PHY407H1 Lab9

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QUESTION 1

a) See Assignment 9_Analysis.pdf by me.
b) See Lab9_q1b.py by me.

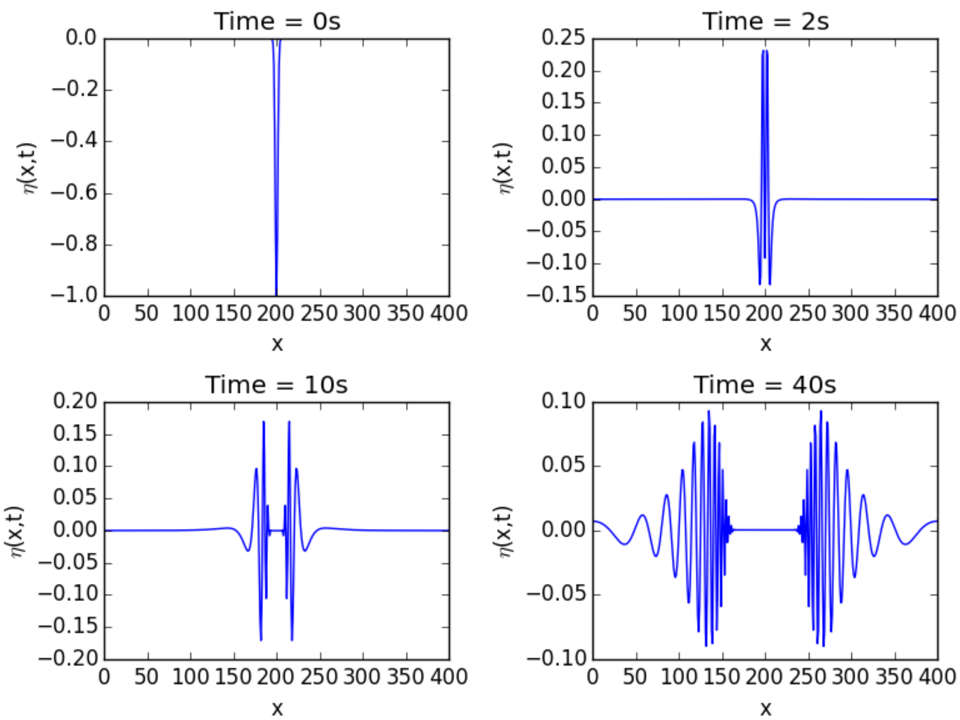

FIG. 1: Plot for η(x,t)

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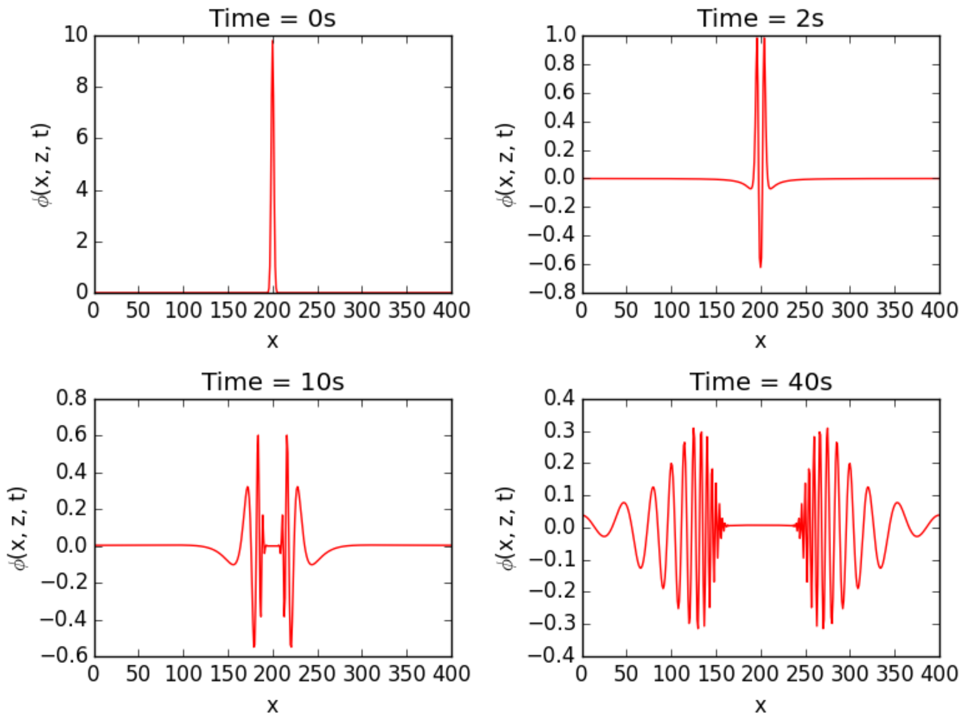


FIG. 2: Plot for ϕ(x,z,t)

The spectral method gives me around the same answers for η(x,t). Regarding speed, this simulation (spectral method) took 0.698999881744 seconds!

The ϕ(x, z, t) looks like η reflected along the x axis. The solutions that Paul gave out showed the contour plots of ϕ. My code accounted only for ϕ when z = 0. I didn't have time to troubleshoot/debug the phi solution, but the solution at $\phi(z = 0)$ seem to be what I would expect.

In comparison with the graphs I got in Lab8, shown below:

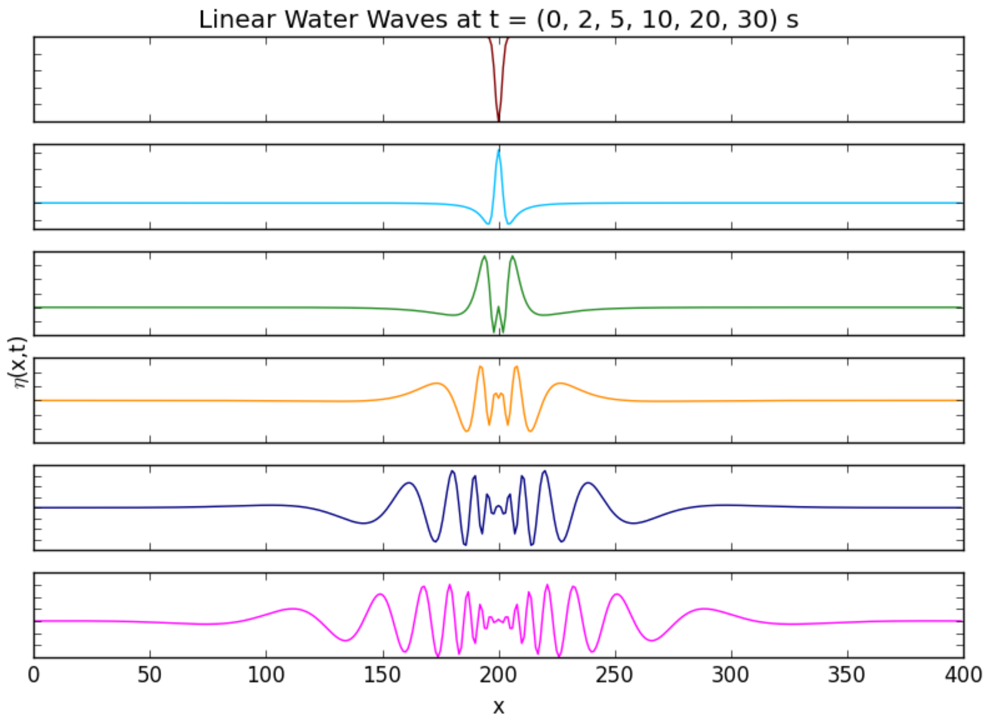


FIG. 3: The figure from the last lab

QUESTION 2

a) b) c)See Lab9_q2abc.py by Chi.

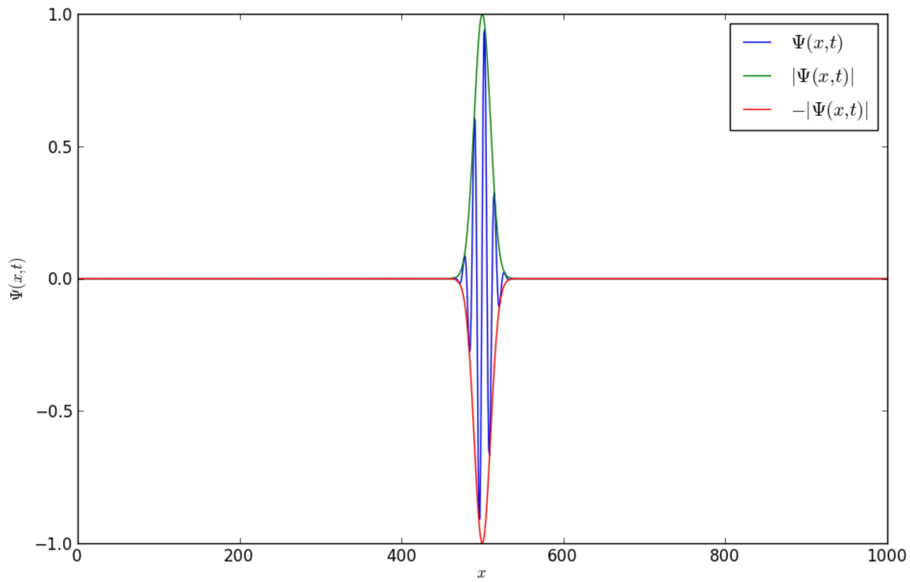


FIG. 4: Wave function from Crank-Nicolson Method

The animation is shown in a vpython window. The time dependent wave function is simply evolving in time from starting in a stationary state. We can see the solution is oscillating between two points (the ends of the potential wall).

d)See Lab9_q2d.py by Chi.

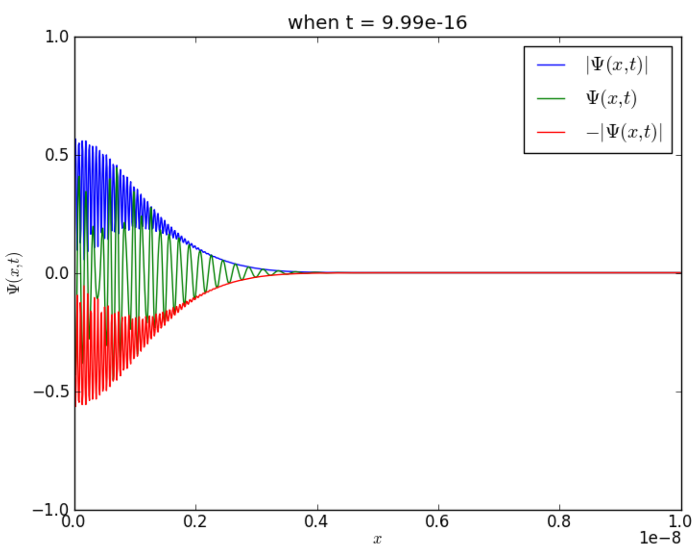


FIG. 5: x0 = 0.4L

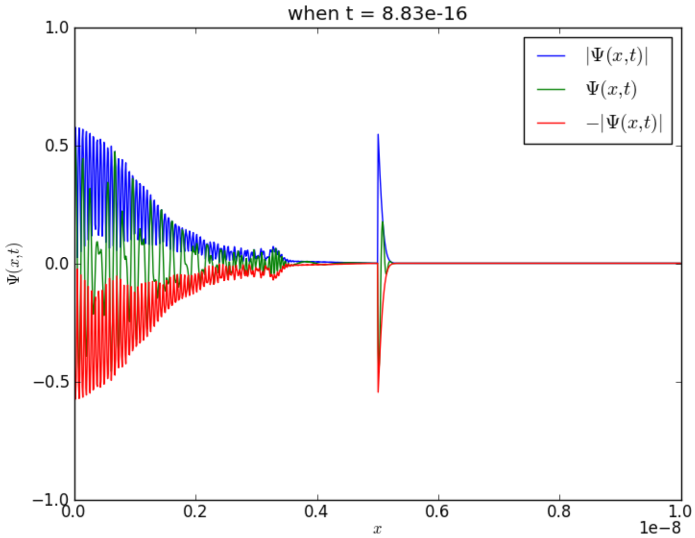
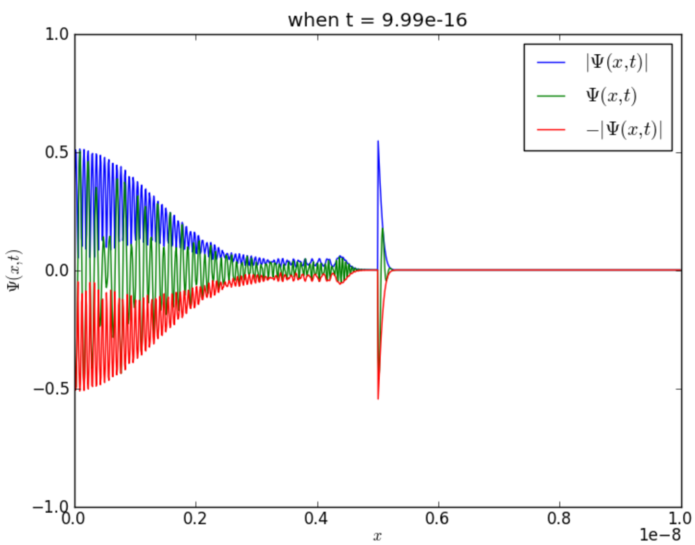


FIG. 6: x0 = 0.49L



With the potential jump, I can see that wave function spikes at around L/2, which is where our potential jump is.

$$\frac{\partial \eta}{\partial t} = \sum_{k=-\infty}^{\infty} e^{ikx} \frac{d\tilde{\eta}_k}{dt} \quad (3)$$

$$\frac{d^2 \hat{\phi}_k(z)}{dz^2} - k^2 \hat{\phi}_k(z) = 0 \quad , \quad \phi_k(z) \sim e^{kz} \quad (1)$$

$$\frac{\partial \phi(z=0)}{\partial t} = -g\eta_k \quad (2)$$

Initial Condition $\eta(x, t=0) = \eta_0(x) = \sum_{k=-\infty}^{\infty} e^{ikx} \tilde{\eta}_k(t=0)$

$$\eta(x,t) = \sum_{k=-\infty}^{\infty} e^{ikx} \hat{\eta}_k(t) \quad (4)$$

$$\phi(x,z,t) = \sum_{k=-\infty}^{\infty} e^{ikx} \hat{\phi}_k(z) \hat{\phi}_k(t) \quad (5)$$

Starting with $\eta(x,t)$,

$\frac{\partial \eta}{\partial t} = \frac{\partial \dot{\phi}}{\partial t}$, It's a wave, Known solution.

$\Rightarrow \eta(x,t) = C_1 \cos(\omega_k t) + C_2 \sin(\omega_k t)$, at rest.

$\frac{\partial \eta}{\partial t}(t=0) = 0 = -C_1 \cancel{\sin(0)}^{\,0} + C_2 \cos(0)$

$\Rightarrow C_2 = 0.$

$\eta(x,t) = C_1 \cos(\omega_k t)$, use another I.C.

$\eta(x,0) = \eta_0(x) = C_1 \cancel{\cos(0)}^{\,1}$

$\rightarrow C_1 = \eta_0(x)$, $\eta(x,t) = \eta_0(x)\cos(\omega_k t)$

$\therefore$ From ④, $\eta_k(x,t) = \sum_{-\infty}^{\infty} e^{ikx} \eta_0(x) \cos(\omega_k t)$

as expected.

Now for ϕ. Already know $\phi_k(z) \sim e^{kz}$

From ②, $\phi(z=0) = -g \int \eta_k \, dt$,

$$\phi_k(t) = -g \int \eta_0(x) \cos(\omega_k t) \, dt \text{, pull } \eta_0(x) \text{ out.}$$

$$= -g \eta_0(x) \left[\frac{1}{\omega_k} \sin(\omega_k t) \right]$$

Therefore, subbing $\phi_k(z) = e^{kz}$ and

$$\phi_k(t) = -\frac{g\eta_0(x)}{\omega_k} \sin(\omega_k t) \text{ into ⑤,}$$

we get $\phi(x,z,t) = -\sum_{-\infty}^{\infty} e^{kz} e^{ikx} \frac{g\eta_0(x)}{\omega_k} \sin(\omega_k t)$

as expected.

When $k=0 \Rightarrow \omega_k = 0$, so the cosines and sines become 1's and 0's which does not depend on time.