Functions ⇒ Takes input and give some output.

$f(a, b) = a^2 + 2ab + b^2$

$f(x) = 2x + 1$

$f(n) = 2n + 1$

$x = 5$ → input

input = 5

output = 11

output $2 * 5 + 1 \Rightarrow 11$

```
public static int sum ( int a, int b) {
    int sum = a+b;
    return sum;
}

public static void fun ( int a) {

}
```

return type → int

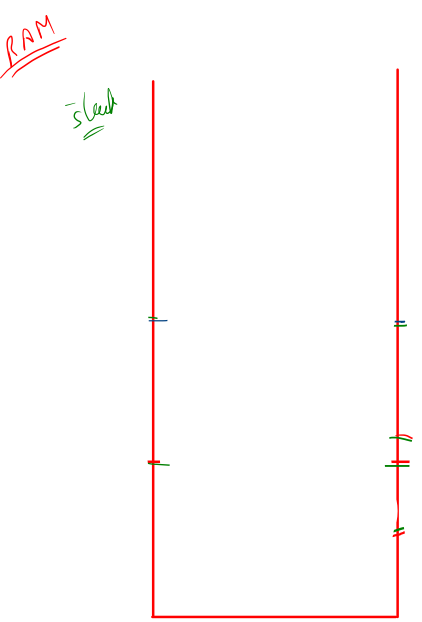
parameters / arguments

no return type → void

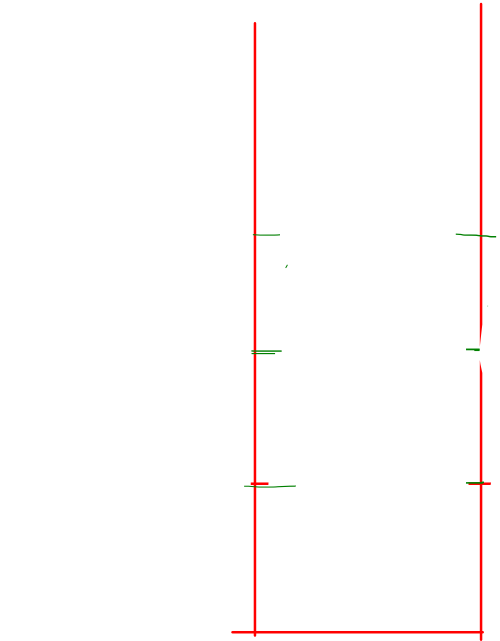
sum (2, 3) = 5

sum (1, 4) = 5

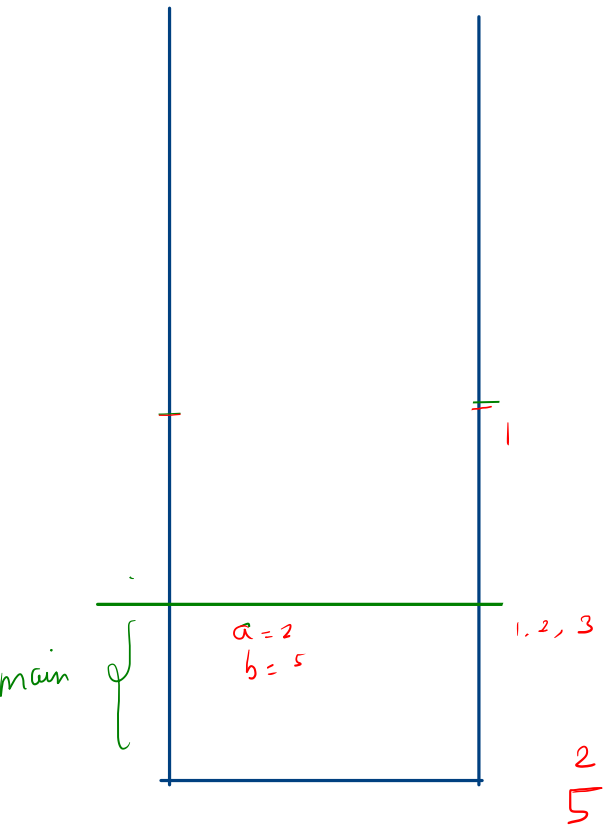
sum (9, 11) = 20



```java
class Main {
    public static int sum(int a, int b){
        int sum = a + b;
        return sum;
    }
    Run | Debug
    public static void main(String[] args) {
        int a= 2;
        int b= 3;

        int ans = sum(a,b);
        System.out.println(ans);
    }
}
```



```java
class Main {
    public static int sum(int a, int b){
        int sum = a + b;
        return sum;
    }

    public static void fun(int a, int b){
        int sum = a+b;
        System.out.println(sum);

        System.out.println("I am inside a void function");
        System.out.println("I am inside a void function");
    }

    public static void main(String[] args) {
        int a= 2;
        int b= 3;

        fun(a,b);
    }
}
```




```java
class Main {
    public static void swap(int a, int b){
        int temp = a;
        a = b;
        b = temp;
    }
    Run | Debug
    public static void main(String[] args) {
        int a= 2;
        int b= 5;

        swap(a, b);

        System.out.println(a);
        System.out.println(b);
    }
}
```

Recursion

→ How to solve any recursion problem

1) Keep a faith that it works for a smaller problem. (faith)
2) Solve for the smallest problem. (base case)
3) Solve for the actual problem (to reach your expectation)

Ques Print decreasing numbers from 'n' to 1 using recursion.

n = 5

Expectation 5 4 3 2 1

```
public static void printDecreasing(int n){
    System.out.println(n);
    printDecreasing(n-1);
}
```

Expectation (n)
function will print numbers from 'n' to 1

Faith (n-1)
function will print numbers from (n-1) to 1

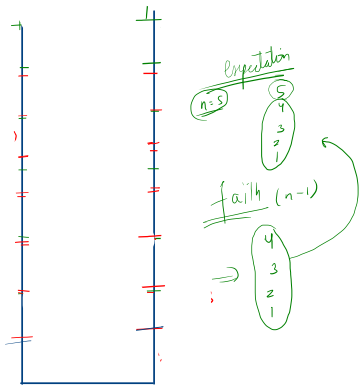
n=5 → 5 4 3 2 1

n-5 → 4 3 2 1

work

you won't be able to learn recursion without dry-run-

stack overflow




```
class Main {
    public static void printDecreasing(int n){
        if(n==0){
            return;
        }
        System.out.println(n);
        printDecreasing(n-1);
    }
    Run | Debug
    public static void main(String[] args) {
        int n = 3;
        printDecreasing(n);
    }
}
```

3
2
1

Ques Print numbers in increasing order using recursion.

Faith (n-1)

It will work for (n-1)
print numbers from 1 to 'n-1'

1
2
3
4

5

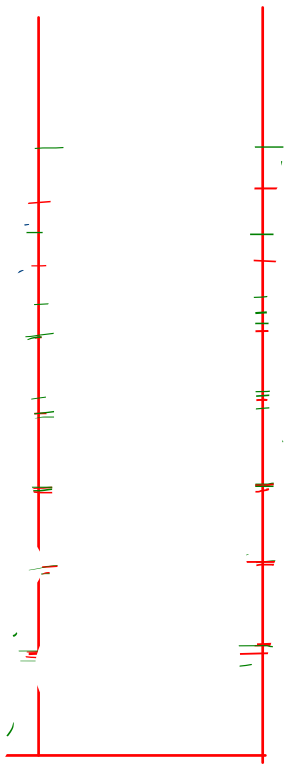
```
psv printIncreasing(int n){

    printIncreasing(n-1);
    System.out.println(n);

}
```

Expectation (n)

Function will print numbers from 1 to n

n=5

1
2
3
4
5



```java
public static void printIncreasing(int n){
    if(n==0){
        return;
    }
    printIncreasing(n-1);
    System.out.println(n);
}
Run | Debug
public static void main(String[] args) {
    int n = 3;
    printIncreasing(n);
}
```

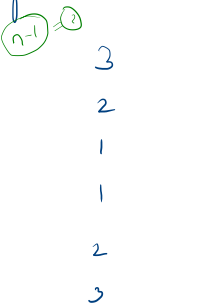
1
2
3

Ques Print numbers in decreasing order, then increasing using recursion.

n=5

Faith (n-1)

It will print numbers from (n-1) to 1 and then 1 to (n-1)



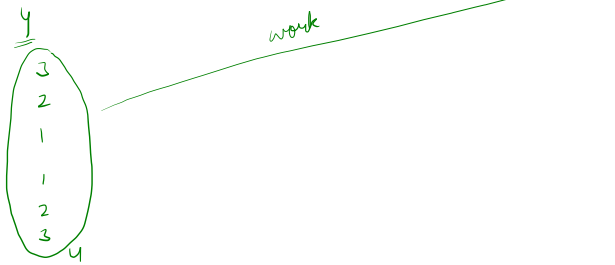

```
psv pdi(int n){
    System.out.println(n);
    pdi(n-1);
    System.out.println(n);
}
```

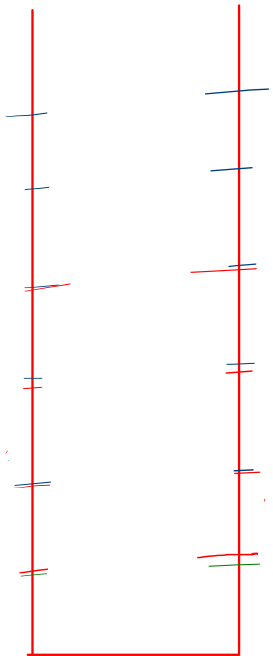
System.out.println(n); → fails

Expectation

It will print numbers in decreasing then increasing order

n=4





```java
public static void pdi(int n){
    if(n==0){
        return;
    }
    System.out.println(n);
    pdi(n-1);
    System.out.println(n);
}
Run | Debug
public static void main(String[] args) {
    int n = 4;
    pdi(n);
}
```

Ques Find factorial of given number 'n'.

Faith (n-1)

It will return factorial (n-1)

1 x 2 x 3 x 4

smallAns

```
public static int fac(int n){
    int smallAns = fac(n-1);
    int ans = smallAns * n;
    return ans;
}
```

faith

work

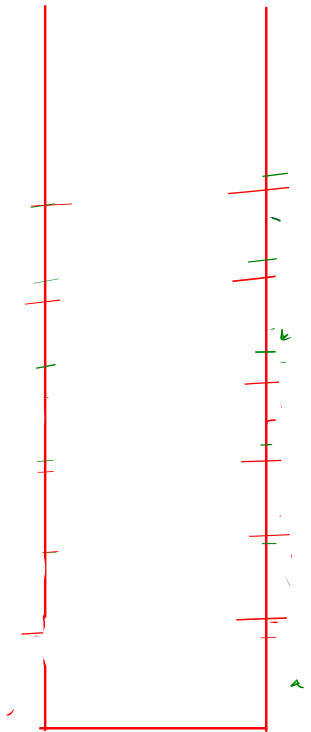
Expectation

It will return factorial of given number

n = 5

1 x 2 x 3 x 4 x 5

work



24

```
public static int factorial(int n){
    if(n==0){
        return 1;
    }
    int smallAns = factorial(n-1);
    int ans = smallAns * n;
    return ans;
}
Run | Debug
public static void main(String[] args) {
    int n = 4;
    int ans = factorial(n);
    System.out.println(ans);
}
```

Q: Find x^n using recursion

Faith: It will give $x^{(n-1)}$

Expectation: It will return x^n

```
public static int power(int x, int n){
    int smallAns = power(x, n-1);
    int ans = smallAns * x;
    return ans;
}
```

$2 \times 2 \times 2 \times 2 \times 2 \times 2$

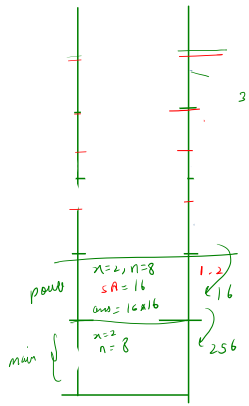
Faith ($n/2$): It will calculate $x^{\frac{n}{2}}$ correctly. $x=2$, $n=6$

```
public static int power(int x, int n){
    int smallAns = power(x, n/2);
    int ans = smallAns * smallAns;
    if(n%2 != 0){
        ans = ans * x;
    }
    return ans;
}
```

Expectation: It will return x^n. $x=2$, $n=6$

$2 \times 2 \times 2 \times 2 \times 2 \times 2$

smallAns



ans = 2*2 → 4

ans = 16 * 16 → 256

```
public static int power2(int x, int n){
    if(n==0){
        return 1;
    }
    int smallAns = power2(x, n/2);
    int ans = smallAns * smallAns;

    if(n%2 != 0){
        ans = ans*x;
    }

    return ans;
}
Run | Debug
public static void main(String[] args) {
    int x = 2;
    int n = 6;
    int ans = power2(x,n);
    System.out.println(ans);
}
```

O(log n)

$x^{\frac{n}{2}}$ x^n

$2^8 \Rightarrow 2^{\frac{8}{2}} * 2^{\frac{8}{2}} \Rightarrow 2^4 * 2^4 \Rightarrow 2^8$

$2^1 \Rightarrow 2^{\frac{1}{2}} * 2^{\frac{1}{2}} \Rightarrow 2^{\frac{3}{2}} * 2^{\frac{3}{2}} \Rightarrow 2^3 * 2^3 \Rightarrow 2^6 * 2$

$n \rightarrow \frac{n}{2} \rightarrow \frac{n}{4} \rightarrow \frac{n}{8} \rightarrow \frac{n}{16} \cdots\cdots 1$ — x steps

$\frac{n}{2^x} \rightarrow \frac{n}{2^{x-1}} \rightarrow \frac{n}{2^{x-2}} \rightarrow \frac{n}{2^{x-3}} \rightarrow \frac{n}{2^4} \frac{n}{2^5} \frac{n}{2^6} \cdots\cdots 1$

$O(x)$

$\frac{n}{2^x} = 1$

$n = 2^x$

$2^x = n$

$\log 2^x = \log n$

$x \log 2 = \log n$

$x \log_2^2 = \log_2^n$

$x * 1 = \log_2^n$

$x = \log n$

$\log a^b = b \log a$

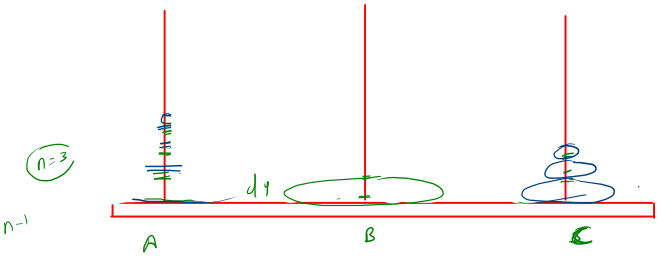
$\log_a^a \Rightarrow 1$

$O(\log n)$

Ques Tower of Hanoi

shift n discs from tower A to tower B with the help of tower C

→ you can move one disc at a time
→ you can not put a longer disc on top of a smaller disc



Faith

move (n-1) disc from A → C using B (n-1 steps to move 2 disc from A to C)

→ move nth disc from A to B (move 3rd disc from A → B)

move (n-1) disc from C to B (move 2 disc from C → B)

psv TOH(n, A, B, C) ✓

→ TOH(n-1, A, C, B) → faith

move nth disc from A to B

→ TOH(n-1, C, B, A)

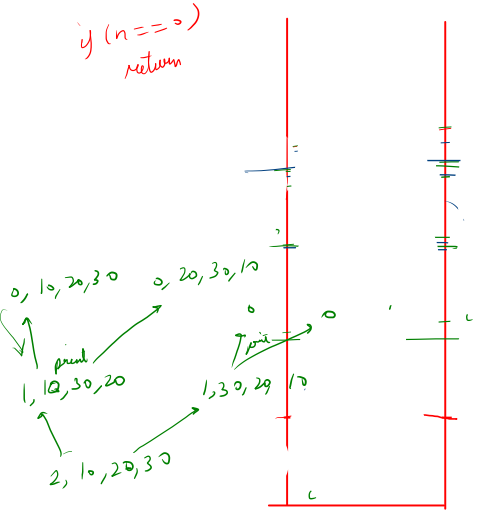
Expectation ✓ ns

function will move n discs from 'A' to 'B' using 'C'

n=3

write steps to move 3 disc from A to B

if (n==0) return

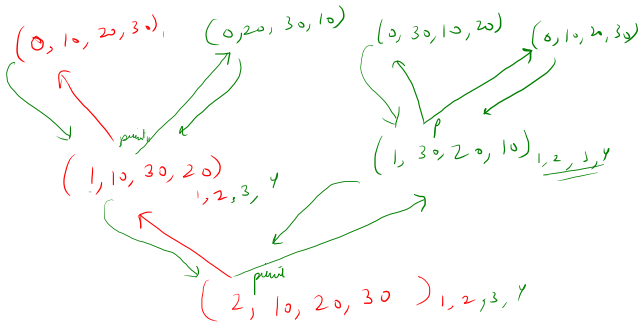



```java
public static void towerOfHanoi(int n, int A, int B, int C){
    if(n==0){
        return;
    }

    towerOfHanoi(n-1, A, C, B);
    // move nth disc from A to b
    System.out.println("Moving ring " + n + "from" + A + " to " + B + " using " + C);
    towerOfHanoi(n-1, C, B, A);
}

Run | Debug
public static void main(String[] args) {
    int n = 3;
    towerOfHanoi(n, A: 10, B: 20, C: 30);
}
```

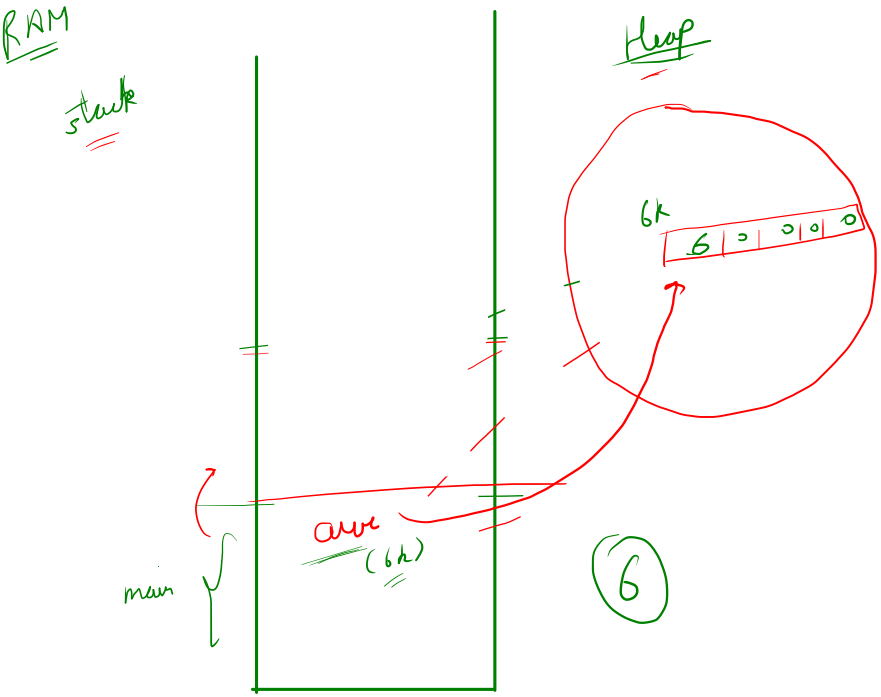
Moving ring 1 from 10 → 30 using 20
Moving ring 2 from 10 → 20
Moving ring 1 from 30 → 20

recursive tree diagram

→ color path



Recursion with Arrays




```java
public class Main {
    public static void increaseArray(int[] arr){
        arr[0]++;
    }

    public static void increaseInteger(int a){
        a++;
    }
    Run | Debug
    public static void main(String[] args) {
        int[] arr = new int[1];
        arr[0] = 5;

        int a = 5;
        increaseArray(arr);
        increaseInteger(a);

        System.out.println(arr[0]);
        System.out.println(a);
    }
}
```

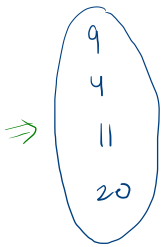
int [] arr = new int [5]

Ques Print array using recursion.

Faith (smaller area)

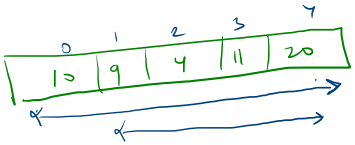
It will print array from idx+1 to end

1 → 4



psv

```
printArray(int[] arr, int idx){
    System.out.println(arr[idx]);
    printArray(arr, idx+1);
}
```

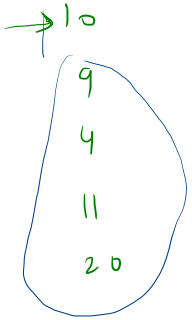


printArray(arr, 0)

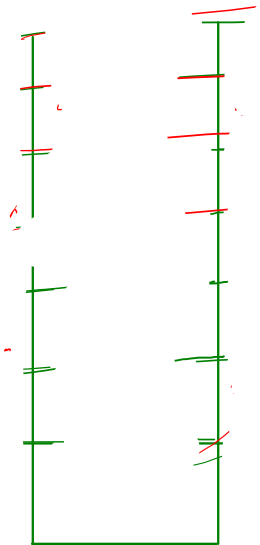
Expectation

function will print array from 'idx' to end (n-1)

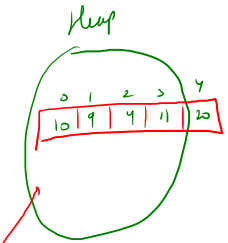
from 0 → 4



Flow



Stack



10
9
4
11
20

```java
public static void printArray(int[] arr, int idx){
    if(idx == arr.length){
        return;
    }

    System.out.println(arr[idx]);
    printArray(arr, idx+1);
}
Run | Debug
public static void main(String[] args) {
    int[] arr = {10,9,4,11,20};
    printArray(arr, 0);
}
```

Ques Print array in reverse

Faith

It will print from (idx+1) to end in reverse.

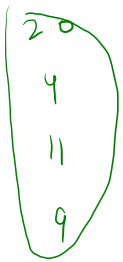
PSV

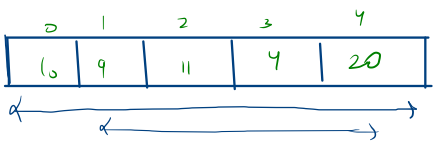
```
printReverse(int[] arr, int idx){
    printReverse(arr, idx+1);
    System.out.println(arr[idx]);
}
```

Expectation

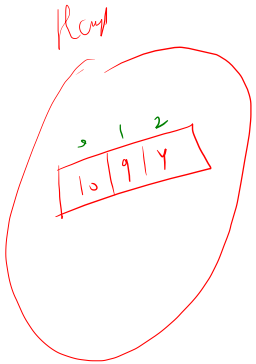
It will print from 'idx' to end in reverse

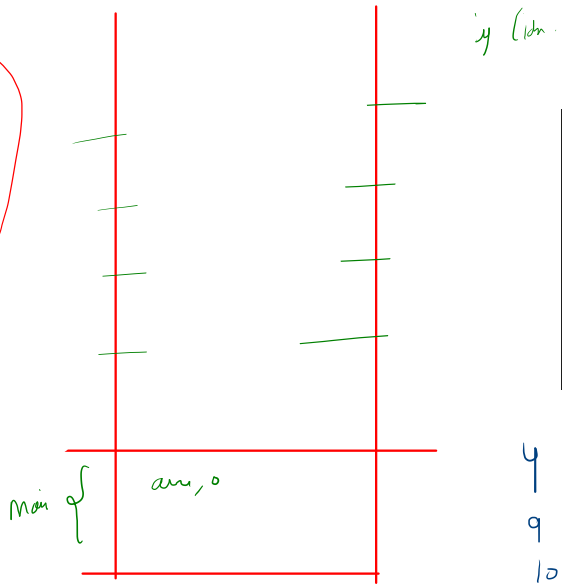
0 → 4 in reverse





printReverse(arr, 0)





if (idx == arr.length)
return;

```
public static void printReverse(int[] arr, int idx){
    printReverse(arr, idx+1);
    System.out.println(arr[idx]);
}
Run | Debug
public static void main(String[] args) {
    int[] arr = {10,9,4,20};
    printReverse(arr,idx: 0);
}
```

4
9
10

Ques Find max of array using recursion.

Faith
It will return max of a smaller array.

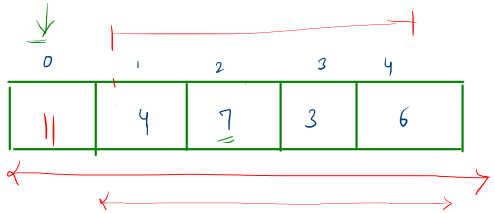
smallAns = 7

ps int maxofArray(int[] arr, int idx){

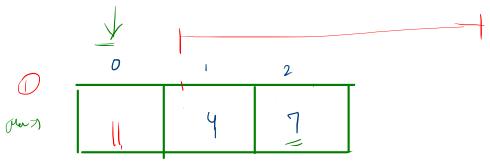
int smallAns = max(arr, idx+1) →
int ans = Math.max(arr[idx], smallAns);
return ans;

}

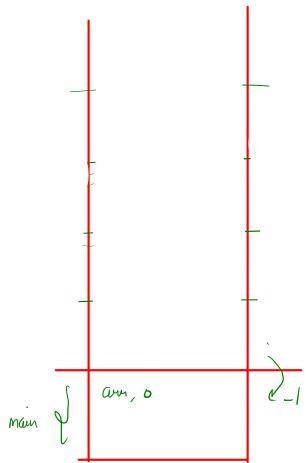
Expectation
It will return 'max' from 'idx' to end.
idx = 0 to n-1
ans = 11



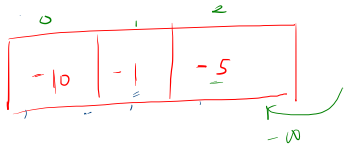
max(arr, 0)
0 → end
idx+1 to end




```
public static int maxofArray(int[] arr, int idx){
    if(idx == arr.length){
        return 0; Integer.MIN_VALUE
    }

    int smallAns = maxofArray(arr, idx+1);

    int ans = Math.max(arr[idx],smallAns);
    return ans;
}
Run | Debug
public static void main(String[] args) {
    int[] arr = {10,9,4,11,20};
    System.out.println(maxofArray(arr,idx: 0));
}
```

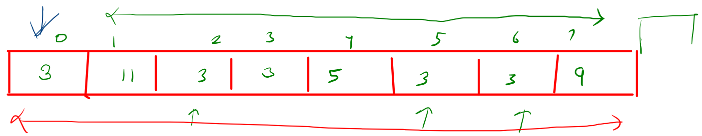


arr =>



Ques Find first index of given target element.

tar = 3



fun (arr, 0, 3) = 0

fun (arr, 8, 3)

Faith

It will return first index of tar from 'idx+1' to end

sA = 2

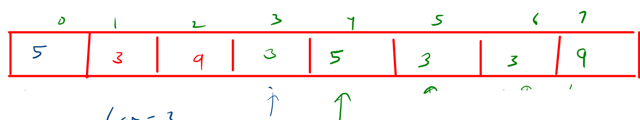
expectation

It will return the first index of the given tar element.

ans = 0

```
public static int firstIndex(int[] arr, int idx, int tar) {
    int smallAns = fi(arr, idx+1, tar)
    if (arr[idx] == target)
        ans = idx;
    else
        ans = smallAns;

    return ans;
}
```

(arr, 8, 3) → -1
(arr, 7, 3) → -1
(arr, 6, 3) → 6
(arr, 5, 3) SA = 6 → 5
(arr, 4, 3) → 5
(arr, 3, 3) → 3
(arr, 2, 3) SA = 3
(arr, 1, 3) → 3
(arr, 0, 3) → 1



tar = 3

recursion on the way down

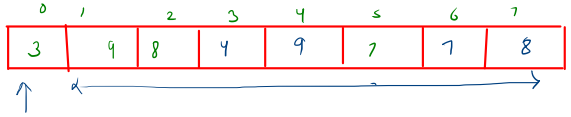
```java
public static int firstIndex(int[] arr, int idx, int tar){
    int smallAns = firstIndex(arr, idx+1, tar);
    int ans = 0;

    if(arr[idx] == tar){
        ans = idx;
    } else {
        ans = smallAns;
    }

    return ans;
}
Run | Debug
public static void main(String[] args) {
    int[] arr = {10,9,4,11,20};
    System.out.println(maxofArray(arr,idx: 0));
}
```

smallest problem

{ if (idx = arr.length)
return -1

Ques Find last index of tar

tar = 3



Faith

from 'idx+1' to end it will find the last index of tar

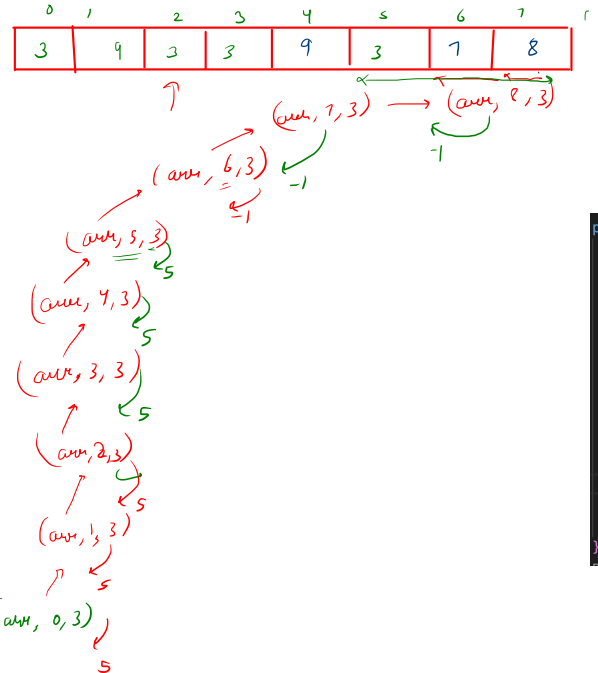
smallAns = -1

```
ps int lastIndex(int[] arr, int idx, int tar){
    int smallAns = lI(arr, idx+1, tar);

    int ans = 0;
    if(smallAns == -1){
        if(arr[idx] == tar)
            ans = idx;
    } else ans = smallAns;

    return ans;
}

lastIndex(arr, 0, tar);
```

Expectation

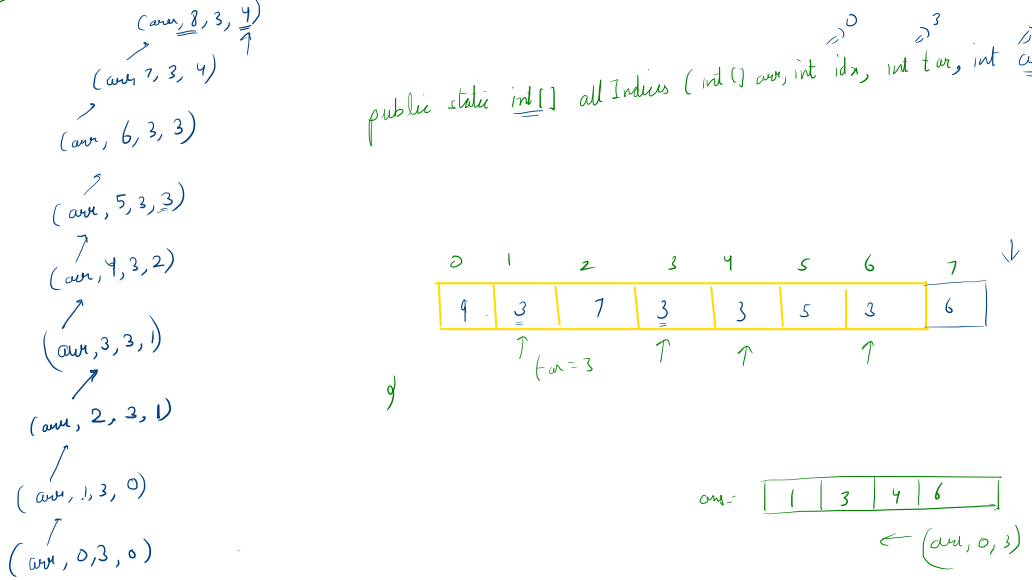
from '0' to end, it will return last index of target.

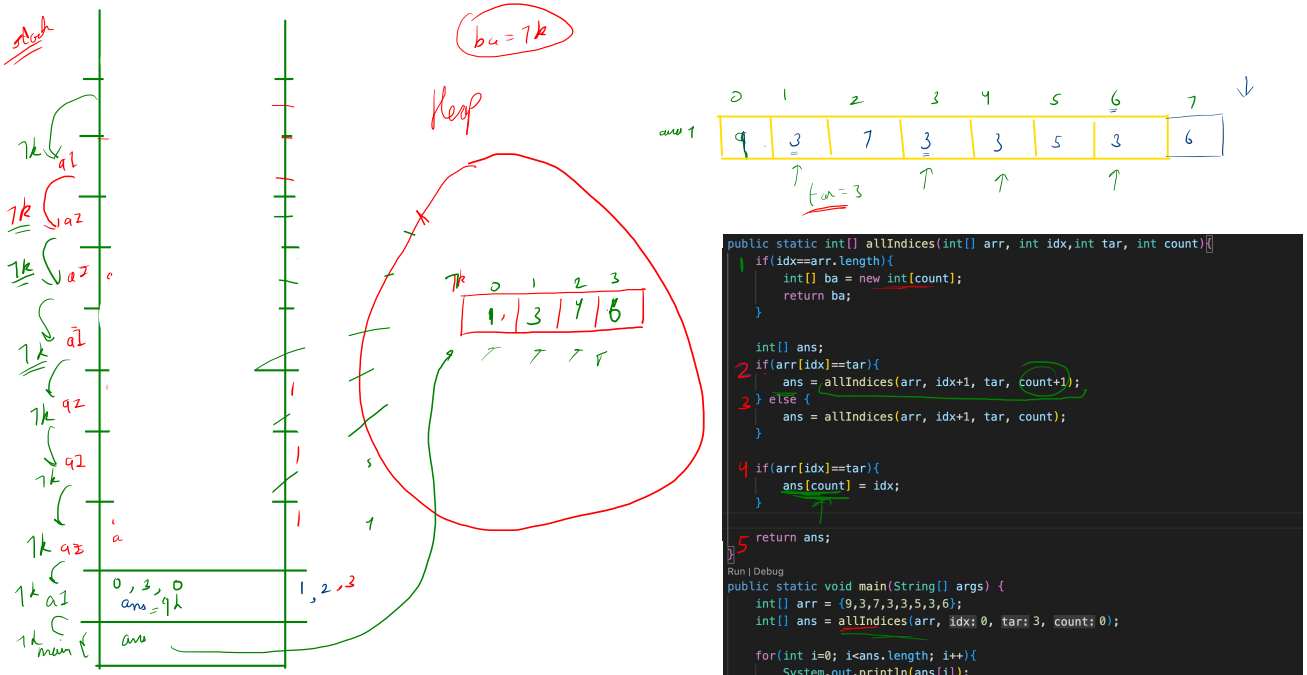
ans = 5




```java
public static int lastIndex(int[] arr, int idx, int tar){
    int smallAns = lastIndex(arr, idx+1, tar);

    int ans = 0;
    if(smallAns != -1){
        if(arr[idx]==tar){
            ans = idx;
        } else {
            ans = smallAns;
        }
    } else {
        ans = smallAns;
    }

    return ans;
}
```

Q) Find all indices of tar in array






```java
public static int[] allIndices(int[] arr, int idx,int tar, int count){
    if(idx==arr.length){
        int[] ba = new int[count];
        return ba;
    }

    int[] ans;
    if(arr[idx]==tar){
        ans = allIndices(arr, idx+1, tar, count+1);
    } else {
        ans = allIndices(arr, idx+1, tar, count);
    }

    if(arr[idx]==tar){
        ans[count] = idx;
    }

    return ans;
}
Run | Debug
public static void main(String[] args) {
    int[] arr = {9,3,7,3,3,5,3,6};
    int[] ans = allIndices(arr, idx: 0, tar: 3, count: 0);

    for(int i=0; i<ans.length; i++){
        System.out.println(ans[i]);
    }
}
```

Ques Find nth fibonacci number

0th	1	2	3	4	5	6	7	8	9	10	11	12
0	1	1	2	3	5	8	13	21	34	55	89	144

$f(n) = f(n-1) + f(n-2)$

$f(6) = f(5) + f(4)$

(n-1)

Fourth

n=6

n-1 = 5

lt = 5

fib(n-1);

fib(n-2);

```
public static int fib ( int n) {
    int lt = fib(n-1);
    int lst = fib(n-2);
    int ans = lt + lst;
    return ans;
}
```

temp estation

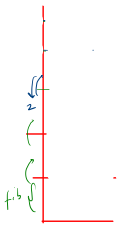
It will calculate nth fibonacci term.

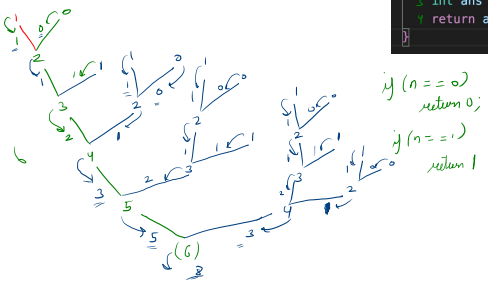
n=6

ans = 8

ans = 1+1 = 2

lt = 2



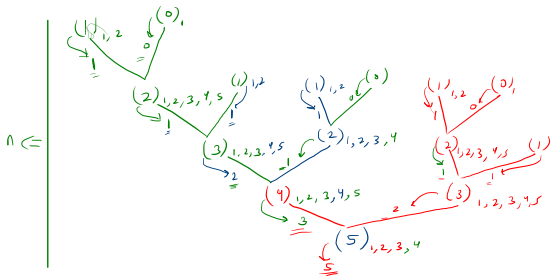


```
if (n == 0)
    return 0;
if (n == 1)
    return 1
```

```
// fibonacci number
public static int fib(int n){
    int lt = fib(n-1);
    int lst = fib(n-2);

    int ans = lt + lst;
    return ans;
}
```

TC => (number of calls) height of tree

2^n



```
// fibonacci number
public static int fib(int n){
    if(n==0){
        return 0;
    }
    if(n==1){
        return 1;
    }
    int lt = fib(n-1);
    int lst = fib(n-2);

    int ans = lt + lst;
    return ans;
}
```

Series

nth even fibonacci number.

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377	610

9-3 ⇒ n-3 ↳ least even fib num
9-6 ⇒ n-6 ↳ second least

1st even fib number ⇒ 3rd fib number.
2nd even fib number ⇒ 6th fib number
3rd even fib number ⇒ 9th fib number.
4th even fib number ⇒ 12th fib
nth even fib number ⇒ 3*n fib number

f(n-3) ↳ least even fib num
f(n-6) ↳ Second even fib num

$f(n) \Rightarrow f(n-1) + f(n-2)$

$f(n-2) + f(n-2) + f(n-3) + f(n-4)$

$f(n-3) + f(n-4) + f(n-3) + f(n-3) + f(n-4)$

$3*f(n-3) + f(n-4) + f(n-4)$

$3f(n-3) + f(n-5) + f(n-6) + f(n-4)$

$\Rightarrow 3f(n-3) + f(n-5) + f(n-4) + f(n-6)$

even

$\Rightarrow 3f(n-3) + f(n-3) + f(n-6)$

$f(n) \Rightarrow 4f(n-3) + f(n-6)$ ⇒ nth even fib number ⇒ 4*(least even fib number) + (second least even fib number)

$f_E(n) \Rightarrow 4f_E(n-1) + f_E(n-2)$

f_E ↑ ↓ ↓

least even fib number. Second least even fib number.

10

$10^4 + 7$

largest previous number (int)

x=8

1 10

ans /.10

ans = n+4 ⇒ ans=12

12 /. 10 ⇒ 2

ans = n /. 5.0

7

n /. 2 → 0 / 1

n /. 9

18 /. 9 = 0

99 /. 9 ⇒ 0

27, 9 ⇒ 0

37, 9 ⇒ 2

least even ⇒ 10^9

last ⇒ 10^9

ans ⇒ $4*10^9 + 10^2$

⇒ $4*10^9$

least even ⇒ 7/10^9

⇒ $3*10^9$

7×10^4

10^9

sum of

-2^{31}

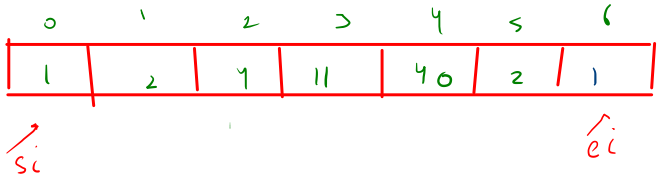
$2^{31}-1$

10^9

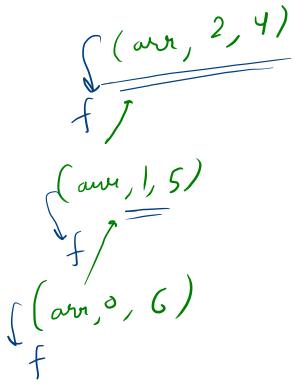
10^{10} /. (mod)

0 $2^{32}-1$

Ques Check whether the given array is palindrome or not.




```java
public static boolean isPalindromic(int[] arr, int si, int ei) {
    if(arr[si]!=arr[ei]){
        return false;
    }

    boolean ans = isPalindromic(arr, si + 1, ei - 1);
    return ans;
}
```



Dynamic array

data type

```
ArrayList<String> al = new ArrayList<>();
```

Dynamic array

size

```
al.size()
```

→ 3

add

```
al.add("abc")
al.add("def")
al.add("abcd")
```

get

```
al.get(idx);
```

	0	1	2
arr →	10	13	19

remove

```
al.remove(idx)
al.remove(1)
```

set

```
al.set(idx, value);
```

set 45 at 1st index

arr[1] = 45

Strings

str = "a b c d e"

substring => Continuous segment of string.

subsequence => discontinuous segment of string

"a b c d"

2^n (n = len of string)

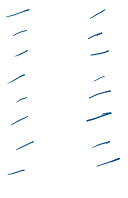
0 len: ""

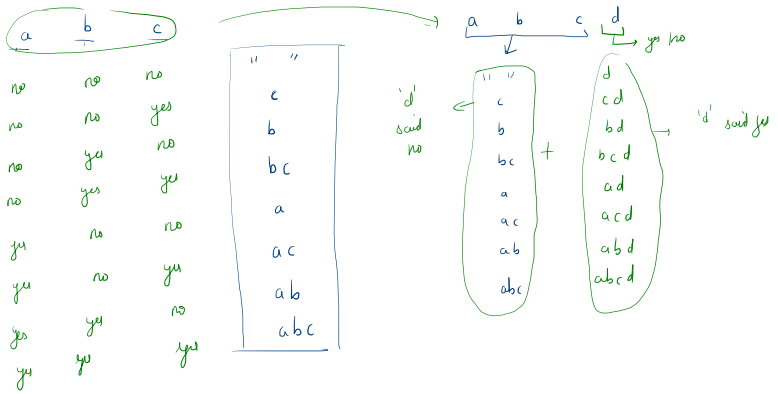
1 len: a, b, c, d

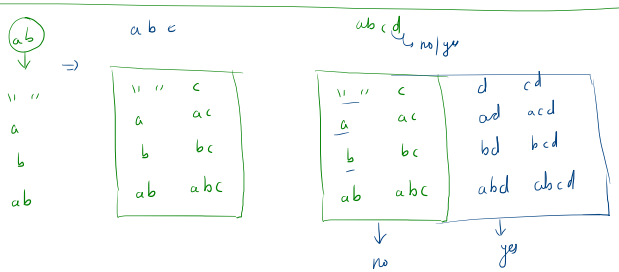
2 len: ab, ac, ad, bc, bd, cd

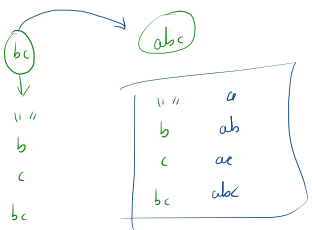
3 len: abc, abd, acd, bcd

4 len: abcd









Ques Get all the subsequences of a given string.

Faith (smaller value)

It will return all the subsequences for 'bc'

smallAns = {"", b, c, bc}

public static ArrayList<String> getSS(String str){

abc

Expectation

It will return all the subsequences of given string

str = "abc"

ans = {"", a, b, c, ab, bc, ac, abc}

work?

("")

{""}

(c)

{"", c}

(bc)

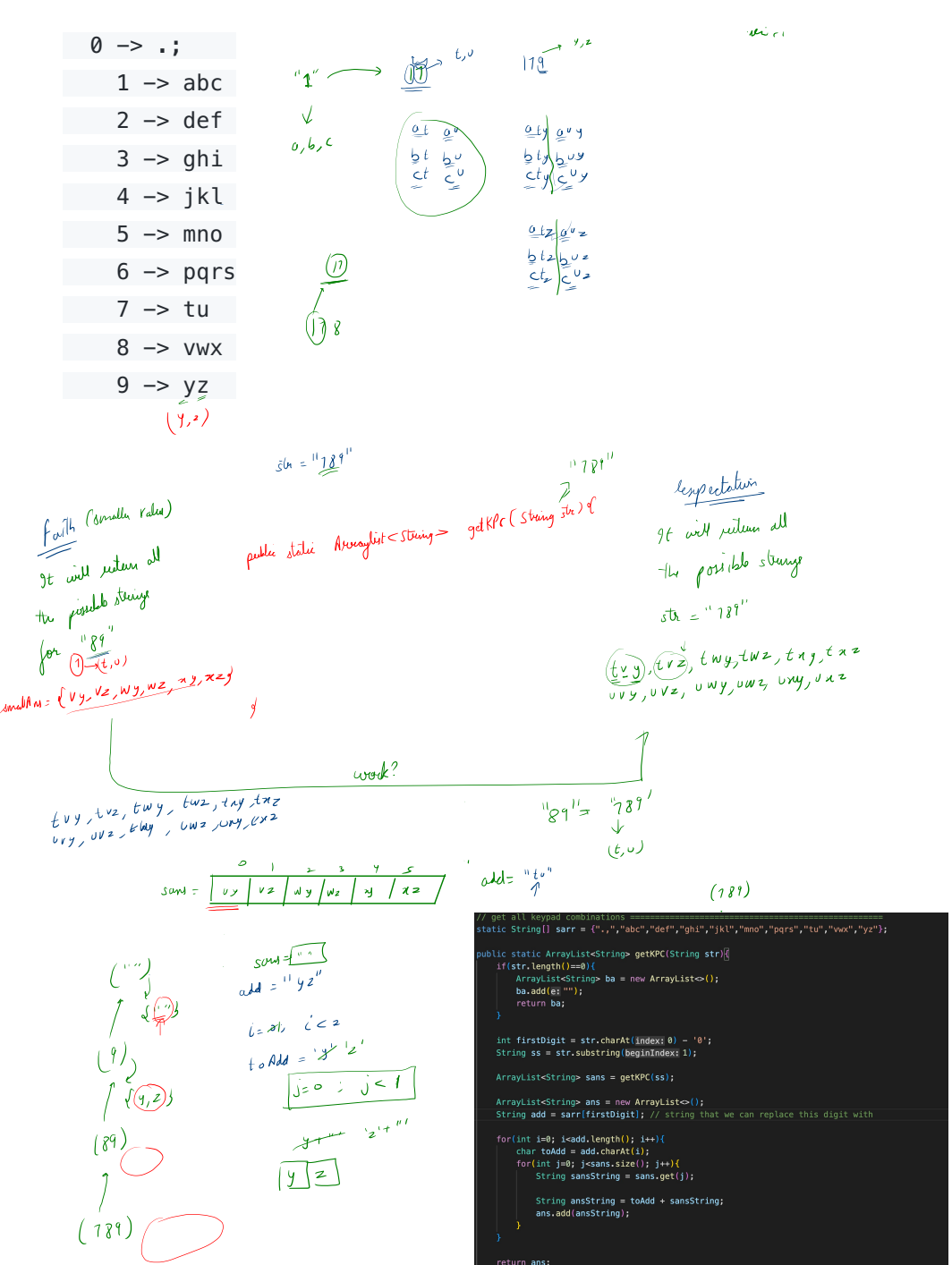
{"", c, b, bc}

(abc)

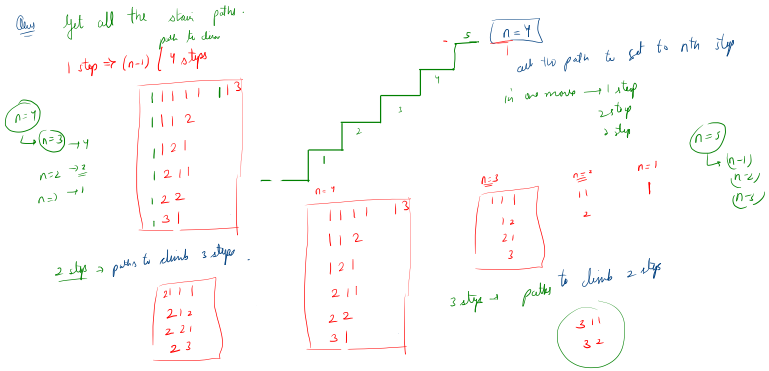
{"", c, b, bc, a, ac, ab, abc}

```java
public static ArrayList<String> getSS(String str){
    char fc = str.charAt(index: 0);
    String ss = str.substring(beginIndex: 1);

    ArrayList<String> sans = getSS(ss);

    ArrayList<String> ans = new ArrayList<>();
    // copy sans
    for(int i=0; i<sans.size(); i++){
        String ssub = sans.get(i);
        ans.add(ssub);
    }

    // add first char
    for(int i=0; i<sans.size(); i++){
        String ssub = sans.get(i);
        String sub = fc + ssub;
        ans.add(sub);
    }

    return ans;
}
```

0 -> .;
1 -> abc
2 -> def
3 -> ghi
4 -> jkl
5 -> mno
6 -> pqrs
7 -> tu
8 -> vwx
9 -> yz



Q: get all the stair paths.



Faith

It will find all the possible path for (n-1) (n-2) (n-3)

public static ArrayList<String> getStairPath(int n){

Expectations

It will return all the possible paths

n = 4

n=3

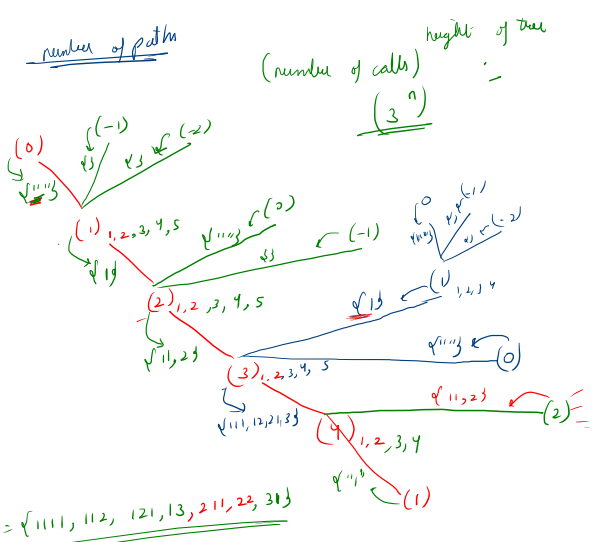
ans1 = {"111", "12", "21", "3"}

n=2 (2)

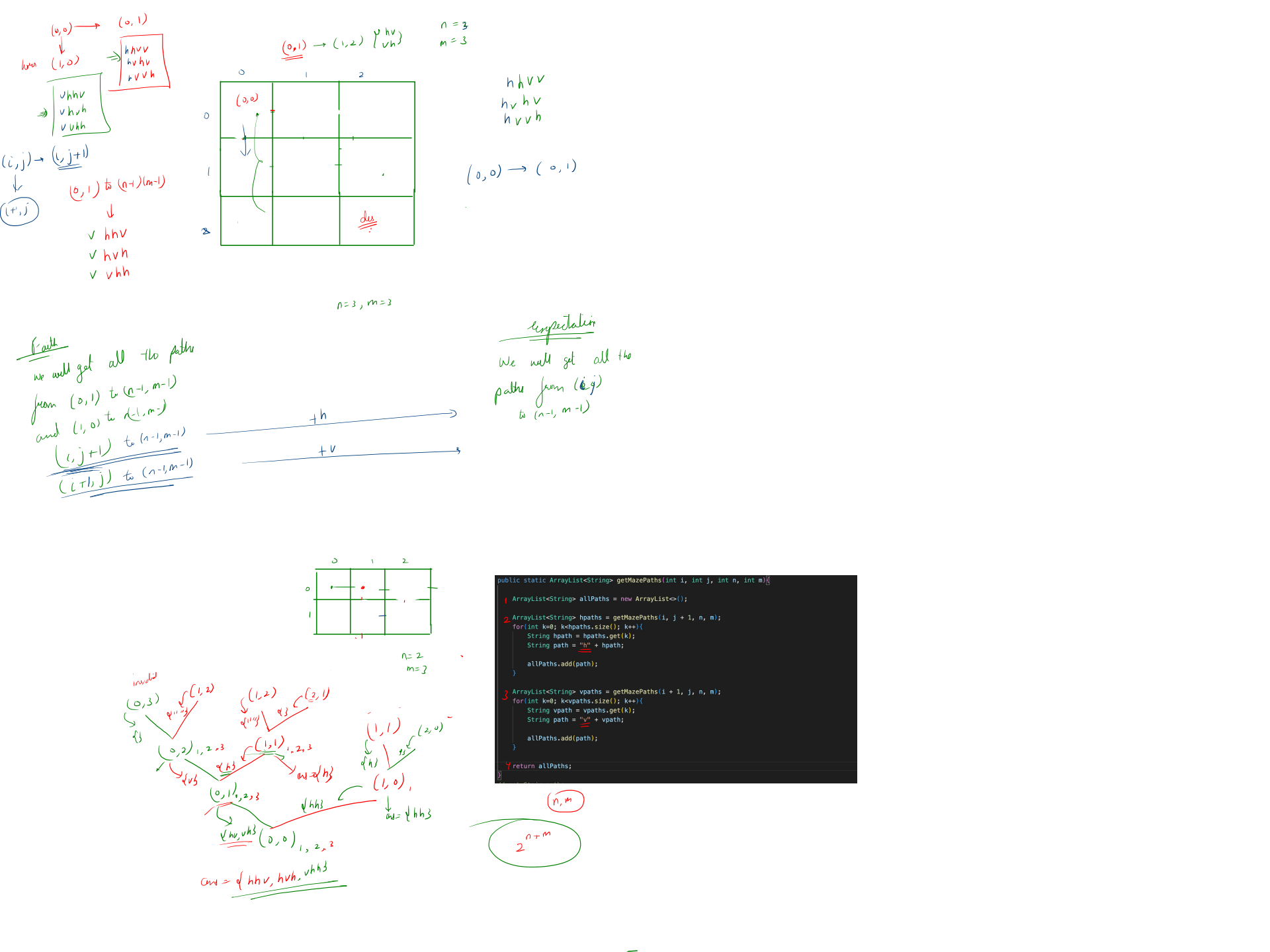
ans2 = {"11", "2"}

n=1 (1)

ans3 = {"1"}




```
1
2
public static ArrayList<String> getStairPaths(int n){
    if(n<0){ // no possible path
        ArrayList<String> ba = new ArrayList<>();
        return ba;
    }

    if(n==0){
        ArrayList<String> ba = new ArrayList<>();
        ba.add(e: ""); // not doing anything , just standing here (reached my destination)
        return ba;
    }
    ArrayList<String> ans = new ArrayList<>();
    ArrayList<String> sans = getStairPaths(n-1);

    for(int i=0; i<sans.size(); i++){ // take a jump of 1
        String spath = sans.get(i);
        String path = "1" + spath;
        ans.add(path);
    }

    sans = getStairPaths(n-2); // take a jump of 2
    for(int i=0; i<sans.size(); i++){
        String spath = sans.get(i);
        String path = "2" + spath;
        ans.add(path);
    }

    sans = getStairPaths(n-3); // take a jump of 3
    for(int i=0; i<sans.size(); i++){
        String spath = sans.get(i);
        String path = "3" + spath;
        ans.add(path);
    }

    return ans;
}
```




```java
public static ArrayList<String> getMazePaths(int i, int j, int n, int m){

1 ArrayList<String> allPaths = new ArrayList<>();

2 ArrayList<String> hpaths = getMazePaths(i, j + 1, n, m);
  for(int k=0; k<hpaths.size(); k++){
      String hpath = hpaths.get(k);
      String path = "h" + hpath;

      allPaths.add(path);
  }

3 ArrayList<String> vpaths = getMazePaths(i + 1, j, n, m);
  for(int k=0; k<vpaths.size(); k++){
      String vpath = vpaths.get(k);
      String path = "v" + vpath;

      allPaths.add(path);
  }

4 return allPaths;
}
```

(n, m)

2^{n+m}

Smaller

relate with how you are dependent on smaller problem