HW 4

(a) hyperplane can be written as:

$$w_1x_1 + w_2x_2 \cdots + w_nx_n + w_0 = 0$$

point x_a can be written as:

$$x_a\,(x_{a1}, x_{a2} \cdots x_{an})$$

Distance between x_a and hyperplane:

$$D = \frac{x_{a1}w_1 + x_{a2}w_2 \cdots + x_{an}w_n + w_0}{\sqrt{w_1^2 + w_2^2 \cdots + w_n^2}}$$

And:

$$\frac{|g(x_a)|}{||w||} = \frac{|w^T x_a + w_0|}{\sqrt{w_1^2 + w_2^2 \cdots + w_n^2}} = \frac{x_{a1}w_1 + x_{a2}w_2 \cdots + x_{an}w_n + w_0}{\sqrt{w_1^2 + w_2^2 \cdots + w_n^2}} = D$$

Thus: $D = \frac{|g(x_a)|}{||w||}$

(b) suppose w is the normal vecor for $g(x)=0$

$$x_p = x_a + tw \quad t \text{ is a scalar}$$

Due to x_p is on the hyperplane:

$$g(x_p) = w^T(x_a + tw) + w_0 = 0$$

$$w^T x_a + w^T t w + w_0 = 0$$

$$g(x_a) + t||w||^2 = 0$$

$$t = -\frac{g(x_a)}{||w||^2}$$

$$x_p = x_a - \frac{g(x_a)}{||w||^2} w$$