Math 214 – Foundations of Mathematics **Homework 4/8** **Your name**

(Due Tuesday, April 22)

Proposition 10.16: If the sequence (x_k) converges to L, then $\lim_{k\to\infty} x_{k+1} = L$.

Proof. let the sequence (x_k) converge to L, Then,
By def of convergence: for each $\varepsilon > 0$ there exists $N \in \mathbb{N}$ such that for each $n \geqslant \mathbb{N}, |x_n - L| < \varepsilon$.
Using the same N as above: $n - 1 \geqslant N - 1$. This gives us $|x_{(n-1)+1} - L| = |x_n - L| < \varepsilon$.
Which we know to be true as given by the def of convergence. so $\lim_{k\to\infty} x_{k+1} = L$. by the def of limit

□

Proposition 10.21(ii): Let $\lim_{k\to\infty} = L$. If $(x_k)_{k=0}^{\infty}$ is decreasing, then $x_k \geqslant L$ for all $k \geqslant 0$.

Proof. Let $\lim_{k\to\infty} = L$. and $(x_k)_{k=0}^{\infty}$ is decreasing. Then,
For the purpose of contradiction, assume that there exists some $x_i < L$.
By the def of a limit: x_k converges to L and the def of convergence: for each $\varepsilon > 0$ there exists $N \in \mathbb{N}$ such that for each $n \geqslant \mathbb{N}, |x_n - L| < \varepsilon$.
Take x_i, By LEMMA 10.37/2, for every $m \geqslant i$, $x_i \geqslant x_m$. This means that once x is less than L it will always be less than L. That is for any N we choose there will be a $n \geqslant N$ such that $x_n < L$ If we then set $N \geqslant i$

□

Proposition 10.27: Given any $r \in \mathbb{R}_{>0}$, the number $\sqrt{r}$ is unique in the sense that, if $x \in \mathbb{R}_{>0}$ and $x^2 = r$, then $x = \sqrt{r}$.