

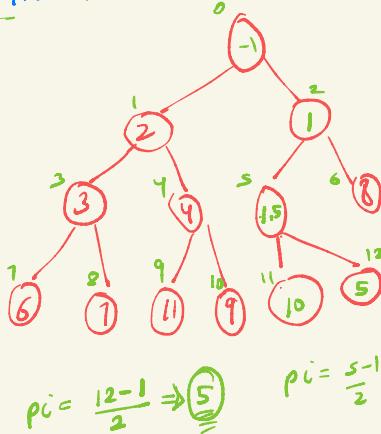

Heap [min pq]

Complete Binary Tree ✓CBT

HOP
Heap order property

why?
what?
how?

12



$$pi = \frac{12-1}{2} \Rightarrow 5$$

(-1)
upHeapify

→ every parent will be smaller than child

{-1, 2, 1, 3, 4, 1.5, 8, 6, 7, 11, 9, 10, 12, 5}

$$pi = 4$$

$$lci = 2 * pi + 1; \\ rci = 2 * pi + 2;$$

$$pi = \frac{5-1}{2} \Rightarrow 2 \\ pi = \frac{2-1}{2} \Rightarrow 0 \\ swap(c[i], c[pi])$$

$$lci = 9 \\ rci = 10$$

$$pi = \frac{ci-1}{2}$$

```
static class Heap {
    ArrayList<Integer> data;
    public Heap(){
        data = new ArrayList<>();
    }

    public void swap(int i, int j){
        int valAtI = data.get(i);
        int valAtJ = data.get(j);

        data.set(i, valAtJ);
        data.set(j, valAtI);
    }

    public void upHeapify(int ci){
        int pi = (ci-1)/2;

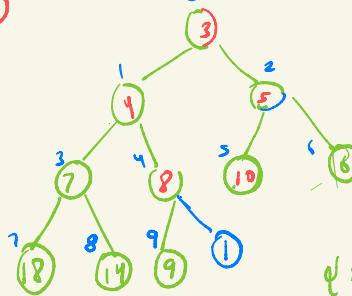
        if(data.get(pi) > data.get(ci)){
            swap(ci, pi);
            upHeapify(pi);
        }
    }
    // add
    public void add(int val){
        data.add(val);

        upHeapify(data.size()-1);
    }
}
```

data ⇒ {3, 4, 5, 7, 8, 10, 6, 18, 14, 9}

(-1)

O(n)



$$pi = 0 \\ lci = 2 * pi + 1 = 1 \\ rci = 2 * pi + 2 = 2$$

$$\{0, 13\} \\ pi_{min} \\ lci = 3 \\ rci = 4$$

{1, 4}

downHeapify

```

private void downHeapify(int pi){
    int lci = 2*pi + 1;
    int rci = 2*pi + 2;

    int mini = pi;

    if(lci < data.size() && data.get(lci) < data.get(mini)){
        mini = lci;
    }

    if(rci < data.size() && data.get(rci) < data.get(mini)){
        mini = rci;
    }

    if(mini != pi){
        swap(pi,mini);
        downHeapify(mini);
    }
}

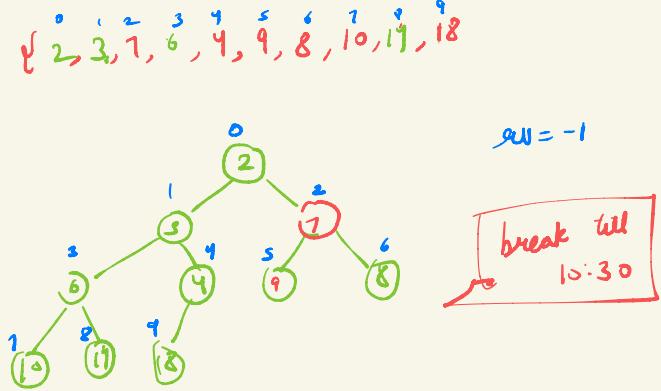
// remove
public int remove(){
    if(data.size()==0){
        System.out.println("There is no element in heap");
        return -1;
    }

    swap(i: 0,data.size()-1);
    int rv = data.remove(data.size()-1);

    downHeapify(pi: 0);
}

return rv;
}

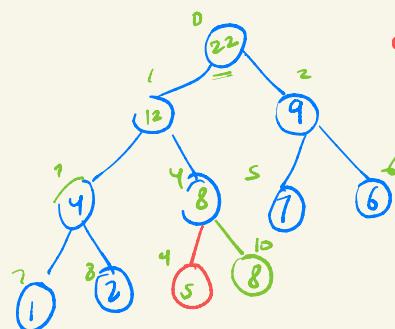
```



$$\text{mini} = \cancel{0} \cancel{1} \cancel{2} \cancel{3}$$

$$\begin{aligned} pi &= \cancel{0} \cancel{1} \cancel{2} 3 \\ lci &= \cancel{0} \cancel{1} 7 \\ rci &= \cancel{0} 4 8 \end{aligned}$$

$$\begin{aligned} ci &= \cancel{0} \cancel{1} \cancel{2} 0 \\ pi &= \cancel{0} \cancel{1} \cancel{2} -1 \end{aligned}$$



$$\begin{aligned} &0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13 \\ &22, 12, 9, 4, 8, 7, 6, 1, 3, 5, 8, 3 \end{aligned}$$

$$6, 4, 1, 3, 10, 9, 11, 8, 13$$

```

private void swap(int i, int j){
    int valAtI = data.get(i);
    int valAtJ = data.get(j);

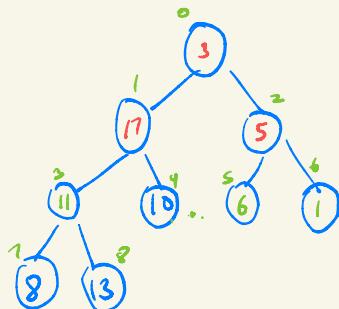
    data.set(i,valAtJ);
    data.set(j,valAtI);
}

private void upHeapify(int ci){
    int pi = (ci-1)/2;
    if(pi>=0 && data.get(pi) < data.get(ci)){
        swap(ci,pi);
        upHeapify(pi);
    }
}

// add
public void add(int val){
    data.add(val);
    upHeapify(data.size()-1);
}

```

Hop



arr ⇒ {
3, 17, 5, 11, 10, 6, 1, 8, 13}
↑ ↑ ↑

(PQ)

```

void swap(int[] data, int i, int j){
    int temp = data[i];
    data[i] = data[j];
    data[j] = temp;
}

void upHeapify(int[] data, int ci){
    int pi = (ci-1)/2;

    if(pi >= 0 && data[pi] > data[ci]){
        swap(data, pi, ci);
        upHeapify(data, pi);
    }
}

void buildHeap(int arr[]){
    for(int i=0; i<arr.length; i++){
        upHeapify(arr, i);
    }
}

```

Priority Queue

Priority Queue < Integer > pq = new Priority Queue<=>;
↳ min pq {min heap}

 pq.add(3)
 pq.add(4)

 pq.peek();

 pq.add(2);

 pq.remove()
 pq.remove

```

class Student {
    int roll-no;
    String name;
}

```

Comparable
↓
Interface
comparator

S1, S2

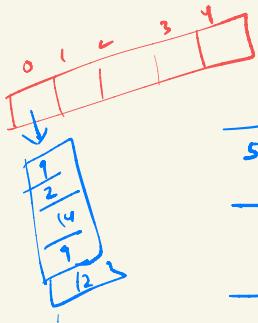
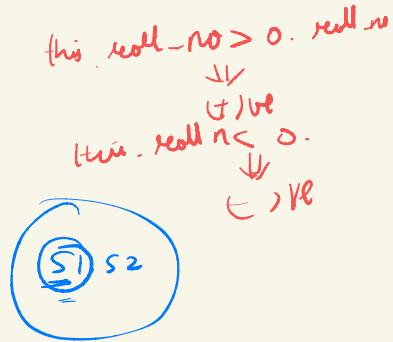
$s1 = \{1, 3\}$

$s2 = \{2, 3\}$

```
public int compareTo(Student o){
    return this.roll_no - o.roll_no;
}
```

$$1 - 2 \Rightarrow -1$$

smaller on the top \Rightarrow this - other
 greater on the top \Rightarrow other - this



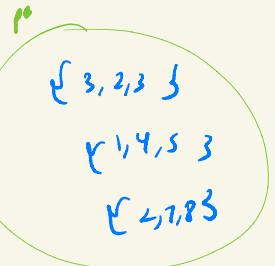
↓ matrix

5	9	2	14	9	12	8
1	1	5	10	11	7	13
4	5	7	15	6	17	18
2	3	1	19	16	5	4
3	4	8	9	1	2	3

5 10 arrays

matrix

$pQ < \underline{\underline{\text{int}[j]}} > pQ$
 = new $pQ < 70;$



```

int[][] matrix={{3,2,3},{1,4,5},{2,7,8}};
PriorityQueue<int[]> pq = new PriorityQueue<>((int[] t, int[] o) ->{
    // return t[0] - o[0]; // min pq
    return o[1] - t[1];
});

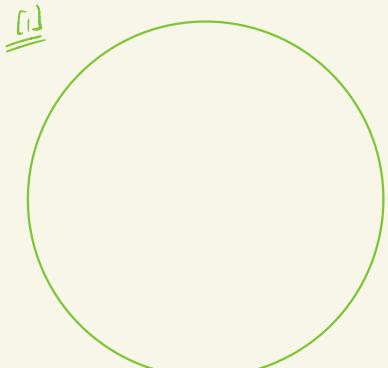
for(int i=0; i<matrix.length; i++){
    pq.add(matrix[i]);
}

int[] top=pq.remove();
for(int i=0; i<top.length; i++){
    System.out.print(top[i]+" ");
}

```

3, 2, 3

{3, 2, 3}
{1, 4, 5}
{2, 7, 8}
{1, 4, 5}



```

public int compareTo(Student other){
    return this.roll_no - other.roll_no; // min pq
    // return other.roll_no - this.roll_no //=> max pq
}

```

...
public static int findKthLargest(int[] arr, int n, int k){
 PriorityQueue<Integer> pq = new PriorityQueue<>();

 for(int i=0; i<n; i++){
 pq.add(arr[i]);

 if(pq.size() > k){
 pq.remove();
 }
 }
 return pq.peek();
}

...
public static int findKthSmallest(int[] arr, int n, int k){
 PriorityQueue<Integer> pq = new PriorityQueue<>(Collections.reverseOrder());

 for(int i=0; i<n; i++){
 pq.add(arr[i]);

 if(pq.size() > k){
 pq.remove();
 }
 }
 return pq.peek();
}

6
7 10 4 3 20 15
4
1, 2, 3 → smallest elements
③

5, 1, 10, 9, 9, 3, 11, 9, 12
↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑

10:30
break

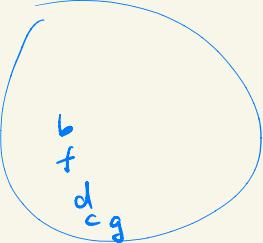
a a a b b c d e f e g e f e e

$\rightarrow a=3$
 $\rightarrow b=2$
 $c=1$
 $d=1$
 $e=5$
 $f=2$
 $g=1$

map.get(other)
map.get(this)

$a > b$

h = b
 $fre=2$
 f



```

public String frequencySort(String s) {
    HashMap<Character, Integer> map = new HashMap<>();
    for(int i=0; i<s.length(); i++){
        char ch = s.charAt(i);
        map.put(ch, map.getOrDefault(ch, defaultValue: 0)+1);
        // if map.containskey(ch){
        //     map.put(ch, map.get(ch)+1);
        // } else {
        //     map.put(ch, 1);
        // }
    }

    ArrayList<Character> keys = new ArrayList<>(map.keySet());
    PriorityQueue<Character> pq = new PriorityQueue<>((Character t, Character o) -> {
        return map.get(o) - map.get(t);
    });

    for(char ch : keys){
        pq.add(ch);
    }

    StringBuilder sb = new StringBuilder();
    while(pq.size() > 0){
        char ch = pq.remove();
        int fre = map.get(ch);

        while(fre-- > 0){
            sb.append(ch);
        }
    }

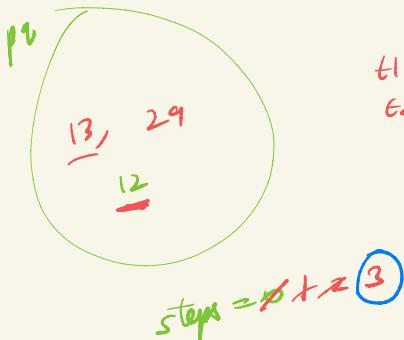
    return sb.toString();
}

```

sb.append
 $sb = e \underline{e} \underline{e} \underline{e} \underline{e} a a a b b \underline{f} \underline{f} \underline{d} \underline{c}$
 $\Rightarrow e e e e a a a f f f d c$

6 7
 $\underline{\underline{h=10}}$

1 2 3 9 10 12



$t_1 = 9$
 $t_2 = 10$
 $1 \times 9 + 2 \times 10$

```

static int solve(int a[], int n, int k){
    PriorityQueue<Integer> pq = new PriorityQueue<>();

    for(int e:a){
        pq.add(e);
    }

    int steps = 0;
    while(pq.size() > 1){
        if(pq.peek() >= k) return steps;

        int t1 = pq.remove();
        int t2 = pq.remove();

        int newSweetness = 1*t1 + 2*t2;
        pq.add(newSweetness);

        steps++;
    }

    if(pq.peek() < k) return -1;
    return steps;
}

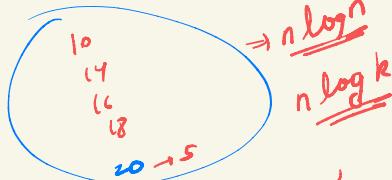
```

$10 < 20$ ans = $\{ 10, 14, 16, 18, 20 \}$ $x=15$
 \downarrow \downarrow \downarrow \downarrow \downarrow
 5 $=$ 5 $k=4$
 Σ $(10, 20)$

$n = 10^8$
 $k = 10, 20$
 $n \log k$

14 \leq 18

element with smaller dist with x



$ans = \{ 14, 16, 18, 10 \}$
 $\{ 10, 14, 16, 18 \}$

```

public List<Integer> findClosestElements(int[] arr, int k, int x) {
    PriorityQueue<Integer> pq = new PriorityQueue<>((Integer t, Integer o) -> {
        if(Math.abs(t-x) == Math.abs(o-x)){
            return t - o;
        }
        return Math.abs(t-x) - Math.abs(o-x);
    });

    for(int e:arr){
        pq.add(e);
    }

    List<Integer> ans = new ArrayList<>();
    while(ans.size() < k){
        ans.add(pq.remove());
    }

    Collections.sort(ans);
    return ans;
}

```

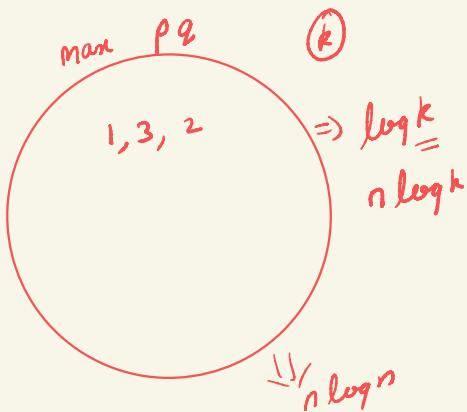
$this - other$
 \downarrow
 $abs(this - x) - abs(other - x)$

$k=3$

$\{ 5, 1, 3, 7, 2, 4, 11 \}$

$\{ 1, 2, 3 \}$

k smallest elements $\Rightarrow n \log n$ (min pq)
 $n \log k$ (min pq)

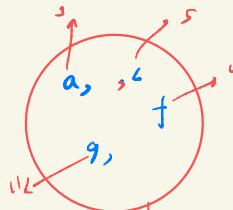


Ques

If some frequent
pref → longer string
pref → smaller freq strings

$a \rightarrow 3$	$e \Rightarrow 2$
$b \rightarrow 3$	$f \Rightarrow 4$
$c \rightarrow 5$	$g \Rightarrow 11$
$d \rightarrow 1$	$gg \Rightarrow 2$
$ba = 3$	

$k = 4$



{a, f, c, g}
{g, c, f, a}

```

public List<String> topKFrequent(String[] words, int k) {
    HashMap<String, Integer> map = new HashMap<>();
    for(String s : words){
        map.put(s, map.getOrDefault(s, defaultValue: 0)+1);
    }
    PriorityQueue<String> pq = new PriorityQueue<>((String t, String o)->
        if(map.get(t) == map.get(o)){
            return o.compareTo(t);
        } else {
            return map.get(t) - map.get(o);
        });
    for(String key:map.keySet()){
        pq.add(key);
        if(pq.size()>k){
            pq.remove();
        }
    }
    List<String> ans = new ArrayList<>();
    while(pq.size()>0){
        ans.add(pq.remove());
    }
    Collections.reverse(ans);
    return ans;
}

```

bricks for smaller changes
ladders for bigger changes

[4, 12, 2, 7, 3, 18, 20, 3, 19]

ladders = 2, bricks = 10

$b = 10 + 8 - 6 \times 2 =$

{1, 2, 3, 4, 5, 8, 10}

```

public int furthestBuilding(int[] heights, int bricks, int ladders) {
    PriorityQueue<Integer> pq = new PriorityQueue<>();
    for(int i=1; i<heights.length; i++){
        int diff = heights[i] - heights[i-1];
        if(diff<=0) continue;
        pq.add(diff);
        if(pq.size()>ladders){
            int min = pq.remove();
            bricks -= min;
        }
        if(bricks < 0) return i-1;
    }
    return heights.length - 1;
}

```

① ↓ ↓
{4, 12, 24, 48, 50, 52, 31, 37, 90}
 $b = 18 + 10 + 8 - 6 \times 2 = 3$
 $l = 2$



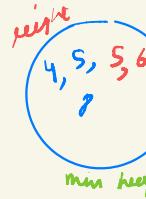
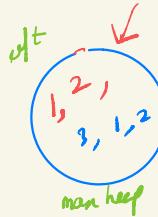
addNum(2)
findMedian()

q², 4, 1, 5, 6, 3, 8, 5, 1, 2

q¹, 2, 3, 4, 5, 5, 6, 8, 9

break till

10:40



3 < 7

left.peek
→ 4

$$\begin{aligned} l &< r \\ ls &= -rs \\ ls &= -rs + 1 \end{aligned}$$

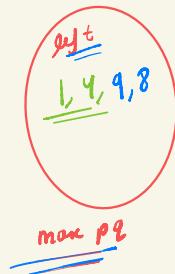
```
public void addNum(int num) { // log(n)
    if(left.size() == 0 || num <= left.peek()){
        left.add(num);
    } else {
        right.add(num);
    }

    int ls = left.size();
    int rs = right.size();

    if(ls - rs == 2){
        right.add(left.remove());
    } else if(rs > ls){
        left.add(right.remove());
    }
}

public double findMedian() { // O(1)
    if(left.size() == right.size()){
        return (1.0 * left.peek() + right.peek()) * 1.0 / (2.0);
    } else {
        return left.peek() * 1.0;
    }
}
```

left ⇒ smaller numbers
right ⇒ larger numbers

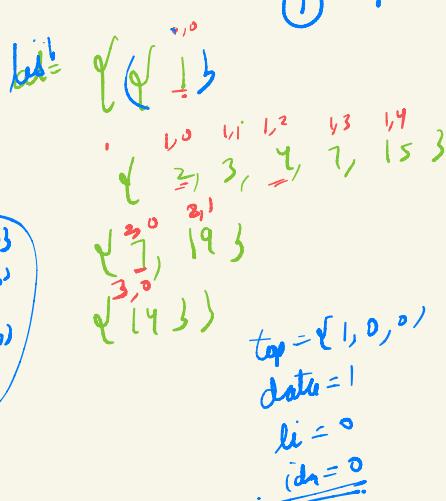
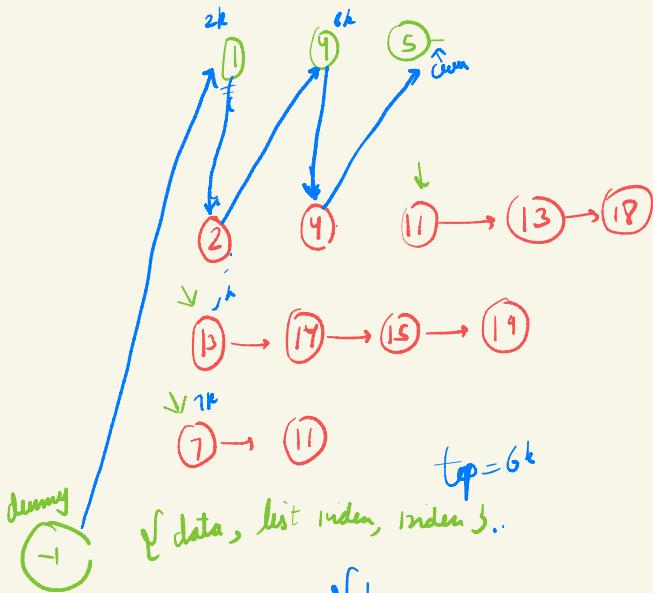


ls = -rs
.

ls = rs + 1
↓
left.peek

1, 4, 8 (9), 10, 13, 15

o



```

public ListNode mergeKLists(ListNode[] lists) {
    PriorityQueue<ListNode> pq = new PriorityQueue<>((ListNode t, ListNode o) -> {
        return t.val - o.val;
    });

    for(ListNode ln : lists){
        if(ln!=null)
            pq.add(ln);
    }

    ListNode dummy = new ListNode(-1);
    ListNode curr = dummy;

    while(pq.size() > 0){
        ListNode top = pq.remove();

        curr.next = top;
        curr = curr.next;
        if(top.next!=null){
            pq.add(top.next);
        }
    }

    return dummy.next;
}

```

```

public static ArrayList<Integer> mergeKSortedLists(ArrayList<ArrayList<Integer>> lists) {
    ArrayList<Integer> ans = new ArrayList<>();

    PriorityQueue<int[]> pq = new PriorityQueue<>((int[] t, int[] o) -> {
        return t[0] - o[0];
    });

    for(int i=0; i<lists.size(); i++){
        int data = lists.get(i).get(0);
        pq.add(new int[]{data, i, 0});
    }

    while(pq.size() > 0){
        int[] top = pq.remove();

        int data = top[0];
        int li = top[1];
        int idx = top[2];

        ans.add(data);
        if(idx + 1 < lists.get(li).size()){
            data = lists.get(li).get(idx+1);
            pq.add(new int[]{data, li, idx+1});
        }
    }

    return ans;
}

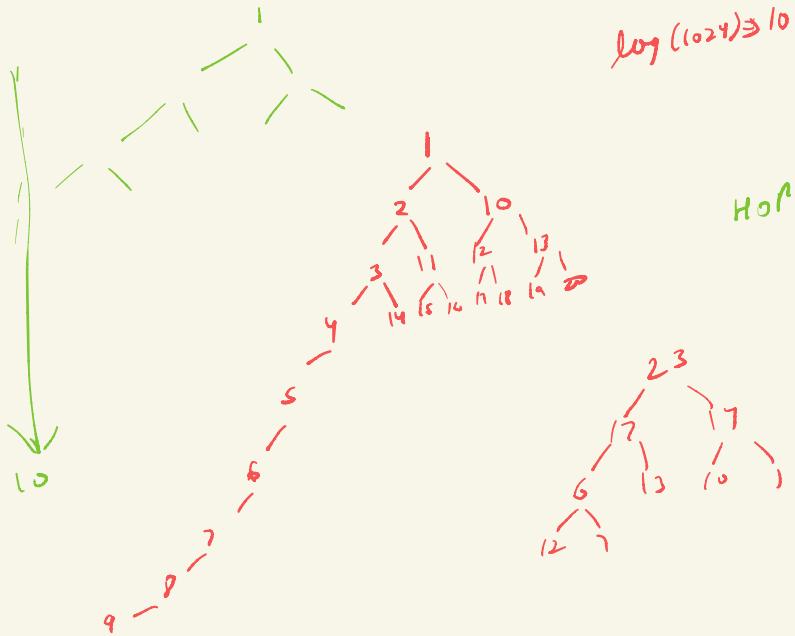
```

$\log_{10} 1023$

1024

$$\cancel{\log_2 9 \approx 3}$$

$$\log_{10} (1024) \geq 10$$



Hop