1a) $\underbrace{y\cos xy}_{N}\, dy + \underbrace{x\cos xy}_{M}\, dx = 0$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \text{— for exact}$$

$$\frac{\partial M}{\partial y} = \frac{\partial(x\cos xy)}{\partial y} = -x^2 \sin xy$$

$$\frac{\partial N}{\partial x} = \frac{\partial(y\cos xy)}{\partial y} = -y^2 \sin xy$$

$\therefore$ Not exact

1b) $y'' + 2y' + y = 0$

$$\lambda^2 + 2\lambda + 1 = 0$$

$$(\lambda+1)^2 = 0$$

$$\lambda = -1$$

$$y_c = c_1 e^{-x} + c_2 x e^{-x}$$

Two linear independent solution $e^{-x}$, $xe^{-x}$

d) $f(z) = z^3 = (x+iy)^3$

$$= x^3 - iy^3 + i\,3xy(x+iy)$$

$$= x^3 - iy^3 + i\,3x^2y - 3xy^2$$

$$= x^3 - 3xy^2 + i(3x^2y - y^3)$$

$Re(z^3) = x^3 - 3xy^2$

$Im(z^3) = 3x^2y - y^3$

$\boxed{n=2}$

e) $R = \frac{1}{(n-1)!} \frac{d^{n-1}}{dz^{n-1}} (z-a)^n f(z)$

$$= 1 \times \frac{d}{dz} \cancel{(z)^2} \frac{1}{\cancel{z^2}}$$

$$= 1 \times 0$$

$$= 0$$

2. (*a*) Solve the ODE $\frac{dy}{dx}+\frac{y}{x}=\frac{y^2}{x}\log(x)$. 4

(*b*) A series RL circuit having a resistance of 20 ohm and inductance of 8H is connected to a DC voltage source of 120V at $t=0$. Find the current in the circuit at $t=6$. 4

Solution

(a) $$\frac{dy}{dx}+\frac{y}{x}=\frac{y^2}{x}\log x$$

$$\frac{dy}{dx}+P(x)\cdot y=g(x)\cdot y^a$$

By compairing we get,

$\boxed{a=2}$ $\quad g(x)=\frac{\log x}{x}$

$P(x)=\frac{1}{x}$

Let,

$$U=y^{1-2}$$

$$U=y^{-1}$$

$$\Rightarrow \frac{dU}{dx}=-1\, y^{-2}\frac{dy}{dx}$$

$$\Rightarrow \frac{dU}{dx}=-y^{-2}\frac{dy}{dx}$$

$$\Rightarrow \frac{dU}{dx}=-U^2.\left(\frac{y^2}{x}\log x-\frac{y}{x}\right)$$

$$\Rightarrow \frac{dv}{dx} = -v^2\left(\frac{1}{v^2x}\log x - \frac{1}{vx}\right)$$

$$\Rightarrow \frac{dv}{dx} = -\frac{\log x}{x} + \frac{v}{x}$$

$$\Rightarrow \frac{dv}{dx} - \frac{v}{x} = -\frac{\log x}{x}$$

$$\text{I.F} = e^{\int -\frac{1}{x}} = e^{-\ln x} = \frac{1}{x}$$

So, $$v \cdot \frac{1}{x} = \int -\frac{\log x}{x} \cdot \frac{1}{x}\, dx$$

$$v \cdot \frac{1}{x} = \int -\frac{\log x}{x^2}\, dx$$

$$v \cdot \frac{1}{x} = \frac{\log x}{x} + \frac{1}{x} + C$$

$$\Rightarrow v = \log x + 1 + Cx$$

$$\Rightarrow y = \left(\frac{1}{\log x + 1 + Cx}\right) \quad \text{(Ans)}$$



(PTO)

b) $L \frac{dI}{dt} + RI = E(t)$

$\Rightarrow 8 \frac{dI}{dt} + 20 I = 120$

$\Rightarrow \frac{dI}{dt} + \left(\frac{20}{8}\right) I = \left(\frac{120}{8}\right)$

$$I = \frac{E(t)}{R}\left(1 - e^{\left(-\frac{R}{L}\right)t}\right)$$

$$I = \frac{120}{20}\left(1 - e^{-\frac{20}{8} \times 6}\right)$$

$$I = 6\left(1 - e^{-\frac{120}{8}}\right)$$

$$\boxed{I = 6 \text{ Amp}} \ (\text{Am})$$

(2)

<u>or</u>

(a) $(x^2-xy)dy+(xy-1)dx=0$

$\Rightarrow (xy-1)dx+(x^2-xy)dy=0$

$\frac{\partial M}{\partial y}=\frac{\partial}{\partial y}(xy-1)=x \quad , \frac{\partial}{\partial x}(x^2-xy)=2x-y=\frac{\partial N}{\partial x}$

$$\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N}=\frac{x-2x+y}{x^2-xy}=\frac{y-x}{-x(y-x)}$$

$$=\frac{-1}{x}$$

$$u=e^{-\int\frac{1}{x}dx}=e^{-\log x}=x^{-1}.$$

Now,

$$\frac{(x^2-xy)}{x}dy+\frac{(xy-1)}{x}dx=0$$

$$\Rightarrow (x-y)dy+\left(y-\frac{1}{x}\right)dx=0$$

<u>Sol<sup></sup>n</u>

$$\int\left(y-\frac{1}{x}\right)dx+\int -y\,dy=c$$

$$\Rightarrow \boxed{xy-\log x-\frac{y^2}{2}=c}$$

(b) $\frac{dT}{T-T_A} = k\,dt$

$\Rightarrow \ln(T-T_A) = kt$

$\Rightarrow T = T_A + e^{kt+c}$

at, $t=0$, $T=40$

$\Rightarrow 40 = 20 + e^{c}$

$\Rightarrow e^{c} = 20$

$\Rightarrow c = \ln 20$

at $t=10$, $T=35$

$\Rightarrow 35 = 20 + e^{10k+\ln 20}$

$\Rightarrow 15 = e^{10k} \cdot 20$

$\Rightarrow e^{10k} = \frac{15}{20} = \frac{3}{4}$

$\Rightarrow 10k = \ln\left(\frac{3}{4}\right)$

$\Rightarrow k = \frac{\ln\left(\frac{3}{4}\right)}{10}$

So,

$30 = 20 + e^{\frac{\ln(3/4)}{10}\times t + \ln 20}$

$\Rightarrow \frac{10}{20} = e^{\frac{\ln(3/4)}{10} \cdot t}$

$\Rightarrow \frac{1}{2} = e^{\frac{\ln(3/4)}{10} t}$ $\Rightarrow \ln\left(\frac{1}{2}\right) = \frac{\ln\left(\frac{3}{4}\right)}{10} \times t$

$\Rightarrow t = 10 \ln(1/2) / \ln(3/4) = 24.094$

3. (a) Solve the non-homogeneous ODE

$$y'' + y = \tan(x).$$ 4

(b) Find the power series solution of

$$(1-x^2)y'' - 2xy' + 2y = 0.$$ 4

**Solution**

(a) $y'' + y = \tan x$

$\Rightarrow$ Make it homogeneous

$$y'' + y = 0$$

$$\lambda^2 + 1 = 0$$

$$\Rightarrow \lambda = \pm i \qquad \text{So, } y_c = e^{0 \cdot x}(C_1 \cos x + C_2 \sin x)$$

$$\Rightarrow y_c = \underset{\Downarrow \atop y_1}{C_1 \cos x} + \underset{\Downarrow \atop y_2}{C_2 \sin x}$$

Wronskian $= \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = 1$

$$y_p = -\cos x \int \frac{\sin x \cdot \tan x \, dx}{1} + \sin x \int \cos x \cdot \tan x \, dx$$

$$= -\cos x \cdot \left[\ln|\tan x + \sec x| - \sin x\right] - \sin x \cdot \cos x + C$$

$\therefore y = y_c + y_p$ (Ans)

(b) $y(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$

$y'(x) = a_1 + 2a_2 x + 3a_3 x^2 + \cdots$

$y''(x) = 2a_2 + 6a_3 x + 12 a_4 x^2 + \cdots$

so,

$(1-x^2) y'' - 2xy' + 2y = 0$

$\Rightarrow (2a_2 + 6a_3 x + 12 a_4 x^2 \ldots) - x^2(2a_2 + 6a_3 x + 12a_4 x^2 + \ldots)$

$- 2x(a_1 + 2a_2 x + 3a_3 x^2 + \ldots) + 2(a_0 + a_1 x + a_2 x^2 + \ldots) = 0$

on simplification

$\Rightarrow 2a_2 + 2a_0 + 6a_3 x + x^2(12a_4 - 4a_2) - 10a_3 x^3 - 2a_4 x^4 = 0$

So, $2a_2 + 2a_0 = 0$

$\Rightarrow \boxed{a_2 = -a_0}$

$\boxed{a_3 = 0}$

$12a_4 - 4a_2 = 0$

$\Rightarrow 3a_4 = a_2$

$\Rightarrow \boxed{a_4 = -\frac{a_0}{3}}$

... function of f(z) of a complex variable z defined in ... bounded domain D is said to be continuous ... choosen positive number

OR

3) a) $x^2y'' - 3xy' + 4y = 0$

characteristic eqn: $\lambda^2 - 4\lambda + 4 = 0$

$\Rightarrow (\lambda-2)^2 = 0$

$\Rightarrow \lambda = 2$

sol solution: $y = Ax^2 + B(\ln x)x^2$ ——— (i)

given $y(1) = 1$

from (i)

$1 = A \cdot 1^2 + B \cdot (\ln 1) x^1$

$\Rightarrow A = 1$

$y' = 2Ax + 2B(\ln x)x + Bx$

given, $y'(1) = 1$

Now,

$1 = 2 \times A \times 1 + 2B(\ln 1) \cdot 1 + B \times 1$

$\Rightarrow 1 = 2 \times 1 \times 1 + B$

$\Rightarrow B = -1$

Now,

$$\boxed{y = x^2 - (\ln x)x^2}$$

3) b)

$$y'' - y' + 6y = 0 \quad \text{——} \ (1)$$

Let

$$y = \sum_{m=0}^{\infty} a_m x^m$$

$$y' = \sum_{m=0}^{\infty} a_m \cdot m\, x^{m-1} = \sum_{m=1}^{\infty} a_m \cdot m \cdot x^{m-1}$$

$$y'' = \sum_{m=0}^{\infty} a_m \cdot m(m-1)\, x^{m-1} = \sum_{m=2}^{\infty} a_m \cdot m(m-1)\, x^{m-2}$$

putting the value of $y, y'$ & $y''$ in eq$^n$ (1) we get

$$\cancel{\sum_{m=0}^{\infty} a_m x^m - \sum_{m=1}^{\infty} a_m m x^{m-1} + 6\sum_{m=2}^{\infty} a_m \cdot m(m-1) x^{m-2} = 0}$$

$$= \cancel{\sum_{m=0}^{\infty} a_m x^m - \sum_{m=0}^{\infty} a_{m+1}(m+1) x^m}$$

$$\sum_{m=2}^{\infty} a_m m(m-1)x^{m-2} - \sum_{m=1}^{\infty} a_m m x^{m-1} + 6\sum_{m=0}^{\infty} a_m x^m = 0$$

$$\Rightarrow \sum_{m=0}^{\infty} a_{m+2}(m+2)(m+1)x^m - \sum_{m=0}^{\infty} a_{m+1}(m+1)x^m + \sum_{m=0}^{\infty} 6a_m$$

$$\Rightarrow \sum_{m=0}^{\infty} \left[a_{m+2}(m+2)(m+1) - a_{m+1}(m+1) + 6a_m\right] x^m :$$

$$\Rightarrow a_{m+2}(m+2)(m+1) - a_{m+1}(m+1) + 6a_m = 0$$

put $m=0$

$$a_2 \times 2 \times 1 - a_1 \times 1 + 6a_0 = 0$$

$$\Rightarrow a_2 = \frac{-6a_0 + a_1}{2}$$

put $m=1$

$$a_3 \cdot 3 \times 2 - a_2 \times 2 + 6a_1 = 0$$

$$\Rightarrow a_3 = \frac{-6a_1 + 2a_2}{6}$$

$$= \frac{-6a_1 + 2 \times \frac{-6a_0 + a_1}{2}}{6}$$

$$= \frac{-6a_1 - 6a_0 + a_1}{6}$$

$$= \frac{6a_0 - 5a_1}{6}$$

$N \geq 0$,

$$y = \sum_{m=0}^{\infty} a_m x^m$$

$$= a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots$$

$$= a_0 + a_1 x + \left(-\frac{6a_0 + a_1}{2}\right) x^2 + \left(\frac{6a_0 - 5a_1}{2}\right) x^3 + \ldots$$

(4)(a) $F = (y^2 - 7y,\ 2xy + 2x)$

C: 

Green's theorem :-

$$\oint_C F_1\,dx + F_2\,dy = \iint_R \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) dx\,dy$$

L.H.S

$$= \oint (y^2 - 7y)\,dx + (2xy + 2x)\,dy$$

Let $y = \sin\theta$, $x = \cos\theta$

$$= \int_0^{2\pi} (\sin^2\theta - 7\sin\theta)(-\sin\theta)\,d\theta + \int_0^{2\pi} (2\sin\theta\cos\theta + 2\cos\theta)\cos\theta\,d\theta$$

$$= \int_0^{2\pi} 7\sin^2\theta\,d\theta - \int_0^{2\pi} \sin^3\theta\,d\theta + \int_0^{2\pi} 2\sin\theta\cos^2\theta\,d\theta + \int_0^{2\pi} 2\cos^2\theta\,d\theta$$

$$= \frac{7}{2}\int_0^{2\pi} (1 - \cos 2\theta) - \frac{1}{4}\int_0^{2\pi} 3\sin\theta - \sin 3\theta - \left[\frac{\cos^3\theta}{3}\right]_0^{2\pi} + \int_0^{2\pi} 1 + \cos 2\theta$$

$$= \frac{7}{2}\left[\theta - \frac{1}{2}\sin 2\theta\right]_0^{2\pi} - \frac{1}{4}\left[-\frac{1}{3}\cos 3\theta + 3\cos\theta\right]_0^{2\pi} - 0 + \left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{2\pi}$$

$$= \frac{7}{2}\times 2\pi + 2\pi$$

$$= 9\pi.$$

$$\iint_R 2(2y+2) - (2y-7)\, dx\, dy$$

$$= 2\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} 9\, dx\, dy$$

$$= 2\times\int_{-1}^{1} 9\left(\sqrt{1-x^2}\right) dy$$

$$= 9\times 2\left[\frac{x\sqrt{1-x^2}}{2} + \frac{1}{2}\sin^{-1}\frac{x}{a}\right]_{-1}^{1}$$

$$= 9\times 2\left[0 + \frac{1}{2}\sin^{-1}1 - 0 - \frac{1}{2}\sin^{-1}(-1)\right]$$

$$= 9\times 2\left(\frac{\pi}{4} + \frac{\pi}{4}\right)$$

$$= 9\times\frac{\pi}{2}\times 2 = 9\pi \text{ (Ans) (Same)}$$

(5)

$F(x, y, z)$

Curl of $F$ = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$

$$= \hat{i}\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right) - \hat{j}\left(\frac{\partial}{\partial x}F_z - \frac{\partial}{\partial z}F_x\right)$$

$$+\hat{k}\left(\frac{\partial}{\partial x}F_y - \frac{\partial}{\partial y}F_x\right)$$

$\Rightarrow$ div (curl F)

$= \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)$ (curl OF F)

$= \frac{\partial}{\partial x}\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right) - \frac{\partial}{\partial y}\left(\frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z}\right) + \frac{\partial}{\partial z}\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)$

$= 0$

or (a) $\int_{(-1,7,0)}^{(3,1,9)} \left(xyz^2\,dx + \frac{1}{2}(2x)^2\,dy + x^2yz\,dz\right)$

$F_x = xyz^2$, $F_y = \frac{1}{2}(2x)^2$, $F_z = x^2yz$

$$\text{curl}(F) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xyz^2 & \frac{1}{2}(2x)^2 & x^2yz \end{vmatrix}$$

$= \hat{i}\left(x^2z - \cancel{\tfrac{1}{2}}\,2x^2z\right) - \hat{j}\left(2x^2yz - 2xyz\right) + \hat{k}\left(z^2x - z^2x\right)$

$= 0$ (so it is a conservative field)

$\frac{\partial f}{\partial x} = xyz^2$, $\frac{\partial f}{\partial y} = \frac{1}{2}(2x)^2$, $\frac{\partial f}{\partial z} = x^2yz$

$\Rightarrow \partial f = xyz^2\,dx$

$\Rightarrow f = \frac{x^2yz^2}{2} + C(y,z)$

$$\frac{\partial f}{\partial y} = \frac{x^2z^2}{2} + \frac{\partial}{\partial y} c(y,z)$$

by comparing $\frac{\partial f}{\partial y}$ and that is given in question

$$\frac{\partial}{\partial y} c(y,z) = 0$$

$$\Rightarrow \partial c(y,z) = 0 . \partial y$$

$$\Rightarrow c(y,z) = c(z)$$

$$f = \frac{x^2yz^2}{2} + c(z)$$

$$\Rightarrow \frac{\partial f}{\partial z} = \frac{x^2y.z}{\cancel{2}} + \frac{\partial}{\partial z} c(z)$$

$$\frac{\partial}{\partial y} c(z) = 0$$

$$\Rightarrow c(z) = c_1$$

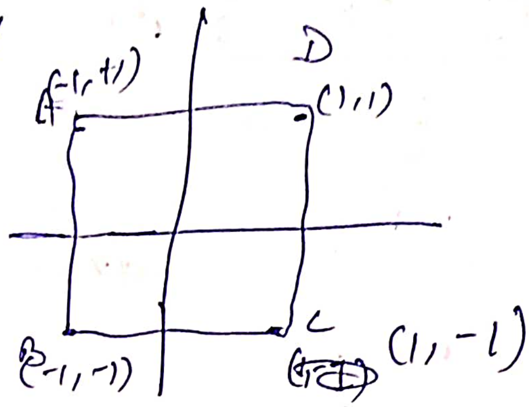
$$\Rightarrow \boxed{f = \frac{x^2yz^2}{2} + c_1}$$

value of integral

$$= \frac{9 \times 1 \times 81}{2} - \frac{1 \times 49 \times 10}{2}$$

$$= \frac{229}{2} \text{ (Ans)}$$

(6) $\iint_R (x^2+y^2)\,dx\,dy$




$= \int_{-1}^{1}\int_{0}^{1}(x^2+1)\,dx\,dy$

eqn of AD

$\Rightarrow \boxed{y=1}$

$= \int_{-1}^{1}(x^2+y)\times 1\,dx$

$= \left[\frac{x^3}{3}+x\right]_{-1}^{1} = \frac{1}{3}+1-\left(\frac{-1}{3}-1\right)$

$= \frac{2}{3}+2 = \frac{8}{3}$

eqn of DC

$x = 1$

$\int_{0}^{-1}\int^{1}(y^2+1)\,dx\,dy$

$= \int_{1}^{-1}(y^2+1)\,dy$

$= \left(\frac{y^3}{3}+y\right)_{1}^{-1} = \frac{-1}{3}-1-\frac{1}{3}-1$

$= -\frac{8}{3}$

eqn of CB

$y = -1$

$\int_{1}^{-1}\int_{0}^{1}(1+x^2)\,dy\,dx = \int_{1}^{-1}(1+x^2)\,dx$

$= \left(x+\frac{x^3}{3}\right)_{1}^{-1} = -\frac{8}{3}$

Lly eq^n of AB

$\boxed{x = -1}$

$$\int_{-1}^{1}\int_{0}^{1} (1+y^2)\, dx\, dy$$

$$\int_{-1}^{1} (1+y^2)\, dy$$

$$= \left(y + \frac{y^3}{3}\right)_{-1}^{1} = \frac{8}{3}$$

$$= \frac{8}{3} + \frac{8}{3} - \frac{8}{3} - \frac{8}{3} = 0$$

OR

(a) Given $f(z) = |z|^2$

We have to check differentiability at $z=0$.

Now

$$\lim_{z \to 0} \frac{f(z) - f(0)}{z - 0}$$

$$= \lim_{z \to 0} \frac{z^2 - 0}{z - 0}$$

$$= \lim_{z \to 0} z$$

$$= 0$$

It finitely exist. Hence $f(z)$ is differentiable at $z=0$. Again

$$f'(z) = 0 \quad \text{at} \quad z = 0.$$

Ans