N·H
ELSEVIER
15 April 1995
OPTICS COMMUNICATIONS
Optics Communications 116 (1995) 43–48

Light propagation in the Fresnel region. New numerical approach

Maciej Sypek
Institute of Physics, Warsaw University of Technology, Koszykowa 75, PL-00-662, Warsaw, Poland

Received 12 August 1994; revised version received 21 November 1994

Abstract

The paper presents the comparison of light propagation simulation methods in the Fresnel region. The theoretical approach is based on the Kirchhof scalar theory of diffraction. In the case of the complex amplitude calculated by using the spectrum of the plane waves a new approach is included. Advantages and disadvantages of the presented methods are compared and discussed. The simulated results are verified in an experimental optical setup.

1. Introduction

In digital holography the light propagation phenomena is usually simulated and calculated by means of a computer, using a Fast Fourier Transformation (FFT) algorithm. A Fresnel hologram reconstruct a signal in a finite distance behind the hologram plane. In comparison to digital Fourier holography the effort to calculate a Fresnel hologram is comprehensive. Although the FFT is the essential tool in digital holography it can not be straightforward applied for the simulation of light propagation within a Fresnel approximation.

A direct practical application of the Fresnel formula [1,6,9] is not useful for a numerical simulation of light propagation in the Fresnel region. Both object and synthetic hologram are sampled in the numerical approach. For a small distance between the object and hologram plane the Fresnel factor reveals fast oscillations. Additionally there is no simple correlation between output and input coordinates in the Fresnel equation.

The other approach to the numerical simulation of the light propagation in the Fresnel region is the decomposition of the field amplitude into plane waves [2,5,7,8]. In this case we can describe the output complex amplitude *as the convolution of the input distribution and an impulse response function for free space* [2,3]. This last function is known in the analytical form and exhibits very slow oscillations for a small distance between object and hologram plane. Additionally output and input coordinates are equivalent. However for an increasing distance (still in the Fresnel region) aliasing errors will occur. It is worth to notice that this approach can be easy extended for nonparaxial cases [2,7].

The method proposed in this paper substantially eliminates the aliasing error. It can be realised by a modification of the previous approach. According to the wavelength, sampling frequency and number of samples, the maximal propagation distance can be calculated. Moreover for the propagation calculations a double length vector for the convolution operation is used. By division of the propagation distance into single steps and repetition of the convolution operation in the extended vector it is possible to eliminate the aliasing error. A significant problem – in this case – is an influence of the vector swapping. This effect is equiv-

0030-4018/95/$09.50
SSDI 0030-4018(95)00027-5

alent to opaque square pupils placed at constant distances along the optical axis in the optical setup. Diffraction on the edges of the pupils can disturb the output distribution for many steps. It is worth to notice that this approach enables to analyse a shift from the optical axis of an output area and can be extended for nonparaxial cases.

2. Direct theoretical approach

The amplitude and phase distribution $u_2(x_2, y_2)$ in the plane at the distance z behind the input plane is expressed by the *integral based on the Fresnel approximation*,

$$u_2(x_2, y_2) = \frac{\exp(ikz)}{i\lambda z} \int\int_{-\infty}^{+\infty} u_1(x_1, y_1) \times \exp\left[\frac{ik}{2z}\left[(x_1 - x_2)^2 + (y_1 - y_2)^2\right]\right] dx_1\, dy_1 . \quad (1)$$

Input signal $u_1(x_1, y_1)$ is illuminated by a plane wave with wavelength λ $(k = 2\pi/\lambda)$.

Direct practical application of the above Fresnel formula is not useful for a numerical simulation. In digital holography the wave propagation is simulated by means of a computer, using a Fast Fourier Transform algorithm (FFT). According to the definition of the Fourier transformation [4], complex amplitude distribution $u(x_2, y_2)$ can be easy expressed in the form of the two-dimensional Fourier transformation of the function $u_1'(x_1, y_1)$,

$$u_1'(x_1, y_1) = u_1(x_1, y_1) \exp\left[\frac{ik}{2z}(x_1^2 + y_1^2)\right] . \quad (2)$$

The spatial frequencies of Fourier transform can be correlated with coordinates x_2 and y_2, then $\nu_x = x_2/\lambda z$ and $\nu_y = y_2/\lambda z$. Thus Eq. (1) can be rewritten in the following form,

$$u_2(x_2, y_2) = \frac{\exp(ikz)}{i\lambda z} \exp\left[\frac{ik}{2z}(x_2^2 + y_2^2)\right] \times \mathscr{F}\left\{ u_1(x_1, y_1) \exp\left[\frac{ik}{2z}(x_1^2 + y_1^2)\right]\right\}_{\nu_x, \nu_y} \quad (3)$$

where $\mathscr{F}\{\ldots\}$ denoted Fourier transform.

Practical application of the above formula is still complicated. The numerical processing requires sampling of all the functions in Eq. (3). The coordinates in the input (x_1, y_1) and output (x_2, y_2) plane are not equivalent. The output coordinates are scaled by a factor which value depends on the propagation distance z. The above facts leads to the conclusion that the output plane should be sampled with a different frequency than the input one. For many cases a resampling operation and interpolation would be necessary.

Moreover the distance parameter z of the Fresnel factor

$$\exp\left[\frac{ik}{2z}(x_1^2 + y_1^2)\right] , \quad (4)$$

is placed in the denominator of the fraction. Small values of z yield a large phase of the Fresnel factor which exhibits rapid oscillations. A large sampling frequency of the input signal is necessary in the numerical analysis.

Fig. 1 presents the cross section (256 points) of the real part of the Fresnel factor (4) for wavelength $\lambda = 0.6328$ μm and sampling distance 50 μm. The total area is composed of 256^2 complex points and corresponds to dimensions 12.8 × 12.8 mm.

It is easy to notice that for distances less than approximately 1400 mm (for the mentioned parameters) this approach fails. In this place it is worth to mention that the general theory (see Eq. (1)) requires the following condition to be fulfilled for the Fresnel region,

$$z^2 \gg (\pi/4\lambda)\left[(x_0 - x_1)^2 + (y_0 - y_1)^2\right]^2_{max} . \quad (5)$$

However this requirement is in general not a necessary one for the Fresnel approximation to remain valid. The Fresnel approximation gives accurate results for *diffraction angles up to 18 deg [11]*. For the above parameters (aperture width 12.8 mm) this last condition is fulfilled for z greater than 20 mm. In this case – in the

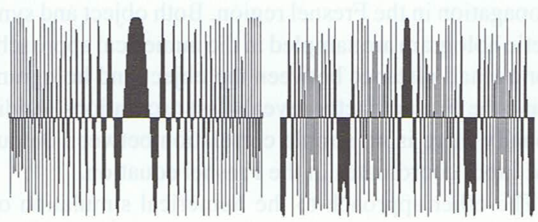

Fig. 1. Oscillations of the Fresnel factor for $z = 1400$ mm (left) and 250 mm (right).

region from 20 mm up to 1400 mm – direct application of the theoretical Fresnel formula is not possible. *It is worth notice that below 20 mm the use of the off-axis theory is necessary.*

3. Convolution approach

The other approach to the numerical simulation of the light propagation in the Fresnel region is the decomposition of the input field amplitude into plane waves. By introducing the following notation: $u_1(x_1, y_1) = u_1(x, y)$ and $u_2(x_2, y_2) = u_2(x, y)$ it is possible to write the output complex field as a convolution of the input field and function $h(x, y)$,

$$u_2(x, y) = u_1(x, y) * h(x, y) . \tag{6}$$

The function $h(x, y)$ can be interpreted as an impulse response function in free space for a given distance z,

$$h(x, y) = \frac{\exp(ikz)}{i\lambda z} \exp\left[\frac{ik}{2z}(x^2 + y^2)\right] . \tag{7}$$

Basing on the convolution theorem [4], we can write a convolution of two functions in the object space as the product of their Fourier transforms in the Fourier space,

$$U_2(\nu_x, \nu_y) = U_1(\nu_x, \nu_y) H(\nu_x, \nu_y) , \tag{8}$$

where

$$U_1(\nu_x, \nu_y) = \mathscr{F}\{u_1(x, y)\} ,$$
$$U_2(\nu_x, \nu_y) = \mathscr{F}\{u_2(x, y)\} .$$

The Fourier transform of the function $h(x, y)$ can be calculated analytically and is written as follows,

$$H(\nu_x, \nu_y) = \mathscr{F}\{h(x, y)\}$$
$$= \exp(ikz) \exp[-i\pi\lambda z(\nu_x^2 + \nu_y^2)] . \tag{9}$$

The mentioned approach has a following interpretation [7]. The input complex amplitude field is decomposed into plane waves. This first operation corresponds to Fourier transformation. The coherent transfer function H known in an analytical form describes the phase retardation of each plane wave. This retardation depends on the propagation angle and the distance z. The output complex amplitude can be obtained as the superposition of the plane waves "corrected"

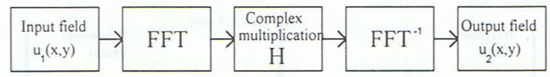

Fig. 2. Convolution approach algorithm.

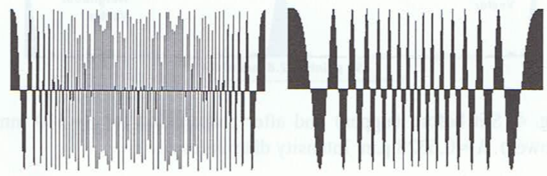

Fig. 3. Oscillations of the function H for distance $z = 1400$ mm (left) and 250 mm (right). Arbitrary units.

by function H. This last operation corresponds to reverse Fourier transformation. Fig. 2 presents the above algorithm.

A practical application of the described approach is convenient. Both direct and reverse Fourier transformation coordinates in the input (x_1, y_1) and output (x_2, y_2) plane are equivalent. Resampling or interpolations operations are not necessary. Moreover the phase of the transfer function $H(\nu_x, \nu_y)$, Eq. (9) linearly depends on the distance parameter z. For small values of the parameter z the phase is close to 0 and the whole factor reveals very slow oscillations.

Fig. 3 presents the cross section in the Fourier space (256 points) of the real part of the function $H(\nu_x, \nu_y)$ for wavelength $\lambda = 0.6328$ μm. The sampling distance in the object plane is equal to 50 μm. According to the properties of the FFT algorithm, the spatial frequency equal to zero is not in the middle of the vector of the samples (it is placed in the first vector element). It is easy to notice that for distances less than 1400 mm (for mentioned parameters) the convolution approach can be easy applied.

4. Errors in the convolution approach

Direct application of the convolution approach leads to two significant effect which can disturb the output plane complex amplitude distribution.

The first effect is caused by the properties of the Discrete Fourier Transformation (DFT). It is obvious that both object and Fourier spaces are sampled. According to the definition of the DFT and FFT [4], the object and spectrum vectors are limited sequences of complex numbers. The mentioned finite sequences

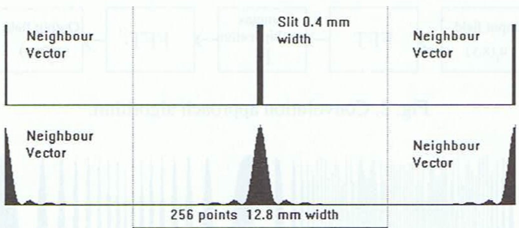

Fig. 4. Slit before (upper) and after propagation for $z=500$ mm (lower). $\lambda=0.6328$ μm. Intensity distributions.

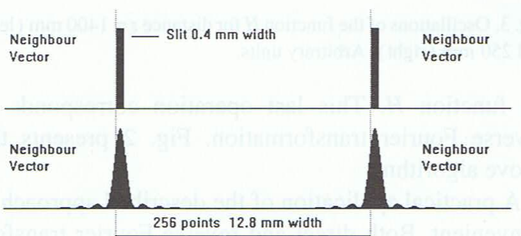

Fig. 5. Slit before (upper) and after propagation for $z=500$ mm (lower). $\lambda=0.6328$ μm. Intensity distributions.

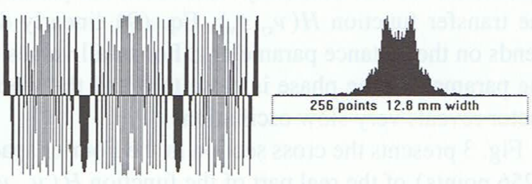

Fig. 6. Oscillations of the fourier transform of the Fresnel factor are to fast (left). This leads to the disturbance of the simulated picture (right).

are repeated infinitely in space. This effect is presented in Fig. 4 where a single vector is composed of 256 points and is surrounded by two neighbour vectors.

Usually during the propagation process the light field expands (this is true except some cases as e.g. convergation or Bessel beams) and misses the processing vector. We should remember that the same process can be also observed in the neighbour vectors. The light field flows to the neighbour vectors. The final result of the propagation simulation stored in one (middle) vector is disturbed. Fig. 5 shows the mentioned effect when the slit is placed at the edge of the vector.

The second effect is connected with the properties of the Fourier transform of the Fresnel factor. For an increasing distance z the function $H(\nu_x, \nu_y)$ described by Eq. (9) reveals fast oscillations different from the oscillations of the Fresnel factor given by Eq. (4). For a constant sampling frequency in the Fourier space the phase distribution of the transform of the Fresnel factor is "corrupted". Details of this phenomena for the sampling frequency too low comparatively to the fast phase oscillations are the same as in the case of low resolution Fresnel-encoded lenses [10].

Fig. 6 (left side) presents the cross section in the Fourier space (256 points) of the real part of the factor (9) for distance $z=3200$ mm and wavelength $\lambda=0.6328$ μm. The sampling distance in the object plane is equal to 50 μm. The oscillations are too fast in comparison to the sampling frequency. The transfer function is distorted and causes high spatial frequency noise in the light intensity distribution (right side, fast oscillations) at the distance $z=3200$ mm behind the slit. The slit width is equal to 3.2 mm.

5. Modified convolution approach

A new method proposed in this paper will remove errors described in the previous section.

To avoid the influence of the neighbours, the main processing vectors (length N) are embedded in another vector (length $2N$) of zero's. An analogical method is often used for computer generated Fourier type holograms [1].

One-dimensional propagation is simulated by a convolution of a vector of twice the length. For two-dimensional propagation the input field is stored in a two-dimensional matrix $N\times N$ (matrices $N\times M$ are also possible). A two-dimensional propagation simulation is realised by N one-dimensional propagation for rows of the matrix. Each row is embedded in the $2N$ length vector of zeros. Next N one-dimensional propagation for columns are realised. Each column is also embedded in the $2N$ length vector or zeros. By this algorithm (see Fig. 7) the computation time increases

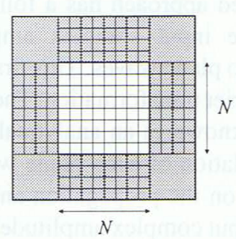

Fig. 7. Modified convolution approach for two dimensions.

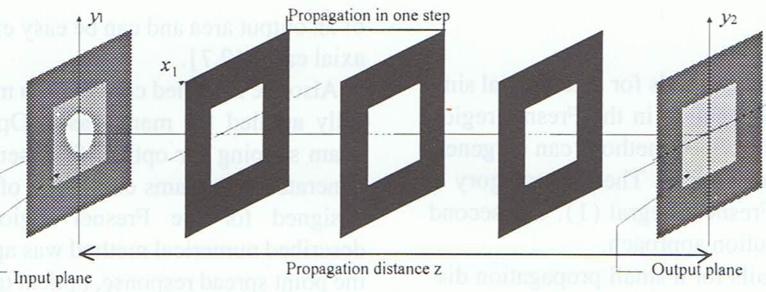

Fig. 8. Experimental model of the embedding of the vectors and division of the propagation distance into single steps.

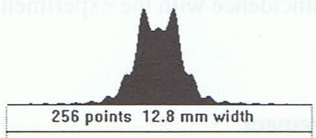

Fig. 9. Light intensity distribution behind the slit (width 3.2 mm) in the distance $z=3200$ mm obtained by modified convolution approach.

little more than twice (*for large N*) in comparison to the standard convolution method.

In the case of fast oscillations of the Fourier spectrum of the Fresnel factor, the whole propagation distance z can be divided into steps. According to the wavelength, sampling frequency and number of samples, the maximal propagation distance in a single step can be calculated.

The two modifications described above are equivalent to a set of opaque square pupils placed at constant distances along the optical axis in the optical setup (Fig. 8). However, diffraction on the edges of the pupils can disturb the output distribution for a large number of steps.

By the use of the modified convolution approach it is possible to improve the propagation simulation process. The following picture (Fig. 9) presents the light intensity distribution at the distance $z=3200$ behind a slit of width 3.2 mm. The slit is illuminated by a plane wave characterised by wavelength $\lambda=0.6328$ µm. The whole propagation distance was divided into two steps. Each step is equal to 1600 mm. In comparison to Fig. 6 fast oscillations are removed.

6. Experimental results

The modified convolution approach was verified in an experimental setup. A phase Fresnel zone plate with diameter 2.8 mm and focal length $f=74$ mm was placed in the arrangement presented below (Fig. 10). The intensity response of the point source was captured for defocusing parameter $W_{20}=4$. For picture analysis a Bischke CCD 500 camera was used. The dimensions of one pixel of the CCD array were equal to 12 µm. A He-Ne laser ($\lambda=0.6328$ µm) was utilised as a light source.

A parallel numerical simulation of the above arrangement was performed in a two-dimensional array composed of 512^2 complex points. For a sampling distance of 12 µm for both x and y directions the whole array corresponds to the square area dimensions 6.1×6.1 mm.

The results obtained in the experiment are in good coincidence with the results of the numerical simulation [12] (Fig. 11).

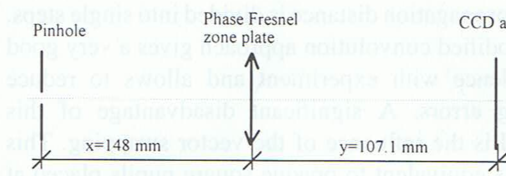

Fig. 10. Experimental setup.

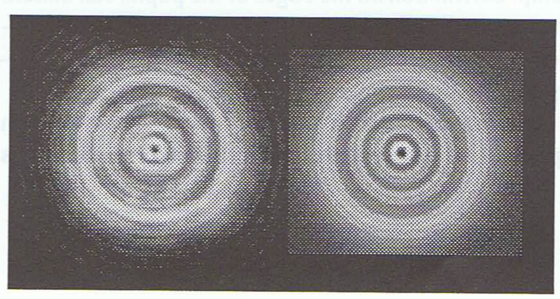

Fig. 11. Experimental (left) and numerical simulation (right) results.

7. Conclusions

In this paper different methods for a numerical simulation of the light propagation in the Fresnel region were compared. The described methods can be generally divided into two categories. The first category is based directly on the Fresnel integral (1). The second is based on the convolution approach.

A direct approach fails for a small propagation distance because of very fast oscillations of the Fresnel factor. Furthermore the output and input plane coordinates are not equivalent, what leads to resampling or interpolation operation. An advantage of this method is a relatively short computation time. Only one FFT operation and two complex vector multiplication are necessary.

The convolution approach also enables numerical calculations of the light complex amplitude in the Fresnel region. It is specially attractive for small and very small distances. A significant advantage of the method is the equivalence of the input and output plane coordinates. However, twice an FFT operation is used which results in a prolongation of the computation time.

Additionally a new modification of the convolution method is proposed. For the propagation calculation a double length vector for the convolution operation is used. According to the wavelength, sampling frequency and number of samples, the maximal propagation distance for one step can be calculated. The whole propagation distance is divided into single steps. The modified convolution approach gives a very good coincidence with experiment and allows to reduce aliasing errors. A significant disadvantage of this method is the influence of the vector swapping. This effect is equivalent to opaque square pupils placed at constant distances along the optical axis in the optical setup. Diffraction on the edges of the pupils can disturb the output distribution for many steps. Long computation time – twice an FFT on the double length vector – is also a considerable disadvantage.

It is worth to notice that the modified convolution approach allows to analyse a shift from the optical axis of an output area and can be easy extended for nonparaxial cases [2,7].

Also the modified convolution method was successfully applied for many cases. Optical structures for beam shaping for optical fibre sensors and computer generated holograms composed of a few planes were designed for the Fresnel region. Additionally a described numerical method was applied for analysing the point spread response, optical transfer function and imaging simulations of the unconventional imaging elements. In all above cases the numerical results gives very good coincidence with the experimental results.

Acknowledgement

The author would like to thank Andrzej Kolodziejczyk and Mariusz Krukowski for valuable discussions and technical help.

References

[1] O. Bryngdahl and F. Wyrowski, in: Progress in Optics, Vol. 28, ed. E. Wolf (North Holland, Amsterdam, 1990) pp. 1–86.
[2] W.T. Cathey, Optical Information Processing and Holography (Wiley, New York, 1974).
[3] J.W. Goodman, An Introduction to Fourier Optics (McGraw-Hill, New York, 1968).
[4] R. Bracewell, The Fourier Transform and Its Applications (McGraw-Hill, New York, 1965).
[5] J.W. Goodman, ed., Holography, SPIE Proc. Vol. 59, No. 9, September 1971 (special issue).
[6] U. Schnars and W. Juptner, Appl. Optics 33 (1994) 179.
[7] D. Leseberg, Appl. Optics 31 (1992) 223.
[8] D. Leseberg, J. Opt. Soc. Am. 6 (1989) 229.
[9] M. Bernardt, F. Wyrowski and O. Bryngdahl, Optics Comm. 77 (1990) 4.
[10] E. Carcole, J. Campos and S. Bosch, Appl. Optics 33 (1994) 162.
[11] K. Gniadek, Optical information processing (PWN, Warsaw 1992) (in Polish).
[12] W.H. Southwell, J. Opt. Soc. Am. 71 (1981) 7.