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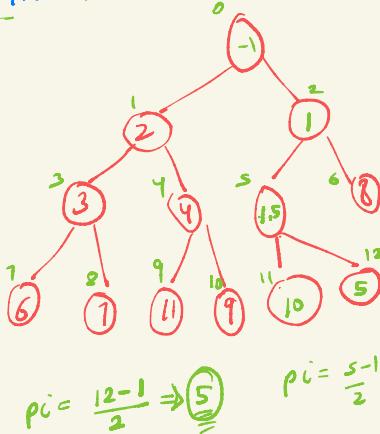
# # Heap [min pq]

Complete Binary Tree ✓CBT

HOP  
Heap order property

why?  
what?  
how?

12



$$pi = \frac{12-1}{2} \Rightarrow 5$$

(-1)  
upHeapify

→ every parent will be smaller than child

{-1, 2, 1, 3, 4, 1.5, 8, 6, 7, 11, 9, 10, 12, 5}

$$pi = 4$$

$$lci = 2 * pi + 1; \\ rci = 2 * pi + 2;$$

$$swap(c_i, p_i)$$

$$lci = 9 \\ rci = 10$$

$$pi = \frac{ci-1}{2}$$

```
static class Heap {
    ArrayList<Integer> data;
    public Heap(){
        data = new ArrayList<>();
    }

    public void swap(int i, int j){
        int valAtI = data.get(i);
        int valAtJ = data.get(j);

        data.set(i, valAtJ);
        data.set(j, valAtI);
    }

    public void upHeapify(int ci){
        int pi = (ci-1)/2;

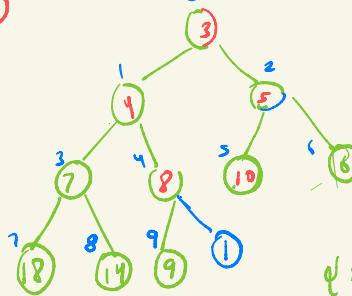
        if(data.get(pi) > data.get(ci)){
            swap(ci, pi);
            upHeapify(pi);
        }
    }
    // add
    public void add(int val){
        data.add(val);

        upHeapify(data.size()-1);
    }
}
```

data ⇒ {3, 4, 5, 7, 8, 10, 6, 18, 14, 9}

(-1)

O(n)



$$pi = 0 \\ lci = 2 * pi + 1 = 1 \\ rci = 2 * pi + 2 = 2$$

$$\{0, 13\} \\ pi_{min} \\ lci = 3 \\ rci = 4$$

{1, 4}

downHeapify

```

private void downHeapify(int pi){
    int lci = 2*pi + 1;
    int rci = 2*pi + 2;

    int mini = pi;

    if(lci < data.size() && data.get(lci) < data.get(mini)){
        mini = lci;
    }

    if(rci < data.size() && data.get(rci) < data.get(mini)){
        mini = rci;
    }

    if(mini != pi){
        swap(pi,mini);
        downHeapify(mini);
    }
}

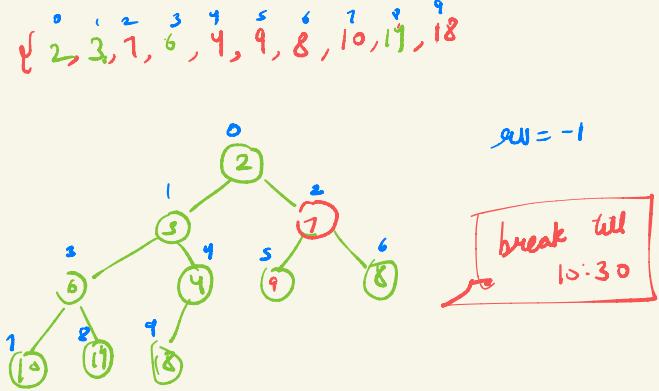
// remove
public int remove(){
    if(data.size()==0){
        System.out.println("There is no element in heap");
        return -1;
    }

    swap(i: 0,data.size()-1);
    int rv = data.remove(data.size()-1);

    downHeapify(pi: 0);
}

return rv;
}

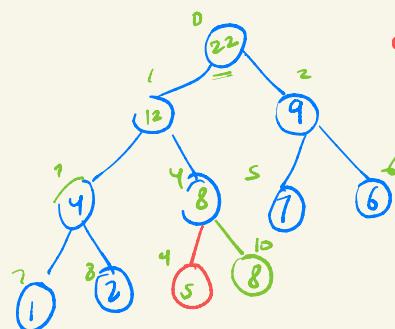
```



$$\min i = \cancel{0} + \cancel{3} + 8$$

$$\begin{aligned} pi &= \cancel{0} + \cancel{3} \\ lci &= \cancel{1} + \cancel{2} + 7 \\ rci &= \cancel{4} + 8 \end{aligned}$$

$$\begin{aligned} ci &= \cancel{0} + \cancel{1} + \cancel{2} + 0 \\ pi &= \cancel{0} + \cancel{1} + \cancel{2} - 1 \end{aligned}$$



6, 4, 1, 3, 10, 9, 17, 8, 13

{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 }

```

private void swap(int i, int j){
    int valAtI = data.get(i);
    int valAtJ = data.get(j);

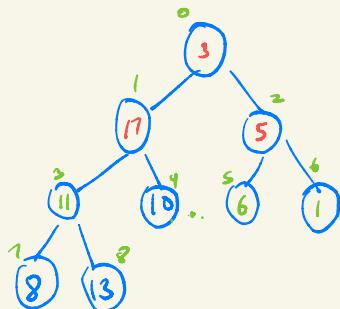
    data.set(i, valAtJ);
    data.set(j, valAtI);
}

private void upHeapify(int ci){
    int pi = (ci-1)/2;
    if(pi>=0 && data.get(pi) < data.get(ci)){
        swap(ci,pi);
        upHeapify(pi);
    }
}

public void add(int val){
    data.add(val);
    upHeapify(data.size()-1);
}

```

Hop



arr ⇒ {3, 17, 5, 11, 10, 6, 1, 8, 13}

(pq)

```

void swap(int[] data, int i, int j){
    int temp = data[i];
    data[i] = data[j];
    data[j] = temp;
}

void upHeapify(int[] data, int ci){
    int pi = (ci-1)/2;

    if(pi >= 0 && data[pi] > data[ci]){
        swap(data, pi, ci);
        upHeapify(data, pi);
    }
}

void buildHeap(int arr[]){
    for(int i=0; i<arr.length; i++){
        upHeapify(arr, i);
    }
}

```

## # Priority Queue

Priority Queue < Integer > pq = new Priority Queue<=>;  
↳ min pq {min heap}

pq.add(3)  
pq.add(4)

pq.peek();

pq.add(2);

pq.remove()  
pq.remove

Comparable  
↓  
Interface  
comparator

s1, s2

```

class Student {
    int roll-no;
    String name;
}

```

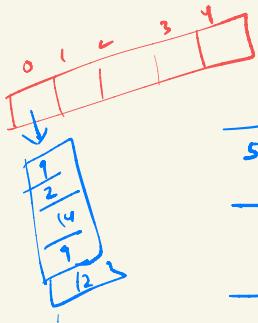
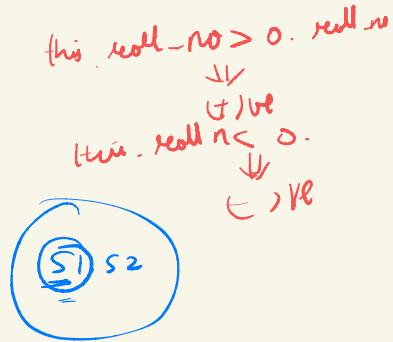
$s1 = \{1, 3\}$

$s2 = \{2, 3\}$

```
public int compareTo(Student o){
    return this.roll_no - o.roll_no;
}
```

$$1 - 2 \Rightarrow -1$$

smaller on the top  $\Rightarrow$  this - other  
 greater on the top  $\Rightarrow$  other - this



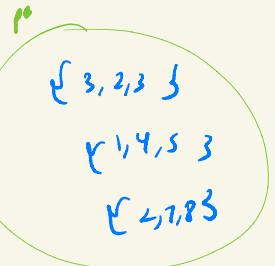
↓ matrix

5	9	2	14	9	12	8
1	1	5	10	11	7	13
4	5	7	15	6	17	18
2	3	1	19	16	5	4
3	4	8	9	1	2	3

5 10 arrays

matrix

$pQ < \underline{\underline{\text{int}[j]}} > pQ$   
 = new  $pQ < 70;$



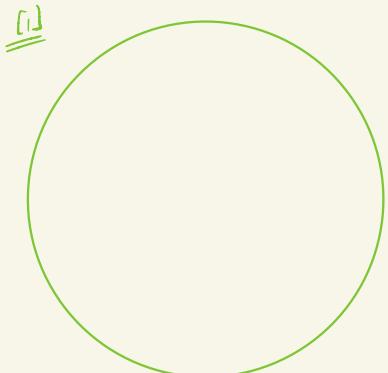
```

int[][] matrix={{3,2,3},{1,4,5},{2,7,8}};
PriorityQueue<int[]> pq = new PriorityQueue<>((int[] t, int[] o) ->{
    // return t[0] - o[0]; // min pq
    return o[1] - t[1];
});
for(int i=0; i<matrix.length; i++){
    pq.add(matrix[i]);
}
int[] top=pq.remove();
for(int i=0; i<top.length; i++){
    System.out.print(top[i]+" ");
}

```

3, 2, 3

{3, 2, 3}  
{1, 4, 5}  
{2, 7, 8}  
{1, 4, 5}



```

public int compareTo(Student other){
    return this.roll_no - other.roll_no; // min pq
    // return other.roll_no - this.roll_no //=> max pq
}

```

...  
public static int findKthLargest(int[] arr, int n, int k){  
 PriorityQueue<Integer> pq = new PriorityQueue<>();  
  
 for(int i=0; i<n; i++){  
 pq.add(arr[i]);  
  
 if(pq.size() > k){  
 pq.remove();  
 }
 }
 return pq.peek();
}

...  
public static int findKthSmallest(int[] arr, int n, int k){  
 PriorityQueue<Integer> pq = new PriorityQueue<>(Collections.reverseOrder());  
  
 for(int i=0; i<n; i++){  
 pq.add(arr[i]);  
  
 if(pq.size() > k){  
 pq.remove();  
 }
 }
 return pq.peek();
}

6  
7 10 4 3 20 15  
4  
1, 2, 3 → smallest elements  
③

5, 1, 10, 9, 9, 3, 11, 9, 12  
↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑

10:30  
break

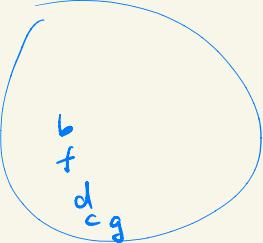
a a a b b c d e f e g e f e e

$\rightarrow a=3$   
 $\rightarrow b=2$   
 $c=1$   
 $d=1$   
 $e=5$   
 $f=2$   
 $g=1$

map.get(other)  
map.get(this)

$a > b$

h = b  
 $fre=2$   
 $f$



```

public String frequencySort(String s) {
    HashMap<Character, Integer> map = new HashMap<>();
    for(int i=0; i<s.length(); i++){
        char ch = s.charAt(i);
        map.put(ch, map.getOrDefault(ch, defaultValue: 0)+1);
        // if map.containskey(ch){
        //     map.put(ch, map.get(ch)+1);
        // } else {
        //     map.put(ch, 1);
        // }
    }

    ArrayList<Character> keys = new ArrayList<>(map.keySet());
    PriorityQueue<Character> pq = new PriorityQueue<>((Character t, Character o) -> {
        return map.get(o) - map.get(t);
    });

    for(char ch : keys){
        pq.add(ch);
    }

    StringBuilder sb = new StringBuilder();
    while(pq.size() > 0){
        char ch = pq.remove();
        int fre = map.get(ch);

        while(fre-- > 0){
            sb.append(ch);
        }
    }

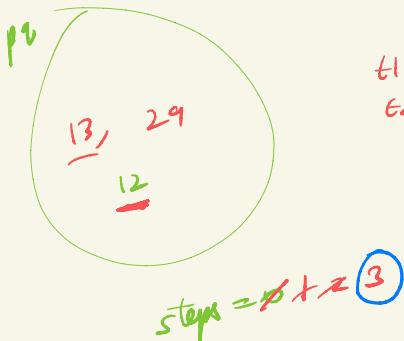
    return sb.toString();
}

```

sb.append  
 $sb = e \underline{e} \underline{e} \underline{e} \underline{e} a a a b b \underline{f} \underline{f} \underline{d} \underline{c}$   
 $\Rightarrow e e e e a a a f f h l g d e$

6 7  
 $\underline{\underline{h=10}}$

1 2 3 9 10 12



$t_1 = 9$   
 $t_2 = 10$   
 $1 \times 9 + 2 \times 10$

```

static int solve(int a[], int n, int k){
    PriorityQueue<Integer> pq = new PriorityQueue<>();

    for(int e:a){
        pq.add(e);
    }

    int steps = 0;
    while(pq.size() > 1){
        if(pq.peek() >= k) return steps;

        int t1 = pq.remove();
        int t2 = pq.remove();

        int newSweetness = 1*t1 + 2*t2;
        pq.add(newSweetness);

        steps++;
    }

    if(pq.peek() < k) return -1;
    return steps;
}

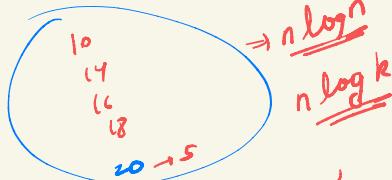
```

$10 < 20$     ans =  $\{ 10, 14, 16, 18, 20 \}$      $x=15$   
 $\downarrow$        $\downarrow$        $\downarrow$        $\downarrow$        $\downarrow$   
 $5$        $=$        $5$        $k=4$   
 $\Sigma$        $(10, 20)$

$n = 10^8$   
 $k = 10, 20$   
 $n \log k$

14  $\leq$  18

element with smaller dist with  $x$



$ans = \{ 14, 16, 18, 10 \}$   
 $\{ 10, 14, 16, 18 \}$

```

public List<Integer> findClosestElements(int[] arr, int k, int x) {
    PriorityQueue<Integer> pq = new PriorityQueue<>((Integer t, Integer o) -> {
        if(Math.abs(t-x) == Math.abs(o-x)){
            return t - o;
        }
        return Math.abs(t-x) - Math.abs(o-x);
    });

    for(int e:arr){
        pq.add(e);
    }

    List<Integer> ans = new ArrayList<>();
    while(ans.size() < k){
        ans.add(pq.remove());
    }

    Collections.sort(ans);
    return ans;
}

```

$this - other$   
 $\downarrow$   
 $abs(this - x) - abs(other - x)$

$k=3$

$\{ 5, 1, 3, 7, 2, 4, 11 \}$

$\{ 1, 2, 3 \}$

$k$  smallest elements  $\Rightarrow n \log n$  (min pq)  
 $n \log k$  (min pq)

