Tutorial - 4

Ques-1) $T(n) = 3T(n/2) + n^2$

$T(n) = aT(n/b) + f(n)$

$a \geq 1, b \geq 1$

on comparing

$a=3, b=2, f(n)=n^2$

Now; $c = \log_b a = \log_2 3 = 1.584$

$n^c = n^{1.584} < n^2$

$f(n) > n^c$

$\boxed{T(n) = \theta(n^2)}$

Ques-2) $T(n) = 4T(n/2) + n^2$

$a \geq 1, b \geq 1$

$a=4, b=2, f(n)=n^2$

$c = \log_2 4 = 2$

$n^c = n^2 = f(n) = n^2$

$\boxed{T(n) = \theta(n^2 \log_2 n)}$

Ques-3) $T(n) = T(n/2) + 2^n$

$a=1$

$b=2$

$f(n) = 2^n$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$f(n) > n^c$

$\boxed{T(n) = (2^n)}$

Ques-4) $T(n) = 2^n + (n/2) + n^n$

$a = 2^n, b=2, f(n) = n^n$

$c = \log_b a = \log_2 2^n = n$

$n^c = n^n$

$f(n) = n^c$

$\boxed{T(n) = \theta(n^2 \log_2 n)}$

Ques-5) $T(n) = 16T\left(\frac{n}{4}\right) + n$.

$a=16, b=4, f(n)=n$

$c = \log_4 16 = 2$

$n^c = n^2$

$f(n) < n^c$

$\boxed{T(n) = \theta(n^2)}$

Ques-6) $T(n) = 2T(n/2) + n\log n$.

$a=2, b=2$

$f(n) = n\log n$

$c = \log 2 = 1$

$n^c = n^1 = n$

$n\log n > n$

$f(n) > n^c$

$\boxed{T(n) = \theta(n \log n)}$

Ques-7) $T(n) = 2T(n/2) + n/\log n$

$a=2, b=2, f(n) = n/\log n$

$c = \log_2 2 = 1$

$n^c = n^1 = n$

$\frac{n}{\log n} < n$

$f(n) < n^c$

$\boxed{T(n) = \theta(n)}$

Que 8) $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$

$a=2, b=4, f(n)=n^{0.5}$

$c = \log_b a = \log_4^2 = 0.5$

$n^c = n^{0.5}$

$n^{0.5} < n^{0.51}$

$f(n) > n^c$

$\therefore$ $\boxed{T(n) = \theta(n^{0.51})}$

Que 9) $T(n) = 0.5T(n/2) + \frac{1}{n}$

$a=0.5, b=2$

$a \geq 1$, but here a is 0.5

So we cannot apply Masters theorem.

Que 10) $T(n) = 16T(n/4) + n!$

$a=16, b=4$ $f(n)=n!$

$c = \log_b a = \log_4^{16} = 2$

$n^c = n^2$

As $n! > n^2$

$\boxed{T(n) = \theta(n!)}$

Que 11) $4T(n/2) + \log n$

$a=4, b=2, f(n)=\log n$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$f(n)$ ~~$\log n$~~ $\log n$.

~~If not~~ $\log n < n^2$

$f(n) < n^c$

$T(n) = \theta(n^c)$

$T(n) = \boxed{\theta(n^2)}$

Que 12) $T(n) = \text{Sqrt}(n) + (n/2) + \log n$

$a = \sqrt{n}, b=2$

$c = \log_b a = \log_2 \sqrt{n} = \frac{1}{2}\log_2 n$

$\because \frac{1}{2}\log_2 n < \log(n)$

$f(n) > n^c$

$\boxed{T(n) = \theta(f(n)) = \theta(\log(n))}$

Que 13) $T(n) = 3T(n/2) + n$

$\rightarrow a=3; b=2; f(n)=n$

$c = \log_b a = \log_2 3 = 1.5849$

$n^c = n^{1.5849}$

$n < n^{1.5849}$

$f(n) < n^c$

$\boxed{T(n) = \theta(n^{1.5849})}$

Que 14) $T(n) = 3T(n/3) + \text{Sqrt}(n)$

$a=3, b=3$

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

As $\text{Sqrt}(n) < n$

$f(n) < n^c$

$\boxed{T(n) = \theta(n)}$

Que 15) $T(n) = 4T(n/2) + n$

$a=4, b=2$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$n < n^2$ (for any constant)

$f(n) < n^c$

$\boxed{f(n) = \theta(n^2)}$

Ques-16) $T(n) = 3T(n/4) + n\log n$

$a=3,\ b=4,\ f(n)=n\log n$

$c=\log_b a = \log_4 3 = 0.792$

$n^c = n^{0.792}$

$n^{0.792} < n\log n$

$\boxed{T(n)=\theta(n\log n)}$

Ques-17) $T(n) = 3T(n/3) + n/2$

$a=3,\ b=3$

$c=\log_b a = \log_3 3 = 1$

$f(n)=n/2$

$\therefore n^c = n^1 = n$

As $n/2 < n$

$f(n) < n^c$

$\boxed{T(n)=\theta(n)}$

Ques-18) $T(n) = 6T(n/3) + n^2\log n$

$a=6,\ b=3,\ f(n)=n^2\log n$

$c=\log_b a = \log_3 6 = 1.6309$

$n^c = n^{1.6309}$

As $n^{1.6309} < n^2\log n$

$\therefore \boxed{T(n)=\theta(n^2\log n)}$

Ques-19) $T(n) = 4T(n/2) + n/\log n$

$a=4,\ b=2,\ f(n)=\frac{n}{\log n}$

$c=\log_b a = \log_2 4 = 2$

$n^c = n^2$

$\frac{n}{\log n} < n^2$

$\boxed{T(n)=\theta(n^2)}$

Ques-20) $T(n) = 64T(n/8) - n^2\log n$

$a=64 \quad b=8$

$c=\log_b a = \log_8 64 = \log_8 (8)^2$

$c=2$

$nf = n^2$

$n^2\log n > n^2$

$\boxed{T(n)=\theta(n^2\log n)}$

Ques-21) $T(n) = 7T(n/3) + n^2$

$a=7,\ b=3,\ f(n)=n^2$

$c=\log_b a = \log_3 7 = 1.7712$

$n^c = n^{1.7712}$

$n^{1.7712} < n^2$

$\boxed{T(n)=\theta(n^2)}$

Ques-22) $T(n) = T(n/2) + n(2-\cos n)$

$a=1,\ b=2$

$c=\log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$n(2-\cos n) > n^c$

$\boxed{T(n)=\theta(n(2-\cos n))}$