Tutorial-6

Ques-1) What do you mean by minimal Spanning Tree? what are the application of MST.

Minimal Spanning Tree is a Sub-set of edges of a Connected edge weighted undirected graph that Connects all the Vertices together without any Cycles and with minimum possible edge weighted.

APPLICATIONS:-

1) Consider n stations are to be linked using a Communication network and laying of communication link b/w any two stations involves a cost. The ideal solution would be extract a Sub group termed as minimal cost Spanning Tree.

2) Designing LAN

3) Highways or railways b/w Cities

4) Laying pipelines Connecting offshore drilling sites, refineries &

Ques-2) Analyze time and space Complexity of Prim, Krushkal, Dijkstra's and Bellman-Ford

Prim's TC - $O(|E| \log|V|)$

SC - $O(|V|)$

Krushkal's TC - $O(|E| \log|E|)$

SC - $O(|V|)$

Dijkstra's TC = $O(V^2)$

SC = $O(V^2)$

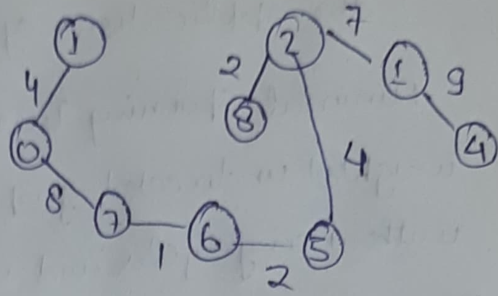
Bellmann Ford's TC - $O(VE)$

SC - $O(E)$

Ques-3) Apply Krushkal's & Prim's Algorithm on given graph to Compare MST & its weight?

Krushkal's.

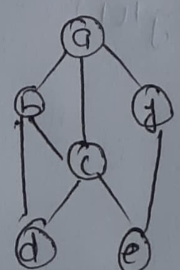
U	V	W	
6	7	1	✓
5	6	2	✓
2	8	2	✓
0	1	4	✓
2	5	4	✓
6	8	6	✓
2	3	7	✓
1	8	7	✓
0	7	8	✓
1	2	8	✓
4	3	9	✓
4	5	10	✗
1	7	11	✗
3	5	14	✗



weight = 1+2+2+4+4+7+8+9
= 37

Prim's. weight = 4+8+2+4+2+7+9+3
= 37

Ques-4) Given a sorted graph,
i) If weight of every edge is increased by 10 units.
ii) If weight of every edge is multiplied by 10 units.

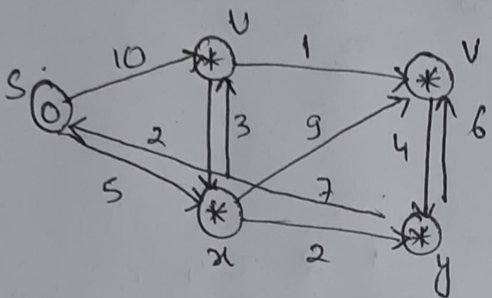


1) The shortest path may change. The reason is that there may be different no. of edges in different paths from 's' to 't'

e.g.- Let the shortest path of weight 15 & has edge 5. Let there be another path with 2 edges & total weight 25. The weight of shortest path is increased by 5*10 & becomes 15+50, weight of other path is increased by 2*10 & b/w 25+20, So shortest path changes to other path with weight as 45.

(ii) If we multiply all edges weight by 10, the shortest path remains the same. The weight of all paths from 'S' to 't' gets multiplied by same unit. The number of edges or path doesn't matter.

Ques 5) Apply Dijkstra's and Bellman Ford Algorithm on graph given right side to compare shortest path to all nodes from node S



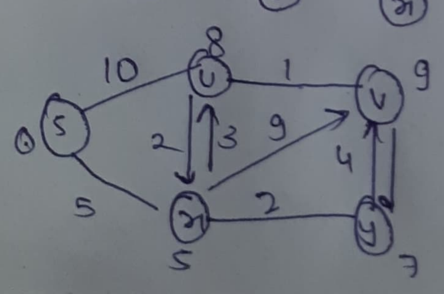
Dijkstra's Algo:-

Node	Shortest dist from Source node
u	8
x	5
v	9
y	7

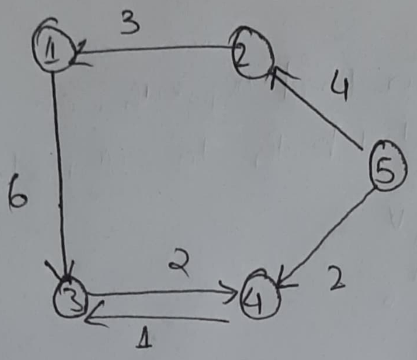
Bellman Ford's Algo

	S	u	v	x	y
1st -	0	~~∞~~ 10	∞	~~∞~~ 5	∞
2nd -	0	10	~~∞~~ 11	5	∞
3rd	0	~~10~~ 8	~~11~~ 9	5	~~∞~~ 7
4th	0	8	9	5	7

Graph does not have -ve cycle.



Ques 6) Apply all pair Shortest path algorithm - Floyd Warshall on below mentioned graph, Also analyze Space & time complexity of it



$$
\begin{array}{c|c}
 & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\
\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} &
\begin{bmatrix}
0 & \infty & 6 & 3 & \infty \\
2 & 0 & \infty & \infty & \infty \\
\infty & \infty & 0 & 2 & \infty \\
\infty & 1 & 1 & 0 & \infty \\
\infty & 4 & \infty & 2 & \infty
\end{bmatrix}
\end{array}
\qquad
\begin{array}{c|c}
 & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\
\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} &
\begin{bmatrix}
0 & \infty & 6 & 3 & \infty \\
2 & 0 & 8 & 5 & \infty \\
\infty & \infty & 0 & 2 & \infty \\
\infty & 1 & 1 & 0 & \infty \\
\infty & 4 & \infty & 2 & 0
\end{bmatrix}
\end{array}
$$

$$
\begin{array}{c|c}
 & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\
\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} &
\begin{bmatrix}
0 & \infty & 6 & 3 & \infty \\
2 & 0 & 8 & 5 & \infty \\
\infty & \infty & 0 & 2 & \infty \\
3 & 1 & 1 & 0 & \infty \\
6 & 4 & 12 & 2 & 0
\end{bmatrix}
\end{array}
\qquad
\begin{array}{c|c}
 & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\
\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} &
\begin{bmatrix}
0 & \infty & 6 & 3 & \infty \\
2 & 0 & 8 & 5 & \infty \\
\infty & \infty & 0 & 2 & \infty \\
3 & 1 & 1 & 0 & \infty \\
6 & 4 & 12 & 2 & 0
\end{bmatrix}
\end{array}
$$

$T.C = O(|V|^3)$

$S.C = O(|V|^2)$