QUESTION 1

November 17, 2015 1:31 AM

(a) $[p(\theta)\,d\theta][p(\phi)\,d\phi] = \frac{\sin\theta}{2}\,d\theta \cdot \frac{1}{2\pi}\,d\phi$

Ranges of θ & ϕ: $\boxed{\begin{array}{c} 0 \leq \phi \leq 2\pi \\ 0 \leq \theta \leq \pi \end{array}}$

Normalisation:

$d\phi = \frac{1}{2\pi} \cdot \quad d\phi = \frac{1}{2\pi} \cdot 2\pi = 1$ ✓

$\sin\theta\,d\theta = -\frac{1}{2}\cos\theta \quad = -\frac{1}{2}\left(\underbrace{\cos(\pi)}_{-1} - \underbrace{\cos(0)}_{1}\right) = \left(\frac{-1}{2}\right)(-2) = 1$ ✓

(b) (*) $\int_0^{\phi(u)} p(\phi')\,d\phi' = \int_0^u du' = u$

$\int_0^{\phi(u)} \frac{1}{2\pi}\,d\phi' = u$

$\Rightarrow \boxed{\phi(u) = 2\pi u}$

(*) $\int_0^{\theta(v)} p(\theta')\,d\theta' = \int_0^v dv' = v$

$\frac{1}{2}\int_0^{\theta(v)} \sin(\theta')\,d\theta' = v$

$-\frac{1}{2}\cos(\theta(v)) = v + \text{constant}$

$\theta(v) = \cos^{-1}(-2v + \text{constant})$

Choose constant $= 1$

$\Rightarrow \boxed{\theta(v) = \cos^{-1}(1 - 2v)}$

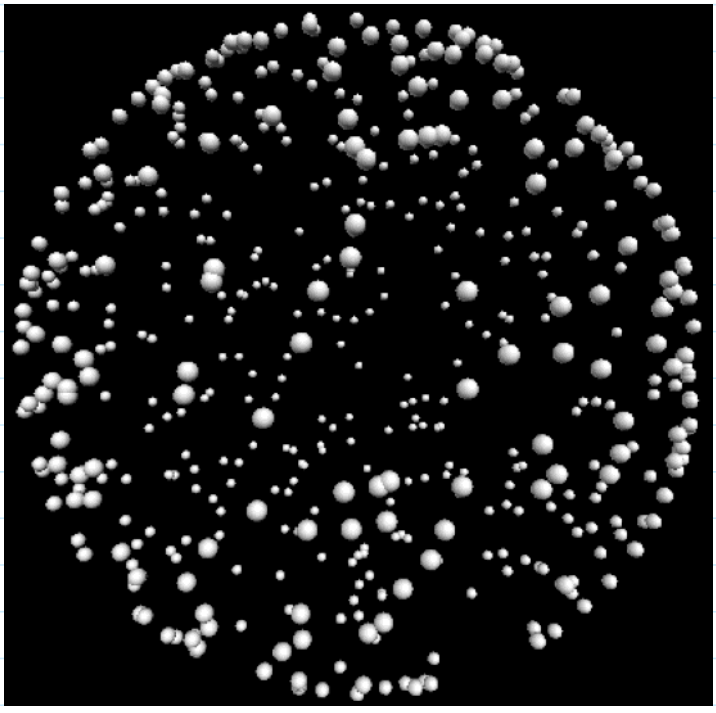
So:

$\theta = \cos^{-1}(1-2v)$

$\phi = 2\pi u$

where u and v are random

c + d



A screen shot showing the result of my Python program.