OI lab 6

Zadatak 1

Matrica plaćanja

Igrač A		Igrač B b_1	b_2	b_3
	a_1	2	5	4
	a_2	3	6	2
	a_3	1	8	6

→ za određivanje vektora p potrebno je riješiti problem lin. prog. iz slj. postavke:

$$\arg\min\ x_1 + x_2 + x_3$$

$$\text{p.o.}\quad 2x_1 + 3x_2 + x_3 \geq 1$$

$$5x_1 + 6x_2 + 8x_3 \geq 1$$

$$4x_1 + 2x_2 + 6x_3 \geq 1$$

$$x_{1,2,3} \geq 0$$

→ da bismo odredili mješovitu strategiju B igrača (q-vektor) rješavamo slj. problem:

$$\arg\max\ y_1 + y_2 + y_3$$

$$\text{p.o.}\quad 2y_1 + 5y_2 + 2y_3 \leq 1$$

$$3y_1 + 6y_2 + 2y_3 \leq 1$$

$$y_1 + 8y_2 + 6y_3 \leq 1$$

$$y_{1,2,3} \geq 0$$

→ sada rješavamo problem simplex metodom:

$$\arg\max\ y_1 + y_2 + y_3$$

$$\text{p.o.}\quad 2y_1 + 5y_2 + 4y_3 + y_4 = 1$$

$$3y_1 + 6y_2 + 2y_3 + y_5 = 1$$

$$y_1 + 8y_2 + 6y_3 + y_6 = 1$$

$$y_{1,2,3,4,5,6} \geq 0$$

	b	y_1	y_2	y_3	y_4	y_5	y_6	
y_4	1	2	5	4	1	0	0	$1:2=0.5$
y_5	1	3	6	2	0	1	0	$1:3=0.33$ ←
y_6	1	1	8	6	0	0	1	
	0	1	1	1	0	0	0	

↑

	b	y_1	y_2	y_3	y_4	y_5	y_6	
y_4	$\frac{1}{3}$	0	1	$\frac{8}{3}$	1	$-\frac{2}{3}$	0	←
y_1	$\frac{1}{3}$	1	2	$\frac{2}{3}$	0	$\frac{1}{3}$	0	
y_6	$\frac{1}{3}$	0	6	$\frac{16}{3}$	$-\frac{1}{3}$	0	1	
	$-\frac{1}{3}$	0	-1	$\frac{1}{3}$	0	$-\frac{1}{3}$	0	

↑

	b	y_1	y_2	y_3	y_4	y_5	y_6
y_3	$\frac{1}{8}$	0	$\frac{3}{8}$	1	$\frac{3}{8}$	$-\frac{1}{4}$	0
y_1	$\frac{1}{4}$	1	$\frac{7}{4}$	0	0	$\frac{1}{2}$	0
y_6	0	0	4	0	$-\frac{7}{3}$	$\frac{4}{3}$	1
	$-\frac{3}{8}$	0	$-\frac{9}{8}$	0	$-\frac{1}{8}$	$-\frac{1}{4}$	0

Konačno rješenje: $(y_1, y_2, y_3) = \left(\frac{1}{4}, 0, \frac{1}{8}\right) \Rightarrow$ igrač B

$(x_1, x_2, x_3) = \left(\frac{1}{8}, \frac{1}{4}, 0\right) \Rightarrow$ igrač A

Vrijednost igre: $v = \hat{v} = \dfrac{1}{y_1+y_2+y_3} = \dfrac{8}{3}$

$q_1 = \hat{V} \cdot y_1 = \frac{8}{3} \cdot \frac{1}{4} = \frac{2}{3}$

$q_2 = \hat{V} \cdot y_2 = \frac{8}{3} \cdot 0 = 0$ $\qquad q = \left(\frac{2}{3} \quad 0 \quad \frac{1}{3}\right)^T$

$q_3 = \hat{V} \cdot y_3 = \frac{8}{3} \cdot \frac{1}{8} = \frac{1}{3}$

$p_1 = \hat{V} \cdot x_1 = \frac{1}{3}$

$p_2 = \hat{V} \cdot x_2 = \frac{2}{3}$ $\qquad p = \left(\frac{1}{3} \quad \frac{2}{3} \quad 0\right)^T$

$p_3 = \hat{V} \cdot x_3 = 0$

→ da bismo vidjeli da li je moguće riješiti problem korištenjem min max principa moramo naći donju i gornju granicu vrijednosti igre, tj. u i w.

	b_1	b_2	b_3	min
a_1	2	5	4	2
a_2	3	6	2	2
a_3	1	8	6	1
max	3	8	6	2

$u = \max\min c_{ij} = \max\{2, 2, 1\} = 2$

$w = \min\max c_{ij} = \min\{3, 8, 6\} = 3$

$\Rightarrow u \neq w \Rightarrow$ ne postoji sedlasta tačka igre, te minmax strategija ne daje rješenje?

→ Dominantna strategija:

$$\begin{pmatrix} 2 & 5 & 4 \\ 3 & 6 & 2 \\ 1 & 8 & 6 \end{pmatrix} \Rightarrow \text{nema dominantnih redova, dominira kolona 2 u odnosu na 1}$$

$$\downarrow$$

$$\begin{pmatrix} 2 & 4 \\ 3 & 2 \\ 1 & 6 \end{pmatrix} \rightarrow \text{finalna reducirana matrica}$$