Week 2: For each of the following, determine if the statement is true or false. If the statement is true, provide a full proof. If the statement is false, provide an explanation including a counter-example. Suppose that a, b, c, and d are integers.

2a. If $a \mid c$ and $b \mid c$, then $(a + b) \mid c$.

2b. If $a \mid c$ and $a \mid d$, then $a \mid cd$

2c. If $a \mid c$ and $b \mid c$, then $ab \mid c$

2d. If $a \mid b$ and $a \mid c$, then $a \mid (b + c)$

Solutions:

2a. **False** For example, if you have $3 \mid 12$ and $2 \mid 12$, then $(3 + 2) \mid 12$, however $12 \mod 5 = 2$.

2b. **True** Assume k, l are integers and $c = ak$ and $d = al$, thus by multiplying the two together we get $cd = a^2(kl)$, $cd = a(akl)$ since a is multiplied by an integer, by the Definition of Divisibility, $a \mid cd$. Q.E.D.

2c. **False** For example, $12 \mod 4 = 0$ and $12 \mod 6 = 0$, however $12 \mod 24 = 12$, not 0.

2d. **True** Assume l, m are integers s.t $b = al$ and $c = am$. By adding the two equations together we get that $b + c = al + am$, which after factoring out an a becomes $b + c = a(l + m)$. Since both l and m are constants, $b + c = a(k)$ where $k = l + m$. Thus $a|(b + c)$ by the Definition of Divisibility. Q.E.D.