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ORIGINAL PAPER

Numerical Simulation of Radiative MHD Sutterby Nanofluid Flow Through Porous Medium in the Presence of Hall Currents and Electroosmosis

K. Ramesh[1] · Madhav Rawal[2] · Aryaman Patel[2]

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Abstract

Analysis of thermal and fluid phenomena based on the fluid dynamics theory leads to understanding of fundamental mechanisms in modern technologies. Thermal/fluid transport is critical to many applications, such as photothermal cancer therapy, solar thermal evaporation and polymer composites. The current study focusses to investigate the effect of magnetohydrodynamics, Hall currents and electroosmosis on the propulsion of Sutterby nanofluids in a porous microchannel. The Brownian motion and thermophoresis effects have also been considered. The governing equations for the momentum, temperature and nanoparticle volume fraction have been modified under the suitable non-dimensional quantities. The resulting dimensionless system of equations have been solved using bvp4c package in computational software MATLAB. The pictorial representations have been presented for various flow quantities with respect to sundry fluid parameters. It is noted from the investigation that, there is a decrease in fluid velocity with an increase in Hartmann number, temperature decreases with the increment in radiation parameter and nanoparticle volume fraction reduces with the increment of Prandtl number and thermophoresis parameter. The results obtained for the Sutterby nanofluid propulsion model reveal many engrossing behaviors and has many applications such as disease diagnostics and cancerous tissues destruction, and that provide a further dimension to investigate the nanofluid flow problems with thermophysical properties in two/three dimensions.

Keyword Sutterby nanofluid · Thermofluiddynamics · Magnetofluiddynamics · Electroosmosis · Hall currents

✉ K. Ramesh
ramesh.katta1@gmail.com

[1] Department of Mathematics, Symbiosis Institute of Technology, Symbiosis International (Deemed University), Pune 412115, India

[2] Department of Mechanical Engineering, Symbiosis Institute of Technology, Symbiosis International (Deemed University), Pune 412115, India

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Introduction

In the 1990s, Rohrer declared that nanoscience and nanotechnology have entered the limelight and showcased the chances and challenges of the "nano-age". In 1993, Choi worked on fluids that could transfer heat efficiently without expensive cryogenic systems or the requirement of a high pumping power. He used the available nanotechnology to synthesize stable suspensions of nano particles in base liquids which are normally heat transfer fluids. In 1995, Choi presented the paper on basic concepts of nanofluids in the American Society of Mechanical Engineers [1]. In 2000, Xuan et al. [2] have proved that the thermal conductivity of the liquid may be increased by 20 percent by adding 1–5% concentration of nanomaterial by volume in the base liquid. The control over the thermal properties of the nanofluids opens up avenues for diverse applications. The nanofluids with low viscosities and high photo-thermal consequences are excellent choices for direct solar absorption collectors [3], they are also highly desirable for engine cooling, lubrications and thermal storage, apart from thermal engineering aspect they are also used for medicine, magnetic sealing, microbial fuel and many other applications. In view of this, Akbar and Nadeem [4] have presented the peristaltic propulsion of Sutterby nanofluid in a tube. Akbar [5] used the shooting method to analyze the peristaltic propulsion of Sutterby nanoliquid in a channel with double diffusive natural convection. Hayat et al. [6] have provided the numerical solutions (using shooting method) to study a peristaltically induced motion of Sutterby nanoliquid in a channel under the uniform magnetic field and the impact of complaint walls on the flow. Sheikholeslami et al. [7] have discussed the MHD nanoliquid motion in the rotating plates with the help of fourth order Runge-Kutta method (FORKM). Sheikholeslami et al. [8] have presented the control volume based finite element method (CVFEM) solutions to study the magnetohydrodynamic flow of nanoliquid in an annulus. Reddy and Makinde [9] have used the Runge-Kutta-Fehlberg integration scheme to discus the propulsion of Jeffrey fluid under the simultaneous effects of thermophoresis, Brownian motion, magnetic field and buoyancy forces through an asymmetrical channel. Hayat et al. [10] have analyzed the MHD boundary layer propulsion of nanofluids with slip condition using the bvp4c package. Pattnaik et al. [11] have discussed the unsteady MHD nanofluid flow past a channel with the Koo-Kleinstreuer-Li model. Majeed et al. [12] have provided a theoretical analysis on the flow of magnetic nanofluid (kerosene, refrigerant-134a, and water-based nanfluid containing magnetite as nanoparticles) under the impact of dipole using shooting method. Zhang et al. [13] have studied the entropy analysis of the Jeffrey nanofluid motion (blood-based nanofluid containing magnetic Zinc-oxide nanoparticles) through the tapered arteries. Some more relevant works can be seen through the references [14,15] and the references therein.

Electroosmosis is the fluid movement in the application of electric field. This phenomenon occurs when a solid surface, after coming in contact with an electrolyte, acquires an electrostatic charge based on its zeta potential. This causes the re-distribution of ions which forms a charged layer that diffuses the surface charges. This layer is known as electric double layer. When the application of tangential electric field to this layer, the EDL (Electric Double Layer) experiences a force that causes movement in the electrolyte solution. This movement is known as electroosmosis. Electroosmotic flow has diverse applications such as soil analysis and filtering, microfluidic devices, chemical analysis involving highly charged surfaces, movement of liquids via phloem in plants and fuel cells for movement of water. Electroosmotic microfluidic devices are very significant in the inspection of dilution techniques in motion of blood, portable kits for disease diagnostics and cancerous tissues destruction [16], characterize cancer cell migration under confinement [17] and ovarian cancer cells [18].

Nath et al. [19] have given the experimental results on the flow behavior of metastatic cancer cells in a microchannel. Ganguly et al. [20] have considered the influence of electroosmosis on the propulsion of MHD nanofluids in a microchannel. Aparajita and Satapathy [21] have presented their analysis on the electroosmotic propulsion of a non-Newtonian nanofluid through a channel. Prakash et al. [22] have estimated the numerical study on the heat and flow characteristics of blood (assumed as a non-Newtonian nanoliquid) flow in micro-vessels with electroosmosis. Tripathi et al. [23] have discussed the heat and mass transfer of nanoliquids in a electrokinetically microchannel under the peristaltic waves. Vasu and De [24] have mathematically modelled the motion of power law fluids under electro-osmotic effects in a rectangular channel.

A porous medium is a material containing voids or pores. Motion of liquids through the porous medium is an interesting area of study in fluid mechanics. Permeability is the property of porous medium that measures the capacity of the medium to transmit fluid. Darcy in 1856 formulated a law that describes the fluid motion thorough a porous medium based on the experiments. The equation of the law is derived from the Naiver-Stokes equation via homogenization. It is analogous to Fourier's law of heat transfer, Fick's law of diffusion and Ohm's law of electrical networks. However, it has long been recognized that the equation is approximately correct only in a specific flow regime and at a very low velocity. The correct approximation was given by Forechheimer in 1901. The few applications of porous medium include sensors of humidity where molecules of water get adsorbed in the pores of the material, change the electrical conductivity of the sensor and in space crafts for a high energy-absorption material to resist high velocity impacts. Hatami and Ganji [25] have studied the propulsion of a generalized nanofluid between coaxial cylinders in a porous medium. Kameswaran et al. [26] have discussed the motion of nanoliquids over a shrinking/stretching sheet in a porous medium. Mahdi et al. [27] have studied the nanofluid motion in porous media under the thermophysical properties of nanofluids. Sheikholeslami et al. [28] have described the viscous nanoliquid motion due to a permeable stretching surface. Ting et al. [29] have discussed the nanoliquid propulsion in a porous channel, and evaluated the entropy generation and thermal performance under local thermal non-equilibrium conditions. Zhang et al. [30] have analyzed the radiative heat transfer and magnetohydrodynamic propulsion of nanoliquids over a flat plate in a porous medium.

Aforesaid studies explained the diverse effects such as MHD, electroosmosis and porous medium on the flow of different nanofluids. To the best of authors knowledge, no work has been made on the study of Sutterby nanoliquid flow in the microchannel under the Hall currents, MHD, electroosmosis and porous medium. To fill the gap in this area, in the current article, the flow of Sutterby nanoliquid in the microchannel in the presence of magnetic field, electroosmosis and Darcy's law has been considered. The flow is considered along the x-direction, between two plates under constant pressure and electroosmosis. A uniform magnetic field of strength B_0 is applied in the direction perpendicular to motion of the fluid. The lower wall is maintained at temperature T_0, concentration of C_0 and the upper wall is maintained at temperature T_1 and concentration C_1 respectively (see Fig. 1). It is also assumed that, the choice of velocity as $(\bar{u}(y), 0, 0)$. The non-dimensional system is solved with the help of bvp4c package in MATLAB software. The next section describes the mathematical formulation of the current study, The "Results and Discussion" section explains the discussion of the results through graphical representations. The last section dedicated to describe the important outcomes of the present study.

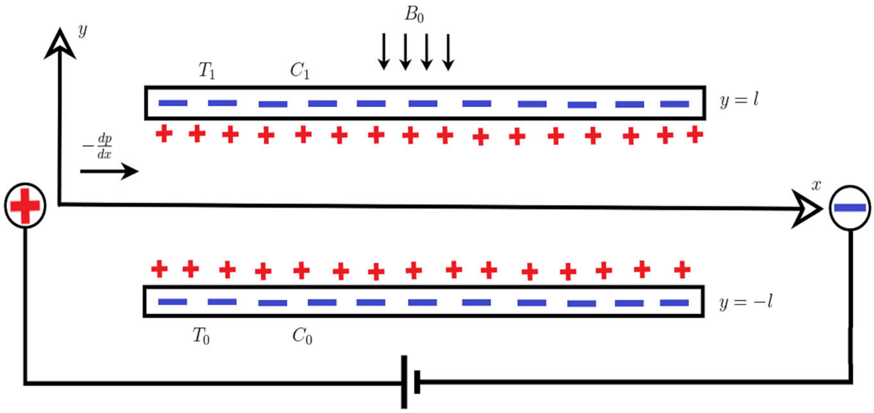

Fig. 1 Physical sketch of the current problem

Mathematical Formulation

The mathematical model (continuity, momentum, energy and nanoparticle volume fraction) for the MHD Sutterby nanoliquid under the electroosmosis and porous medium may be put in the form [31,32]:

$$\nabla.\bar{q} = 0, \tag{1}$$

$$\rho_f \left(\frac{\partial \bar{q}}{\partial t} + (\bar{q}.\nabla)\bar{q} \right) = -\nabla \bar{p} + \nabla.\bar{\tau} + \bar{R} + \bar{J} \times \bar{B} + \rho_e E_x, \tag{2}$$

$$(\rho C)_f \left(\frac{\partial \bar{T}}{\partial t} + (\bar{q}.\nabla)\bar{T} \right) = k_f \nabla^2 \bar{T} - \nabla.q_r + (\rho C)_p D_B \nabla \bar{T}.\nabla \bar{C} + (\rho C)_p \frac{D_T}{T_m} \nabla \bar{T}.\nabla \bar{T}, \tag{3}$$

$$\left(\frac{\partial \bar{C}}{\partial t} + (\bar{q}.\nabla)\bar{C} \right) = D_B \nabla^2 \bar{C} + \frac{D_T}{T_m} \nabla^2 \bar{T}, \tag{4}$$

The stress tensor for the Sutterby nanoliquid may be written as [33]

$$\bar{\tau} = \frac{\mu_f}{2} \left(\frac{\sinh^{-1}(\beta_1 \gamma)}{\beta_1 \gamma} \right)^n \bar{A}, \tag{5}$$

$$\bar{A} = \triangledown \bar{q} + (\triangledown \bar{q})^T, \tag{6}$$

$$\gamma = \sqrt{\sum_i \sum_j \gamma_{ij} \gamma_{ji}} = \sqrt{\frac{\Lambda}{2}}, \tag{7}$$

where $\Lambda (= \mathrm{Tr}(\bar{A})^2)$ is the second invariant strain tensor, $\bar{A}$ is the Rivlin-Erickson tensor, γ is the shear rate, μ_f is the dynamic viscosity and β_1 and n are the material constants. Using the expansion of $\sinh^{-1}$ by Taylor's series approximation, after some simplifications and with the choice of velocity, the interpretation for the stress tensor can be written as:

$$\bar{\tau} = \mu_f \left(\frac{\partial \bar{u}}{\partial \bar{y}} - \frac{n\beta_1^2}{6} \left(\frac{\partial \bar{u}}{\partial \bar{y}} \right)^3 \right). \tag{8}$$

The generalized Ohm's law can be written as:

$$\bar{J} + \frac{w_e \tau_e}{B_o}(\overline{J} \times \overline{B}) = \sigma \left(E + \mu_e(\overline{q} \times \overline{B}) + \frac{\nabla P_e}{en_e} \right), \tag{9}$$

where cyclotron efficiency (w_e), electron collision time (τ_e), electrical conductivity (σ), magnetic permeability (μ_e), electron pressure (P_e), electric charge (e) and electron density (n_e). It is well known that the electron pressure is negligible for weakly ionized fluids. The thermoelectric effects, external field effects and polarization effects are considered to be zero. With these assumptions the Ohm's law equation becomes:

$$\bar{J} \times \bar{B} = \left(-\frac{\sigma B_o^2}{1+m^2}\bar{u}, 0, 0 \right). \tag{10}$$

Using Darcy's law, the pressure drop and velocities can be related through the equation

$$\nabla \bar{p} = \left(\frac{-\mu_f \phi}{k_0} \right) \bar{q}, \tag{11}$$

with the mentioned choice of velocity we get,

$$\bar{R} = \left(\frac{-\mu_f \phi}{k_0}\bar{u}, 0, 0 \right). \tag{12}$$

The well known poisson equation can be noted as

$$\nabla^2 \bar{\phi} = \frac{-\rho_e}{\epsilon}, \tag{13}$$

where ϵ is the dielectric permittivity and $\bar{\phi}$ is the electric potential. The density of electric charge follows the Boltzmann distribution and which is given by

$$\rho_e = -2n_o ez \sinh \left(\frac{ez\bar{\phi}}{k_B T_v \epsilon} \right), \tag{14}$$

where bulk concentration n_o, average temperature T_v, valency of ions z, elementary charge valency e and Boltzmann constant k_B. For the assumption $\left(\left| \frac{ez\bar{\phi}}{k_B T_v} \right| < 1 \right)$, the Eq. (14) can be written as

$$\rho_e = -\frac{2n_o e^2 z^2 \bar{\phi}}{k_B T_v \epsilon}. \tag{15}$$

Using the aforementioned theory and the choice of velocity; the momentum, energy and nanoparticle volume fraction equations for the Sutterby nanoliquid may be written as:

$$\mu_f \frac{d^2\bar{u}}{d\bar{y}^2} - \frac{\mu_f n \beta_1^2}{2} \left(\frac{d\bar{u}}{d\bar{y}} \right)^2 \left(\frac{\mathrm{d}^2\bar{u}}{d\bar{y}^2} \right) - \left(\frac{\sigma B_o^2}{1+m^2} + \frac{\mu\phi}{k_0} \right) \overline{u} - \left(\frac{2ne^2 z^2 \overline{\phi}}{k_B T_v} \right) E_x + \overline{G} = 0, \tag{16}$$

$$k_f \left(\frac{d^2\bar{T}}{d\bar{y}^2} \right) + (\rho C)_p D_B \left(\frac{d\bar{C}}{d\bar{y}} \frac{d\bar{T}}{d\bar{y}} \right) + \frac{(\rho C)_p D_T}{T_m} \left(\frac{d\bar{T}}{d\bar{y}} \right)^2 + \frac{16\sigma^* T_0^3}{3\alpha^*} \frac{d^2\bar{T}}{d\bar{y}^2} = 0, \tag{17}$$

$$D_B \left(\frac{d^2\bar{C}}{d\bar{y}^2} \right) + \frac{D_T}{T_m} \left(\frac{d^2\bar{T}}{d\bar{y}^2} \right) = 0, \tag{18}$$

with the corresponding boundary equations

$$\bar{u} = 0,\ \bar{T} = T_o,\ \bar{C} = C_o \ \text{at}\ y = -l, \tag{19}$$

$$\bar{u} = 0,\ \bar{T} = T_1,\ \bar{C} = C_1 \ \text{at}\ y = l. \tag{20}$$

The dimensionless quantities can be expressed as:

$$u = \frac{\bar{u}}{U},\quad y = \frac{\bar{y}}{l},\quad G = \frac{l^2 \bar{G}}{\mu_f U},\quad M = \sqrt{\frac{\sigma}{\mu_f}}\, B_o l,$$

$$\beta = \frac{n \beta_1^2 U^2}{l^2}, Da = \frac{k_0}{\phi\, l^2},\quad \phi = \frac{\bar{\phi}}{\xi},$$

$$U_{HS} = \frac{-E_x\, \epsilon\, \xi}{\mu_f\, U},\quad k = lez\sqrt{\frac{2n_o}{\epsilon k_B T_v}},\quad \tau = \frac{(\rho C)_p}{(\rho C)_f},$$

$$\theta = \frac{\bar{T} - T_0}{T_1 - T_0},\quad \sigma = \frac{\bar{C} - C_0}{C_1 - C_0},$$

$$Nb = \frac{\rho_f\ \tau\ D_B\ (C_1 - C_0)}{\mu_f},\quad Nt = \frac{\rho_f\ \tau\ D_T\ (T_1 - T_0)}{T_m\ \mu_f},$$

$$Pr = \frac{\mu\ C_f}{k_f},\quad Rn = \frac{16\, \sigma^*\ T_o^3}{3\ \mu\ \alpha^*\ C_f}.$$

Using aforesaid quantities, the governing equations can be reduced to

$$\frac{d^2u}{dy^2} - \frac{\beta}{2}\left(\frac{du}{dy}\right)^2\left(\frac{d^2u}{dy^2}\right) - \left(\frac{M^2}{1+m^2} + \frac{1}{Da}\right)u + k^2 U_{HS}\phi + G = 0, \tag{21}$$

$$(1 + RnPr)\frac{d^2\theta}{dy^2} + NbPr\left(\frac{d\sigma}{dy}\frac{d\theta}{dy}\right) + NtPr\left(\frac{d\theta}{dy}\right)^2 = 0, \tag{22}$$

$$\frac{d^2\sigma}{dy^2} + \frac{Nt}{Nb}\frac{d^2\theta}{dy^2} = 0, \tag{23}$$

with the suitable non-dimensional boundary conditions

$$u = 0,\ \theta = 0,\ \sigma = 0 \ \text{at}\ y = -1, \tag{24}$$

$$u = 0,\ \theta = 1,\ \sigma = 1 \ \text{at}\ y = 1. \tag{25}$$

From Eq. (13), the non-dimensional Poission-Boltzmann equation is given by :

$$\frac{d^2\phi}{dy^2} - k^2\phi = 0, \tag{26}$$

under the non-dimensional condition $\phi = \xi_1$ at $y = -1$ and $\phi = \xi_2$ at $y = 1$, the resulting electric potential is noted as

$$\phi = \left(\frac{\xi_1 + \xi_2}{2\cosh(k)}\right)(\cosh(ky)) - \left(\frac{\xi_1 - \xi_2}{2\sinh(k)}\right)(\sinh(ky)). \tag{27}$$

Results and Discussion

The system of equations (21–25) are highly nonlinear. The bvp4c package is one of the well accepted approach to determine the solutions of the boundary value problems (BVP).

Table 1 Comparison of velocity profiles for the current study with existing literature

y	Lodhi and Ramesh [32] when $\alpha = 0$ and $\lambda_1 = 0$	Present study when $\beta = 0$
−1	0.0000	0.0000
−0.8	0.1934	0.1948
−0.6	0.3383	0.3411
−0.4	0.4401	0.4442
−0.2	0.5019	0.5072
0	0.5253	0.5316
0.2	0.5102	0.5171
0.4	0.4550	0.4620
0.6	0.3560	0.3624
0.8	0.2074	0.2116
1	0.0000	0.0000

For this reason, we chose the bvp4c package in MATLAB software to solve the equations (21–23) along with the conditions (24, 25) numerically. Table 1 represents the comparison of the present investigation with the earlier published literature Lodhi and Ramesh [32]. For the comparison the common parameters have been considered as $M = 0.5, m = 2, Da = 3, k = 2, U_{HS} = 1, \xi_1 = 0.1, \xi_2 = 0.2, G = 1$. The corresponding parameters with respect to the fluids have been presented in the Table 1. It is depicted from the table that, the provided velocity profile is in close agreement with the exact solution of existing literature [32]. It proves that, the presented numerical solutions (MATLAB built-in bvp4c package) are valid with respect to the existing literature.

The figures (which are presented in this section) are plotted to study the relation of various flow quantities such as, velocity of the fluid with respect to the parameters Darcy number Da, pressure gradient G, electroosmosis parameter k, Hartmann number M and Hall parameter m, and the nanoparticle temperature and volume fraction with the diverse parameters Brownian motion parameter Nb, thermophoresis parameter Nt, radiation parameter Rn and Prandtl number Pr. It is observed from Fig. 2(a–e) that, all the figures are symmetric about the central axis and the maximum velocities are achieved in the centre of the channel which is an expected behavior in a flow between parallel plates. It is clear from the Fig. 2(a) that the fluid velocity increases with the significant increment of Darcy Number. This works well with the physical observation that a fluid faces lesser resistance in a clear medium (higher Da) as compared with the porous medium (lower Da). It is observed from Fig. 2(b) that, there is an increment in velocity with the rising values of pressure gradient. It is obvious that, when we apply more pressure to the fluid, more fluid takes place in the system, in view of this higher velocities have been observed for higher pressure gradient. The electroosmosis parameter is inversely proportional to the thickness of Electrical Double Layer (EDL). An increase in electroosmosis parameter would signify a reduction in the EDL which in turn causes an increase in the velocity which is clearly visible in the Fig. 2(c). The increase in Hall current parameter m causes an increase in the velocity (see Fig. 2d). It is clear form the Fig. 2(e) that the velocity reduces for the higher values of Hartmann number. Hartmann number is a dimensionless number which gives the ratio between electromagnetic forces to viscous forces. An increase in the Hartmann number would mean an increase in the Lorentz force, which is a resistive force. Due to this reason there is a decrement in velocity with increase of Hartmann number.

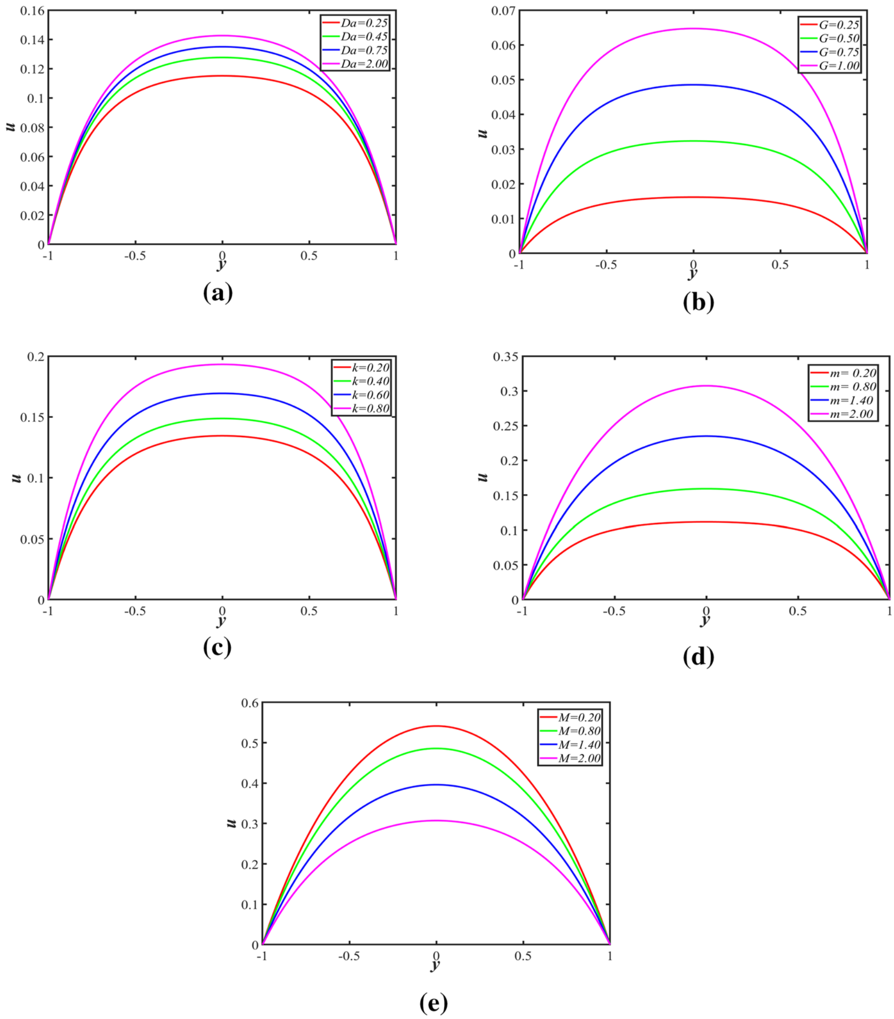

Fig. 2 Variations in velocity for diverse parameters of interest

Brownian motion parameter signifies the degree of random motion of particles in the fluid. An increase in the Brownian motion parameter would mean an increase in collisions due to the increase in random motion. These increased number of collisions would result in an increase in the internal energy of the system and hence result in an increase in temperature. This phenomenon is clearly observed in Fig. 3(a), which shows an increase in the nanoparticle temperature curve with an increase in Brownian parameter Nb. It is depicted from Fig. 3(b) that, the nanoparticle temperature increases with thermophoresis parameter Nt. It is physically justifies that, thermophoresis parameter signifies a force that is generated by a temperature gradient which causes the motion of particles. An increase in the thermophoresis parameter tends to an increase in the internal energy and consequently, an increase in temperature. It is inspected that nanoparticle temperature enhances by increasing the Prandtl number Pr (see Fig. 3c). It is physically interpreted that temperature strongly

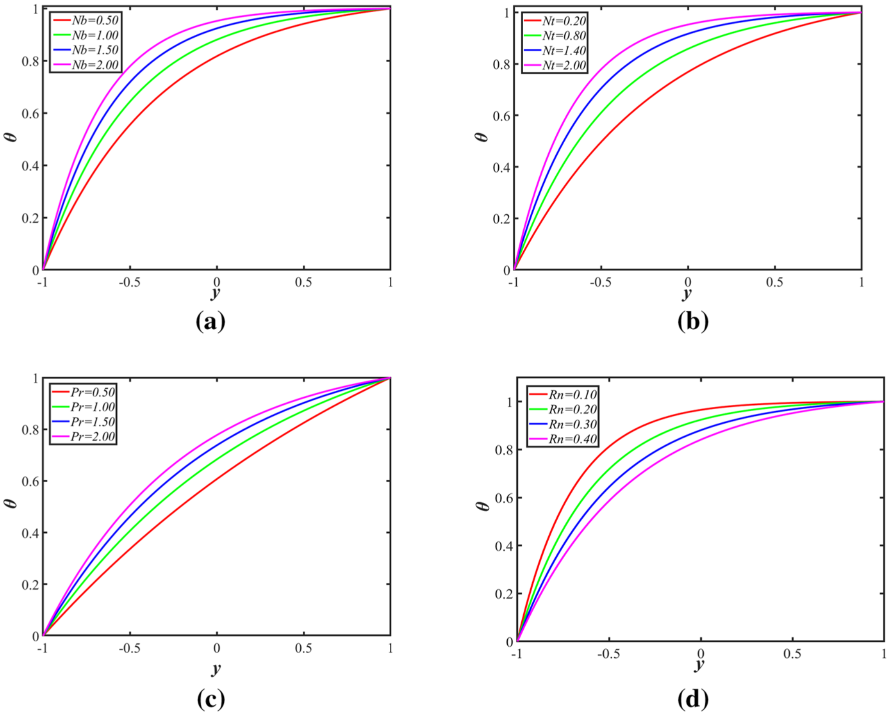

Fig. 3 Variations in nanoparticle temperature for diverse parameters of interest

depends on momentum diffusivity and thermal diffusivity. From Fig. 3(d), it is clear that an increase in the radiation parameter Rn decreases the nanoparticle temperature. It is noticed from Fig. 4(a) that, the nanoparticle volume fraction increases with increase in Brownian Parameter Nb, and the opposite trend is noticed with respect to thermophoresis parameter Nt and Prandtl number Pr (see Fig. 4b, c). It is clear from Fig. 4(d) that, the nanoparticle volume fraction rises with the rising values of radiation parameter Rn.

Conclusions

In the proposed problem, we have presented the effects of MHD, Hall currents and electroosmosis on the propulsion of Sutterby nanoliquid in the porous microchannel. The effects such as thermophoresis and Brownian motion have also been considered into account. The resulting system is highly non-linear and are successfully solved using the bvp4c package in MATLAB. The main findings of the present investigation are summarized as follow:

- The velocity of the fluid enhances with respect to Darcy number.
- There is a decrease in fluid velocity with the rising values of Hartmann number.
- Temperature decreases with the increment of radiation parameter.
- Nanoparticle volume fraction decreases with the increment of Prandtl number and thermophoresis parameter.

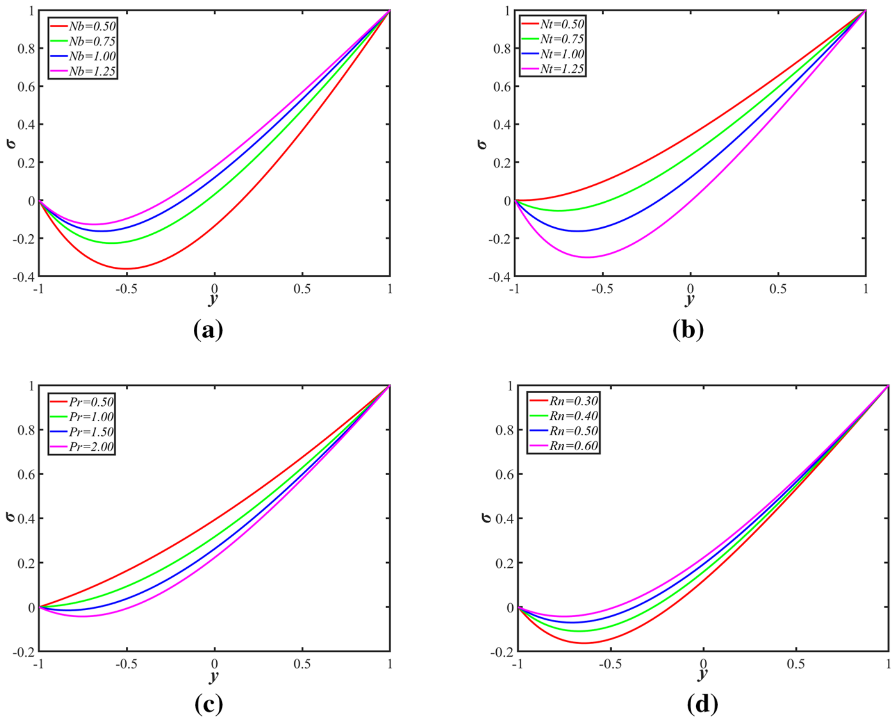

Fig. 4 Variations in nanoparticle volume fraction for diverse parameters of interest

- The results for Newtonian fluid, Eyring fluid, pseudo-plastic (shear thinning) fluid and dilatant (shear thickening) fluid properties can be captured by setting $n = 0$, $n = 1$, $n < 0$ and $n > 0$, respectively.

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