PHY407H1 Lab7

Eric Yeung*
Department of Physical Sciences, University of Toronto, Toronto M1C 1A4, Canada
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QUESTION 1

a) b) See Lab7_q1ab.py by Me. The two questions were combined for convenience.
For the case (l = 0, n = 1):

```
The ground state E_1 is -13.4999966366 eV for l = 0
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For the case (l = 0, n = 2):

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The first excited state E_2 is -3.38780488905 eV for l = 0
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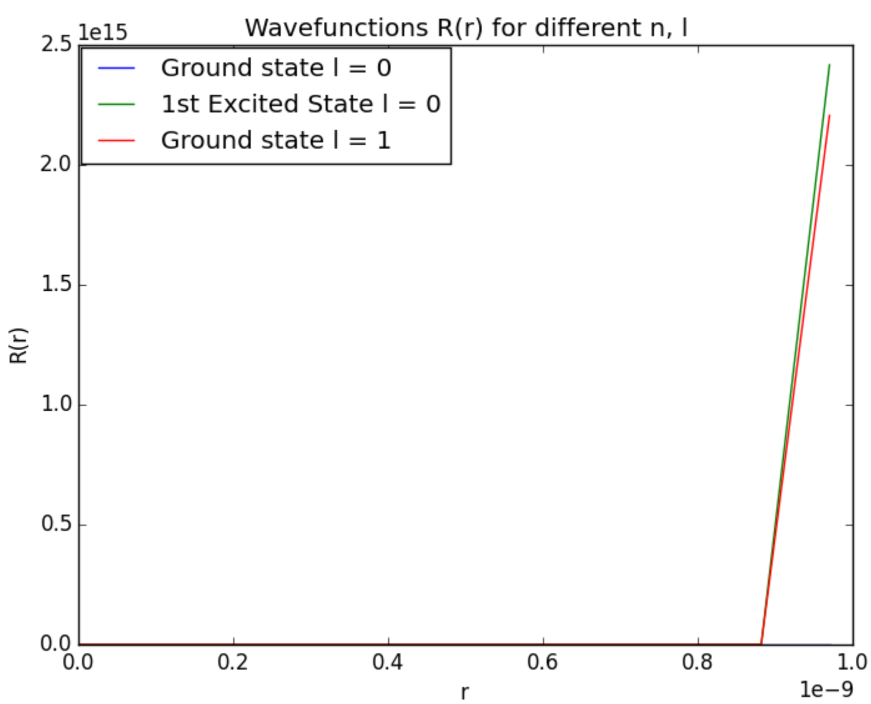
For the case (l = 1, n = 1):

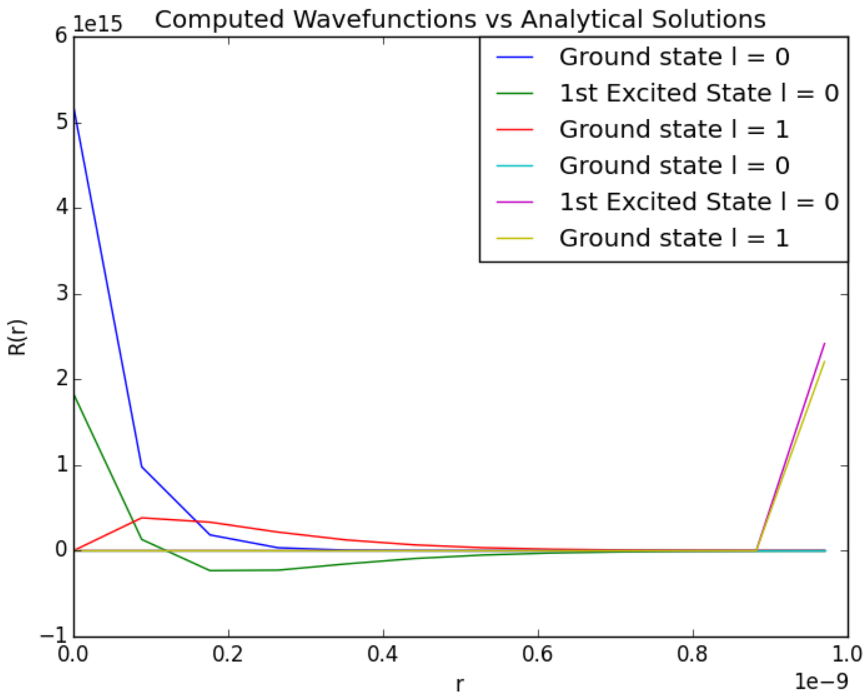
```
The ground state E_1 is -3.40127691697 eV for l = 1
```

As one can see, the ground energy at l = 1 is almost the same as the first excited state for l = 0. As the parameters are adjusted, the values mentioned earlier become closer to one another.
c) See Lab7_q1c.py by Me.
This question was tricky, there was overflow.



d) See Lab7_q1d.py by Me.
Compared and plotted our results with analytical ones at http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/hydwf.html
Was not even close.

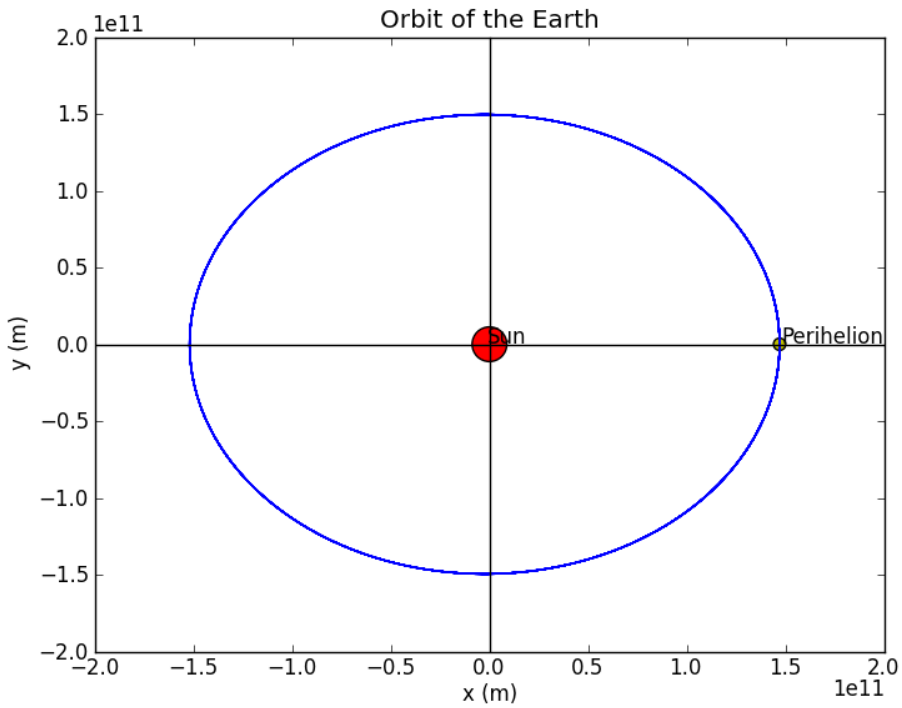
* eric.yeung@mail.utoronto.ca



Computed Wavefunctions vs Analytical Solutions
1e15
6
5
4
3
2
1
0
−1
R(r)
Ground state l = 0
1st Excited State l = 0
Ground state l = 1
Ground state l = 0
1st Excited State l = 0
Ground state l = 1
0.0
0.2
0.4
0.6
0.8
1.0
r
1e−9

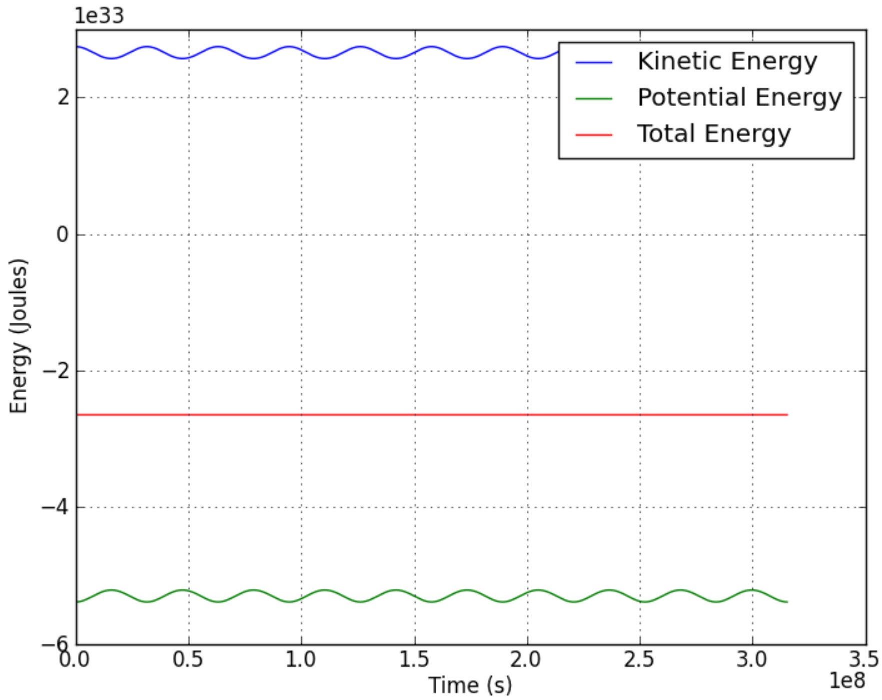
QUESTION 2

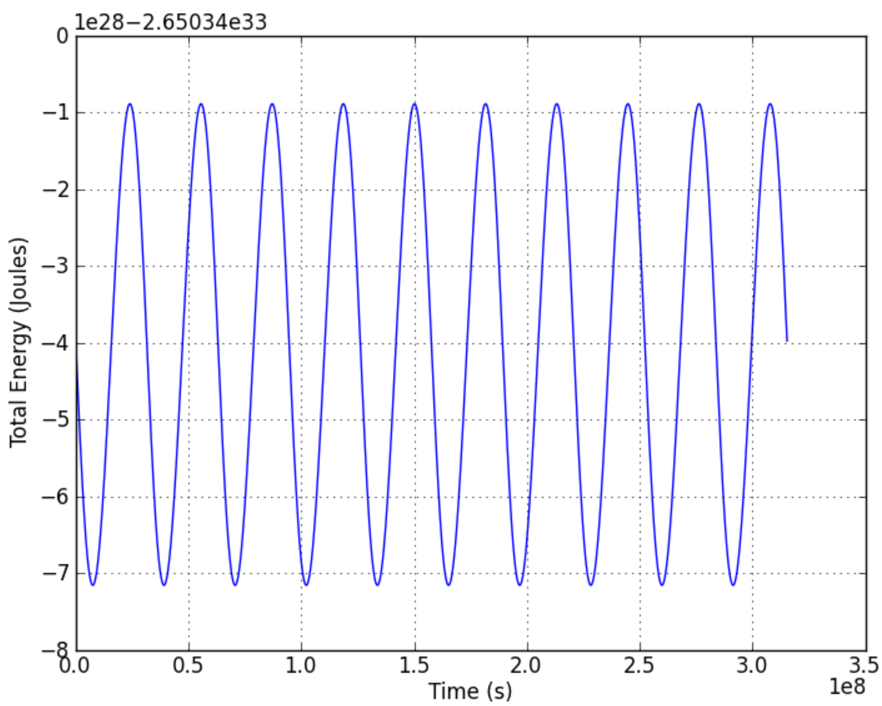
a) See Lab7_q2a.py by Chi. Orbit was solved using the Verlet method. Which can be seen below.



The kinetic and potential energy are compared with each other. They were both oscillating while the total energy is constant. The plot is shown below.

The the total energy was plotted using a very small scale to see the oscillating behaviour.

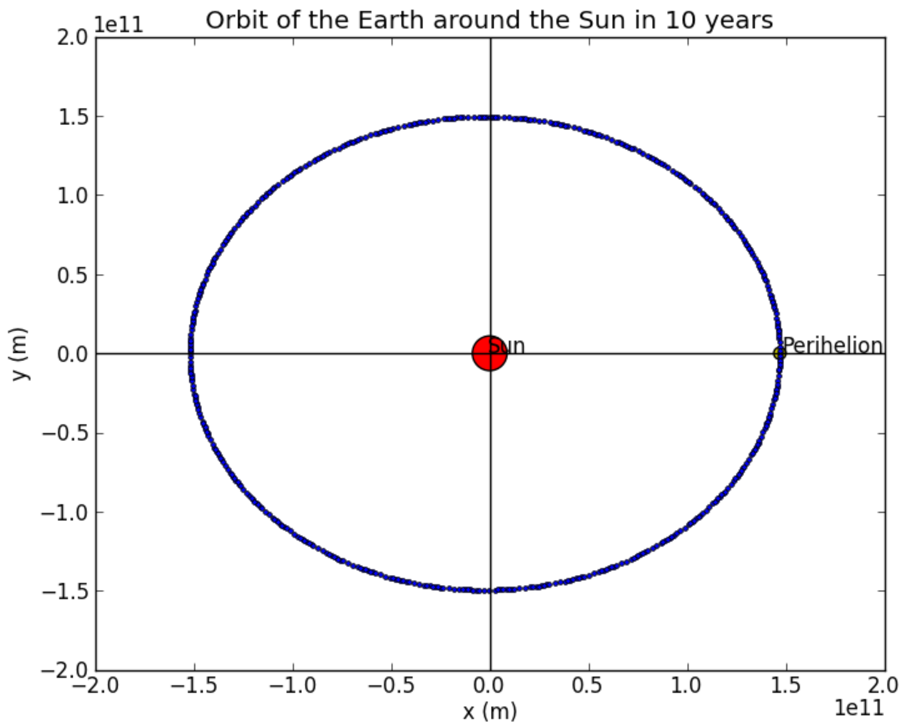

1e33
Kinetic Energy
Potential Energy
Total Energy
Energy (Joules)
Time (s)
1e8

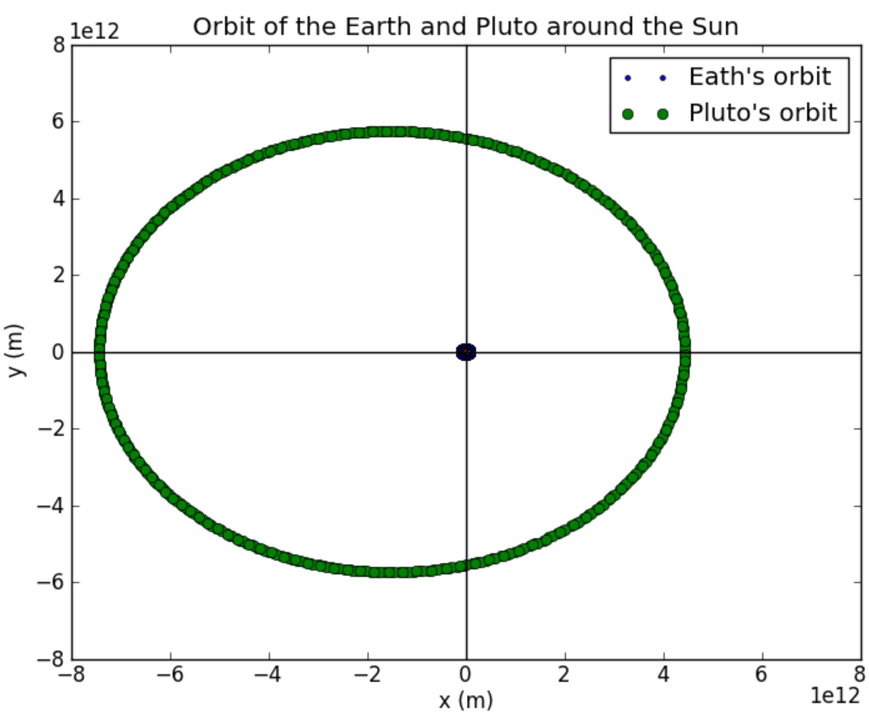

1e28−2.65034e33
Total Energy (Joules)
Time (s)
1e8

b) See Lab7_q2b.py by Chi.

This question used the Bulirsch-Stoer method to solve the exact same problem as the last question.

But this time, not only the Earth's orbit was solved and plotted; the orbit of Pluto was also solved. The two graphs are below.





QUESTION 3

a) Majority of the work in a) is from Chi.
Starting from:

$$\begin{aligned} y_{k+1} &= y_k + h_k f(y_{k+1}) \\ f(y_{k+1}) &= \lambda y_{k+1}, \forall \lambda (\text{ Not necessarily real}) \\ y_{k+1} &= y_k + h_k \lambda y_{k+1} \\ y_{k+1} - h_k \lambda y_{k+1} &= y_k \\ \therefore y_{k+1} &= \frac{y_k}{1 - h_k \lambda} \end{aligned}$$

Our condition for stability of the method is

$$\frac{1}{|1 - h_k \lambda|} \leq 1$$
$$|1 - h_k \lambda| \geq 1$$

So for Re(λ) ¡ 0, the backwards Euler is unconditionally stable. And for Re(λ) ¿ 0, the method is conditionally stable. For this case, we also require a minimum step size h_k as our condition.

b) Taken from Wikipedia:

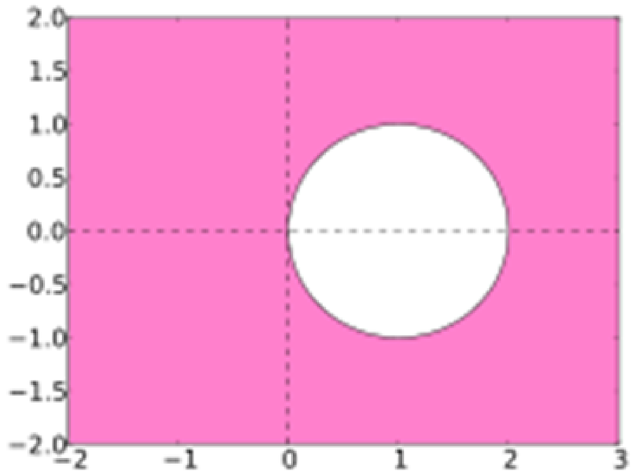


FIG. 1: Region of Stability for Backwards Euler

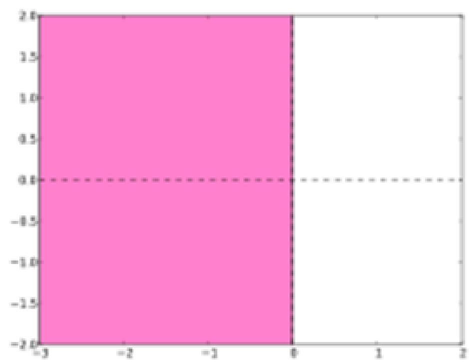


FIG. 2: Region of Stability for Trapezoid

Where the pink regions denote stability.