

Dynamic Programming | Set 20 (Maximum Length Chain of Pairs)

You are given n pairs of numbers. In every pair, the first number is always smaller than the second number. A pair (c, d) can follow another pair (a, b) if $b < c$. Chain of pairs can be formed in this fashion. Find the longest chain which can be formed from a given set of pairs. Source: [Amazon Interview | Set 2](#)

For example, if the given pairs are $\{\{5, 24\}, \{39, 60\}, \{15, 28\}, \{27, 40\}, \{50, 90\}\}$, then the longest chain that can be formed is of length 3, and the chain is $\{\{5, 24\}, \{27, 40\}, \{50, 90\}\}$

This problem is a variation of standard [Longest Increasing Subsequence](#) problem. Following is a simple two step process.

- 1) Sort given pairs in increasing order of first (or smaller) element.
- 2) Now run a modified LIS process where we compare the second element of already finalized LIS with the first element of new LIS being constructed.

The following code is a slight modification of method 2 of [this post](#).

```
#include<stdio.h>
#include<stdlib.h>

// Structure for a pair
struct pair
{
    int a;
    int b;
};

// This function assumes that arr[] is sorted in increasing order
// according the first (or smaller) values in pairs.
int maxChainLength( struct pair arr[], int n)
{
    int i, j, max = 0;
    int *mcl = (int*) malloc ( sizeof( int ) * n );

    /* Initialize MCL (max chain length) values for all indexes */
    for ( i = 0; i < n; i++ )
        mcl[i] = 1;

    /* Compute optimized chain length values in bottom up manner */
    for ( i = 1; i < n; i++ )
        for ( j = 0; j < i; j++ )
            if ( arr[i].a > arr[j].b && mcl[i] < mcl[j] + 1)
                mcl[i] = mcl[j] + 1;

    // mcl[i] now stores the maximum chain length ending with pair i
```

```
/* Pick maximum of all MCL values */
for ( i = 0; i < n; i++ )
    if ( max < mcl[i] )
        max = mcl[i];

/* Free memory to avoid memory leak */
free( mcl );

return max;
}

/* Driver program to test above function */
int main()
{
    struct pair arr[] = { {5, 24}, {15, 25},
                          {27, 40}, {50, 60} };
    int n = sizeof(arr)/sizeof(arr[0]);
    printf("Length of maximum size chain is %d\n",
           maxChainLength( arr, n ));
    return 0;
}
```

[Run on IDE](#)

Output:

```
Length of maximum size chain is 3
```

Time Complexity: $O(n^2)$ where n is the number of pairs.

The given problem is also a variation of [Activity Selection problem](#) and can be solved in $(n \log n)$ time. To solve it as a activity selection problem, consider the first element of a pair as start time in activity selection problem, and the second element of pair as end time. Thanks to Palash for suggesting this approach.

Please write comments if you find anything incorrect, or you want to share more information about the topic discussed above.

[56 Comments](#) Category: [Arrays](#) Tags: [Dynamic Programming](#)

Related Posts:

- [Find four elements a, b, c and d in an array such that \$a+b = c+d\$](#)
- [Find three closest elements from given three sorted arrays](#)
- [How to check if a given array represents a Binary Heap?](#)
- [Find the longest path in a matrix with given constraints](#)
- [Given a Boolean Matrix, find \$k\$ such that all elements in \$k\$ 'th row are 0 and \$k\$ 'th column are 1.](#)
- [Find maximum value of \$\text{Sum}\(i \cdot \text{arr}\[i\]\)\$ with only rotations on given array allowed](#)
- [Given an array of pairs, find all symmetric pairs in it](#)