Logic Proposition

A proposition is a declaration or declative sentence which is either true or false but not both.

Ex: Three plus three equals to six (True proposition)

Three plus three equals to seven. (False proposition)

$x+y>1$ is not a statement.

$x=-3, y=1$

→ Questions, exclamatory and commands are not statements.

Ex: The sun rises in the east. ✓ (It is a statement)

Open the door. ✗ (Not a statement)

Do you know Hindi? ✗ (Not a statement)

Compound Statement:

Compound statement is a combination of two or more proposition.

They are combined by logical connectivities.

Logical Connectivities:

→ Negation ($\sim, \neg$)

→ Conjunction ($\wedge$) - and

→ Disjunction ($\vee$) - or

→ Conditional ($\rightarrow, \Rightarrow$) if ... then

→ Bi-conditional ($\leftrightarrow, \Leftrightarrow$) if and only if

Negation:

If the proposition P is true, then $\sim P$ is false.

If the proposition P is false, then $\sim P$ is true.

Truth Table

p	$\sim p$
T	F
F	T

Conjunction ($\wedge$):

If p and q are two statements, then conjunction of p and q is called compound statement denoted by $p \wedge q$ and is true when both p and q are true otherwise false.

Truth Table

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction ($\vee$):

$p \vee q$ is true if atleast one of the statement is true.

Truth Table

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Conditional Statement ($\rightarrow$):

If two statements are combined by using the logical statement if .... then, then that is called conditional statement.

$$p \rightarrow q$$

nction
enoted
re true

Truth Table

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Bi-conditional statement:

The bi-conditional statement is true when p and q ~~are true for same~~ have the same truth table and false otherwise.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Proposition Equivalence:

. true.

→ A compound proposition is always true is known Tautology.

→ A compound statement is always false is known Contradiction.

→ A compound statement which is either true or false is known as proposition.

Ex:

p	$\sim p$	$(p \vee \sim p)$	$(p \wedge \sim p)$
T	F	T	F
F	T	T	F.

gical

$p \vee \sim p$ - tautology

$p \wedge \sim p$ - contradiction.

→ Construct the truth table

$p \wedge (\sim q \vee q)$

p	q	$\sim q$	$(\sim q \vee q)$	$p \wedge (\sim q \vee q)$
T	T	F	T	T
T	F	T	T	T
F	T	F	T	F
F	F	T	T	F

Assignment: 07/01/25

Construct the truth table

~~$p \wedge (\sim q \vee q)$~~ $(\sim p \wedge (\sim q \wedge r)) \vee ((q \wedge r) \vee (p \wedge r))$

Logical ~~values~~ equivalence:

If two propositions $P(p, q, \ldots)$ and $Q(p, q, \ldots)$ where $p, q, \ldots$ are propositional variables have the same truth values in every possible case, then the propositions are called equivalent and denoted by $P \equiv Q$

~~$P \leftrightarrow$~~

Ex:

Using truth table, show that

$\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$.

L.H.S

p	q	$(p \wedge q)$	$\sim(p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

RHS

$\sim p$	$\sim q$	$\sim p \vee \sim q$
F	F	F
F	T	T
T	F	T
T	T	T

LHS = RHS

$P \equiv Q$.

Assignment:

Show that (1) $\sim(p \vee q)$, $\sim p \wedge \sim q$ are equivalent.

(2) $p \rightarrow q$ and $\sim p \vee q$ are equivalent.

(3) $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are equivalent.

Algebra of preoposition

Name	Logical equivalence
1) Identity Law	$P \wedge T \equiv P$ $P \vee F \equiv P$
2) Domination Law	$P \vee T \equiv T$ $P \wedge F \equiv F$
3) Double Negation	$\sim(\sim P) \equiv P$
4) Idempotent Law	$P \vee P \equiv P$ $P \wedge P \equiv P$
5) Associative Law	$P \vee (Q \vee R) \equiv (P \vee Q) \vee R$ $P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$
6) Distributive Law	$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$ $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$
7) De Morgan's Law	$\sim(P \wedge Q) \equiv \sim P \vee \sim Q$ $\sim(P \vee Q) \equiv \sim P \wedge \sim Q$.
8) Absorption Law	$P \vee (P \wedge Q) \equiv P$ $P \wedge (P \vee Q) \equiv P$.

$P \wedge T$	P
T T	= T
F T	= F

$P \wedge T = P$.

9) Commutative Laws $\quad P \vee Q \equiv Q \vee P$

$P \wedge Q \equiv Q \wedge P$.

10) Negation Laws $\quad P \vee \sim P \equiv T$

$P \wedge \sim P \equiv F$

11) Implication Law $\quad P \to Q \equiv \sim P \vee Q$.

8)

P	Q	$P \wedge Q$	$P \vee (P \wedge Q)$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

$P \equiv P \vee (P \wedge Q)$. [Proved].

12) Law of contrapositive $\quad P \to Q \equiv \sim Q \to \sim P$.

Ex

Show that $\sim(p \to q)$ and $(p \wedge \sim q)$ logically equivalent without using the truth table.

LHS

$\sim(p \to q) \equiv \sim(\sim p \vee q)$ (By implication law)

$\equiv \sim(\sim p) \wedge (\sim q)$

$\equiv p \wedge \sim q$.

Logical ~~Implication~~ Equivalence Involving Implication

1) $p \to q \equiv \sim p \vee q$

2) $p \to q \equiv \sim q \to \sim p$

3) $p \vee q \equiv \sim p \to q$

4) $p \wedge q \equiv \sim(p \to \sim q)$

5) $\sim(p \to q) \equiv p \wedge \sim q$

6) $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

7) $(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$

8) $(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$

9) $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

10) $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

11) $p \leftrightarrow q \equiv \sim p \leftrightarrow \sim q$

12) $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

13) $\sim (p \leftrightarrow q) \equiv p \leftrightarrow \sim q$.

~~Without~~ with making truth table, prove 1 and 2.

1) $p \to q \equiv \sim p \vee q$.

p	q	$p \to q$	~~$p \vee q$~~	$\sim p$	$\sim p \vee q$
T	T	T	~~T~~	F	T
T	F	F	~~T~~	F	F
F	T	T	~~T~~	T	T
F	F	T	~~F~~	T	T

$p \to q \equiv \sim p \vee q$ [Proved]

2) $p \to q \equiv \sim q \to \sim p$

p	q	$p \to q$	$\sim q$	$\sim p$	$\sim q \to \sim p$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

$p \to q \equiv \sim q \to \sim p$ [Proved]

Q) show $\sim (p \vee (\sim p \wedge q))$ and $(\sim p \wedge \sim q)$ are logically equivalent, without using truth table.

$\sim (p \vee (\sim p \wedge q)) \equiv \sim p \wedge (p \vee \sim q)$ (De-Morgan's law)

$\equiv (\sim p \wedge p) \vee (\sim p \wedge \sim q)$ (De-Morgan's law)

$\equiv F \vee (\sim p \wedge \sim q)$ (Negation law)

$\equiv \sim p \wedge \sim q$

Normal Form:

The standard forms are called normal form or canonical forms.

They are of two types:-

1) Disjunctive normal form (DNF) ($\vee$)

2) Conjunctive normal form (CNF) ($\wedge$).

$\wedge$ - conjunction → product - and

$\vee$ - disjunction → sum - or

Elementary Product:

A product of the variables and their negations in a formula is called an elementary product.

Let p and q be any two variables then

$\sim p \wedge q$, $\sim q \wedge p \wedge \sim p$, $p \wedge \sim p$, $q \wedge r$

are examples of elementary product.

Elementary Sum:

A sum of the variable and their negation called an elementary sum.

If p, q → variables, then

$\sim p \vee q$, $\sim q \vee p \vee \sim p$, $p \vee \sim p$, $q \vee r$ are examples of elementary sum.

Disjunctive Normal Form:-

A formula which is equivalent to a given formula that consists of a sum of elementary product is called a disjunctive normal form.

Ex: $\underbrace{(p \wedge q)}_{\text{product}} \underbrace{\vee}_{\text{sum}} q \underbrace{\vee}_{\text{sum}} \underbrace{(\sim p \wedge q)}_{\text{product}}$

separated by sum → when product are separated by sum

Ex:
obtain disjunctive normal form of $p \wedge (p \Rightarrow q)$.

Imp property
$p \Rightarrow q \equiv \sim p \vee q$
$p \Leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$\equiv p \wedge (\sim p \vee q)$

$\equiv \underbrace{(p \wedge \sim p)}_{\text{product}} \underset{\text{sum}}{\vee} \underbrace{(p \wedge q)}_{\text{product}}$.

↳ Product is separated by sum.

Ex: Is it a DNF
$(p \Rightarrow q) \wedge (\sim p \wedge q)$.

$\equiv (\sim p \vee q) \wedge (\sim p \wedge q)$.

~~$\equiv \sim p \wedge \sim p \vee q \wedge \sim p \wedge \sim p \wedge q \vee q \wedge q$.~~

$\equiv \sim p \vee (\sim p \wedge q) \vee q \wedge (\sim p \wedge q)$

$\equiv \underbrace{(\sim p \wedge q)}_{\text{Product}} \underset{\text{sum.}}{\vee} \underbrace{(q \wedge \sim p)}_{\text{Product}}$

Conjuctive Normal Form :-
A formula which consists of a product of elementarey sum, is called a conjuctive normal form.

Ex: $(p \vee q) \wedge (\sim p \vee q)$.

Ex: obtain conjuctive normal form of
$(p \Rightarrow q) \wedge (q \vee (p \wedge r))$

$\equiv (\sim p \vee q) \wedge (q \vee (p \wedge r))$

~~$\equiv \sim p \wedge (q \vee (p \wedge r)) \vee q (q \vee (p \wedge r))$~~

$\equiv \underbrace{(\sim p \vee q)}_{\text{sum}} \underset{\text{product}}{\wedge} (\underbrace{q \vee p}_{\text{sum}}) \underset{\text{product}}{\wedge} \underbrace{(q \vee r)}_{\text{sum}}$

~~$\equiv (\sim p \vee q)$~~

Predicate:

Statements involving variables such as "$x > 3$", "$x = y + 3$", "computer is functioning properly".

Hence statement $x > 3$ has two parts.

1st part: variable x → subject.

2nd part: is the predicate.

$x > 3$

$P(x): x > 3$

↓

subject

$P(4): 4 > 3$

↳ T

Ex: $Q(x, y): x = y + 3$

$Q(1, 2)$ and $Q(3, 0)$.

Quantifier:

Quantifier are words that refer to quantifies such as 'some' or 'all' or "there exist" and indicate how frequently or certain statement is true.

There exist two types of quantifier:

→ Universal quantifier

↳ Existential quantifier.

Universal Quantifier: ($\forall$) (for all).

Ex All students are smart

Let $P(x)$ denotes students, x denotes smart.

$\forall x\ P(x)$.

Existential quantifier:

The phrase "there exist" denoted by $\exists$ is called the existence quantifier.

Ex: Let $\exists x$ such that $x^2 = 9$. Let $P(x): x^2 = 9$

$\exists x\ P(x)$

Mathematical Induction:

The mathematical induction is used to prove statements that assumes $P(n)$ is true for all +ve integers where $P(n)$ is a proportional function.

Working Procedure:

step 1: $P(1)$ is true.

Step 2: Assume $P(K)$ is true (inductive hypothesis)

step 3: To prove $P(K+1)$ is true.

Ex:

using mathematical induction, prove that

$$1+2+2^2+\dots+2^n=2^{n+1}-1$$

step 1: $n=1$, LHS

$$1+2=3$$

RHS

$$2^2-1=4-1=3.$$

$$LHS = RHS$$

$$\Rightarrow P(1) \text{ is true.}$$

step 2: Let $P(K)$ is true.

$$1+2+2^2+\dots+2^k=2^{k+1}-1$$

step 3: for $P(k+1)$

$$1+2+2^2+\dots+2^K+2^{k+1}$$

$$=2^{k+1}-1+2^{k+1}$$

$$=2\cdot 2^{k+1}-1$$

$$=2^{k+2}-1.$$

i.e. The result is true for $n=k+1$

Hence the statement is true for all n.

Ex:

using mathematical induction, $1^2+2^2+3^2+\dots+n^2$

$$=\frac{n(n+1)(2n+1)}{6}.$$

Step 1: $n=1$

$LHS = 1^2 = 1$

$RHS = \frac{1(2)(3)}{6} = 1$

$LHS = RHS$

$P(1)$ is true.

Step 2: $n=k$

Let us assume that $P(k)$ is true.

i.e. $1^2+2^2+\dots+k^2 = \frac{k(k+1)(2k+1)}{6}$

Step 3: $n=k+1$

$1^2+2^2+\dots+k^2+(k+1)^2$

$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$

$= (k+1)\left[\frac{k(2k+1)}{6} + (k+1)\right]$

$= (k+1)\left(\frac{2k^2+k+6k+6}{6}\right)$

$= (k+1)\left(\frac{2k^2+7k+6}{6}\right)$.

$= (k+1)\left(\frac{2k^2+4k+3k+6}{6}\right)$

$= (k+1)\left[\frac{2k(k+2)+3(k+2)}{6}\right]$

$= \frac{(k+1)(k+2)(2k+3)}{6}$

$\therefore P(k+1)$ is true.

Assignment

Using mathematical induction, prove that

1) $6^{n+2} + 7^{2n+1}$ is divisible by 43.

2) $n < 2^n$

Permutation

The study of ... with determ... arranging ... number of ...

→ An arra... called ...

→ An arra... called ...

→ The n... takin...

→ The ... k o...

Funda...

If ar... and ... diffe... Then ... wa...

Ex:

P(A)

Permutation And Combination

The study of permutation and combination concerned with determining the number of different ways of arranging and selecting objects out of a given number of objects.

→ An arrangement where order is important is called a permutation.

→ An arrangement where order is not important is called combination.

→ The number of distinct permutation of n objects taking k at a time

$$^nP_k = \frac{n!}{(n-k)!}$$

$$= \frac{n(n-1)(n-2)\cdots(n-k+1)\cancel{(n-k)!}}{\cancel{(n-k)!}}$$

$$= n(n-1)(n-2)\cdots(n-k+1).$$

→ The number of combinations of n objects taken k at a time

$$^nC_k = \binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

Fundamental Counting Principle:

If an incident be performed in m different ways and another incident performed independently in n different ways.

Then the incident one and two performed in $(m \times n)$ ways i.e. mn ways.

Ex: There are 8 horses run in a race. How many ways the first three places filled up?

Soln: $\frac{3rd}{6} \times \frac{2nd}{7} \times \frac{1st}{8}$ (For 1st time there we have 8 horses)

$= 336$ ways

Ex: In how many ways ~~coin~~ in crosses of a coin turned out? (can occur in 2 ways i.e. head & tail)

Q) In how many different ways a committee of 5 can be performed from a class of 30 students?

$$\frac{30!}{5! \times 25!}$$

Permutation with Restriction

Ex How many 9-letter words can be formed from the letter "HYPERBOLA" if the letters 'P' 'E' 'R' must be together.

$7!\,3!$

Ex: How many 9-letters words can be formed from the letter "HYPERBOLA" if the letters 'H' 'Y' 'P' 'E' 'R' word can't be together.

$9!\,5!$

Positional Restriction

C H A N G E S.

Q) No. of permutations of the letters without any restriction. $7!$

Q) No. of 6 letters arrangements. ${}^7P_6$

Q) No. of arrangements starting with S. $6!$

Q) No. of arrangements which begin with exactly one consonants.

No. of ways consonant can be chosen. 5

Q) No. of ways a vowel can be chosen from A, E

No. of ways a vowel can be chosen from A, E since second letter can't be a consonant. $2!$

Q) No. of ways the rest of the five letter can be arranged in $5!$

Here they can be arranged in $5 \times 2! \times 5!$

Q) In how many ways, four red and four black balls can be arranged in a row so that they are alternate in colour? (Assignment) – 28/05/25

Q) Solve $^nP_2 = 30$. find value of n.

$$\frac{n!}{(n-2)!} = 30 \Rightarrow n(n-1) = 30$$

$$\Rightarrow n^2 - n - 30 = 0$$

$$\Rightarrow n = 6.$$

Q) $^nP_5 = 10\ ^{n-1}P_4$.

$n = 10$.

Circular Permutation

Q) In how many ways, can six people be seated at a round table?

$(6-1)! = 5!$ ways.

Q) find the no. of ways in which A, B, C, D, E can be seated at a round table such that

(a) A & B must always be together. – $2 \times 3!$ ways

(b) C & D must not sit together.

AB C D E

$\rightarrow (4-1)! = 3!$

AB → both are arranged in 2 ways

As $2 \times 3!$

$4 - 2 \times 3!$ ways.

Q) In how many ways can 10 boys & 5 girls be seated around a table such that no 2 girls sit together?

(Assignment) – 28/05/25

Combination with restriction:

Q) From a class of ten students (six girls and four boys), a committee of three is to be formed with

(i) No restriction → $^{10}C_3$ ways

(ii) atleast one girl → $^{10}C_3 - {^6C_0}\,{^4C_3}$

(iii) a particular boy chosen → $^9C_2$

(iv) more boys than girls. → $^4C_3 \times {^6C_0} + {^4C_2} \times {^6C_1}$

(v) atmost two girls. → $^6C_0 \times {^4C_3} + {^6C_1} \times {^4C_2} + {^6C_2} \times {^4C_1}$.

Q) The vertices of a polygon determined 56 triangles. How many sides does the polygon had?

$^nC_3 = 56$

$\Rightarrow n = 8$

Q) How many diagonals are there in a hexagon?

$^6C_2 - 6$.

Assignment 28/05/25

Q) How many numbers can be formed from digit 2, 3, 4, & 5 if no digit is repeated?

Q) In how many ways you can set 15 students in a 3-seater, 4-seater and 8-seater car?

Pigeonhole Principle:-

It states that if 'n' objects are to be placed in 'k' boxes (k<n), then there must be at least one box which contains more than one object (pigon).

Ex → Assume a drawer contain a mixture of black socks and blue socks. Consider the colours of the socks as pigeonholes.

→ By pigeonhole principle we must have to pick atleast three socks to get two socks of the same colour.

Ex: From a set of randomly chosen people what is the probability that two of them have the same birthdays.

By pigeonhole principle, if there are 367 people in the room atleast two of them have the same birthday.

Assignment 31/05/2025

1) In how many ways an animal trainer can arrange five lions and four tigers such that no lions are together?

2) How many arrangements of the word 'BENGALI' can be made

(i) If two vowels are never together?

(ii) If the vowels are to occupy odd places?

3) Find the number of subsets of the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$ having 4 elements.

4) Twenty persons are invited to a party. In how many ways they can be seated such that two particular persons sit on either side of the host?

5) Given 7 consonants, four vowels, how many 5 letter arrangements can be made if each arrangement consist of three different consonants and two different vowels.

6) How many three digit numbers divisible by 5 can be formed from the digits 2, 3, 5, 6, 7 and 9.