Math 214 – Foundations of Mathematics   Homework 3/20   Sam Harrington

(Due Thursday, April 3)

Proposition 7.3: For all $n \in \mathbb{N}$, $\mathbf{v}(n) = k$ if and only if $10^{k-1} \leqslant n < 10^k$.

Proof. Let $n \in N$
We begin by assuming $\mathbf{v}(n) = k$, by def of $\mathbf{v}(n)$ k is the min of the set $\{t \in \mathbb{N} : n < 10^t\}$
Because k is in the set, $n < 10^k$, Also b/c k is the min, $k \leqslant t$ for all t.
For the purpose of contradiction, assume $10^{k-1} > n$.
This implies $k - 1$ is in the set above; however we stated k is the minium of the set a contradiction leading to $10^{k-1} \leqslant n < 10^k$

Now assume $10^{k-1} \leqslant n < 10^k$.
Because $n < 10^k$, this implies k is in the set defined above.
For the purpose of contradiction, let $x = \mathbf{v}(n)$ in which $k \neq x$, that is $x \leqslant t$ for all t.
Leading to $k > x$. and $x \leqslant k - 1$:
$10^{k-1} \leqslant n < 10^x \leqslant 10^{k-1}$: $10^{k-1} < 10^{k-1}$ a contradiction. $\mathbf{v}(n) = k$.
Proving $\mathbf{v}(n) = k$ if and only if $10^{k-1} \leqslant n < 10^k$ □

Proposition 7.12(vi). Let $n = \sum_{i=0}^{\mathbf{v}(n)-1} x_i 10^i$, where each x_i is a digit. Prove

$$n \equiv x_0 + x_1 + \cdots + x_{\mathbf{v}(n)-1} \pmod 9.$$

Proof. Let $n = \sum_{i=0}^{\mathbf{v}(n)-1} x_i 10^i$, where each x_i is a digit.

$$n = \sum_{i=0}^{\mathbf{v}(n)-1} x_i 10^i$$

$$n \equiv \sum_{i=0}^{\mathbf{v}(n)-1} x_i 10^i \pmod 9$$

$$1 \equiv 10 \pmod 9$$

$$1 \equiv 10^i \pmod 9$$

$$n \equiv \sum_{i=0}^{\mathbf{v}(n)-1} x_i (1) \pmod 9$$

$$n \equiv x_0 + x_1 + \cdots + x_{\mathbf{v}(n)-1} \pmod 9.$$

□