Sea $A(X_1, X_2, X_3, \dots, X_n)$, donde $A \sim N(\mu, \sigma)$. Mostrar estimadores máximo verosímiles

$$L(\vec{x};(\mu,\sigma)) = \prod_{i=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}} = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n e^{-\sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\sigma^2}}$$

$$\Rightarrow \ln(L(\vec{x};(\mu,\sigma))) = n\ln\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right) - \sum_{i=1}^{n}\frac{(x_i-\mu)^2}{2\sigma^2}$$

$$\Rightarrow \frac{\partial \ln(L(\vec{x};(\mu,\sigma)))}{\partial \mu} = \frac{-1}{2\sigma^2}\sum_{i=1}^{n}(-2)(x_i-\mu) = \left(\frac{1}{\sigma^2}\right)\left(\sum_{i=1}^{n}(x_i-\mu)\right) = \left(\frac{1}{\sigma^2}\right)\left(-n\mu + \sum_{i=1}^{n}x_i\right)$$

$$\left(\frac{1}{\sigma^2}\right)\left(-n\mu + \sum_{i=1}^{n}x_i\right) = 0 \Rightarrow n\mu = \sum_{i=1}^{n}x_i \Rightarrow \underbrace{\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n}x_i = \overline{X}}$$

$$\Rightarrow \frac{\partial \ln(L(\vec{x};(\mu,\sigma)))}{\partial \sigma^2} = -\frac{n}{2}\frac{2\pi}{2\pi\sigma^2} + \frac{1}{2(\sigma^2)^2}\sum_{i=1}^{n}(x_i-\mu)^2$$

$$\frac{-n}{2\sigma^2} + \frac{1}{2(\sigma^2)^2}\sum_{i=1}^{n}(x_i-\mu)^2 = 0 \Rightarrow \frac{n}{2\sigma^2}\cdot 2(\sigma^2)^2 = \sum_{i=1}^{n}(x_i-\mu)^2 \Rightarrow$$

$$\Rightarrow n\sigma^2 = \sum_{i=1}^{n}(x_i-\mu)^2 \Rightarrow \underbrace{\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\hat{\mu})^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\overline{X})^2}$$