Statistical Inference Part 1

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Part I: Simulation

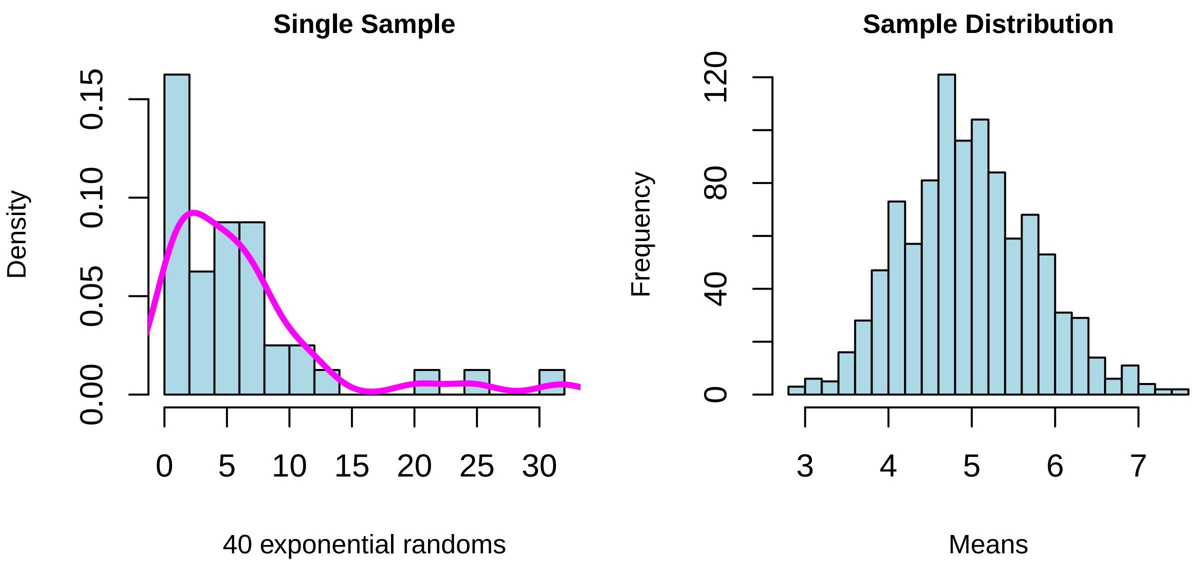
Summary

The simulation and discussion below illustrate the Law of Large Numbers (LLN) and the Central Limit Theorem (CLT) in action. The LLN states that as n increases, statistical estimates approach the true population's value . The CLT states that as n increases, the averages of variables become normally distributed. As the basis for statistical inference, this means that even for distributions that are not normal, as n increases, the sample means will be normally distributed. This is demonstrated below by simulating an exponential distribution 1000 times.

Simulation

First, let's generate and visualize the data.

```
set.seed(301)
lambda<-0.2; # given parameters
n<-40
Ksims<-NULL; # using code from the example provided
for (i in 1 : 1000) Ksims = c(Ksims, mean(rexp(n, lambda)))
par(mfrow=c(1,2), mar=c(4,4,2,1))
singlesample<-rexp(n, lambda)
hist(singlesample, prob=TRUE, breaks=20, main="Single Sample",
     cex.main=.8, col="lightblue", xlab="40 exponential randoms", cex.lab=.8)
lines(density(singlesample), col="magenta", lwd=3, lty=1)
hist(Ksims, col="lightblue", breaks=20, main="Sample Distribution",
     cex.main=.8, xlab="Means", cex.lab=.8)
```



In the first graph, I show a single sample to illustrate the skew of an exponential distribution. The point is that the data are not normally distributed. The second graph shows the distribution of the means of 1000 simulated exponential samples, and the data appear normally distributed.

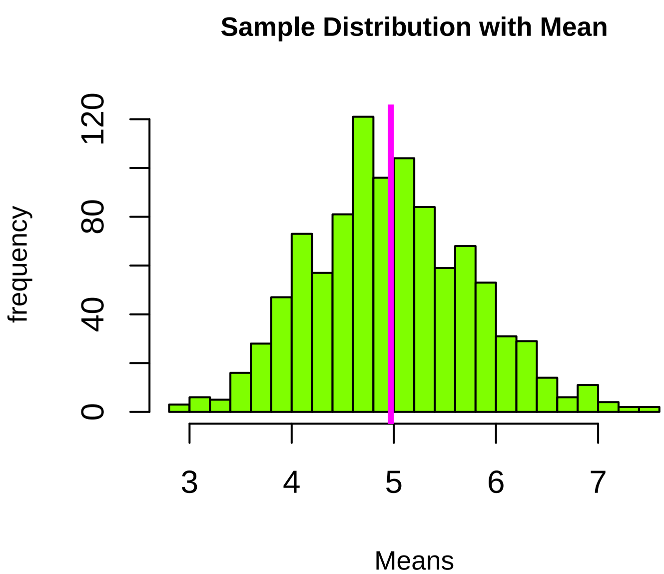
Question 1. Show the sample mean and compare it to the theoretical mean of the distribution.

The theoretical sample mean for an exponential distribution is 1/lambda. Lambda was given at 0.2, so the theoretical sample mean is $1/.2 = 5$. Summarizing the data from our simulation, which is the distribution of the means of 1000 exponentials shows both the range and average of those means. The central limit theorem suggests that when n is large enough, the mean of the sample mean should approach the true mean.

```
summary(Ksims)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##   2.858   4.444   4.922   4.970   5.477   7.557
```

```
par(mfrow=c(1,1))
hist(Ksims, col="chartreuse", breaks=20, main="Sample Distribution with Mean",
     cex.main=.8, xlab="Means", ylab="frequency", cex.lab=.8)
abline(v=mean(Ksims), lwd=3, col="magenta")
```



Indeed, the observed sample mean is approximately the theoretical mean; or, we may say that 'the estimator is consistent.' In the figure below, the pink verticle line represents the sample mean; we see that it is drawn approximately where x = 5.

Question 2. Show how variable the sample is (via variance) and compare it to the theoretical variance of the distribution.

The LLN works for other statistics too. Given that the CLT suggests that the sample mean is normally distributed, and given that we found that the sample mean is a consistent estimate, we should expect that the variance of the sample mean is consistent as well. We can compare the theoretical variance to our sample variance

The theoretical variance of a random exponential sample is (1/lambda)^2 = 25. The theoretical variance of the sample mean (standard error squared) is:

```r
((1/lambda)/sqrt(n))^2
```

```
## [1] 0.625
```

Which we can compare to the variance of our simulation:

```r
var(Ksims)
```

```
## [1] 0.6205073
```

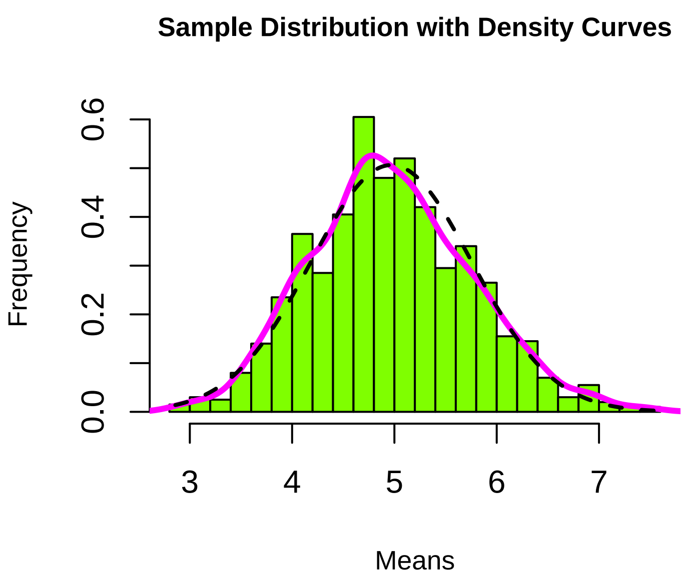
We see that these are *vary* similar. Like with the theoretical and observed mean, the central limit theorem suggests that when n is large enough the observed variance should approach the true variance.

Question 3. Show that the distribution is approximately normal

Finally, the central limit theorem states that as n increases, the sample mean is normally distributed. Below, I compare the density curve of our simulated means to a theoretical normal distribution (see http://www.statmethods.net/graphs/density.html).

```r
hist(Ksims, prob=TRUE, col="chartreuse", breaks=20,
     main="Sample Distribution with Density Curves", cex.main=.8,
     xlab="Means", ylab="Frequency", cex.lab=.8)
lines(density(Ksims), col="magenta", lwd=3, lty=1)

xfit<-seq(min(Ksims),max(Ksims),length=n^2)
yfit<-dnorm(xfit, mean=mean(Ksims),sd=sd(Ksims))
lines(xfit, yfit, col="black", lty=2, lwd=2)
```



Our sample distribution appears approximately normal.

Conclusion

The LLN and CLT provide the basis for statistical inference. When n is large enough, we can use our statistical estimates to make informative comparisons. Because we can assume normality of sample means, we can use the properties of the normal distribution to examine the probability that two samples are meaningfully different. Importantly, this is true for distributions that are non-normal in the first place.