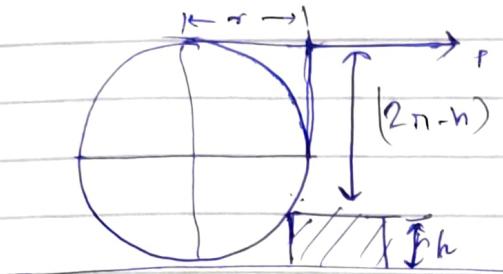
$$\sqrt{3}R_A = \frac{250}{2}$$

$$\boxed{R_A = 125\sqrt{3}}$$

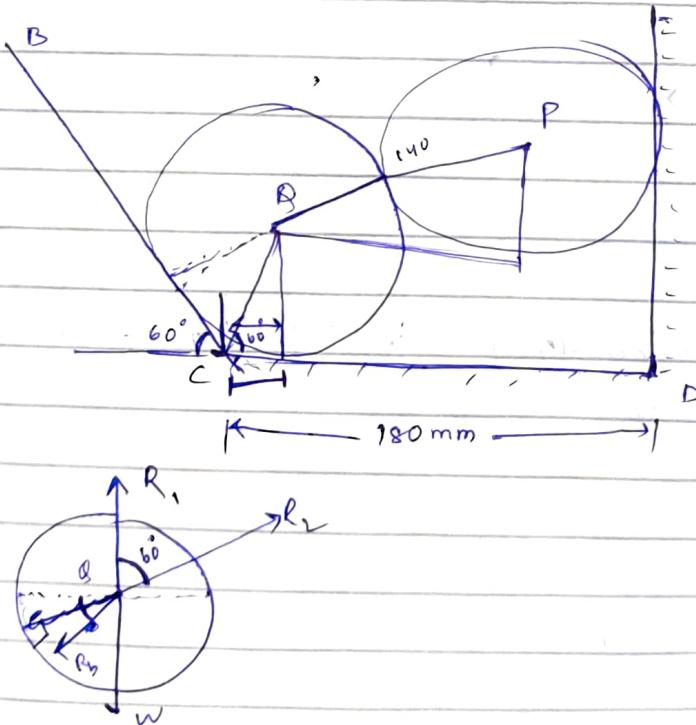
$$R_C = \frac{125\sqrt{3}}{2} + \frac{100\sqrt{3}}{2} = \frac{225\sqrt{3}}{2}$$

$$\boxed{R_C = \frac{225\sqrt{3}}{2}}$$

- Q. A roller of weight of 500 N and radius of 120 mm and is pulled over a step of height 60 mm by horizontal force 'P'. Find the magnitude of 'P' to just start the roller over the step.



- Q. Two cylinder p and q rest in a channel as shown , the diameter of p and q are 100mm and 180 mm respectively and weight of 200N and 500N respectively . Find the support reactions develop at the points of contact.

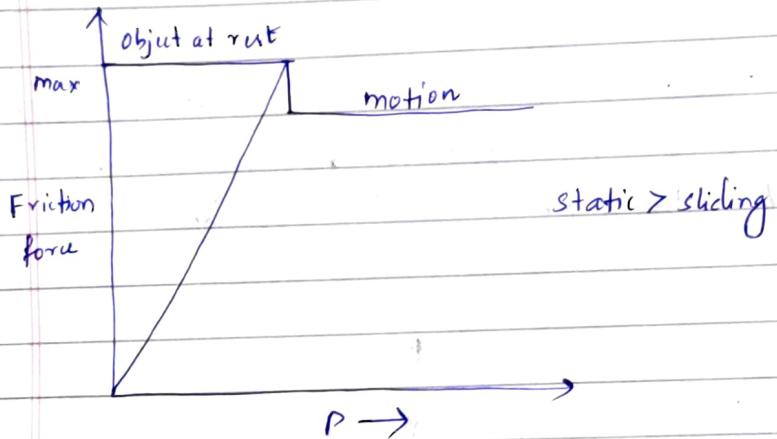


Friction

- Whenever two bodies are in contact there will be limited amount of resistance to sliding between them which is called friction.

Types of Friction

- static friction
- Kinetic friction - (a) sliding friction
(b) rolling friction.



Laws of friction:

- Total friction that can be developed is independent of magnitude of apparent area of contact.
- Total friction that can be developed proportional to the normal force.

$$F = \mu N$$

F = frictional force

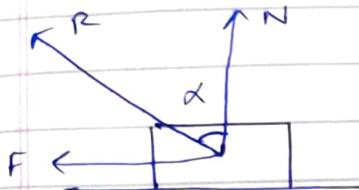
N = Normal force

μ = Co-efficient of friction.

- Total friction is independent of velocity.

Angle of Friction :-

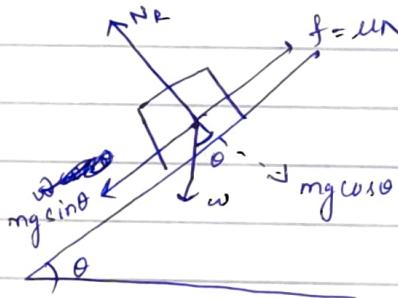
Angle between the resultant of frictional force and reaction normal reaction to the normal.



$$\tan \alpha = \frac{F}{N} = \mu$$

Angle of Repose :-

The minimum angle of inclined force on ~~an~~ inclined plane on which the body is about to slide down is called as angle of repose.



From figure :-

$$N = mg \cos \theta \quad \text{--- (i)}$$

$$F = mg \sin \theta$$

$$\Rightarrow \mu N = mg \sin \theta \quad \text{--- (ii)}$$

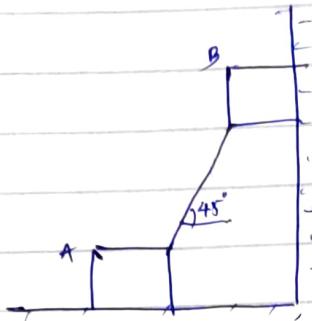
∴

Dividing eqn (i) & (ii),

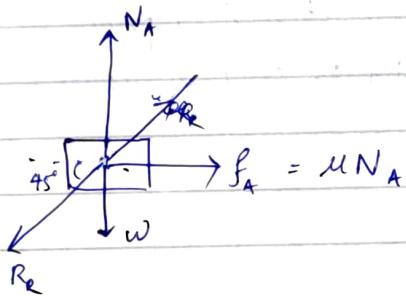
$$\frac{\mu N}{N} = \frac{mg \sin \theta}{mg \cos \theta}$$

$$\Rightarrow \tan \theta = \mu$$

- Q. Two identical blocks of weight 'w' are supported by rod inclined at 45° with the horizontal. If both blocks are limiting equilibrium. Find the coefficient of friction, assuming same at surface and wall.



FBD of A



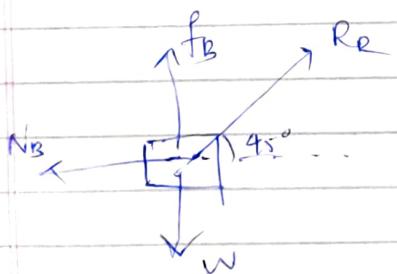
$$N_A = w + R_R \sin 45^\circ$$

$$R_R \cos 45^\circ = -\mu N_A$$

$$\Rightarrow R_R \frac{\sqrt{2}}{2} = \mu \left(w + R_R \frac{\sqrt{2}}{2} \right)$$

$$\Rightarrow \mu = \frac{R_R \times \sqrt{2}}{\sqrt{2}} \left(\sqrt{2}w + R_R \right)$$

$$\mu = R_R \left(\sqrt{2}w + R_R \right)$$



$$f_B + R_R \sin 45^\circ = w$$

$$= \mu N_B + R_R \sin 45^\circ = w \quad \text{--- (1)}$$

$$\mu \left(R_R \frac{\sqrt{2}}{2} \right) = w$$

$$N_B = R_R \cos 45^\circ \quad \text{--- (2)}$$

$$= \frac{R_R}{\sqrt{2}}$$

~~$$\mu \frac{R_R}{\sqrt{2}} = w$$~~

$$\mu = \frac{w}{R_R}$$

$$\mu = \frac{\sqrt{2}w}{R_R}$$

$$= \frac{\sqrt{2}w}{R_R}$$

$$= \frac{\sqrt{2}}{R_R} \left(\sqrt{2}w - R_R \right)$$

$$R_R(\sqrt{2}w + R_R) = \frac{w \times \sqrt{2}}{2 R_R}$$

$$2 R_R^2 (\sqrt{2}w + R_R) = \sqrt{2} w$$

$$\Rightarrow 2 R_R^2 +$$

$$R_R(\sqrt{2}w + R_R) = \frac{\sqrt{2}w}{R_R} - 1$$

$$\Rightarrow R_R \sqrt{2}w + R_R^2 = \frac{\sqrt{2}w - R_R}{R_R}$$

$$w R_R + \frac{R_R}{\sqrt{2}} = w$$

$$w \frac{R_R}{\sqrt{2}} = w - \frac{R_R}{\sqrt{2}}$$

$$w = \frac{\sqrt{2}}{R_R} \left(w - \frac{R_R}{\sqrt{2}} \right)$$

$$w = R_R(\sqrt{2}w + R_R)$$

$$\frac{\sqrt{2}}{R_R} \left(w - \frac{R_R}{\sqrt{2}} \right) = R_R(\sqrt{2} + R_R)$$

$$\Rightarrow \frac{\sqrt{2}}{R_R} \left(w - \frac{R_R}{\sqrt{2}} \right) = \sqrt{2} R_R + R_R^2$$

$$\Rightarrow \frac{\sqrt{2}}{R_R} \left(\frac{\sqrt{2}w - R_R}{\sqrt{2}} \right) = \sqrt{2} R_R + R_R^2$$

$$\Rightarrow \frac{\sqrt{2}w}{R_R} - 1 = \sqrt{2} R_R + R_R^2$$

$$N_A = S \sin 45^\circ + \omega \quad \text{--- (i)}$$

$$\mu N_A = S \cos 45^\circ \quad \text{--- (ii)}$$

$$\omega = \mu N_b + S \sin 45^\circ \quad \text{--- (iii)}$$

$$N_b = S \cos 45^\circ$$

~~$\frac{N_A}{N_B} = \frac{S}{\mu \sqrt{2}}$~~

$$\mu \left(\frac{S}{\sqrt{2}} + \omega \right) = \frac{S}{\sqrt{2}}$$

$$\Rightarrow \mu \left(\frac{S + \sqrt{2}\omega}{\sqrt{2}} \right) = \frac{S}{\sqrt{2}}$$

$$\Rightarrow \mu = \frac{S}{S + \sqrt{2}\omega}$$

$$\omega = \mu \cdot \frac{S}{\sqrt{2}} + \frac{S}{\sqrt{2}}$$

$$\omega = \frac{S}{\sqrt{2}} (\mu + 1) \quad \text{--- (iv)}$$

$$\Rightarrow \frac{\sqrt{2}\omega}{S} = \mu + 1$$

∴ ~~RE~~

$$\frac{\sqrt{2}\omega}{\mu(S + \sqrt{2}\omega)} = \mu + 1$$

$$\Rightarrow \frac{\sqrt{2}\omega}{S + \sqrt{2}\omega} = \mu(\mu + 1)$$

$$\Rightarrow \frac{S + \sqrt{2}\omega}{\sqrt{2}\omega} = \frac{1}{\mu(\mu + 1)}$$

$$\Rightarrow \frac{S}{\sqrt{2}\omega} + 1 = \frac{1}{\mu(\mu + 1)}$$

$$N_A = \frac{S}{\mu \sqrt{2}}$$

$$\frac{S}{\mu \sqrt{2}} = \frac{S}{\sqrt{2}} + \omega$$

$$\omega = \frac{S}{\mu \sqrt{2}} - \frac{S}{\sqrt{2}}$$

$$\omega = S \left(\frac{1}{\mu \sqrt{2}} - \frac{1}{\sqrt{2}} \right) \quad \text{--- (v)}$$

from comparing (i) & (ii)

$$\frac{S}{\sqrt{2}} (\mu + 1) = S \left(\frac{1}{\mu \sqrt{2}} - \frac{1}{\sqrt{2}} \right)$$

$$\Rightarrow \frac{\mu + 1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \left(\frac{1}{\mu} - 1 \right)$$

$$\Rightarrow (\mu + 1)\mu = 1 - \mu$$

$$\Rightarrow \mu^2 + \mu + \mu - 1 = 0$$

$$\Rightarrow \mu^2 + 2\mu - 1 = 0$$

$$\mu = -2 \pm \sqrt{4 - 4(1)(-1)} \quad \text{--- 8}$$

$$= -2 \pm \sqrt{4+4} \quad \text{--- 21/22}$$

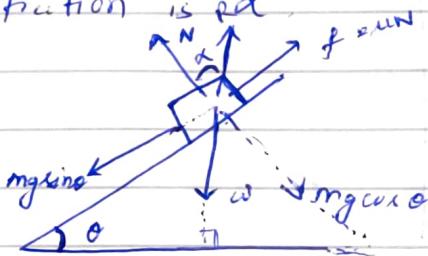
$$= \frac{-2 \pm 2\sqrt{2}}{2} = \boxed{\frac{-1 \pm \sqrt{2}}{2}}$$

$$\mu = -1 + \sqrt{2} \quad \text{or} \quad \mu = -1 - \sqrt{2}$$

→ neglected as it is negative

$$\boxed{\mu = 0.414}$$

- Q A flat stone slab rest on an inclined skidway that makes an angle α with the horizontal. What is the condition of equilibrium if angle of friction is $\mu\alpha$



$$\tan \alpha = \frac{f}{N}$$

$$f = N \tan \alpha$$

$$\begin{aligned} f &= mg \sin \theta \\ \Rightarrow N \tan \alpha &= mg \sin \theta \end{aligned}$$

$$N = mg \cos \theta \quad \text{--- (2)}$$

$$\Rightarrow mg \cos \theta \cdot \tan \alpha = mg \sin \theta$$

$$\Rightarrow \tan \alpha = \tan \theta$$

for equilibrium,

$$\mu N \geq w \sin \theta$$

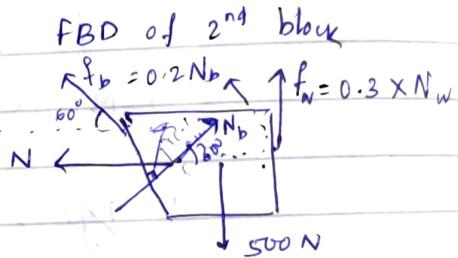
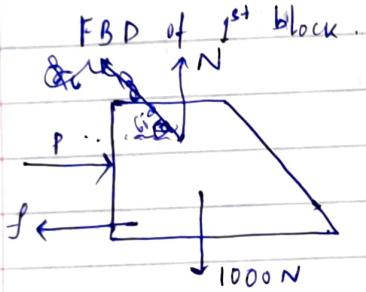
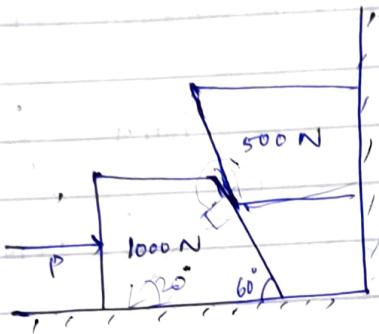
$$w \cos \theta \geq w \sin \theta$$

$$\mu \geq \tan \theta$$

$$\tan \alpha \geq \tan \theta$$

$$\boxed{\alpha \geq \theta}$$

Q. Two blocks of 1000N and 500N are arranged as shown in the figure, if the coefficient of friction at floor is 0.25, coefficient of the wall is 0.3 and coefficient of friction b/w the blocks is 0.2 then find the minimum force 'P' to be applied on the lower block that will hold the system in equilibrium

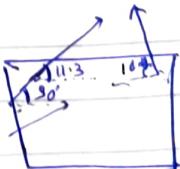
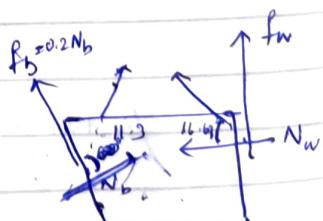


$$\tan \alpha_w = \frac{0.3 N_w}{N_w}$$

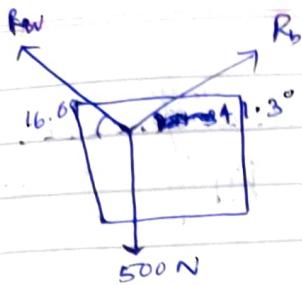
$$\alpha_w = 16.69^\circ$$

$$\tan \alpha_b = 0.2$$

$$\alpha_b = 11.3^\circ$$



41.3
 16.69
 131.3
 57.99
 180.00
 57.99



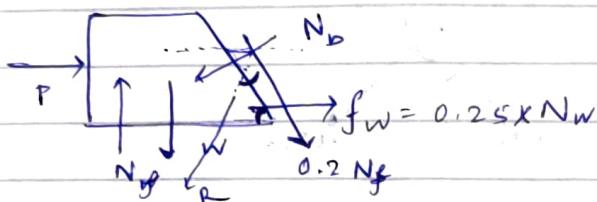
By Lami's theorem,

$$\frac{500}{\sin(122.01)} = \frac{R_b}{\sin(16.69)} = \frac{R_w}{\sin(131.3)}$$

$$R_b = 562.9 \text{ N.}$$

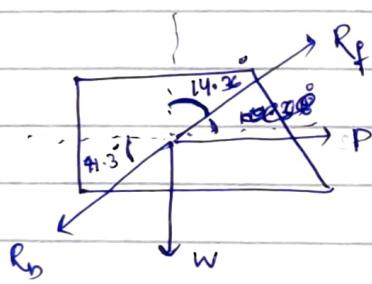
FBD of 1st

$\frac{2}{105}$



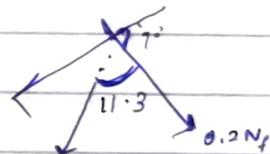
$$\tan \alpha = \frac{0.2 N_b}{N_g}$$

$$\alpha = \tan^{-1}(0.2) = 11.30^\circ$$



$$\tan \alpha_f = \frac{0.25 N_f}{N_f}$$

$$\alpha_f = \tan^{-1}(0.25) \\ = 14.36^\circ$$



$$\sum f_x = 0$$

$$R_f \sin(14.36^\circ) + P = R_b \cos(41.3^\circ) \quad \text{--- (1)}$$

$$\sum f_y = 0$$

$$W + R_b \sin(41.3^\circ) = R_f \cos(14.36^\circ) \quad \text{--- (2)}$$

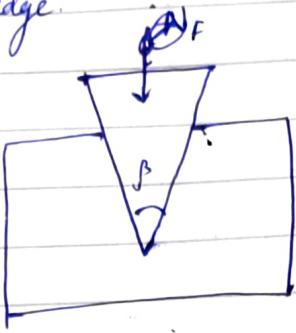
$$R_f \times 0.24 + P = 562.9 \times$$

$$1000 + 562.9 \sin(41.3^\circ) = R_f \cos(14.36^\circ)$$

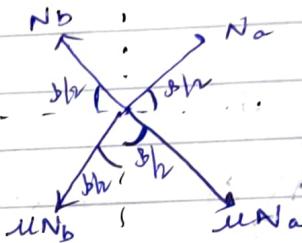
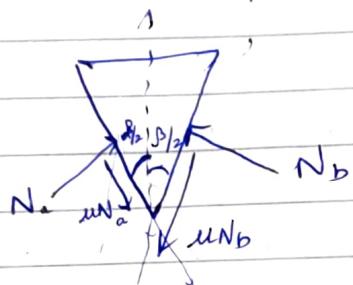
$$1000 + 562.9 \times 0.66 = R_f \times 0.968$$

$$R_f = \frac{1371.514}{0.968} = 1416.85 \text{ N.}$$

- Q. What must be the angle of plane face of steel sledge used for splitting logs if there is no danger of wedge slipping out after each blow of sledge.



Let μ be the coefficient of friction b/w wedge and wood.



$$\begin{array}{c}
 \uparrow N_b \sin \beta/2 + N_a \sin \beta/2 \\
 \leftarrow \mu N_b \sin \beta/2 + N_b \cos \beta/2 \\
 \rightarrow N_a \cos \beta/2 + \mu N_a \sin \beta/2 \\
 \downarrow \mu N_b \cos \beta/2 + \mu N_a \cos \beta/2
 \end{array}$$

$$\sum F_y = 0$$

$$\Rightarrow N_b \sin \beta/2 + N_a \sin \beta/2 = \mu N_b \cos \beta/2 + \mu N_a \cos \beta/2$$

$$\Rightarrow \sin \beta/2 (N_a + N_b) = \mu \cos \beta/2 (N_a + N_b)$$

$$\Rightarrow \boxed{\mu = \tan \beta/2}$$

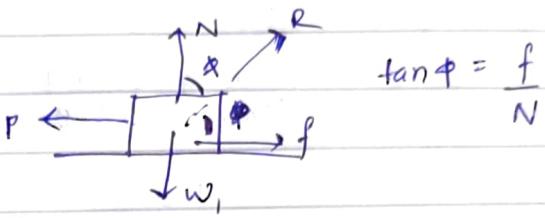
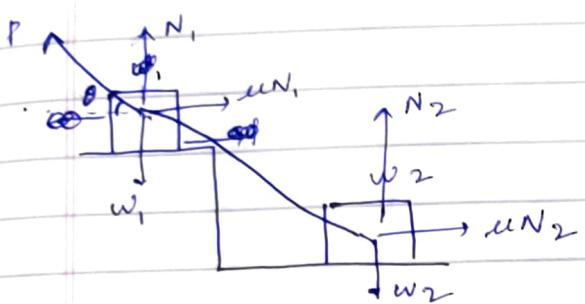
$$\sum F_y = 0$$

$$\mu N_b \sin \beta/2 + N_b \cos \beta/2 = N_a \cos \beta/2 + \mu N_a \sin \beta/2$$

$$\Rightarrow N_b (\mu \sin \beta/2 + \cos \beta/2) = N_a (\cos \beta/2 + \mu \sin \beta/2)$$

$$\mu \sin \beta/2 + \cos \beta/2 \neq 0$$

8. Two bodies having weight w_1 & w_2 are connected by a string and rest on horizontal plane as shown, if the angle of friction of each block is ' ϕ '. Find the magnitude and direction of least applied force P on the upper block that will induce sliding.



$$\tan \phi = \frac{f}{N}$$

$$\sum F_y = 0$$

$$P \sin \theta + N_1 + N_2 = w_1 + w_2$$

$$N_1 + N_2 = w_1 + w_2 - P \sin \theta. \quad \text{---(i)}$$

$$\sum F_x = 0$$

$$P \cos \theta = \mu N_1 + \mu N_2$$

$$P \cos \theta = \mu (N_1 + N_2)$$

$$P \cos \theta = \mu (w_1 + w_2 - P \sin \theta)$$

$$P = \frac{\mu (w_1 + w_2)}{\cos \theta + \mu \sin \theta}$$

$$P \cos \theta + \mu \sin \theta = \mu (w_1 + w_2)$$

$$P (\cos \theta + \mu \sin \theta) = \mu (w_1 + w_2)$$

$$P = \frac{\mu (w_1 + w_2)}{\cos \theta + \mu \sin \theta} \quad \text{---(2)}$$

The minimum value of P is when denominator is max^m.

$$\frac{dP}{d\theta} = \frac{\mu (w_1 + w_2)}{\cos^2 \theta + \mu^2 \sin^2 \theta} \frac{d}{d\theta} (\cos \theta + \mu \sin \theta) = 0$$

$$-\sin \theta - \mu \cos \theta = 0$$

$$\Rightarrow \mu = \tan \theta$$

$$\therefore \tan \phi = \tan \theta$$

$$\boxed{\phi = \theta}$$

$$P = \frac{\tan \phi (w_1 + w_2)}{\cos \phi + \sin \phi}$$

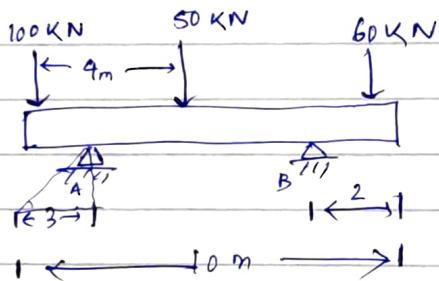
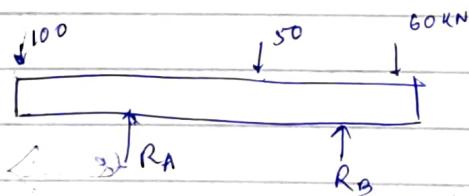
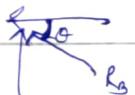
$$\frac{ws\phi + ws^2\phi}{\sin \phi}$$

$$P = \frac{w_1 + w_2}{\frac{ws\phi}{\tan \phi} + \frac{ws^2\phi}{\tan \phi}}$$

$$= \frac{w_1 + w_2}{\frac{ws^2\phi}{\sin \phi} + \sin \phi}$$

$$= \frac{(w_1 + w_2) \sin \phi}{\sin^2 \phi + ws^2 \phi} = (w_1 + w_2) \sin \phi.$$

Q.

Find R_A & R_B .

$$M_A = 0$$

$$(100 \times 3) + (R_A \times 0) - (50 \times 1) + (R_B \times 5) - (60 \times 7) = 0$$

$$R_B = 34 \text{ kN}$$

~~M_B = 0~~

$$R_A + R_B = 100 + 50 + 60$$

$$R_A = 210 - 34$$

$$= 176 \text{ kN.}$$

Truss

- A truss is made up of several bars called members joined together by hinges or rivets.
- The joints of a truss are called nodes.
- A truss is designed to carry the loads at the nodes only.

* Plane Truss:

If centre line on the member of the truss lie in one plane is called plane Truss.

* Space Truss:

If the centre line of the member of the truss doesn't lies in one plane is called perfect truss.

* Perfect Truss:

If the centre lines of a member of a truss lie in one plane.

If the truss contains the least no. of members required to prevent distortion of its shape, when loaded, it is called perfect truss.

If the no. of nodes or joints in a perfect truss is "J" then the no. of members is " $n = 2J - 3$ "

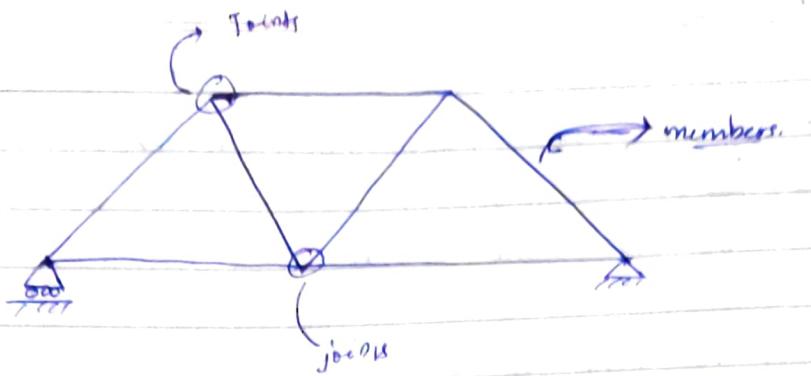
* Deficit

Deficient Truss:-

If the no. of members is less than required i.e., $n < 2J - 3$ is called deficient truss.

Redundant Truss:

$$n > 2J - 3$$



Assumptions to solve Plane Truss :-

1. All the members are in one plane.
2. The members are connected at the ends by smooth hinges.
3. The loads are applied at the joints only.
4. Statically determinant the structure.

Analysis of Truss

1. Determine the reaction at ~~surface~~ supports
2. Determination of the force in member of truss.

Note:- The reaction are determined by the condition that the applied load system and the induced reaction at the supports are in equilibrium.

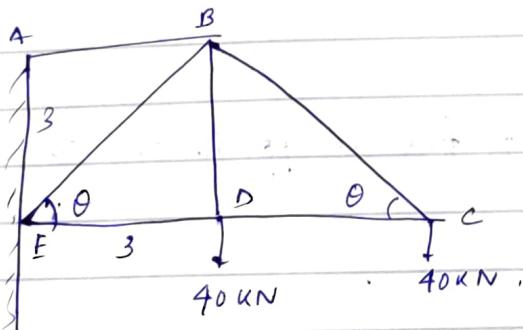
3. The force in the member of the truss are determined by the condⁿ that every point should be in equilibrium.

A Truss can be analysed by the following methods:

- 1) Method of joints
- 2) Method of sections.

1) Method of joint:

In this method after determining the reaction at the supports we consider equilibrium of every joints, where no. of unknown members should be maximum '2'.

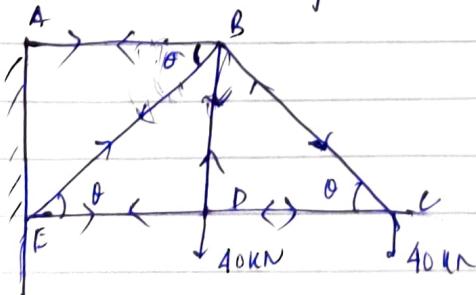


Step-1: Find the angle of inclination of all the inclined members.

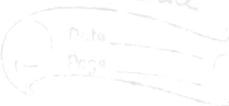
$$\tan \theta = \frac{3}{3}$$

$$\theta = 45^\circ$$

Step-2: Look for a joint at which there are two unknowns. If such a joint is not found, determine the reactions at supports and then add the support these unknowns may be reduced to two. Also ~~assume~~ the members whether tensile or compressive forces are applied

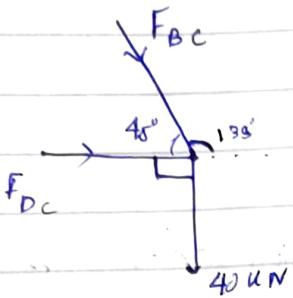


Force towards the joint is compressive.



Step - 3

Joint C



$$\sum f_y = 0$$

$$F_{Bc} \sin 45^\circ + 40 \text{ kN} = 0$$

$$F_{Bc} = -40\sqrt{2} \text{ kN}$$

~~40 kN~~

$$\sum f_x = 0$$

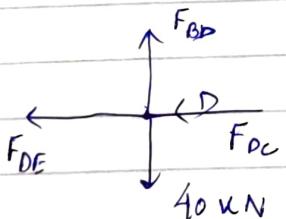
$$F_{Dc} + F_{Bc} \cos 45^\circ = 0$$

$$F_{Dc} = 40 \text{ kN}$$

→ If the assumed direction of force is opposite then the value will be negative.

Step - 4:

Joint - D.



$$\sum f_x = 0$$

$$F_{DE} + F_{DC} = 0$$

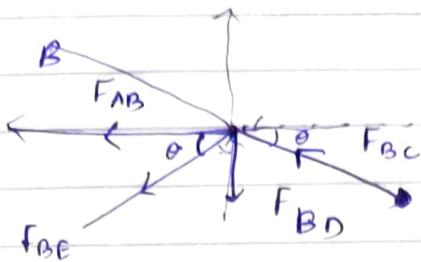
$$\Rightarrow F_{DE} = -40 \text{ kN}$$

$$\sum f_y = 0$$

$$\Rightarrow F_{BD} = 40 \text{ kN}$$

Step - 5

Joint B



$$\sum F_x = 0$$

$$\Rightarrow F_A + F_{BE} \cos 45^\circ + F_{BC} \cos 45^\circ = 0$$

$$\Rightarrow F_{AB} + \frac{F_{BE}}{\sqrt{2}} + \frac{F_{BC}}{\sqrt{2}} = 0$$

$$\sum F_y = 0$$

$$\Rightarrow f_{BD} + F_{BE} \sin 45^\circ = - F_{BC} \sin 45^\circ$$

$$\Rightarrow 40 + \frac{F_{BE}}{\sqrt{2}} = - 40\sqrt{2} \times \frac{1}{\sqrt{2}}$$

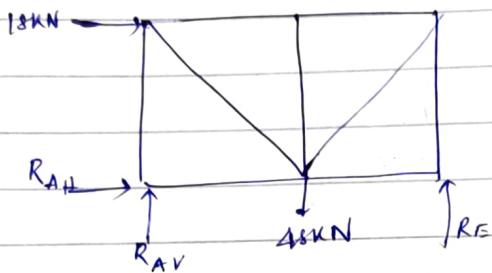
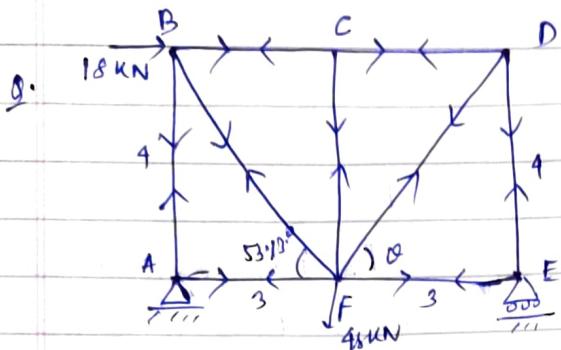
$$\Rightarrow 40 + 40 = - \frac{F_{BE}}{\sqrt{2}}$$

$$\Rightarrow -80\sqrt{2} = F_{BE}$$

$$\Rightarrow F_{AB} = -80 - 40 = 0$$

$$\Rightarrow \boxed{F_{AB} = 120 \text{ kN}}$$

| Member | Magnitude | Nature |
|--------|--------------|---------------------|
| AB | 120 kN | Tensile |
| BC | $40\sqrt{2}$ | Tensile |
| CD | 40 | Tensile/Compressive |
| DE | 40 kN | Compressive |
| BE | $80\sqrt{2}$ | Compressive |
| BD | 40 | Tensile |



Considering whole body under equilibrium

$$\sum M_A = 0$$

$$18 \times 4 - 6R_E + 48 \times 3 = 0$$

~~$$6R_E = 18 \times 4$$~~

$$R_E = \frac{18 \times 4}{6} = 12 \text{ kN}$$

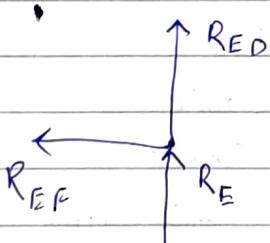
$$RAV + R_E = 48 \text{ kN}$$

$$RAV = -18 \text{ kN}$$

$$72 + 144 = 6R_E$$

$$\frac{216}{6} = R_E = 36 \text{ kN}$$

At joint E



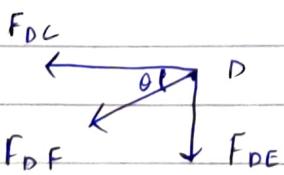
$$R_{ED} \neq R_E = 0$$

$$R_{ED} = -36 \text{ kN}$$

$$R_{EF} = 0$$

At joint D,

$$F_{DE} + F_{DF} \sin(53.13^\circ) = 0$$



$$-36 + F_{DF} \times 0.79 = 0$$

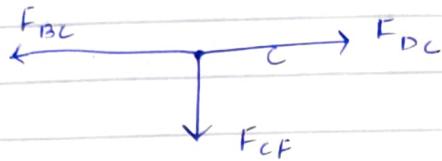
$$F_{DF} = \frac{36}{0.79} = 45.5 \text{ kN}$$

$$F_{DC} + F_{DF} \cos(53.13^\circ) = 0$$

$$F_{DC} + 45.5 \times 0.6 = 0 \quad F_{DC} = 27.3 \text{ N}$$

At point C,

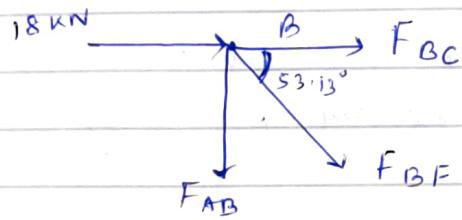
$$F_{CF} = 0$$



$$F_{DC} = F_{BC}$$

$$F_{BC} = 27.3 \text{ N}$$

At point B,



$$18 + F_{BC} + F_{BF} \cos(53.13^\circ) = 0$$

$$18 + 27.3 + F_{BF} \times 0.6 = 0$$

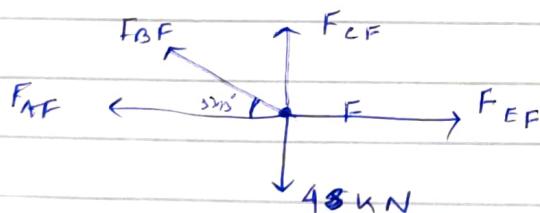
$$F_{BF} \times 0.6 = -45.3$$

$$F_{BF} = -75.5 \text{ N}$$

$$F_{AB} + F_{BF} \sin(53.13^\circ) = 0$$

$$\begin{aligned} F_{AB} &= -(-75.5) \times 0.79 \\ &= 59.645 \end{aligned}$$

At point F,



$$F_{BF} \sin(53.13^\circ) = 48 \text{ N}$$

$$\frac{48}{0.799}$$

$$F_{AF} + F_{BF} \cos 53.13^\circ = F_{EF}$$

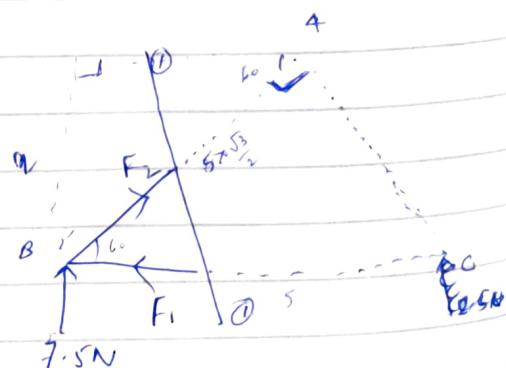
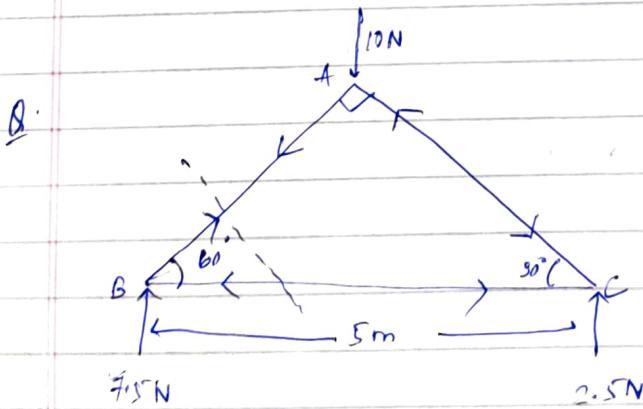
Method of Sections :-

Method of section especially useful in those sections where forces in only a few intermediate members are required.

This method is followed by dividing the truss into two parts by an imaginary section drawn in such a way as not to cut more than three ~~members~~ members with unknown forces in them.

Then we apply the conditions of equilibrium to one of the parts which is in loads and forces exerted upon it by members cut by the section.

- While considering whole frame of truss, the internal forces in the members are not taken into account.
- After taking the section we can consider the internal forces in the cut members as external force to any one part of the section.



cut the section ①①, cutting section AB & BC, as shown, considering the left part only taking moment about c.

$$M_c = 0$$

$$-(7.5 N \times 5) - (F_2 \sin 60^\circ \times 5) = 0$$

$$37.5 + F_2 \times \frac{\sqrt{3}}{2} \times 5 = 0$$

$$F_2 \times \frac{5\sqrt{3}}{2} = -37.5$$

$$F_2 = \frac{15}{\sqrt{3}} = -8.66 \text{ (compressive)}$$

$$\cos 60^\circ = \frac{b}{n}$$

~~$$\cos b = \cos 60^\circ \times n$$~~

$$M_A = 0$$

$$-7.5 \times \cos^2 60^\circ \times 5 - F_1 \frac{5\sqrt{3}}{4} = 0$$

$$\cos 60^\circ = \frac{b}{5}$$

$$b = 5 \cos 60^\circ$$

$$-7.5 \times \frac{1}{4} \times 5 - F_1 \frac{5\sqrt{3}}{4} = 0$$

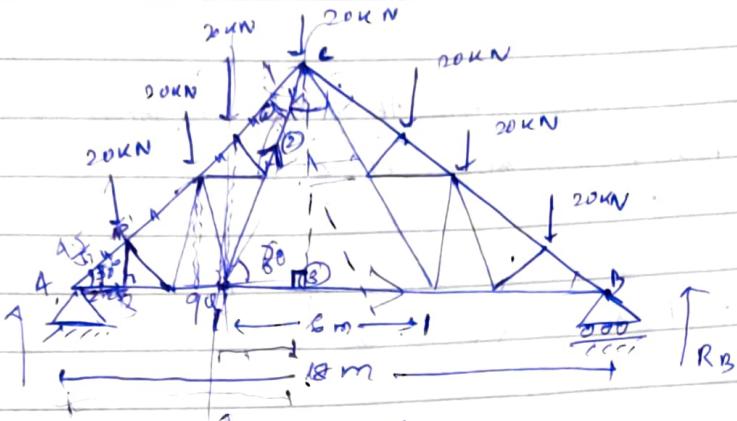
$$\sin 60^\circ = \frac{x}{5 \cos 60^\circ}$$

$$F_1 \times \frac{5\sqrt{3}}{4} = -7.5 \times \frac{5}{4} \quad \therefore -4.33 \cdot N \text{ (T)}$$

$$\frac{\sqrt{3}}{2} \times \frac{1}{2} \times 5 = x$$

$$\frac{5\sqrt{3}}{4} = x$$

Q.



The given figure is
French truss, find out
the forces in a truss ①, ②
(3)

$$\cos 30^\circ = \frac{9}{AC}$$

$$AC = \frac{9 \times 2}{\sqrt{3}} = \frac{18}{\sqrt{3}}$$

$$4A' = \frac{18 \times 4.5}{\sqrt{3} \times 4} \\ = \frac{4.5}{\sqrt{3}}$$

$$\cos 30^\circ = \frac{4.5 \times \sqrt{3}}{4.5}$$

$$\frac{\sqrt{3}}{2} \times 4.5 = 4.5$$

$$4.5 = 2.25$$

$$\text{Joint } ① = \frac{h\sqrt{3}}{9} = \frac{1}{2}$$

$$\Rightarrow h = \frac{9}{2\sqrt{3}}$$

Since it is a symmetrical French truss., $R_V = R_B$

$$R_V + R_B = 20 \times 7$$

$$2R_B = 140 \text{ kN}$$

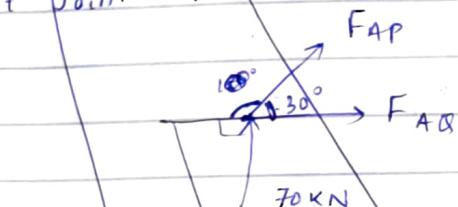
$$R_V = 70 \text{ kN}$$

$$R_H = 0$$

$$\sum M_B = 0$$

$$R_B \times 18 = 20$$

At Joint A



$$F_{AP} \sin 30^\circ + 70 = 0$$

$$F_{AP} = -70 \times 2 = -140 \text{ kN (c)}$$

~~$$F_{AQ} = 0$$~~

~~$$F_{AQ} + F_{AP} \cos 30^\circ = 0$$~~

~~$$\Rightarrow F_{AQ} = -F_{AP} \times \frac{\sqrt{3}}{2}$$~~

$$= -\frac{70}{140} \times \frac{\sqrt{3}}{2} = -121.24 \text{ kN (c)}$$

At P,



$$\sum M_B = 0$$

$$-70 \times 18 - F_{AP} \cdot \sin 30^\circ \times 18 = 0$$

$$F_{AP} \cdot \frac{1}{2} \times 18 = -70 \times 18$$

$$F_{AP} = -\frac{70 \times 18}{\frac{1}{2} \times 18} = -80 \text{ kN} - 140 \text{ kN}$$

By method of sections:

$$\sum M_A = 0$$

$$\Rightarrow 20 \times 2.25 + 20 \times \frac{9\sqrt{3}}{2} + 20 \times 13.5 \times \cos 36^\circ$$

$$F_2 \sin 60^\circ \times 9 = 0$$

$$\Rightarrow F_2 = 49.88 \text{ KN} \cdot 51.9 \text{ KN (T)}.$$

$$\sum M_C = 0$$

$$70 \times 9 - 20 \times 6.75 - 20 \times 9.5 - 20 \times 2.25 - F_3 \times \frac{9}{\sqrt{3}} = 0$$

$$360 = F_3 \frac{9}{\sqrt{3}}$$

$$\Rightarrow F_3 = \frac{360 \times \sqrt{3}}{9} \quad F_3 = 69.28 \text{ N.}$$

$$\sum M_B$$

$$-70 \times 18 + 20 \times 15.75 + 20 \times 13.5 + 20 \times 11.25 - F_1 \sin 30 \times 18 - F_2 \sin 70 \times 12 = 0$$

$$-450 - F_2 \sin 70 \times 12 = F_1 \sin 30 \times 18$$

$$-480 - 539.9 = F_1 \sin 30 \times 18$$

$$\cancel{F_2 \sin 70 \times 12}.$$

$$-1019.9 = F_1 \quad F_1 = -109.98 \text{ KN.}$$

$$\underline{\underline{F_1 = -109.98}}$$



~~SKB~~

$$\tan 30^\circ = \frac{b}{4.5}$$

→ ~~Q.B.A.Y~~

$$\frac{\sqrt{3}}{2} \times \frac{9.5}{\sqrt{3}} = b$$

Varingon Theorem

Principle of transmissibility.

Lami's theorem

Friction (limiting friction, sliding friction), static, Rolling).

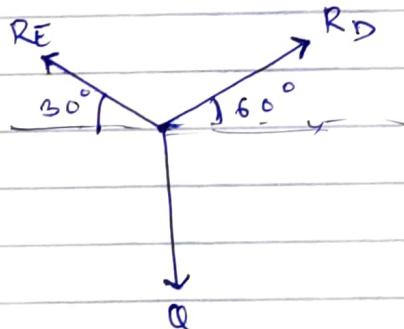


$$Q = 53.4 N$$

$$\frac{57.4}{106.8}$$

To find: $\underline{\underline{R_D}}$ & $\underline{\underline{R_E}}$

f.b.d.:



$$R_D \sin 60^\circ + R_E \sin 30^\circ = 53.4$$

$$\Rightarrow R_D \frac{\sqrt{3}}{2} + R_E \frac{1}{2} = 53.4$$

$$\Rightarrow \sqrt{3} R_D + R_E = 106.8$$

$$\Rightarrow 3 R_D + R_E = 106.8$$

$$\Rightarrow 4 R_E = 106.8$$

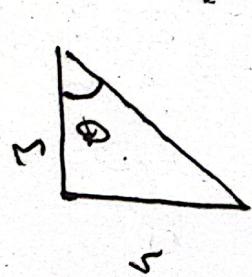
$$R_E = \frac{106.8}{4} = 26.7 N$$

$$R_E \cos 30^\circ = R_D \cos 60^\circ$$

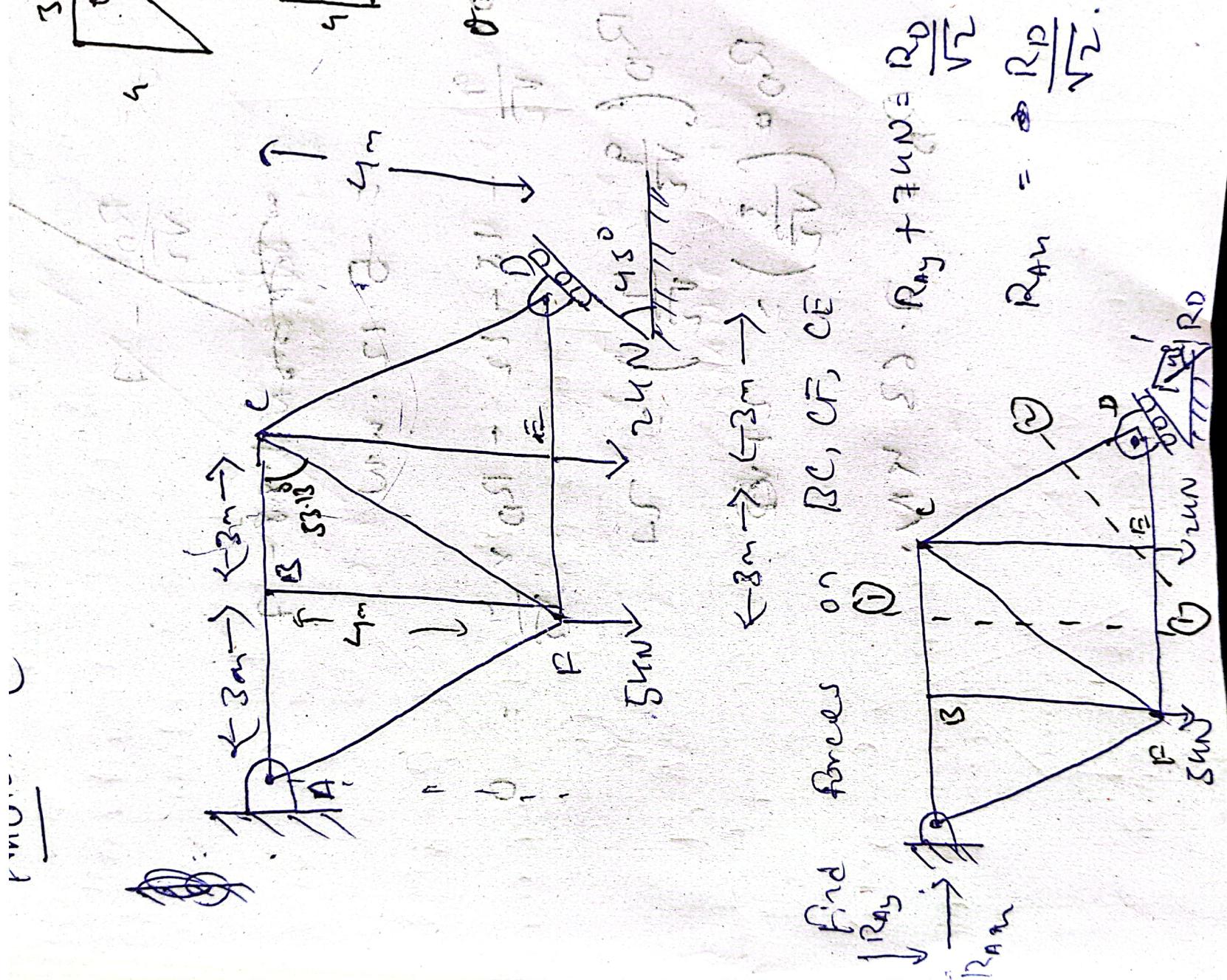
$$\Rightarrow R_E \frac{\sqrt{3}}{2} = \frac{R_D}{2}$$

$$\Rightarrow R_D = \sqrt{3} \times 26.7 N$$

$$= 1.732 \times 26.7 = 46.244 N.$$



13-53



$$\sum M_A = 0$$

$$R_D \sin 45^\circ$$

$$R_D \sin 45^\circ - 5 \times 3 + 266 \leftarrow R_D \cos 45^\circ \times 4 = 0$$

$$R_D \frac{9}{\sqrt{2}} = 12 + 16$$

$$R_D = \frac{17 \times \sqrt{2}}{9} = 2.67 \text{ kN}$$

$$R_{Ay} = 1.88 \text{ kN} = 1.88 \text{ kN}$$

$$R_{Ay} = \frac{R_D}{\sqrt{2}} - 7$$

$$= 1.88 - 7$$

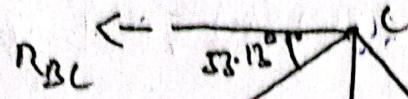
$$= -5.12 \text{ kN}$$

$$R_D \times \frac{9}{\sqrt{2}} - 15 - 12 - R_D \times \frac{7}{\sqrt{2}} = 0$$

$$R_D \left(\frac{9}{\sqrt{2}} - \frac{7}{\sqrt{2}} \right) = 24$$

$$R_D \left(\frac{2}{\sqrt{2}} \right) = 24 \text{ kN}$$

$$R_D = 7.63 \text{ kN}$$



$$\sum M_C = 0$$

$$-R_{EF} \times 4 + 7.63 \sin 45 \times 3 - \cancel{7.63 \cos 45} \\ - 7.63 \cos 45 \times 4 = 0$$

$$-R_{EF} \times 4 = 7.63 \times \frac{3}{\sqrt{2}} - 7.63 \times \frac{3}{\sqrt{2}}$$

$$R_{EF} = - \left(\frac{7.63 \times 4/\sqrt{2} - 7.63 \times 3/\sqrt{2}}{4} \right) \\ = - \left(\frac{21.58 - 16.18}{4} \right)$$

$$R_{EF} = -1.35 \text{ (C)}$$

$$\sum F_y = 0$$

~~$$R_{CF} \sin 53.13 + 2 = 7.63 \sin 45$$~~

$$R_{CF} \sin 53.13 = 7.63 \sin 45 - 2$$

$$R_{CF} = \frac{7.63 \sin 45 - 2}{\sin 53.13}$$

$$R_{CF} = 4.24 \text{ kN (T)}$$

$$\sum F_h = 0$$

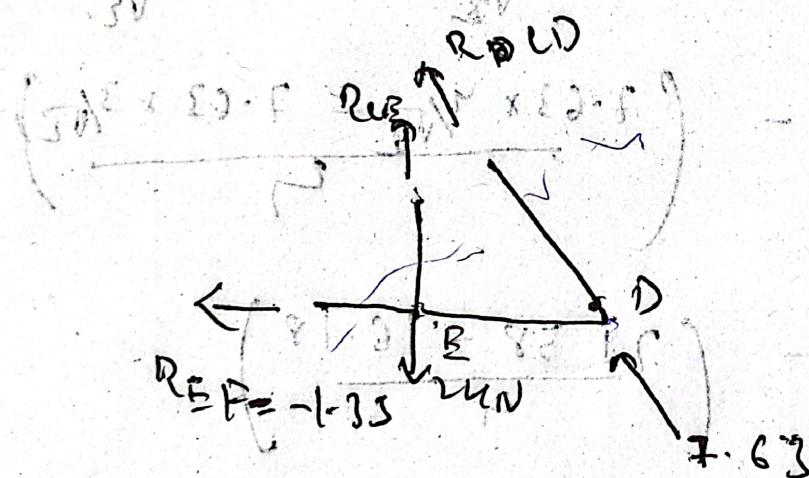
53.13

$$R_{BC} + R_{EF} \cos 53.13^\circ + R_{EP} + 2.63 \cos 45^\circ = 0$$

$$R_{BC} + 4.24 \cos 53.13^\circ - 1.35 + 2.63 \cos 45^\circ = 0$$

$$R_{BC} = -7.63 \cos 45^\circ + 1.35 - 4.24 \cos 53.13^\circ$$

$$R_{BC} = -6.58 \text{ kN (C)}$$



$$\sum M_D = 0$$

$$-R_{CD} \times 3 + 2 \times 2$$

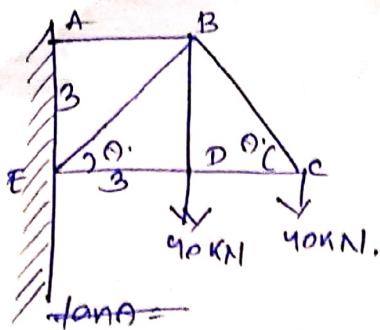
$$R_{CD} = 2 \text{ kN}$$

$$S = 2P \times 2L \times 0.5 = 81.6 \text{ kNm}$$

$$S = 2P \times 2L \times 0.5 = 36$$

$$81.6 \text{ kNm}$$

$$(7) \text{ min } 15^\circ = 129$$



Step-1
Determine the angle of inclination of all the inclined members.

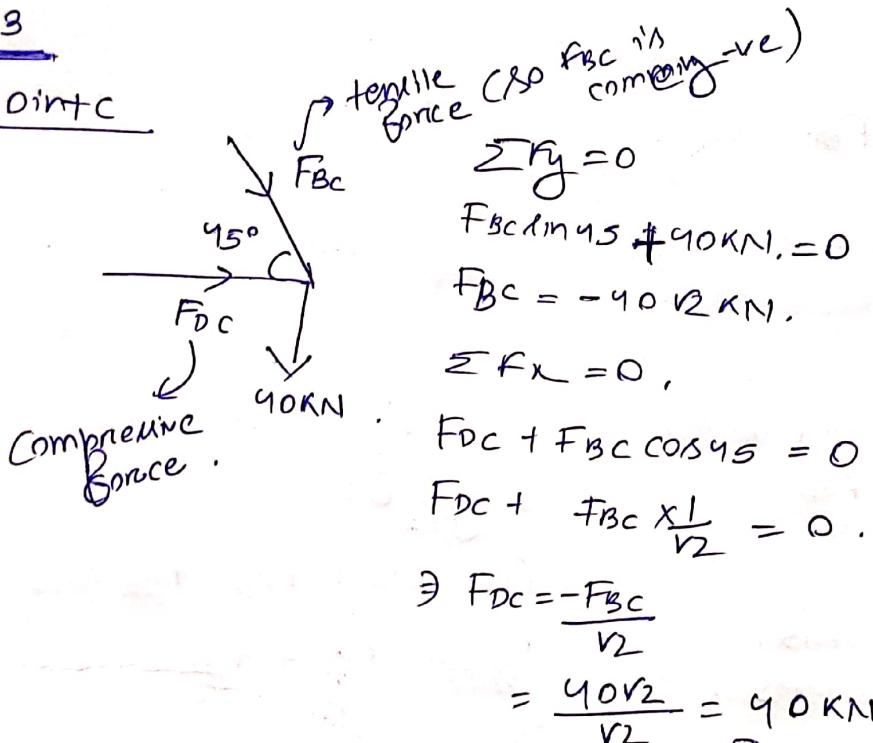
$$\tan \theta = \frac{3}{3} \\ \theta = 45^\circ$$

Step-2

Look for a joint off which there are only two unknowns.
If such a joint is not found determine the reactions at supports and then off the supports these unknowns may be reduced to two. Also mark the members whether tensile or compressive forces are applied. (assume)

Step-3

Joint C



tensile force (as F_{BC} is coming -ve)

$$\sum F_y = 0$$

$$F_{BC} \sin 45^\circ + 90\text{KN} = 0$$

$$F_{BC} = -90\sqrt{2} \text{ KN.}$$

$$\sum F_x = 0$$

$$F_{DC} + F_{BC} \cos 45^\circ = 0$$

$$F_{DC} + F_{BC} \times \frac{1}{\sqrt{2}} = 0$$

$$\Rightarrow F_{DC} = -\frac{F_{BC}}{\sqrt{2}}$$

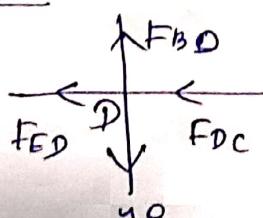
$$= \frac{90\sqrt{2}}{\sqrt{2}} = 90 \text{ KN.}$$

If the assumed direction of unknown forces is opposite the value will be -ve.

Opposite forces are shown in pairs.

Step-4

Joint D



$$\sum F_y = 0$$

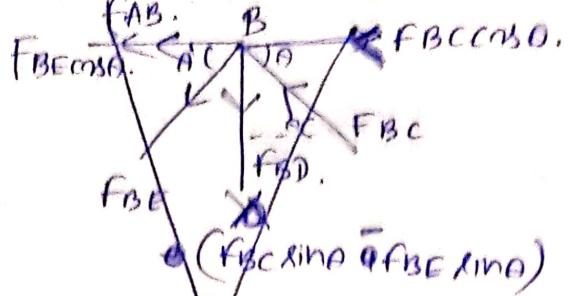
$$F_{BD} = 90 \text{ KN.}$$

$$\sum F_x = 0$$

$$F_{BD} + F_{DC} = 0$$

$$\Rightarrow F_{BD} + 90 = 0$$

$$\Rightarrow F_{BD} = -90 \text{ KN.}$$



$$\sum F_x = 0$$

$$F_{AB} + F_{BE} \cos 45^\circ + F_{BC} \cos 45^\circ = 0.$$

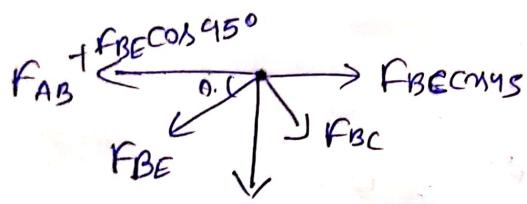
$$\Rightarrow -40\sqrt{2} \times \frac{1}{\sqrt{2}} + F_{BE} \times \frac{1}{\sqrt{2}} + 40\sqrt{2} = 0.$$



$$\sum F_y = 0$$

$$\frac{F_{BC}}{\sqrt{2}} - \frac{F_{BE}}{\sqrt{2}} - F_{BD} = 0.$$

$$\Rightarrow -\frac{40\sqrt{2}}{\sqrt{2}} - \frac{40\sqrt{2}}{\sqrt{2}} - 40 = 0$$



$$F_{BD} + F_{BE} \sin 45^\circ + F_{BC} \sin 45^\circ = 0.$$

$$\sum F_y = 0.$$

$$F_{BD} + \frac{F_{BE}}{\sqrt{2}} + \frac{F_{BC}}{\sqrt{2}} = 0.$$

$$\Rightarrow 40 + \frac{40\sqrt{2}}{\sqrt{2}} + \frac{F_{BE}}{\sqrt{2}} = 0.$$

$$\Rightarrow F_{BE} = -80\sqrt{2} \text{ KN.}$$

$$\sum F_x = 0$$

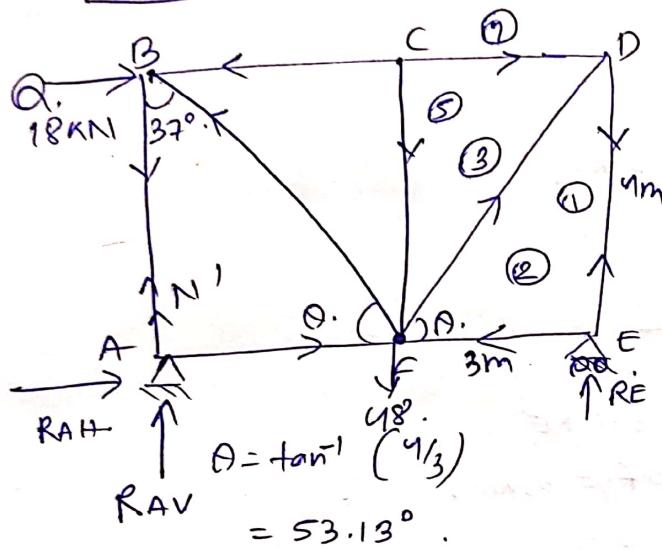
$$F_{BC} \cos 45^\circ = F_{AB} + F_{BE} \cos 45^\circ$$

$$\frac{F_{BC}}{\sqrt{2}} = F_{AB} + \frac{F_{BE}}{\sqrt{2}}$$

$$\frac{40\sqrt{2}}{\sqrt{2}} = F_{AB} - 80\sqrt{2}$$

$$\Rightarrow F_{AB} = 40 + 80 \\ = 120 \text{ KN.}$$

| member | magnitude (kN) | Nature |
|--------|----------------|-------------|
| AB | 120 kN | Tensile |
| BC | 40\sqrt{2} kN | Tensile |
| CD | 40 kN | Compressive |
| DE | 40 kN | Compressive |
| BE | 80\sqrt{2} kN | Compressive |
| BD | 40 kN | Tensile |



At Joint E

$$F_{DE} = -R_E \\ = -32 \text{ kN.}$$

$$\sum F_x = 0 \\ F_{EF} = 0.$$

At Joint B

$$F_{BF} \times \frac{1}{S} \\ F_{BC} + F_{BF} \times \frac{3}{S} \\ F_{AB}$$

At Joint C.

$$F_{BC} = F_{CD}$$

$$F_{CF} = 0.$$

$$\sum F_y = 0. \quad \frac{F_{BF} \times 1}{S} = F_{AB} \\ F_{BF} = \frac{S F_{AB}}{1}$$

At Joint F

~~F~~

$$\sum F_y = 0.$$

$$F_{BF} \times \frac{4}{S} + F_{FD} \times \frac{1}{S} = 0.$$

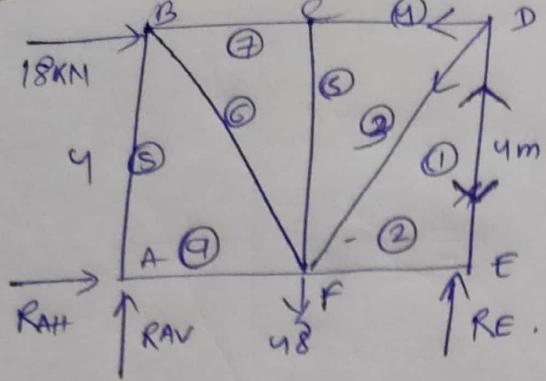
~~+ F_{AF} \rightarrow F~~

$$\Rightarrow F_{BF} \times \frac{4}{S} + F_{FD} \times \frac{1}{S} = 0$$

$$\Rightarrow \frac{4 S F_{AB}}{4} + F_{FD} \times \frac{1}{S} = 0.$$

$$\Rightarrow F_{AB} + \frac{4 F_{FD}}{S} = 0.$$

$$N' = -F_{AB}.$$



$$RAH = -18 \text{ kN}.$$

$$\sum A = -4 \times 18 + 6 \times RE = 0. -48$$

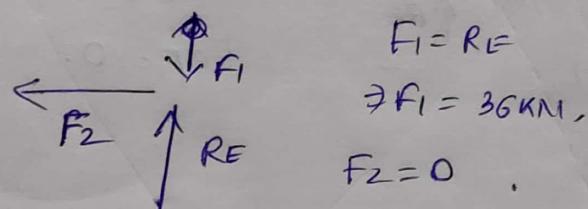
$$\Rightarrow 72 = 6RE$$

$$\Rightarrow RE = \frac{72 + 48 \times 3}{6} = 12 \text{ kN}, \quad 12 + 12 = 36 \text{ kN}.$$

$$F_{DE} = -36 \text{ kN}.$$

$$F_{EF} = 0.$$

At Joint E

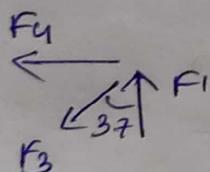


$$F_1 = RE$$

$$\Rightarrow F_1 = 36 \text{ kN},$$

$$F_2 = 0.$$

At Joint D



$$\sum F_x = 0.$$

$$\Rightarrow F_2 = 0$$

$$R_1, R_3 \text{ and } 37 = 0$$

$$\textcircled{Q} \quad R_3 = \frac{36 \times 5}{9} = 20 \text{ kN}$$

$$36 \text{ N}$$

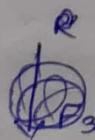
$$\sum F_y = 0.$$

$$R_3 = -36 \times \frac{5}{9} = -20 \text{ kN}$$

$$R_3 = -15 \text{ kN}$$

$$R_1 = 36 \text{ kN}.$$

At Joint F



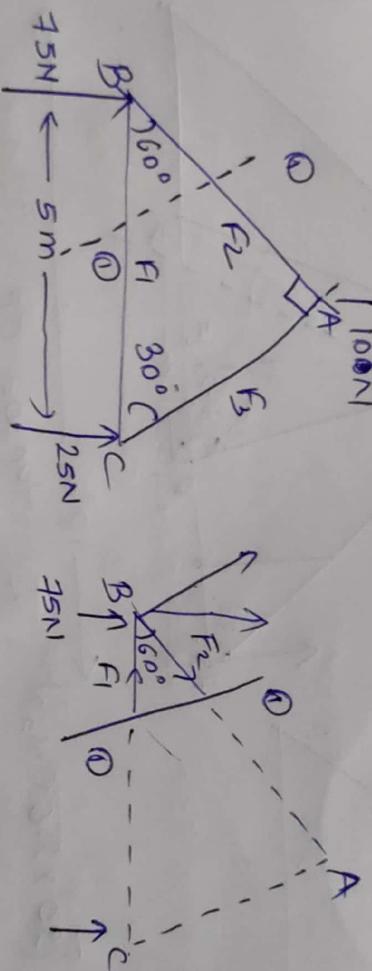
$$RS$$

Method of Section is useful in those cases where forces in intermediate members are required. This method is followed by dividing the truss into 2 parts by an imaginary section drawn in such a way as not to cut more than 3 members, with unknown forces in them. Then we apply the conditions of equilibrium to one of the parts, which is subjected to loads and forces exerted upon it by the members cut by the section.

Note While considering the whole frame or truss the internal forces of the members are not taken into account.

- After taking the section we can consider the internal forces in the cut members as external force to any one part of the section.

Method of Section



Cutting member AB and BC as shown.

Considering the left part only
Taking moment about C.

$$I_m_c = 0$$

$$-(7.5 \times 5) + (F_2 \cos 60^\circ \times 10) - (F_2 \sin 60^\circ \times 5) = 0.$$

$$\Rightarrow -F_2 \times 3\sqrt{2} = 7.5 \times 5.$$

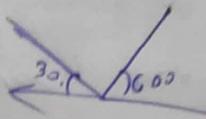
$$\Rightarrow F_2 = \frac{-7.5 \times 5}{3\sqrt{2}} = \frac{-15}{\sqrt{3}} = -8.66$$

$m_A = 0$

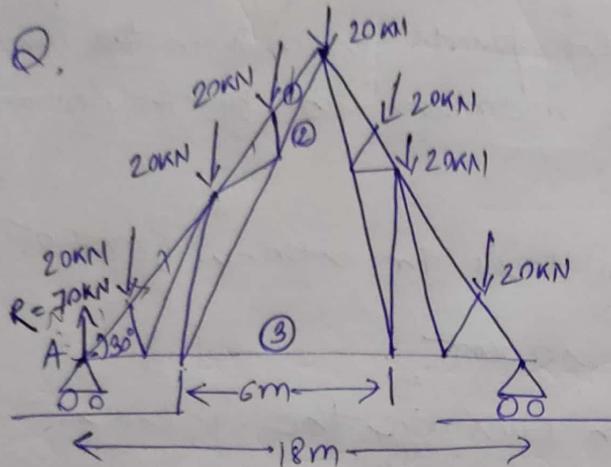
$-(5 \times \frac{1}{2}x^2) - (F_1 \times r_{32}x) = 0.$

$F_1 = \frac{7.5 \times 5}{2 \times \sqrt{3}}$

$= \frac{75}{4\sqrt{3}} = 4.330 \text{ N.}$

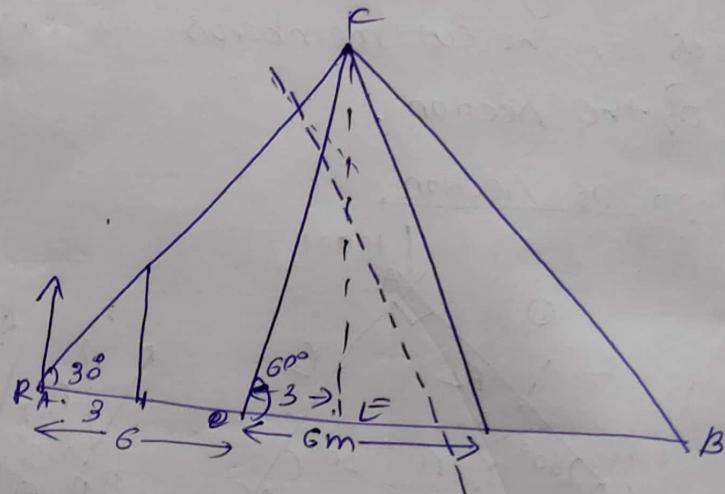
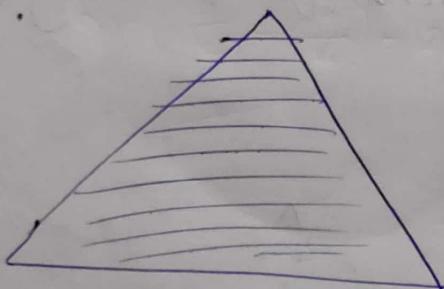


Q.



Find out the Forces in members 1, 2, 3?

Ans.



$m_A = 0$

$(F_2 \sin 60^\circ \times 6) + (20 \times 2.6 \cos 30^\circ)$

$- (20 \times 2 \times 2.6 \cos 30^\circ) -$

$(20 \times 3 \times 2.6 \cos 30^\circ) = 0$

$F_2 = 51.96 \text{ kN (T)}$

$\tan 30^\circ = \frac{CE}{AE}$

$CE = 9\sqrt{3}$

$\tan 30^\circ = \frac{CE}{DE}$

$\frac{\sqrt{3}}{3} = 60^\circ$

$\theta = 60^\circ$

$m_C = 0$

$F_3 + \tan 60^\circ \times 3 + (20 \times 2.6 \cos 30^\circ) + (20 \times 2 \times 2.6 \cos 30^\circ)$

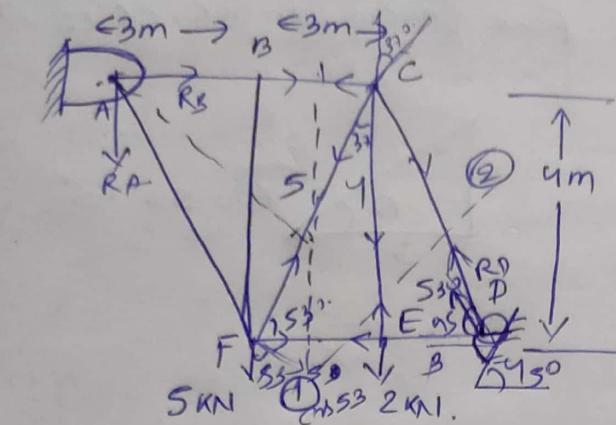
$+ 20 \times 3 \times 2.6 \cos 30^\circ - (70 \times 9) = 0$

$F_3 = 69.28 \text{ kN.}$

Lami's theorem

Friction (Rolling Friction, Static Friction, Limiting Friction).

Q.



Find Balances on BC, CF, CE.

$$R_D + R_{D\text{lim}} = R_{x_B} + R_{x_C}$$

$$R_{x_A} = R_{D\text{lim}}$$

$$\therefore R_{x_B} + R_{D\text{lim}} = 7 \text{ kN}$$

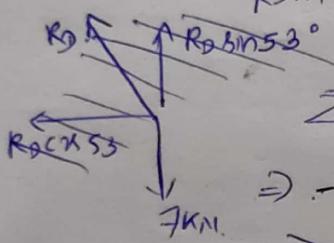
$$R_{x_B} = 7 - \frac{R_D}{\sqrt{2}}$$

$$+ R_{D\text{lim}} \times 9 = 0$$

$$\Rightarrow R_D = \frac{27\sqrt{2}}{5} \text{ kN}$$



$$kNm \rightarrow | \leftarrow 3m \rightarrow |$$



$$\sum M_A = 0$$

$$\Rightarrow -15 - 12 \tan 53^\circ = R_D \cos 53^\circ \times 9$$

$$\Rightarrow \cancel{7 \tan 53^\circ} = \cancel{273}$$

$$\Rightarrow R_D = \frac{3 \times 5}{\sqrt{2}} = 15\sqrt{2}$$

② $\sum F = 0$. $\sum M_A = 0$.

$$-3 \times 5 - F_{FC} \cos 37^\circ = 0$$

$$\Rightarrow F_{FC} = \frac{-15}{\sin 37^\circ} = \frac{-15 \times 5}{4} = -\frac{75}{4} \text{ N}$$

$$\sum M_D = 0$$

$$+ 30 + 9 \times R_A - 9 \times R_B = 0$$

$$\Rightarrow 9R_A - 9R_B = -36$$

$$\Rightarrow 9R_B - 9R_A = 36$$

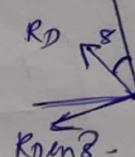
$$\sum M_C = 0$$

$$6R_A + 5 \times 5 \cos 53^\circ - R_D \sin 8^\circ = 0 \Rightarrow R_D = \frac{56.88}{9} = 6.32 \text{ N}$$

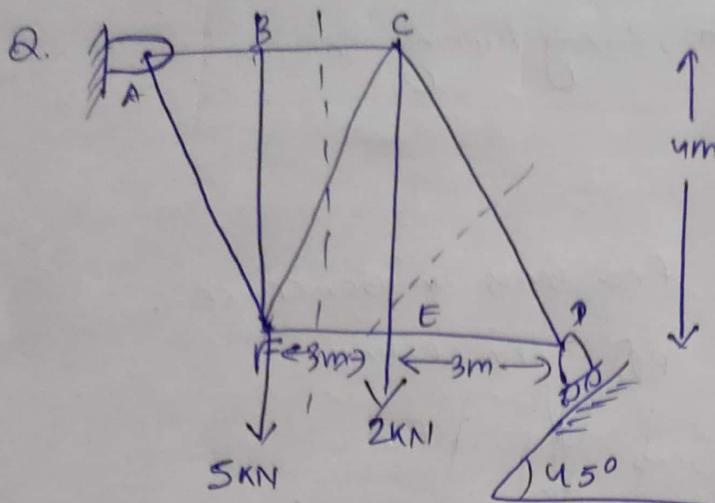
$$\Rightarrow 6R_A + 25 \times \frac{3}{5} - \frac{27\sqrt{2}}{5}$$

$$= 6R_A = \frac{5.31}{5} - 15$$

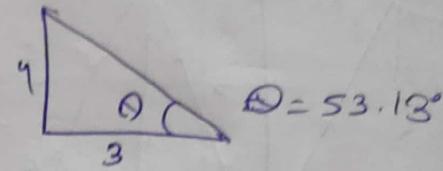
$$= 1.06 - 15 = \frac{13.94}{6} = 2.32 \text{ N}$$



$$\textcircled{2} \quad M_C = 0$$



Find forces on BC, CF, CE?



$$\sum M_A = 0$$

$$R_D \sin 45^\circ \times 9 - 5 \times 3 - 2 \times 6 = R_D \sin 45^\circ \times 4 = 0,$$

$$R_D \left(\frac{q}{V_2} - \frac{y}{V_2} \right) = 27$$

$$R_D \left(\frac{S}{V_2} \right) = 27$$

$$R_D = \frac{27\sqrt{2}}{5} = 7.03 \text{ kN}$$

$$\sum M_{k=0} = 0$$

$$-R_E k_1 + 7.63 \sin 45^\circ k_3 = 7.64 \cos 45^\circ k_4$$

$$-R_{\text{EFM}} = 7.63 \times \frac{4}{V_2} - 7.63 \times 3 = 0 \quad R_E$$

$$R_{EF} = - \left(\frac{\frac{V_2}{7.63} - \frac{X_3}{12}}{\frac{7.63 \times V_2}{12} - \frac{7.63 \times 3}{12}} \right)$$

$$= - \left(\frac{21.58 - 16.18}{9} \right)$$

$$k_{EF} = -1.35$$

$$\sum R_j = 0$$

$$R_{\text{eff}} \text{ for } S3 \cdot 13^2 + 2 = 7.63 \text{ pmyr}^{-1}$$

$$R_{CF} = \frac{7.63 \text{ m/s} - 2}{8 \text{ m/s B}}$$

$$R_{CP} = 9.24 \mu M \quad (7)$$

$\Sigma M_y = 0$.

$$R_{BC} + R_{RF} \cos 53.13 + R_{CF} + 7.03 \cos 95 = 0.$$

$$R_{BC} + 4.24 \cos 53.13 - 1.35 + 7.03 \cos 95 = 0.$$

$$R_{BC} = -7.03 \cos 95 + 1.35 - 4.24 \cos 53.13$$

$$R_{BC} = -6.58 \text{ kN (C)}$$

$\Sigma M_y = 0$.

$$\rightarrow R_{CE} \times 3 + 2 \times 3 = 0$$

$$R_{CE} = 2 \text{ kN.}$$

$$\frac{R_{CE} \times 3}{2}$$