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### Association Rules - Informally

- Let item {fish} occur in 5% of sales transactions and set {fish, white wine} occur in 4% of them. This information allows us to derive an association rule stating that:
- 4 out of 5 *customers*; that is, 80% of *customers* who buy fish also buy white wine.
- In order to derive such rules we need to know how many transactions support respective sets of items (or itemsets).





#### Support of Itemsets

- Let dataset D be a set of *transactions*, where each transaction is a subset of items in *I*.
- Support of an itemset X, denoted by sup(X), is the number of transactions in D that contain all items in X; that is,

$$sup(X) = |\{T \in D \mid X \subseteq D\}|.$$

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#### Example: Supports of Itemsets

#### Example dataset D

Id	Transaction
$T_1$	<i>ABCDEG</i>
$T_2$	<i>ABCDEF</i>
$T_3$	<b>ABC</b> DEH
$T_4$	ABDE
$T_5$	ACDEH
$T_6$	BCE

- sup(ABC) = 3.
- sup(EH) = 2.
- Supports of all supersets of EH are not greater than 2 either.
- Supports of all subsets of EH can be greater than 2.

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# Relative Support of Itemsets

 Relative support of an itemset X, denoted by rSup(X), is the ratio of the number of the transactions in D that contain all items in X to the number of all transactions in D:

$$rSup(X) = sup(X) / |D|$$

 Remark: rSup(X) can be regarded as an estimation of the probability of the occurrence of itemset X in D.

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### **Example: Relative Supports**

#### Example dataset D

Id	Transaction
$T_1$	<i>ABCDEG</i>
$T_2$	<i>ABCDEF</i>
$T_3$	<i>ABCDEH</i>
$T_4$	ABDE
$T_5$	ACDEH
$T_6$	BCE

- rSup(ABC) = 3/6 = 50%,
- $rSup(EH) = 2/6 \approx 33\%$ .

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#### Frequent Itemsets

• X is defined a frequent itemset if sup(X) > minSup,

where minSup is the user-defined threshold value.

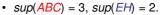
· Basic property of itemsets: Supports of supersets of an itemset X are not greater than sup(X).

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Example:	In(frequent) Itemsets
Example dataset D	• $sup(ABC) = 3$ , $sup(EH) = 2$ .

ABDE

 $T_6$  BCE

*ACDEH* 





- Let minSup = 2. Then: ABC is frequent, EH is not frequent.
- Supports of all supersets of EH are not greater than 2 either, hence supersets of EH are not frequent.
- However, supports of subsets of EH can be greater than 2. Thus, it may happen that subsets of EH are frequent.





#### Association Rules (ARs)

• An association rule is an expression associating two itemsets:

$$X \rightarrow Y$$
, where  $\emptyset \neq Y \subseteq I$  and  $X \subseteq I \setminus Y$ .

- X is called an antecedent of  $X \rightarrow Y$ .
- Y is called a *consequent* of  $X \rightarrow Y$ .
- $X \rightarrow Y$  is said to be *based on*  $X \cup Y$ , and  $X \cup Y$  is called the *base* of  $X \rightarrow Y$ .





### Support of Association Rule

• Support of  $X \rightarrow Y$  is defined as the number of transactions that contains the base of  $X \rightarrow Y$ : that is.

$$sup(X \rightarrow Y) = sup(X \cup Y).$$

• Relative support of  $X \rightarrow Y$  is defined as the relative support of its base:

$$rSup(X \rightarrow Y) = rSup(X \cup Y).$$

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#### Confidence of Association Rule

• Confidence of  $X \rightarrow Y$  is defined as the ratio of the number of transactions that contain the base  $X \cup Y$  to the number of transactions containing the antecedent *X*:

$$conf(X \rightarrow Y) = sup(X \rightarrow Y) / sup(X).$$

• **Remark:**  $conf(X \rightarrow Y)$  can be regarded as an estimation of the conditional probability that Yoccurs in a transaction T provided X occurs in T.





# Example: Association Rules

Example dataset D

Id	Transaction
$T_{I}$	<mark>A</mark> BCDEG
$T_2$	<mark>A</mark> BCDEF
$T_3$	<mark>A</mark> BCDEH
$T_4$	ABDE
$T_5$	ACDEH

 $T_6$  BCE

sup(ABC) = 3, sup(A) = 5.

#### Hence:

- $sup(\{A\} \rightarrow \{BC\}) =$  $sup(\{ABC\}) = 3,$
- $conf(\{A\} \rightarrow \{BC\}) =$  $sup(\{ABC\}) / sup(\{A\}) = 3/5.$





# Strong Association Rules

 Strong association rules (AR) are defined as those rules in AR whose support is above minSup and confidence is above minConf; that is,

$$AR = \{X \rightarrow Y \in AR | sup(X \rightarrow Y) > minSup \land conf(X \rightarrow Y) > minConf\},$$

where  $minSup \in [0, |D|)$  and  $minConf \in [0, 1)$ .





# Strong ARs and Frequent Itemsets

$$AR = \{X \rightarrow Y \in AR | sup(X \rightarrow Y) > minSup \land conf(X \rightarrow Y) > minConf\}$$

= 
$$\{X \rightarrow Y \in AR | sup(X \cup Y) > minSup \land conf(X \rightarrow Y) > minConf\}$$

= 
$$\{X \rightarrow Y \in AR | (X \cup Y) \text{ is frequent } \land conf(X \rightarrow Y) > minConf\}$$





### Discovery of Strong **Association Rules**

AR is discovered in two steps:

- · Find frequent itemsets F and their supports in dataset D.
- Generate **AR** only from **F**: Let  $Z \in F$ ,  $Z \neq \emptyset$ and  $Y \subseteq Z$ . Then, any candidate rule  $Z \setminus Y \rightarrow Y$  is a strong association one if:  $sup(Z) / sup(Z \setminus Y) > minConf.$







#### References

- Agrawal R., Imielinski T., Swami A.: Mining Associations Rules between Sets of Items in Large Databases. In: Proc. of the ACM SIGMOD Conference on Management of Data, Washington, USA (1993) 207-216
- Rakesh Agrawal, Ramakrishnan Srikant: Fast Algorithms for Mining Association Rules in Large Databases. VLDB 1994: 487-499
- Fernando Berzal Galiano, Ignacio J. Blanco, Daniel Sánchez, María Amparo Vila Miranda: A New Framework to Assess Association Rules. IDA 2001: 95-104