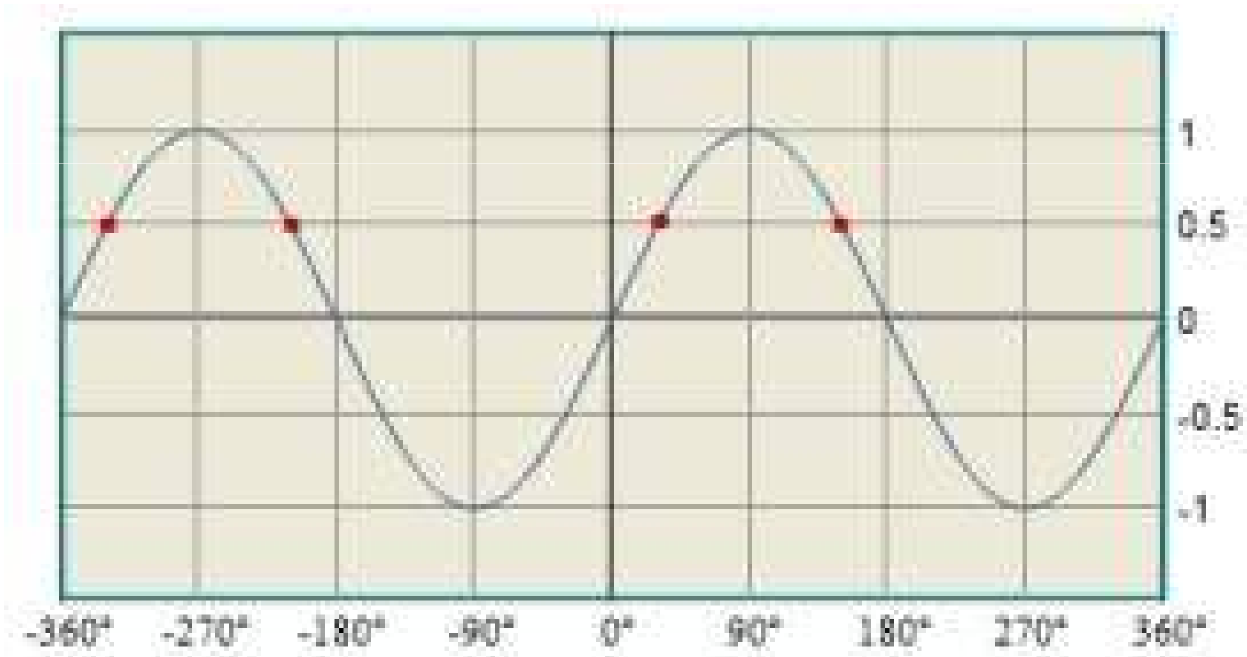
Periodic motion

Definition: The motion which repeates after certain interval of the time, along the same path is called periodic motion. For eg: motion of the planet around sun, motion of piston in car/bike, motion of hands of watch etc.



Simple harmonic motion

➢This is the simplest type of the harmonic motion in which motion will always have constant amplitude and unique frequency.

➢The most fundamental requirement is that acceleration is directed towards the mean position and,

Acceleration α displacement

or ***acceleration = -k * displacement***

Where k is the constant and negative sign shows that the acceleration is directed in opposite to the motion.

➢ This motion can be described interms of time period and amplitude.

➢ Time period: it is the time taken by the particle to complete one oscillation in SHM.

- It is given by, $$\boxed{T = 2\pi\sqrt{\frac{Displacement}{Acceleration}}}$$

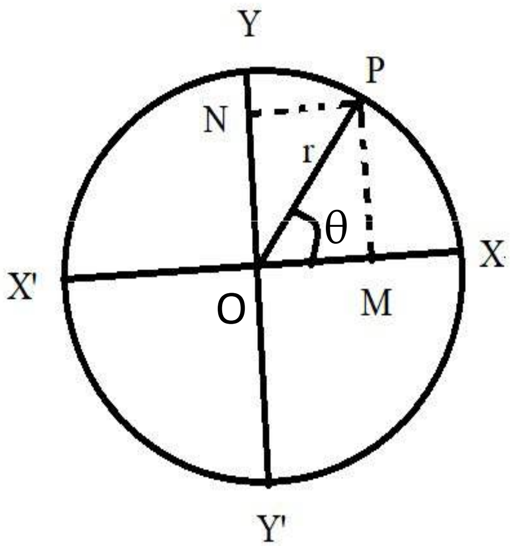
- Frequency: the frequency of the oscillating particle in SHM is defined as the total number of oscillation completed in one second. It is denoted by f and expressed as.

$$\boxed{f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{Acceleration}{Displacement}}}$$

- Amplitude: the maximum displacement of the particle from mean position is called amplitude.

Equation of SHM:

- **Displacement equation:**



SHM can be described by taking the uniform circular motion.

In the ΔONP,

$$\sin\theta = ON/OP$$

$$ON = OP \sin\theta \quad \ldots\ldots.(i)$$

Here, $ON=y$ is called the displacement of particle in SHM and $OP= r$ is the radius of circle. Therefore,

$$y=r \sin\theta$$

$$y=r \sin\omega t, \; \theta= \omega t \quad \ldots\ldots\ldots..(ii)$$

This is the displacement equation in SHM and is periodic, sinusoidal function of time.

➢ **Velocity:** Velocity is simply the time rate of change of displacement. So, we can write,

$$\boxed{v = \frac{dy}{dt}}$$

» $$\boxed{v = \frac{d}{dt}\left(r \sin \omega t\right)}$$

» $$\boxed{v = r\omega \cos \omega t}$$

» $$\boxed{v = r\omega\sqrt{1-\sin^2 \omega t}}$$

» $$\boxed{v = r\omega\sqrt{1-\frac{y^2}{r^2}} = \omega\sqrt{r^2 - y^2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(iii)}$$

From this equation we can say that the velocity is not uniform.

For **y=0, v= rω** . It means that the velocity is maximum at mean position.

Similarly, for **y=r** (at extreme point), **v=0**. It means that the velocity is zero at extreme position for SHM.

- **Acceleration:** The time rate of change of velocity is called acceleration and is given by, for SHM, as

a= dv/dt

$$= \frac{d\left(r\omega\cos\omega t\right)}{dt} = r\omega * -\omega\sin\omega t$$

$$= -\omega^2 * r\sin\omega t$$

$$= -\omega^2 * y$$

This gives the acceleration of particle obeying SHM. Here , for mean position, y=0 so a=0 i.e. acceleration is zero at mean position. Also for y=r (at extreme point), a=- ω^2r. It means that the acceleration is maximum at extreme point.

Simple pendulum

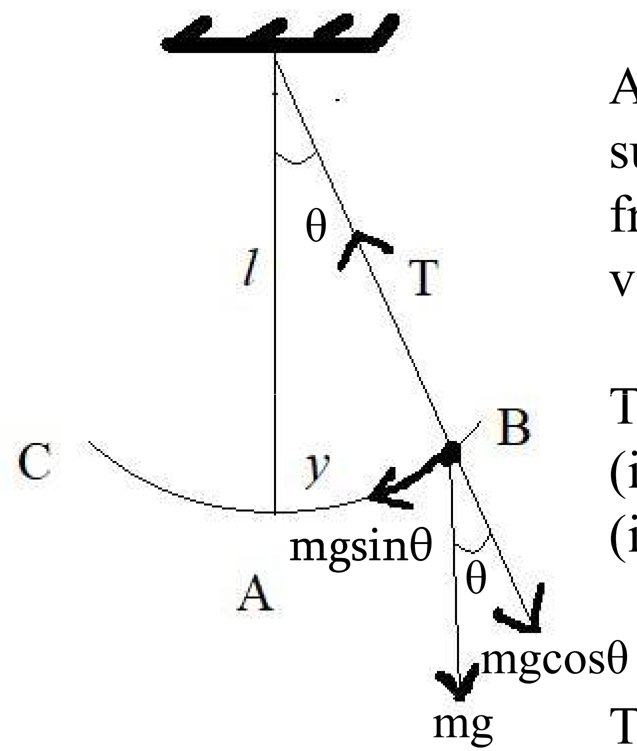


Fig: simple pendulum

A simple pendulum is heavy point mass object suspended by inextensible string of negligible mass from the rigid support which is free to oscillate in a vertical plane.

There are two forces acting on the bob.

(i) First is force due its weight.

(ii) Second is tension T acting on the string along its length towards the point of suspension.

The weight of bob can be resolved into two components. One is mgcosθ acting opposite to the tension on the string and another is mgsin θ as in fig.

- The component mgsinθ represents the restoring force and can provide acceleration of the bob during motion.

Therefore, we write,
$$\boxed{\begin{aligned}&F = ma = -mg\sin\theta\\&\Rightarrow a = -g\sin\theta\end{aligned}}$$

For very small θ, sin θ≈ θ= arc AB/l

= y/l

So, a=-g. y/l=-(g/l).y ………………(v)

here the term (g/l) is constant for any pendulum at given place. Therefore,

a α y

It shows that acceleration is proportional to displacement (y) and is directed towards the mean position (due to presence of –ve sign in equation (v). Hence motion of simple pendulum is simple harmonic in nature.

- **Time period of simple pendulum:** we have for simple pendulum,

a=-g. y/l ………………(vi)

Also , for SHM, acceleration is,

$$\boxed{a = -\omega^2 y} \qquad \text{…………………….(vii)}$$

From (vi) and (vii),

$$\boxed{\begin{aligned} -\omega^2 &= -\frac{g}{l} \\ \Rightarrow \omega &= \sqrt{\frac{g}{l}} \end{aligned}} \quad \text{..................(viii)}$$

Again, time period ,

$$\boxed{T = \frac{2\pi}{\omega}} \quad \text{.......................(ix)}$$

So we have,

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}} \quad \text{......................(x)}$$

This gives the time period of simple pendulum. Here we can see that time period depends on,

(i) length of pendulum.

(ii) acceleration due to gravity.

(iii) But it doesn’t depend on the mass of bob.