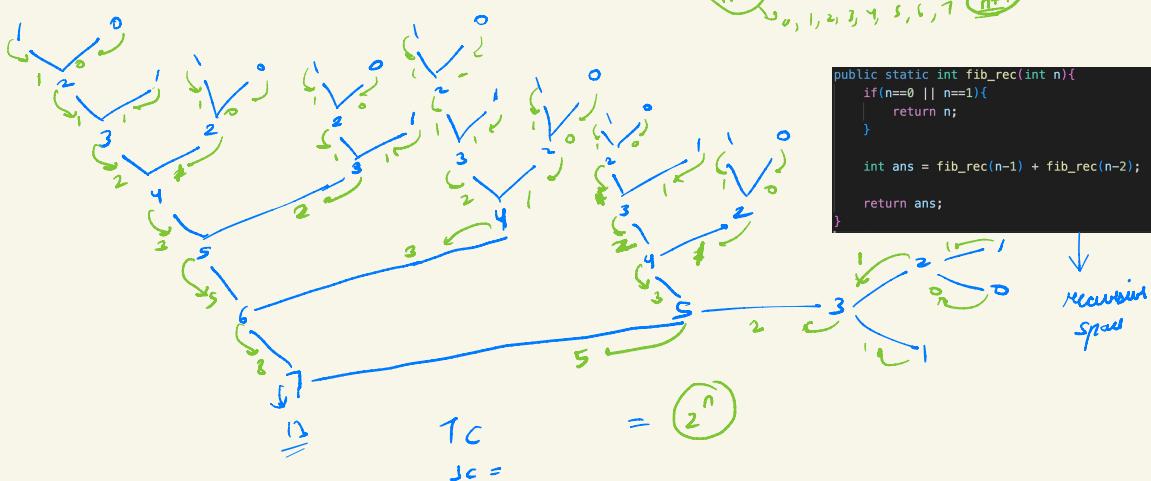
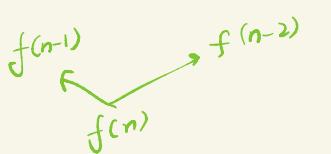



Get the n^{th} term
 $\Rightarrow 0, 1, 1, \textcircled{2}, 3, 5, 8, 13, 21, 34, 55, 89, \dots$
 $2 \rightarrow 1, 3 \rightarrow 2$ memorization



```
public static int fib_rec(int n){
    if(n==0 || n==1){
        return n;
    }
    int ans = fib_rec(n-1) + fib_rec(n-2);
    return ans;
}
```

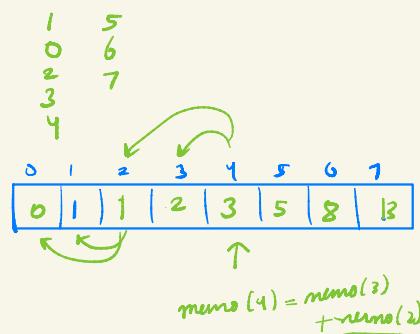
overlapping subproblems



recursion space
 $O(n)$



$T_C = O(n)$
 $SC = O(n) + \text{rec. sp.}$



```
public static int fib_memo(int n, int[] memo){
    if(n==0 || n==1){
        return memo[n]=n;
    }
    if(memo[n]!=0) return memo[n];
    int ans = fib_memo(n-1,memo) + fib_memo(n-2,memo);
    return memo[n]=ans;
}
```

- 1) Find the flow, write the for loop
- 2) Replace return with continue;
- 3) Remove memorization line {why?}
- 4) Replace calls with df.

```

public static int tab(int N, int[] memo){
    for(int n=0; n<=N; n++){
        if(n==0 || n==1){
            memo[n]=n;
            continue;
        }

        // if(memo[n]!=0) return memo[n];

        int ans = memo[n-1] + memo[n-2]; //fib_memo(n-1,memo) + fib_memo(n-2,memo)

        memo[n]=ans;
    }

    return memo[N];
}

```

$$n=0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12$$

$$Tc = O(n)$$

$$Sc = O(1)$$

0	1	2	3	4	5	6	7	8	9	10	11	12
0	1	1	2	3	5	8	13	21	34	55	89	144

```

public static int tabulation(int n){
    int[] dp = new int[n+1];

    dp[0] = 0;
    dp[1] = 1;

    for(int i=2; i<=n; i++){
        dp[i] = dp[i-1] + dp[i-2];
    }

    return dp[n];
}

```

Recursion (↴)

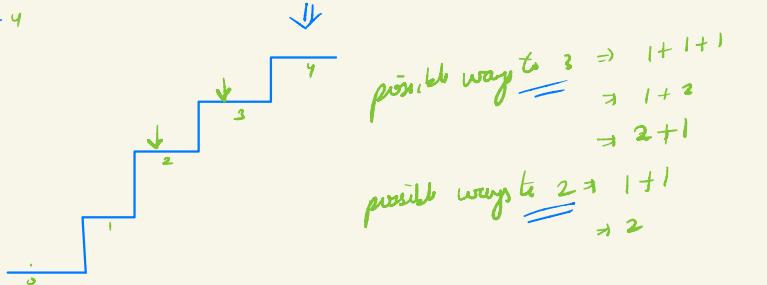


Memorization () add two lines

↓
Tabulation ()

$n=4$

0, 1, 1, 2, 3, 5, 8



$$\text{ways}(n) = \text{ways}(s) + \text{ways}(w)$$

↓ ↓

1 2

$$f(n) = f(n-1) + f(n-2)$$

↳ fibonacci

possible ways to 4 \Rightarrow

1+1+1+1
1+2+1
2+1+1

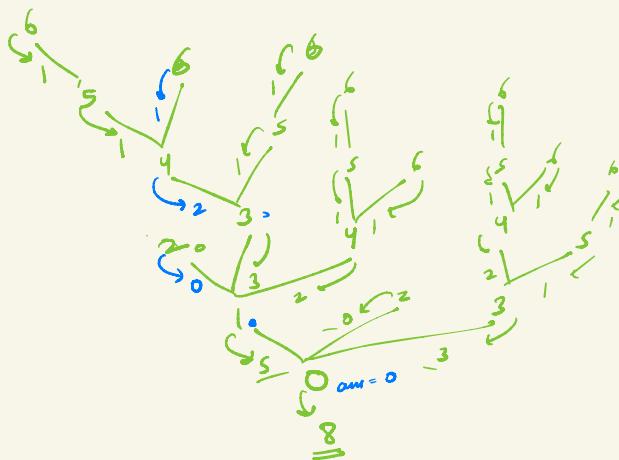
1+1+2
2+2

3	3	0	2	2	3
0	1	2	3	4	5

6

min(ans(i))ⁿ

memoiz²



idm = 5
jump = 12
 $\underline{\underline{5+2}}$

```
public int countWays(int idx, int[] arr){  

    if(idx == arr.length){  

        return 1;  

    }  
  

    int ans = 0;  

    for(int jump=1; jump<=arr[idx]; jump++){  

        if(idx + jump <= arr.length)  

            ans += countWays(idx+jump, arr);  

    }  
  

    return ans;  
}
```

3	3	0	2	2	3
0	1	2	3	4	5

2	3
6	1
5	0
4	

0	1	2	3	4	5	6
8	5	0	3	2	1	1



```
public static int countWays_memo(int idx, int[] arr, int[] memo){
    if(idx == arr.length){
        return memo[idx]=1;
    }

    if(memo[idx]!=0) return memo[idx];

    int ans = 0;
    for(int jump=1; jump<=arr[idx]; jump++){
        if(idx + jump <= arr.length)
            ans += countWays_memo(idx+jump, arr,memo);
    }

    return memo[idx] = ans;
}
```

0	1	2	3	4	5
3	3	0	2	2	3

dp(s)

memo(4)
5
6

memo(s)

number of ways
to reach n from s

break till
10:45

0	1	2	3	4	5	6
8	5	0	3	2	1	1

idn = 6543210
jump = 123

ans = 5 + 0 + 3 = 8

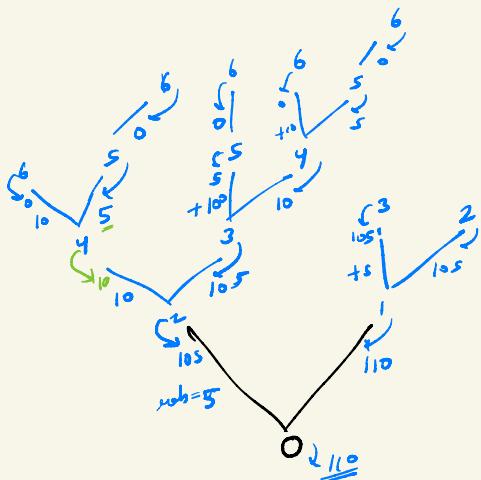
```
public int countWays_tabulation(int idx, int[] arr, int[] memo){
    for(int id=arr.length; id>=0; id--){
        if(id == arr.length){
            memo[id]=1;
            continue;
        }

        int ans = 0;
        for(int jump=1; jump<=arr[id]; jump++){
            if(id + jump <= arr.length)
                ans += memo[id+jump]; //countWays_memo(id+jump, arr,memo);
        }

        memo[id] = ans;
    }

    return memo[0];
}
```

6	1	2	3	4	5	6
5	5	10	100	10	5	6



165

```
public static int findMax(int idx, int[] arr){  
    if(idx == arr.length){  
        return 0;  
    }  
  
    int rob=arr[idx];  
    int notRob=0;  
  
    if(idx+2<=arr.length)  
        rob += findMax(idx+2, arr);  
  
    notRob = findMax(idx+1, arr);  
  
    int ans = Math.max(rob,notRob);  
  
    return ans;  
}
```

$$\text{Ans} \rightarrow 5, \underset{\uparrow}{5}, 10, 100, 10, \underset{\uparrow}{5} =$$

dry sun → 9
why

rec
↓
num ()
↓
tab ()

	↓		↓		↓		
0	1	2	3	4	5	6	
manos	110 110 105 105 10 5 0						
	→	↑	↑				↑

$\delta p(i)$
↓
maximum choice
if at start from i

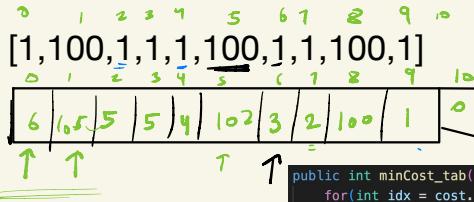
$$\mu = 5 + 105$$

$$nR \geq 110$$

$d\varphi(i)$

```
public int findMax_tabulation(int idx, int[] arr, int[] memo){  
    for(idx=arr.length; idx>=0; idx--){  
        if(idx == arr.length){  
            memo[idx]=0;  
            continue;  
        }  
  
        int rob=arr[idx];  
        int notRob=0;  
  
        if(idx+2<=arr.length)  
            rob += memo[idx+2];//findMax_memo(idx+2, arr,memo);  
  
        notRob = memo[idx+1]; //findMax_memo(idx+1, arr,memo);  
  
        int ans = Math.max(rob,notRob);  
  
        memo[idx]=ans;  
    }  
  
    return memo[0];  
}
```

$dp(idm)$
 \downarrow
 min cost from
 idm to end



(10) idm
 $g5 = 1$
 $g5 = 0$

```
public int minCost_tab_mostOptimized(int[] cost){  

    int n = cost.length;  

    int idxp1 = cost[n-1];  

    int idxp2 = 0;  

    int ansIdx = cost[0] + Math.min(idxp1, idxp2);  

    for(int idx = n-2; idx>=0; idx--){  

        int ansIdx = cost[idx] + Math.min(idxp1, idxp2);  

        idxp2 = idxp1;  

        idxp1 = ansIdx;  

    }  

    return Math.min(idxp1, idxp2);  

}
```

```
public int minCost_tab(int[] cost, int[] dp){  

    for(int idx = cost.length; idx>=0; idx--){  

        if(idx == cost.length){  

            dp[idx]=0;  

            continue;  

        }  

        int oneStep = dp[idx+1]; //minCost_memo(cost,idm+1,dp);  

        int twoStep = Integer.MAX_VALUE;  

        if(idx + 2 <= cost.length){  

            twoStep = dp[idx+2]; //minCost_memo(cost,idm+2,dp);  

        }  

        int ans = cost[idx] + Math.min(oneStep,twoStep);  

        dp[idx]=ans;  

    }  

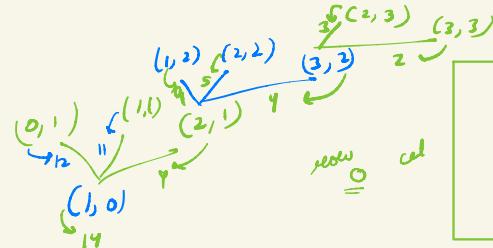
    return Math.min(dp[0],dp[1]);  

}
```

0 {{1, 3, 1, 5},
 1 {2, 2, 4, 1},
 2 {5, 0, 2, 3},
 3 {0, 6, 1, 2}};

ans = 3 + 9

$0 \leq i \leq n$
 $0 \leq j \leq m$
 $n \times m$



row 0 col

(0,3)
 5
 (1,3)
 1
 (2,3)

(0,2)
 9
 (1,2)
 9
 (2,2)

(0,1)
 12
 (1,1)
 11
 (2,1)

(0,0)
 13
 (1,0)
 12
 (2,0)

```
public static int getMax_rec(int[][] mine, int i, int j){  

    if(j==mine[0].length-1){  

        return mine[i][j];  

    }  

    int up = 0;  

    int same = 0;  

    int down = 0;  

    if(i-1 >= 0){  

        up = getMax_rec(mine, i-1, j+1);  

    }  

    same = getMax_rec(mine, i, j+1);  

    if(i+1<mine.length){  

        down = getMax_rec(mine, i+1, j+1);  

    }  

    int ans = mine[i][j] + Math.max(up,Math.max(same,down));  

    return ans;  

}
```

```

{{1, 3, 1, 5},
{2, 2, 4, 1},
{5, 0, 2, 3},
{0, 6, 1, 2}};

```

	0	1	2	3
0	13	12	6	5
1	14	11	4	1
2	16	9	5	2
3	17	11	4	2

```

public static int goldmine_tab(int[][] mine, int[][] dp){
    for(int j = mine[0].length-1; j>=0; j--){
        for(int i=0; i<mine.length; i++){
            if(j==mine[0].length-1){
                dp[i][j] = mine[i][j];
                continue;
            }

            int up = 0;
            int same = 0;
            int down = 0;

            if(i-1 >= 0){
                up = dp[i-1][j+1]; //getMax_memo(mine, i-1, j+1,dp);
            }

            same = dp[i][j+1];//getMax_memo(mine, i, j+1,dp);

            if(i+1<mine.length){
                down = dp[i+1][j+1]; //getMax_memo(mine, i+1, j+1,dp);
            }

            int ans = mine[i][j] + Math.max(up,Math.max(same,down));
            dp[i][j] = ans;
        }
    }

    int ans = 0;
    for(int i=0; i<mine.length; i++){
        ans = Math.max(ans,dp[i][0]);
    }

    return ans;
}

```

	0	1	2	3
0	4	9	11	1
1	2	1	9	5
2	8	2	4	3
3	7	1	2	1

