Math 214 – Foundations of Mathematics **Homework 1/30** **Sam Harrington**

(Due Thursday, February 13)

Proposition 1.22(i): For all $m \in \mathbb{Z}$, $-(-m) = m$. COMPLETED

Proof. Let m be an integer. Then,

$$\begin{aligned}
(-m) + m &= 0 && \text{(Prop. 1.8)} \\
(-m) + (-(-m)) &= 0 && \text{(Axiom. 1.4)} \\
-(-m) &= m && \text{(Prop. 1.10)}
\end{aligned}$$

□

Proposition 1.25(i): For all $m, n \in \mathbb{Z}$, $-(m+n) = (-m) + (-n)$.

Proof. Let m, n be integers. Then,

$$\begin{aligned}
(m+n) + (-(m+n)) &= 0 && \text{(Axiom. 1.4)} \\
(m+n) + ((-m) + (-n)) &= m + ((n + (-m)) + (-n)) && \text{(Axiom. 1.1(ii)) twice} \\
&= m + (((-m) + n) + (-n)) && \text{(Axiom. 1.1(i))} \\
&= (m + (-m)) + (n + (-n)) && \text{(Axiom. 1.1(ii)) twice} \\
&= 0 && \text{(Axiom. 1.4) twice} \\
-(m+n) &= (-m) + (-n) && \text{(Prop. 1.10)}
\end{aligned}$$

□

Proposition 1.27(iii): For all $m, n, p, q \in \mathbb{Z}$, $(m-n)(p-q) = (mp + nq) - (mq + np)$.

Proof. Let m, n, p, q be integers. Then,

$$\begin{aligned}
(m-n)(p-q) &= (m+(-n))(p+(-q)) && \text{definition of subtraction}\\
&= (mp+(-n)p)+(m(-q)+(-n)(-q)) && \text{(Prop 1.11(i))}\\
&= (mp+(-n)p)+(m(-q)+nq) && \text{(Prop 1.20)}\\
&= mp+(((-n)p+m(-q))+nq) && \text{(Axiom 1.1(ii)) twice}\\
&= mp+(nq+(m(-q)+(-n)p)) && \text{(Axiom 1.1(i)) twice}\\
&= (mp+nq)+(m(-q)+(-n)p) && \text{(Axiom 1.1(ii))}\\
&= (mp+nq)+((-(mq))+(-(np))) && \text{(Prop 1.25(iii)) twice}\\
&= (mp+nq)+(-(mq+np)) && \text{(Prop 1.25(i))}\\
&= (mp+nq)-(mq+np) && \text{definition of subtraction}
\end{aligned}$$

$\square$