1. algebra lineal

$$X_{n+1} = 4x_n - x_n^2 \qquad x_0 = 4\sin^2\theta$$

Implica $x_{n+1} = 4\sin^2(2^{n+1}\theta)$ $\theta \in [0, \pi/2]$

Caso base:

$n=0$

$x_1 = 4x_0 - x_0^2$

$x_1 = 4 \cdot 4\sin^2\theta - 16\sin^4\theta$

$= 16\sin^2\theta - 16\sin^4\theta$

$= 16\sin^2\theta\,(1 - \sin^2\theta)$

$= 16\sin^2\theta\cos^2\theta$

$= 44\sin^2\theta\cos^2\theta$

$= 4 \cdot (2 \cdot \sin\theta\cos\theta)^2$

$= 4(\sin(2\theta))^2$

$= 4 \cdot \sin^2(2^{0+1}\theta)$

- Supongamos que $x_{n+1} = 4x_n - x_n^2$ tal que

$x_n = 4\sin^2(2^n\theta)$

$x_{n+1} = 4x_n - x_n^2$

$x_{n+1} = 4(4\sin^2(2^n\theta)) - (4\sin^2(2^n\theta)^2)$

$= 16\sin^2(2^n\theta) - 16\sin^4(2^n\theta))$

$= 16\sin^2(2^n\theta)(1 - \sin^2(2^n\theta))$

$= 16\sin^2(2^n\theta)\cos^2(2^n\theta))$

$= 4.(2 \cdot \sin(2^n\theta) \cdot \cos(2^n\theta))^2$

$= 4(\sin(2 \cdot (2^n\theta))^2$

Así $4(\sin(2^{n+1}\theta))^2$

Si $- x_{n+1} = 4x_n - 4x_n^2$

- $x_0 = \sin^2\theta$

Implica que $x_{n+1} = \sin^2(2^{n+1}\theta)$

$\theta \in [0, \pi/2]$

Caso base: $n=0$

$x_1 = 4x_0 - 4x_0^2$

$x_1 = 4.\sin^2\theta - 4\sin^4\theta$

$x_1 = 4\sin^2\theta\,(1 - \sin^2\theta)$

$x_1 = 4\sin^2\theta\cos^2\theta$

$x_1 = (2 \cdot \sin\theta \cdot \cos\theta)^2$

$x_1 = (\sin(2\theta))^2$

$\Rightarrow x = \sin^2(2^n\theta)$

Ahora Suponga $x_{n+1} = 4x_n - 4x_n^2$

$x_n = \sin^2(2^n\theta)$

$x_{n+1} = 4x_n - 4x_n^2$

$x_{n+1} = 4(\sin^2(2^n\theta) - \sin^4(2^n\theta))$

$x_{n+1} = 4\sin^2(2^n\theta)(1 - \sin^2(2^n\theta))$

$x_{n+1} = 4\sin^2(2^n\theta)(\cos^2(2^n\theta))$

$x_{n+1} = (2\sin(2^n\theta)\cos(2^n\theta))^2$ ✓