Math 214 – Foundations of Mathematics Homework 2/25 Sam Harrington

(Due Tuesday, March 11)

Proposition 4.30: Let $(f_j)_{j=1}^{\infty}$ denote the sequence of Fibonacci numbers. For all $k, m \in \mathbb{N}$, $m \geqslant 2$, $f_{m+k} = f_{m-1}f_k + f_m f_{k+1}$.
NOTE: Do *not* use Proposition 4.29 to prove Proposition 4.30!

Proof. Let $k, m \in \mathbb{N}$, $m \geqslant 2$, Then by Strong Induction on m,
Before we begin lets define the first three elements in f: $f_1 = 1, f_2 = 1, f_3 = f_1 + f_2 = 2$ all by definition of f
Base cases: $m = 2$

$$\begin{aligned} f_{2+k} &= f_{k+2-1} + f_{k+2-2} && \text{def of fib} \\ &= f_{k+1} + f_k \\ &= 1(f_{k+1}) + 1(f_k) \\ &= f_2(f_{k+1}) + f_1(f_k) && \text{def of fib} \\ &= f_{m-1}f_k + f_m f_{k+1} \end{aligned}$$

$m = 3$

$$\begin{aligned} f_{3+k} &= f_{k+3-1} + f_{k+3-2} && \text{def of fib} \\ &= f_{k+2} + f_{k+1} \\ &= f_{k+1} + f_k + f_{k+1} && \text{def of fib} \\ &= 2(f_{k+1}) + 1(f_k) \\ &= f_3(f_{k+1}) + f_2(f_k) && \text{def of fib} \\ &= f_{m-1}f_k + f_m f_{k+1} \end{aligned}$$

We can now assume the $f_{m+k} = f_{m-1}f_k + f_m f_{k+1}$ is true for j such that $2 \leqslant j \leqslant n$

$$\begin{aligned} f_{n+1+k} &= f_{k+n} + f_{k+n-1} && \text{def of fib} \\ &= f_{n-1}f_k + f_n f_{k+1} + f_{n-2}f_k + f_{n-1}f_{k+1} && \text{inductive assumption} \\ &= f_{k+1}(f_n + f_{n-1}) + f_k(f_{n-1} + f_{n-2}) \\ &= f_{n+1}f_{k+1} + f_n f_k && \text{def of fib} \\ &= f_{(n+1)}f_{k+1} + f_{(n+1)-1}f_k \end{aligned}$$

By strong induction, we prove For all $k, m \in \mathbb{N}$, $m \geqslant 2$, $f_{m+k} = f_{m-1}f_k + f_m f_{k+1}$ $\square$

Proposition 5.7: The empty set is a subset of every set. That is, for every set S, $\varnothing \subseteq S$.

Proof. Let S be any set,
We begin by contradiction:
For purpose of contradiction let $\varnothing \not\subseteq S$

By def of subset, there must be some $x \in \varnothing$ in which $x \notin S$

Since by def of $\varnothing$: $x \notin \varnothing$.

We have reached contradiction, implying $\varnothing \subseteq S$

$\square$