


functions

$$f(x) = x + 5;$$

input = 2

output = 7

$$f(a, b) = a + b;$$

input = 2, 3

output = 5

⇒ public static int sum(int a, int b){
 int sum = a + b;
 return sum;

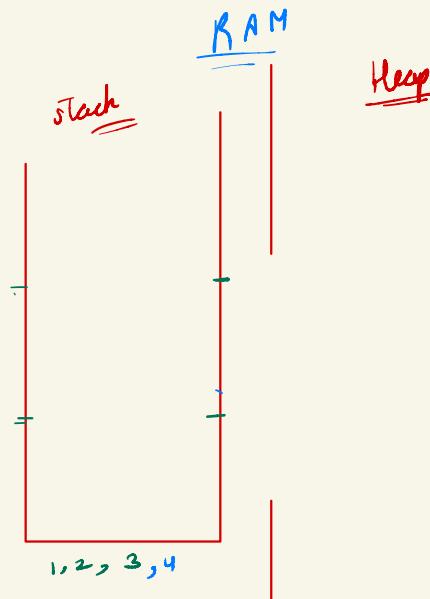
}

~~public static~~ action-type void fun-name (parameters) {
 // logic

3

```
public class Main {  
    public static int sum(int a, int b){  
        int sum = a + b;  
        return sum;  
    }
```

```
    public static void main(String[] args) {  
        1 int a=2;  
        2 int b=3;  
  
        3 int ans=sum(a, b);  
        4 System.out.println(ans);  
    }  
}
```



```

public class Main {
    public static void printSomething(){
        System.out.println("Hi!!!");
        printSomething(),
        int a=5;
        a++;
        a--;
        a=11;
    }

    public static int sum(int a, int b){
        printSomething();

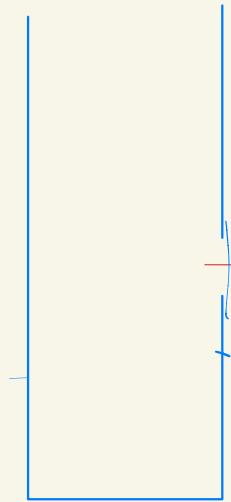
        int sum = a + b;
        return sum;
    }

    Run | Debug
    public static void main(String[] args) {
        int a=2;
        int b=3;

        int ans=sum(a, b);
        System.out.println(ans);
    }
}

```

Hi!! ↘
5 ↘



PNI Principal of Mathematical Induction

sum of first n natural numbers \Leftarrow

$$= \frac{n(n+1)}{2}$$

$n=0, n=1 \rightarrow$ base case
 suppose $\underline{\underline{n=k}} \rightarrow$ smaller problem
 $\underline{\underline{n=k+1}} \rightarrow$ solving

Recursion

a function calling itself

- 1) solve for the smallest problem
- 2) keep a faith that it will work for a smaller problem. (faith)
- 3) solve for rest of the problem. (to reach your expectation)

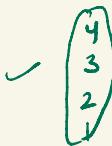
print decreasing

- 1) $n=1$ is the smallest problem
 - 2) $n=s$, $(n-1)$
-

if ($n==1$)
 System.out.println(1);
 return;

Faith (smaller value)

$$n-1 \Rightarrow 4$$



Expectation
If I provide $n=5$, it will

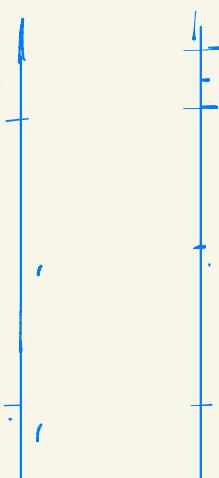


dry run is very important

10 lines \rightarrow 2 pages of notebook

```
public static void pd(int n){
    if(n==0){
        return;
    }

    System.out.println(n);
    pd(n-1);
}
```



Ans print increasing recursively
 $n=5$

1) you won't be able to learn recursion if you don't dry run.

Faith
It will print numbers
for (n-1);
 $n=4 \Rightarrow \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$

```
public static void PI(int n){  
    if(n==0) return;  
    1) PI(n-1);  
    2) System.out.println(n);  
}
```

expectation
1) my function should print increasing numbers.
2) for $n=5$, it should print
1
2
3
4
5

```
public static void PI(int n){  
    if(n==0){  
        return;  
    }  
  
    PI(n-1);  
    System.out.println(n);  
}
```

break till
 $\rightarrow [10:15]$

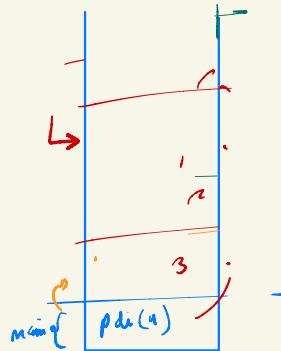
print decreasing numbers

faith

$n-1$

$n=3$
3
2
1
1
2
3
4
3
2
1
1
2
3

```
public static void PDI(int n){  
    if(n==0) return;  
    1) System.out.println(n);  
    2) PDI(n-1);  
    3) System.out.println(n);  
}
```



expectation
It will print numbers in decreasing, then increasing.

$n=4$

4
3
2
1
1
2
3
4

Find factorial of a number

Faith

it will give me
 $\text{fac}(n-1);$

```
public static int fac(int n){  
    1 if(n==1) return 1;  
    2 int sAns = fac(n-1);  
    3 int ans = n * sAns;  
    4 return ans;  
}
```

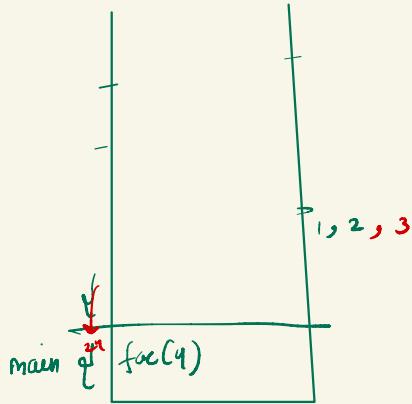
$$4 \times 3 \times 2 \times 1$$

Expectation

My function will return
factorial of n

$$\underline{\underline{n=5}}$$

$$\overbrace{5 \times 4 \times 3 \times 2 \times 1}^{\rightarrow}$$



$$\text{ans} = 2 \times 1 \Rightarrow 2$$

$$\text{ans} = 3 \times 2 \Rightarrow 6$$

$$\text{ans} = 4 \times 6 \Rightarrow 24$$

Calculate x raised to power n . Recursively

Faith

My function
will calculate

x^{n-1} perfectly

$$2^4 \Rightarrow 2 \times 2 \times 2 \times 2$$

```
public static int power(int x, int n){  
    1 if(n==0) return 1;  
    2 int sAns = power(x,n-1);  
    3 int ans = x * sAns;  
    4 return ans;  
}
```

Expectation

My function will calculate
 x raised to power $\underline{\underline{x^n}}$

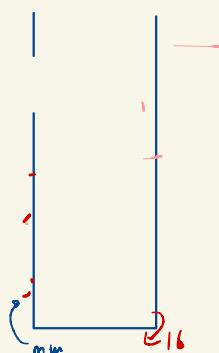
$$2 \times 2 \times 2 \times 2 \times 2$$

$$\text{ans} = x * 1 = 2 * 1$$

$$\text{ans} = 2 * 2 = 4$$

$$\text{ans} = 2 * 4 = 8$$

$$\text{ans} = 2 * 8 = 16$$



```

public static int power(int x, int n){
    1) if(n==0){
        return 1;
    }

    2) int sAns = power(x,n-1);
    3) int ans = x * sAns;

    4) return ans;
}

```

Faith

My function will calculate $x^{\frac{n}{2}}$ perfectly.

$$3^{\frac{6}{2}} \Rightarrow 3^3$$

$$sAns = 3 \times 3 \times 3$$

$$ans = sAns \times sAns$$

$$x^{\frac{n}{2}} \Rightarrow 3^{\frac{1}{2}} \Rightarrow 3^2$$

$$sAns = 3 \times 3 \times 3$$

$$ans = \underline{sAns \times sAns \times x}$$

Expectation

My function will calculate x^n .

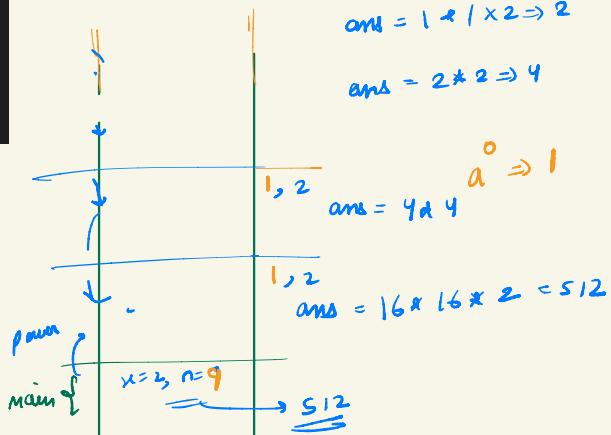
```

public static int power_log(int x, int n){
    1) if(n==0) return 1;
    2) int sAns = power_log(x, n/2);
    3) int ans=0;
        if(n%2==0){ // if n is even
            ans = sAns * sAns;
        } else { // if n is odd
            ans = sAns * sAns * x;
        }
    4) return ans;
}

```

$$3^6 \Rightarrow \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}$$

$$3^1 \Rightarrow \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3} \times \underline{x}$$



On schedule
1 class

10:10 break

revision in string

$$\frac{n}{2^2} = 1 \rightarrow \text{solve this for } n$$

Ours sum of first n natural numbers recursively.

Fault

$$\begin{array}{l} (\underline{n-1}) \\ 1+2+3+4 \end{array}$$

```
// sum of first n natural numbers
public static int sum_natural(int n){
    if(n==0){
        return 0;
    }

    int sAns = sum_natural(n-1);
    int ans = n + sAns;

    return ans;
}
```

Expectation

$$n=5$$

$$1+2+3+4+\underline{5}$$

Recursion in Arrays

Ours Print the array recursively.

Fault

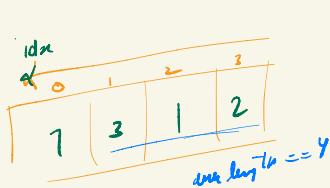
My fun. will print array from $idx+1$ to last.

```
psv print(int idx, int[] arr)
    System.out.println(arr[idx]);
    print(idx+1, arr);
```

Expectation

My function will print array from $idx=0$ to end

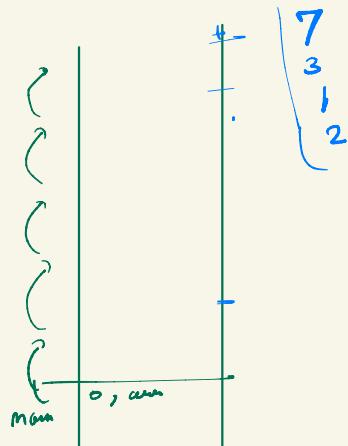
of
3
1
2



Recursion on the way up

```
public static void printArray(int idx, int[] arr){
    if(idx == arr.length){
        return;
    }
    System.out.println(arr[idx]);
    printArray(idx+1, arr);
}
```

why?



quality >> quantity

Java at point

4 hours

15 min

b a b a b

y (down s), ch(e)

y (s > e)
return 1;

return 0;