Integración

1) $$P_1(x) = \frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b)$$

$$\int_a^b f(x)\,dx \approx \int_a^b P_1(x)\,dx$$

$$= \int_a^b \left(\frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b) \right) dx$$

$$= f(a) \int_a^b \frac{x-b}{a-b}\,dx + f(b) \int_a^b \frac{x-a}{b-a}\,dx$$

$$= f(a) \left(\frac{1}{a-b} \int_a^b x\,dx - \frac{b}{a-b} \int_a^b dx \right) + f(b) \left(\frac{1}{b-a} \int_a^b x\,dx - \frac{a}{b-a} \int_a^b dx \right)$$

$$= f(a) \left(\frac{bx}{b-a} \Big|_a^b - \frac{x^2}{2(b-a)} \Big|_a^b \right) + f(b) \left(\frac{x^2}{2(b-a)} \Big|_a^b - \frac{ax}{b-a} \Big|_a^b \right)$$

$$= f(a) \left(\frac{b^2 - 2ab + a^2}{2(b-a)} \right) + f(b) \left(\frac{b^2 - 2ab + a^2}{2(b-a)} \right)$$

$$= f(a) \left(\frac{(b-a)}{2} \right) + f(b) \left(\frac{(b-a)}{2} \right)$$

$$= \frac{(b-a)}{2} (f(a) + f(b))$$

3) $$P_2(x) = \frac{(x-b)(x-x_m)}{(a-b)(a-x_m)} f(a) + \frac{(x-a)(x-b)}{(x_m-a)(x_m-b)} f(x_m) + \frac{(x-a)(x-x_m)}{(b-a)(b-x_m)} f(b)$$

$$\int_a^b f(x)\,dx \approx \int_a^b P_2(x)\,dx$$

$$= \int_a^b \frac{(x-b)(x-x_m)}{(a-b)(a-x_m)} f(a) + \frac{(x-a)(x-b)}{(x_m-a)(x_m-b)} f(x_m) + \frac{(x-a)(x-x_m)}{(b-a)(b-x_m)} f(b) \Big)\,dx$$

$$= f(a) \int_a^b \left(\frac{(x-b)(x-x_m)}{(a-b)(a-x_m)} \right) dx + f(x_m) \int_a^b \left(\frac{(x-a)(x-b)}{(x_m-a)(x_m-b)} \right) dx + f(b) \int_a^b \left(\frac{(x-a)(x-x_m)}{(b-a)(b-x_m)} \right) dx$$

$$= \frac{f(a)}{(a-b)(a-x_m)} \int_a^b (x^2 - x x_m - bx + b x_m)\,dx + \frac{f(x_m)}{(x_m-a)(x_m-b)} \int_a^b (x^2 - xb - xa + ab)\,dx + \frac{f(b)}{(b-a)(b-x_m)} \int_a^b (x^2 - x x_m - ax + a x_m)\,dx$$

$$= \frac{f(a)}{(a-b)(a-x_m)} \left(\frac{x^3}{3} - \frac{x^2}{2} x_m - \frac{x^2}{2} b + x b x_m \right) \Big|_a^b + \frac{f(x_m)}{(x_m-a)(x_m-b)} \left(\frac{x^3}{3} - \frac{x^2}{2} b - \frac{x^2}{2} a + x a b \right) \Big|_a^b + \frac{f(b)}{(b-a)(b-x_m)} \Big(\frac{x^3}{3} - \frac{x^2}{2} x_m$$

$$- \frac{x^2}{2} a + x a x_m \Big) \Big|_a^b$$

$$= \frac{f(a)}{(a-b)(a-x_m)} \left(\frac{2b^3 - 2a^3}{6} - \frac{3b^2 x_m - 3a^2 x_m}{6} - \frac{3b^3 - 3a^2 b}{6} + \frac{6b^2 x_m - 6abx_m}{6} \right) + \frac{f(x_m)}{(x_m-a)(x_m-b)} \Big(\frac{2b^3 - 2a^3}{6} - \frac{3b^3 - 3a^2 b}{6}$$

$$- \frac{3b^2 a - 3a^3}{6} + \frac{6ab^2 - 6a^2 b}{6} \Big) + \frac{f(b)}{(b-a)(b-x_m)} \left(\frac{2b^3 - 2a^3}{6} - \frac{3b^2 x_m - 3a^2 x_m}{6} - \frac{3b^2 a - 3a^3}{6} + \frac{6abx_m - 6a^2 x_m}{6} \right.$$

$$= \frac{f(a)(b-a)}{(a-b)^2 (2a - 2x_m)} \left(\frac{2a^3 - 3a^2 x_m - 3a^2 b + 6abx_m + b^3 - 3b^2 x_m}{3} \right)$$

$$= \frac{f(a)(b-a)}{(a-b)^3} \left(\frac{a^3 - 3a^2 b + 3ab^2 - b^3}{3} \right)$$

$$= \frac{f(a)(b-a)}{(a-b)^3} \left(\frac{(a-b)^3}{3} \right) = \frac{f(a)(b-a)}{3} = \frac{h f(a)}{3}$$

$$\frac{4f(x_m)}{(2x_m-2a)(2x_m-2b)}\left(\frac{2b^3-2a^3-3b^3+3a^2b-3b^2a+3a^3+6ab^2-6a^2b}{6}\right)$$

$$=\frac{(b-a)4f(x_m)}{(b-a)^3}\left(\frac{b^3-3ab^2+3a^2b-a^3}{3}\right)$$

$$=\frac{h4f(x_m)}{(b-a)^3}\;\frac{(b-a)^3}{3}$$

$$=\frac{h4f(x_m)}{3}$$

$$\frac{f(b)(b-a)}{(b-a)^2(2b-2x_m)}\left(\frac{2b^3-3b^2x_m-3b^2a+a^3+6abx_m-3a^2x_m}{3}\right)$$

$$=\frac{f(b)(b-a)}{(b-a)^3}\left(\frac{b^3-3ab^2+3a^2b-a^3}{3}\right)=\frac{f(b)(b-a)}{(b-a)^3}\left(\frac{(b-a)^3}{3}\right)$$

$$=\frac{f(b)(b-a)}{3}$$

$$=\frac{hf(b)}{3}$$

entonces

$$=\frac{hf(a)}{3}+\frac{h4f(x_m)}{3}+\frac{hf(b)}{3}$$

$$=\frac{hf(a)}{3}+\frac{h4f(x_m)}{3}+\frac{hf(b)}{3}$$

$$=\frac{h}{3}\left(f(a)+4f(x_m)+f(b)\right)$$