3'5

$$f^2(x_{i+1}) = f^2(x_i) + hf^3(x_i) + \frac{h^2 f^4(x_i)}{2} + \frac{h^3 f^5(x_i)}{3!} + O(h^4)$$

$$f^2(x_{i-1}) = f^2(x_i) - hf^3(x_i) + \frac{h^2 f^4(x_i)}{2} - \frac{h^3 f^5(x_i)}{3!} + O(h^4)$$

$$f^2(x_{i-1}) + f^2(x_{i+1}) = 2f^2(x_i) + \frac{2h^2 f^4(x_i)}{2} + O(h^4)$$

$$\frac{f^2(x_{i-1}) + f^2(x_{i+1}) - 2f^2(x_i)}{h^2} + O(h^4) = f^4(x_i)$$

$$\frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i) - 2f(x_{i+1}) + 4f(x_i) - 2f(x_{i-1}) + f(x_i) - 2f(x_{i-1}) + f(x_{i-2})}{h^4} + O(h^4) = f^4(x_i)$$

$$\frac{f(x_{i+2}) - 4f(x_{i+1}) + 6f(x_i) - 4f(x_{i-1}) + f(x_{i-2})}{h^4} + O(h^2) = f^4(x_i)$$

3'8) a) $\Omega = (x_0, f(x_0)), (x_1, f(x_1)), (x_2, f(x_2))$

Interpolación lagrange:

$$P_2(x) = f(x_0)L_0(x) + f(x_1)L_1(x) + f(x_2)L_2(x)$$

Donde sabemos que:

$$L_i(x) = \frac{(x-x_n)(x-x_m)}{(x_i-x_n)(x_i-x_m)} \quad \text{tal que} \quad n \neq m \neq i$$

Así:

b) $$P_2(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f(x_0) + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1) + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f(x_2)$$

$$P_2(x) = \frac{f(x_0)}{(x_0-x_1)(x_0-x_2)} \frac{d}{dx} x^2 - x_2 x - x_1 x + x_2 x_1 + \frac{f(x_1)}{(x_1-x_0)(x_1-x_2)} \frac{d}{dx} x^2 - x_2 x - x_0 x + x_2 x_0 + \frac{f(x_2)}{(x_2-x_0)(x_2-x_1)} \frac{d}{dx} x^2 - x_1 x - x_0 x + x_1 x_0$$

$$\frac{f(x_0)}{(x_1-x_0)(x_2-x_0)} 2x - x_2 - x_1 + \frac{f(x_1)}{-(x_1-x_0)(x_2-x_1)} 2x - x_2 - x_0 + \frac{f(x_2)}{(x_2-x_0)(x_2-x_1)} 2x - x_1 - x_0$$

$$\frac{f(x_0)(2x_0 - x_2 - x_1)}{h \cdot 2h} + \frac{f(x_1)(2x_0 - x_2 - x_0)}{-h \cdot h} + \frac{f(x_2)(2x_0 - x_1 - x_0)}{2h \cdot h}$$

$$\frac{f(x_0)(x_0 - x_2 + x_0 - x_1) + f(x_2)(x_0 - x_1 + x_0 - x_0)}{2h^2} - \frac{f(x_1)(x_0 - x_2 + x_0 - x_0)}{h^2}$$

$$\frac{f(x_0)(x_1 - x_0 + x_2 - x_0) + f(x_2)(x_1 - x_0 + x_0 - x_0)}{2h^2} - \frac{f(x_1)(x_2 - x_0 + x_0 - x_0)}{h^2}$$

$$\frac{f(x_0)(h + 2h) + f(x_2)(h)}{2h^2} - \frac{f(x_1)(2h)}{h^2}$$

$$\frac{3f(x_0) + f(x_2)}{2h} - \frac{4f(x_1)}{2h} = \frac{1}{2h}(3f(x_0) + f(x_2) - 4f(x_1))$$