

Pay for bits: Probably a Better Method for Rewarding Forecasters

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Motivation

In [Alignment Problems With Current Forecasting Platforms](#), Sempere and Lawsen outline a variety of problems with current forecasting platforms, whose scoring rules are found to either not be proper—as in the case of Good Judgment Open or CSET-Foretell (now INFER)—or incentivize distorting one’s true probabilities to maximize the chances of placing in the top few positions which earn a monetary reward—as in the case of Metaculus. In addition, in almost all cases, forecasting platforms—or, for that matter, prediction markets—disincentivize collaboration.

In this working paper, we outline an incentivization method, “paying for bits”, which combines features of prediction markets and forecasting platforms to produce a scoring rule that pays forecasters well, is proper, and nonetheless incentivizes collaboration. This scoring rule has both a discrete form—where resolution is either a yes or a no, and a continuous form—where resolution can be any probability between 0 and 1.

In essence, we start out with the logarithmic scoring rule, and we add a few bells and whistles to make it collaborative. We also reinterpret it using the concept of a prediction market with an automatic market maker, which is a better abstraction because it deals more elegantly with forecasters not being able to input arbitrarily low probabilities in a contest which uses the logarithmic scoring rule, and it offers a few escape hatches.

In particular, the practical innovations we suggest are:

- Instead of comparing forecasters against other forecasters, compare them against against the initial probability of the stakeholder who sponsors the question.

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- Create a prediction market for each forecaster with a logarithmic market scoring rule, and allow each forecaster to move only the probability of their own prediction market.
- Require forecasters to put their money where their mouth is by betting their own money, so that if their forecast is worse than the houses' own forecast, they lose money. Optionally, sponsor forecasters once who start out without many funds themselves.

In two accompanying papers, Amplify Samostvety and Amplify Rootclaim, we use this continuous form like a lego-block: we combine it with other methods to build more powerful and general incentive schemes.

A few limitations of previous approaches, and their solutions

Market scoring rules overcome the limitations of Brier and logarithmic scoring rules

The Brier score has limitations stemming from its deep theoretical inelegance. Sure, it is proper, but it also doesn't have desirable properties such as:

- Composability: The Brier score on "A" and the Brier score on "B given A" can't be straightforwardly related to the Brier score on "A and B"
- Comparability: The cumulative relative Brier score doesn't correspond to any particular meaning.
- Comparability II: The Brier score of the average guess will always be at least as good as the average of the Brier scores, and usually better. This incentivizes forecasters to forecast the average crowd guess.

In contrast, the logarithmic scoring rule has an array of desirable properties.

The logarithmic scoring rule is proper and has desirable properties:

- Composability: The logarithmic score on "A&B" is the sum of the log scores on "A" and "B given A"
- Comparability: The cumulative relative log score corresponds to bits of information which would be added or removed
- Comparability II: The arithmetic mean of the log scores is the log score of the geometric mean of the guesses. If forecasters are rewarded in proportion to their log score vs the crowd, there is no advantage to forecasting the current crowd.

More general, for many nice theoretical properties, the log score will have them, whereas the Brier score will not. And yet, the Brier score offers bounded payoffs, whereas the logarithmic score does not. This is why platforms such as Cultivate Labs still use the Brier score to reward their members. In contrast, platforms such as Metaculus solve this problem by forbid their members from inputting probabilities lower than 1% or higher than 99%.

However, a logarithmic market scoring rule makes the logarithmic score bounded, by only allowing players to lose as much as they have previously accumulated. Conversely, the more money—or points in the case of a play-money prediction market—a participant has accumulated, the more extreme they are allowed to make the market. This preserves the desirable properties of the logarithmic scoring rule in approximate form, while making payoffs bounded without having to institute a sharp cut-off.

Competing against the house provides an incentive for collaboration in a way which competing against other forecasters doesn't.

To determine the relative quality of forecasters, forecasting platforms currently score and compare forecasters in either of two ways: - Against the aggregate of other forecasters. For instance, a forecaster's score for a question could be difference between his Brier score and the average Brier score of other forecasters who participated in the same question. And his total score could be the sum of those differences. - In absolute terms, e.g., a forecaster's score could be their Brier score averaged across all questions he has participated in.

Both comparison methods do indeed produce a ranking. But they also introduce a clear zero sum component: forecasters are not incentivized to share information because that would hurt their relative positioning. The second method further introduces the distortion that forecasters are incentivized to choose easier questions, i.e., questions which are easier to predict on.

The alternative suggested in this paper is to reward forecasters monetarily in proportion to their improvement over a question's prior weighted by importance. By the question prior, I mean an initial guess provided by the question creator or sponsor. And by weighing by importance, we incentivize forecasters to predict on more important questions. So overall, we incentivize forecasters to predict on important questions for which the sponsoring stakeholder's initial guess is worse.

This creates a more meaningful metric for comparison: importance-adjusted bits of information added to the decision-maker's initial forecast. From Bayesian probability theory, we get the strong hint that probabilities without priors don't really make sense, so that might be a hint that we want comparisons vis a vis a prior.

There is still a residual incentive to compete with other forecasters to produce the most importance-adjusted bits of information. However, if monetary incentives are large enough, the efficiencies through collaboration and some lower degree of latent altruism might outweigh those incentives. Conversely, relative positional results could be made anonymous, by rewarding forecasters using a privacy-preserving crypto-currency, such as Monero.

Putting your money where your mouth is has a salutatory effect.

Description of the method.

In the interest of brevity, we shall outline our proposed method by means of an example, and the example shall be the question “Will the People’s Republic of China have annexed at least half of Taiwan by 2050?”, as operationalized by [Metaculus](#). Parts which justify or explain the use of other parts are explained first, even if they would come later in terms of time.

The patron determines a rough prior to reduce potential forecasting reward.

Taiwan has been independent of mainland China since the 25th of October 1945, i.e., 76 years into the past. Per Laplace’s law, the chances that this will change by 2050 is $1 - (1 - \frac{1}{(2021-1945)+1})^{2050-2021} \approx 32\%$. Lets take this 32% as the market’s initial probability. Note that per the [reference class problem](#), other reference classes might have been chosen, so the point of this prior is not to be definitive, but rather to provide a starting point less arbitrary than 50% from which forecaster reward might be computed in the next steps.

By sharing this initial step, the question creator saves the time of the many forecasters which may participate in the question.

In the case of a patron aiming to learn from sponsoring a forecasting tournament, the prior might represent the patron’s initial probability. If the patron is unsophisticated and the question is not amenable to base-rate analysis, we may use a 50%, the percent of questions which resolve positively on the site, or some other such point. Alternatively, the patron may sub-contract the creation of the prior, for instance by paying other forecasters to quickly do so or by creating low-liquidity (and thus low-potential reward) prediction markets.

Forecasters predict on the question

Forecasters each get access to a prediction market governed by a logarithmic market scoring rule, and whose initial probability is the prior determined in the previous step. This prediction market is also characterized by a further parameter: liquidity. Liquidity is the amount of money which is available to take bets on both sides. The more liquidity there is, the more money it takes to moves a market’s odds, and the more money that can be made if those odds are wrong. For this reason, question creators would put more liquidity in markets they care more about.

If an individual forecaster thinks that the initial probability is wrong, he can move it to a probability which he thinks is more correct by betting some of his funds. Further, we might hope that the different forecasters might each research

different aspects of the question and then reveal their information to each other, so that they can collectively correct their own markets more efficiently. Because each interacts with his own market. We explore whether this assumption might break down in Appendix A.

When contemplating this scheme, two issues become apparent after some amount of reflection. First, in a prediction market in which the outcome becomes more and more apparent as time goes on, the question creator loses all the funds he put up as liquidity. Secondly, forecasters

Within the automatic market-maker abstraction, both of these issues have a clear solution. First, stop trading before the outcome is known. One particularly advisable point would be to cut trading when the stakeholder interested in the question makes the decision that the market was created to influence. Secondly, slowly reduce the amount of liquidity there is in the markets as time goes on, so that forecasters have a small incentive to predict sooner rather than later.

The question gets a binary resolution and forecasters get rewarded.

Taiwan either gets invaded or doesn't. Forecasters get rewarded in proportion to the number of shares of the winning outcome that they accumulated while interacting with their prediction market.

The question gets a probabilistic resolution

Sometimes, the resolution of a question is uncertain. In cases of uncertainty about how a question should resolve, current forecasting platforms tend to resolve questions as ambiguous. Prediction markets tend to break to one side (as 0% or as 100%), but they also have the option of resolving questions probabilistically, so that shares pay out X cts and $(100-X)$ cts rather than \$0 or \$1.

Some common reasons for uncertain resolutions might be:

Because of uncertain ontologies

The Metaculus question as currently written resolves only if the People's Republic of China takes over at least half of Taiwan. However, this is somewhat arbitrary. If the question had been on a prediction market, it could also have been structured such that an invasion of $n\%$ of Taiwan pays out n cts for each share.

Because of probabilistic knowledge

At the time of question resolution, Taiwan could be enmeshed in a civil war, such that it's unclear how much territory each side controls. If the question had to be resolved, one way to do so would be to pay shares out in proportion to the likelihood that the PRC controls more than 50% of the territory.

Because the question hasn't resolved yet

The original question asked about an invasion of Taiwan by 2050, which is fairly far in the future. So forecasters might not be motivated to predict in question for which the payoff might be very far away. In the two accompanying papers, Amplify Samotsvety and Amplify Rootclaim, we outline two methods for providing forecasters with a reward now for questions whose resolution is far away. But that reward will be probabilistic.

That is, shares for the prediction market are paid out according to the probability estimated by a different forecasting system. If that forecasting system estimates an $X\%$ probability, shares of yes are paid out X cts.

Conclusion

We outlined three improvements which: - Assign a prediction market to each forecaster - Compare forecasters against the stakeholder which - Make forecasters risk their own money

This solves a majority of the problems identified in Alignment Problems.. [some conclusion here.]

Notes:

<https://www.cs.cmu.edu/~sandholm/liquidity-sensitive%20automated%20market%20maker.teac.pdf>

A useful shibboleth: If there are no stakeholders that can give an initial probability, and they are not willing to pay proportionally to the number of bits, there is no point in giving better probabilities.