Math 214 – Foundations of Mathematics **Homework 2/4** **Sam Harrington**

(Due Tuesday, February 18)

Proposition 2.7(iv): Let $m, n, p \in \mathbb{Z}$. If $m < n$ and $p < 0$, then $np < mp$.

Proof. Let m, n, p be integers. And suppose $m < n$ and $p < 0$ Then,

$$\begin{aligned} n - m &\in \mathbb{N} && \text{definition of } < \\ 0 - p &\in \mathbb{N} && \text{definition of } < \\ (0 - p)(n - m) &\in \mathbb{N} && \text{(Axiom. 2.1(ii))} \\ (mp - np) &\in \mathbb{N} && \text{Chapter 1} \\ np &< mp && \text{definition of } < \end{aligned}$$

□

COMPLETED Proposition 2.8: Let $m, n \in \mathbb{Z}$. Exactly one of the following is true: $m < n$, $m = n$, $m > n$.

Proof. Let m, n be integers. Then,
Prop 2.2 states that one and only one of the following is true:
$x \in \mathbb{N}$, $-x \in \mathbb{N}$, or $x = 0$
if we set $x = m - n$ then we get the following three cases:
case(1):

$$m - n \in \mathbb{N} = m > n \qquad \text{definition of } <$$

case (2):

$$\begin{aligned} -(m - n) \in \mathbb{N} = n - m &\in \mathbb{N} && \text{Chapter 1} \\ = m &< n && \text{definition of } < \end{aligned}$$

case(3):

$$\begin{aligned} m - n &= 0 \\ m &= n && \text{Chapter 1} \end{aligned}$$

Proving that exactly one of the following is true: $m < n$, $m = n$, $m > n$. □

Proposition 2.12(ii): Let $m, n, p \in \mathbb{Z}$. If $p > 0$ and $mp < np$, then $m < n$.

Proof. Let m, n, p be integers. And suppose $p > 0$ and $mp < np$ Then,

$$\begin{aligned} np - mp &\in \mathbb{N} && \text{definition of } < \\ p - 0 = p &\in \mathbb{N} && \text{definition of } < \\ (p)(n - m) &\in \mathbb{N} && \text{Chapter 1} \\ (n - m) &\in \mathbb{N} && \text{(Prop 2.11)} \\ m &< n && \text{definition of } < \end{aligned}$$

$\square$