Indian Academy of Sciences, Bengaluru
Indian National Science Academy, New Delhi
The National Academy of Sciences India, Prayagraj

SUMMER RESEARCH FELLOWSHIPS — 2021

Format for the Final-week Report*

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Application Registration no. : MATS295

Date of Commencement of work : 01-06-2021

Mode of work : From Home: ✓ Guide's Laboratory: ☐

Date of completion : 31-07-2021

Total no. of days worked : 61 days

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If, NO, Please specify reason : NA

Samriddhi

Signature of the candidate

Date: 30.07.2021

SNandy

Signature of the guide

Date: 31.07.2021

*The final report could be anywhere between 20 and 25 pages including tables, figures etc.
This format should be the first page of the report and should be stapled with the main report.

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PFMS Unique Code:	A/c holder's name:
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SUMMER RESEARCH FELLOWSHIP 2021

Final-Week Report

REAL LIFE APPLICATION OF BAYES FILTER

Name- Samriddhi Soni

Application Number- MATS295

INTRODUCTION

Statistics is the science of numbers; it is about collection of data and interpreting the results on the basis of the analysis of data. The data can be divided into two parts i.e., quantitative and qualitative data. Statistics has a wide range of applications in engineering such as quality control, design of experiments, system identification, probabilistic design and many more.

One of the major part of statistics is probability and its application such as model building and generating probability distributions. The concept of bayes filter based on the bayes theorem which is a part of the probability theory has many applications. Some of the real-life applications of the Bayesian estimation are mobile robot localization, mapping, motion estimation and many more.

In recent times, the use of probabilistic models and bayes theorem in robotics have grown manifold. This application has given rise to the field of probabilistic robotics.

PROBABILISTIC ROBOTICS

Probabilistic robotics has endowed field robots with unparalleled levels of autonomy and robustness. Building on the field of mathematical statistics, probabilistic robotics equip robots with a new level of robustness in real-world situations. So, what exactly is the probabilistic approach to robotics? At its core probabilistic robotics is the idea of estimating quantities from sensor data. State estimation addresses the problem of estimating quantities from sensor data that are not directly observable, but that can be inferred. Probabilistic state estimation algorithms compute belief distributions over possible world states.

STATE

State is the collection of details of the environment which can impact the robot in future. Environments are characterized by state. Let us denote state through x and the state at time t will be denoted by x_t.

Dynamic State- The state that changes is called dynamic state.

Static State- The state that is non-changing is called static state.

Markov Assumption- The state x_t is called complete if it is the best predictor of the future, the future maybe stochastic but no variable prior to x_t may influence the stochastic evolution of future states, unless this dependence is mediated through the state x_t.

A typical state variable which will be used for discussing the localization of a robot is robot pose:

Robot pose: Robot pose is the location and orientation of the robot relative to a global coordinate frame. For rigid mobile robots which are confined to planar environments, the pose is usually given by three variables, its two location coordinates in the x-y cartesian plane and its Euler angle which is its heading direction (yaw).

BAYES RULE AND ITS APPLICATION IN PROBABILISTIC ROBOTICS

Bayes rule is very important in probabilistic robotics because it relates p(x | y) to its inverse, p(y | x). Following is the bayes rule:

$$p(x|y) = \frac{p(y|x).p(x)}{p(y)} = \frac{p(y|x).p(x)}{\sum_{x'} p(y|x).p(x')} \qquad (discrete\ quantity) \qquad (1)$$

$$p(x|y) = \frac{p(y|x).p(x)}{p(y)} = \frac{p(y|x).p(x)}{\int p(y|x').p(x').dx'} \qquad (continuous\ quantity) \qquad (2)$$

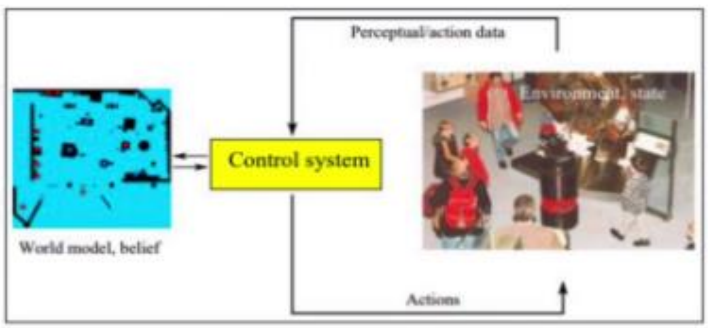
Application-

- If x is a quantity that we would like to infer from y, the probability p(x) will be referred to as prior probability distribution, and y is called the data (e.g., a sensor measurement). The distribution p(x) summarizes the knowledge we have regarding X prior to incorporating the data y, since prior probability is the probability as assessed before making reference to relevant observations.
- In statistics, posterior probability is the probability that a hypothesis is true calculated in the light of relevant observation, therefore the probability p(x | y) is called the posterior probability distribution over X. As (1) suggests, Bayes rule provides a convenient way to compute a posterior p(x | y) using the “inverse” conditional probability p(y | x) along with the prior probability p(x).

- In other words, if we are interested in inferring a quantity x from sensor data y, Bayes rule allows us to do so through the inverse probability, which specifies the probability of data y assuming that x was the case. In robotics, this inverse probability is often coined "generative model," since it describes, at some level of abstraction, how state variables X cause sensor measurements Y.

ENVIRONMENT INTERACTION

Robots are inherently uncertain about the state of their environments. Robot gathers information regarding the environment either by using its sensors or by performing an action to change the state of its environment. However, sensors are noisy and there are many things that cannot be sensed directly, therefore robot maintains an internal belief regarding the state of the environment.



Following are the two fundamental kinds of interactions between a robot and its environment:

I. **Sensor Measurements-** Sensor measurements are the input measurements obtained by the sensors of the robot about the state of the environment, this process is called perception.
II. **Control Actions-** Control actions change the state of the world, by actively asserting forces on the robot's environment. Even if the robot does not perform any action itself, state usually changes, because of the constant change in the environment.

Practically, the robot continuously executes controls actions and measurements are made simultaneously. The collection of all the past measurements and control actions are referred to as data. According to the above-mentioned types of environments interactions, a robot has two different kinds of data streams:

i. **Measurement data-** It provides information about the state of the environment at a particular time. The measurement data at time t will be denoted by:

$$\mathbf{z_t} \tag{3}$$

For denoting the set of all measurements acquired from time t_1 to time t_2; $t_1 \leq t_2$

$$\mathbf{z_{t1:t2} = z_{t1}, z_{t1+1}, z_{t1+2}, \ldots, z_{t2}} \tag{4}$$

ii. **Control data-** Control data carry information about the change of state in the environment. Change of state in the time interval (t-1:t] will be denoted by:

$$\mathbf{u_t} \tag{5}$$

For denoting sequence of control data $u_{t1:t2}$ for $t_1<t_2$

$$\mathbf{u_{t1:t2} = u_{t1}, u_{t1+1}, u_{t1+2}, \ldots, u_{t2}} \tag{6}$$

PROBABILISTIC GENERATIVE LAWS

The state x_t is generated stochastically, thus it is necessary to specify the probability distribution from which x_t is generated. The state x_t depends on all past states, measurements and controls. Hence, the probabilistic law characterizing the evolution of state might be given by a probability distribution of the following form:

$$\mathbf{p(x_t \mid x_{0:t-1}, z_{1:t-1}, u_{1:t})} \tag{7}$$

If the state x is complete then it is a sufficient summary of all that happened in previous time steps. Thus, x_{t-1} is a sufficient statistic for all previous controls and measurements up to this point, i.e., $u_{1:t-1}$ and $z_{1:t-1}$. From all the variables in the expression above, only the control u_t matters if we know the state x_{t-1}. Therefore, due to conditional independence we obtain the following expression:

$$\mathbf{p(x_t \mid x_{0:t-1}, z_{1:t-1}, u_{1:t}) = p(x_t \mid x_{t-1}, u_t)} \tag{8}$$

For monitoring the model of the process by which measurements are being generated, we will consider an important conditional independence (if x_t is complete):

$$\mathbf{p(z_t \mid x_{0:t}, z_{1:t-1}, u_{1:t}) = p(z_t \mid x_t)} \tag{9}$$

State transition probability- The probability $\mathbf{p(x_t \mid x_{t-1}, u_t)}$ is the state transition probability, it specifies how environmental state evolves over time as a function of robot controls u_t.

Measurement probability- The measurement probability $\mathbf{p(z_t \mid x_t)}$ specifies the probabilistic law according to which measurements z are generated from the environment state x.

The state transition probability and the measurement probability together describe the dynamical stochastic system of the robot and its environment. State at time t is stochastically dependent on the state at time t − 1 and the control u_t. The measurement z_t depends stochastically on the state at time t. Such a temporal generative model is also known as hidden Markov model (HMM) or dynamic Bayes network (DBN).

BELIEF DISTRIBUTIONS

Belief- A belief is the robot's internal knowledge about the state of the environment, belief or state of knowledge usually depends on the former data available to the robot.

Probabilistic robotics represents beliefs through conditional probability distributions. A belief distribution assigns a probability (or density value) to each possible hypothesis with regards to the true state. Belief distributions are posterior probabilities over state variables conditioned on the available data. We will denote belief over a state variable x_t by $bel(x_t)$, which is an abbreviation for the posterior:

$$\mathbf{bel(x_t) = p(x_t \mid z_{1:t}, u_{1:t})} \tag{10}$$

For calculating a posterior without incorporating z_t, just after executing the control u_t. Such a posterior will be denoted as follows:

$$\overline{bel}\,(x_t) = p(x_t \mid z_{1:t-1}, u_{1:t}) \tag{11}$$

Calculating $\mathbf{bel(x_t)}$ from $\overline{\mathbf{bel}}\,(\mathbf{x_t})$ is called correction or the measurement update, because the probability $\overline{bel}\,(x_t)$ is predicting the state at time t as measurement at time t is not taken under consideration.

BAYES FILTER

Bayes filter or recursive Bayesian estimation is general probabilistic approach for estimating an unknown probability density function recursively over time using incoming measurements and a mathematical process model.

Algorithm Bayes_filter$(bel(x_{t-1}), u_t, z_t)$**:**
for all x_t do
Control update step -------➔ $\overline{bel}(x_t) = \int p(x_t \mid u_t, x_{t-1})\, bel(x_{t-1})\, dx$
Measurement update step-➔ $bel(x_t) = \eta\, p(z_t \mid x_t)\, \overline{bel}(x_t)$
endfor
return $bel(x_t)$

Figure 1

- ✦ The Bayes filter is recursive, that is, the belief $bel(x_t)$ at time t is calculated from the belief $bel(x_{t-1})$ at time t−1. Its input is the belief bel at time t−1, along with the most recent control u_t and the most recent measurement z_t. Its output is the belief $bel(x_t)$ at time t.

- In Line 3, it processes the control u_t. It does so by calculating a belief over the state x_t based on the prior belief over state x_{t-1} and the control u_t. In particular, the belief $\overline{bel}(x_t)$ that the robot assigns to state x_t is obtained by the integral (sum) of the product of two distributions: the prior assigned to x_{t-1}, and the probability that control u_t induces a transition from x_{t-1} to x_t. The similarity of this update step can be recognized because of the equation $p(x) = \int p(x|y).p(y).dy$. This update step is called the control update, or prediction.
- The second step of the Bayes filter is called the measurement update. In Line 4, the Bayes filter algorithm multiplies the belief $\overline{bel}(x_t)$ by the probability that the measurement z_t may have been observed. The resulting product is generally not a probability, that is, it may not integrate to 1. Hence, the result is normalized, by virtue of the normalization constant η. This leads to the final belief $bel(x_t)$, which is returned in Line 6 of the algorithm.

Violation of Markov Assumption in practical application of Bayes filter

The Bayes filter makes a Markov assumption that specifies that the state is a complete summary of the past. This assumption implies the belief is sufficient to represent the past history of the robot. In robotics, the Markov assumption is usually only an approximation. There are several conditions under which it is violated, such as; unmodeled dynamics in environment, inaccuracies in probabilistic models and approximation errors.

APPLICATION OF BAYES FILTER: SINGLE TO MULTIPLE DOF SYSTEM

Bayes filter can be used for determining the position of bob of a simple pendulum at a particular time. Further, bayes filter can be used for motion estimation of x-y table (2 DOF system). These are some of the ways in which the knowledge of bayes filter and bayes theorem will be applied to further enrich our study in this spectrum of Probability and Bayesian estimation.

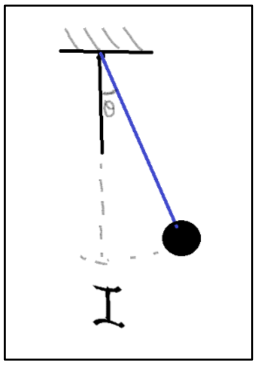
SIMPLE PENDULUM

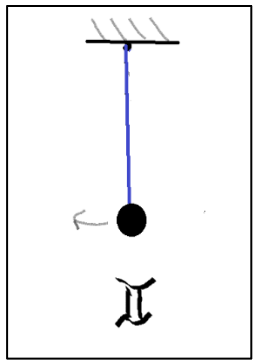
The simple pendulum has a time period which can be defined by small approximation formula. The time period of one oscillation in a simple pendulum is the amount of time a bob of mass m takes to go from position 1 to position 5. The formula which is being used for calculating the time period of one complete oscillation is following:

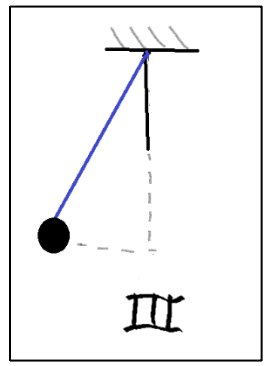
T	Small angle approximation $T=2\pi\sqrt{\frac{L}{g}}$

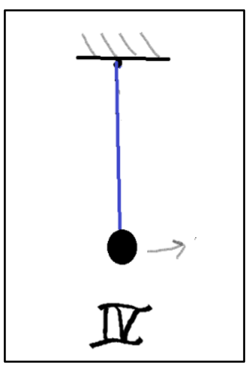
Here the length of the string is taken to be 1m and the gravitational acceleration is taken to be 9.8 m/s^2. The distance covered by the bob during the oscillation can be presented by a formula where s is the distance covered by the bob, L is the length of the string and ϴ is the angular displacement.

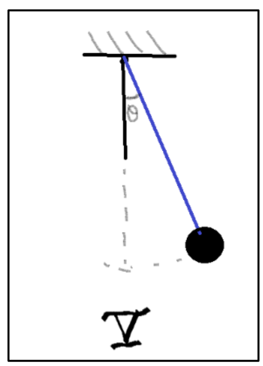
$$s=L\Theta$$











INSTANTANEOUS POSITION OF THE BOB

The position of the bob during oscillation can be determined by both the cartesian coordinates and polar coordinate system. The cartesian coordinates can be (x,y), but for convenience we will convert them into polar coordinates, where $x=r\cos\Theta$ and $y=r\sin\Theta$. Here the value of r would be equal to 1, as the length of string is taken to be 1m and the value of ϴ (instantaneous angular displacement) would be the angle between the mean position and the string.

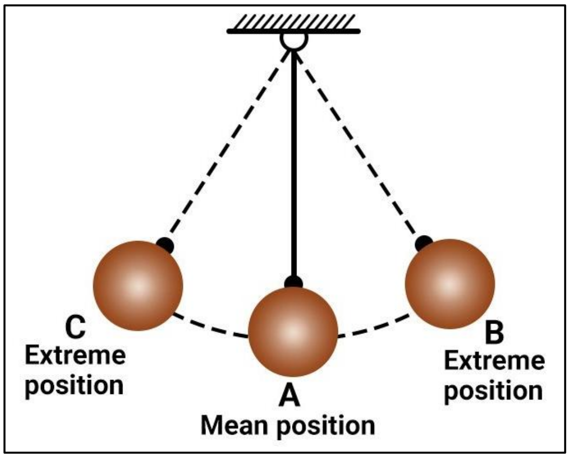
Since, we are using the small angle approximation for determining the time period of oscillation, the initial angular displacement should be less than 15^o. Thus, the initial angular displacement (Θ_o) is taken to be 10^o which is 0.174603175 radians.

For any value of initial angular displacement, the time period of the oscillation will be 2s. Because the value of T does not depend on the initial angular displacement, if the small angle approximation is used.

Initial Angular displacement	**Small angle approximation**
Θ_0 in radians	**T**
0	2.007897776
0.1	2.007897776
0.2	2.007897776
0.3	2.007897776
0.4	2.007897776
0.5	2.007897776
0.6	2.007897776
0.7	2.007897776
0.8	2.007897776

Thus, the time period of the oscillation is fixed to be 2s. If the total time period is 2s, then it can be divided into 20 parts, therefore the value of time would be as follows:

time (in seconds)	**t**	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2



SIGN CONVENTION

According to the time which we have divided into several parts, the respective value of ϴ can also be identified. One thing which should be kept in mind while going through the following table is that the positive value assigned to the angle is because of the convention that the angle is formed on the right side of the mean position, and negative value is assigned to some angles because those angles are formed from the left of the mean position. Following are the various points of time along the instantaneous angular displacement at that time in both degrees and radian:

time (in seconds)	**Instantaneous position**	**Instantaneous angular displacement**	
t	**(r,ϴ)**	**ϴ (in degrees)**	**ϴ (in radians)**
0	$(1,10^o)$	10	0.174603175
0.1	$(1,8^o)$	8	0.13968254
0.2	$(1,6^o)$	6	0.104761905
0.3	$(1,4^o)$	4	0.06984127
0.4	$(1,2^o)$	2	0.034920635
0.5	$(1,0^o)$	0	0
0.6	$(1,-2^o)$	-2	-0.034920635
0.7	$(1,-4^o)$	-4	-0.06984127
0.8	$(1,-6^o)$	-6	-0.104761905
0.9	$(1,-8^o)$	-8	-0.13968254
1	$(1,-10^o)$	-10	-0.174603175
1.1	$(1,-8^o)$	-8	-0.13968254
1.2	$(1,-6^o)$	-6	-0.104761905
1.3	$(1,-4^o)$	-4	-0.06984127
1.4	$(1,-2^o)$	-2	-0.034920635
1.5	$(1,0^o)$	0	0
1.6	$(1,2^o)$	2	0.034920635
1.7	$(1,4^o)$	4	0.06984127
1.8	$(1,6^o)$	6	0.104761905
1.9	$(1,8^o)$	8	0.13968254
2	$(1,10^o)$	10	0.174603175

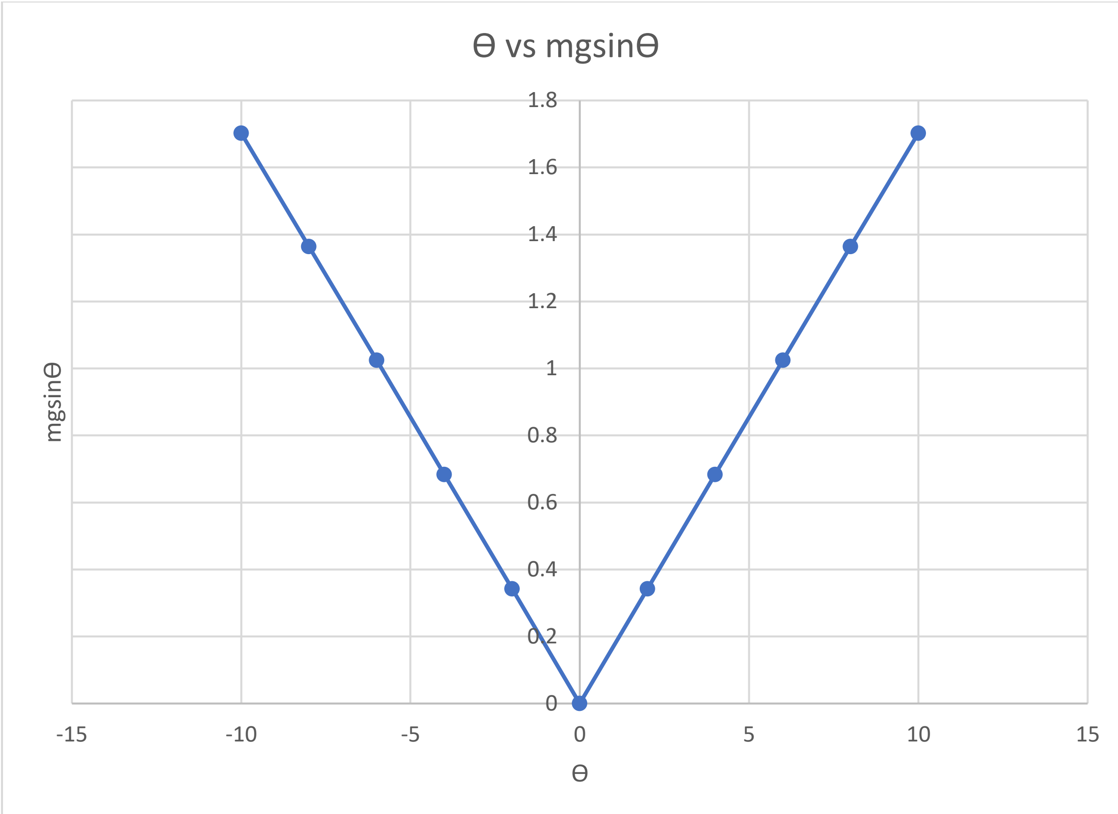
THE GRAVITATIONAL COMPONENT

The gravitational component which is mgsinϴ is responsible for the oscillation of the simple pendulum about the mean position. Here we take the mass of the bob to be the standard i.e.,

m=1 kg.

Following are the various values of the component mgsinϴ along the respective values of ϴ:

Instantaneous angular displacement		g	gsinϴ	m	mgsinϴ
ϴ (in degrees)	**ϴ (in radians)**			**in kg**	
10	0.174603175	9.8	1.70243	1	1.70243
8	0.13968254	9.8	1.364442	1	1.364442
6	0.104761905	9.8	1.02479	1	1.02479
4	0.06984127	9.8	0.683888	1	0.683888
2	0.034920635	9.8	0.342153	1	0.342153
0	0	9.8	0	1	0
-2	-0.034920635	9.8	0.342153	1	0.342153
-4	-0.06984127	9.8	0.683888	1	0.683888
-6	-0.104761905	9.8	1.02479	1	1.02479
-8	-0.13968254	9.8	1.364442	1	1.364442
-10	-0.174603175	9.8	1.70243	1	1.70243
-8	-0.13968254	9.8	1.364442	1	1.364442
-6	-0.104761905	9.8	1.02479	1	1.02479
-4	-0.06984127	9.8	0.683888	1	0.683888
-2	-0.034920635	9.8	0.342153	1	0.342153
0	0	9.8	0	1	0
2	0.034920635	9.8	0.342153	1	0.342153
4	0.06984127	9.8	0.683888	1	0.683888
6	0.104761905	9.8	1.02479	1	1.02479
8	0.13968254	9.8	1.364442	1	1.364442
10	0.174603175	9.8	1.70243	1	1.70243



Interpretation-

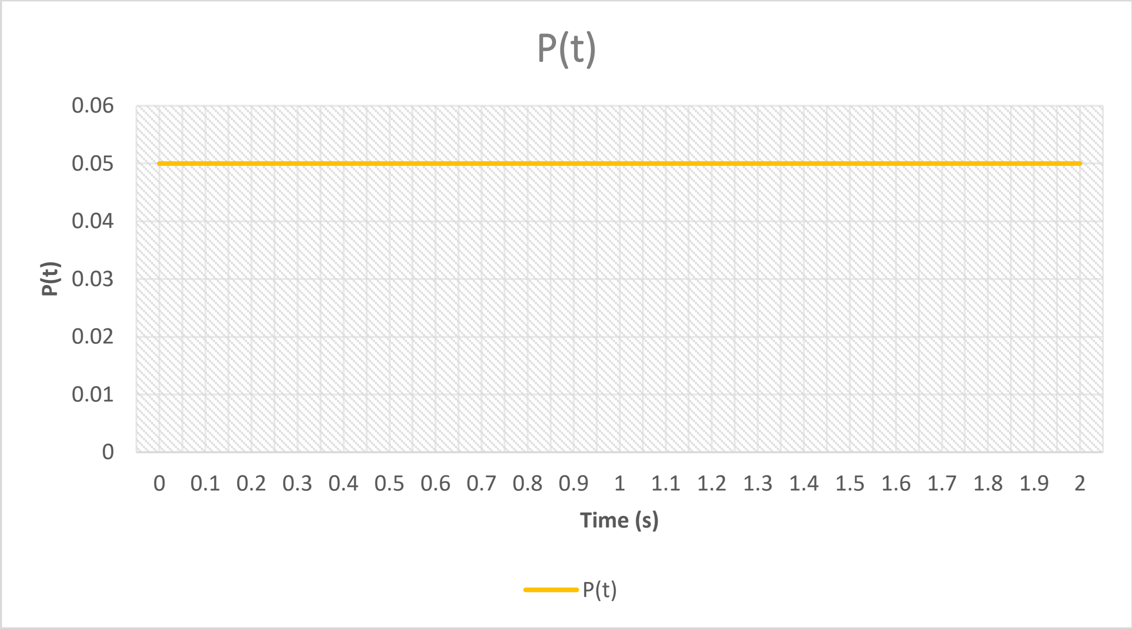
The value of the component mgsinϴ is maximum when it is at the extreme positions. The extreme positions in this case is either 10^o or -10^o. The value of the component mgsinϴ decreases when it comes from the extreme position to the mean position, and it increases when the bob goes from the mean position to the extreme position. The value of the component is minimum and close to 0 when the bob is at its mean position, and the mean position in this case is 0^o.

PROBABILITY DISTRIBUTIONS

Probability distribution of time [P(t)]

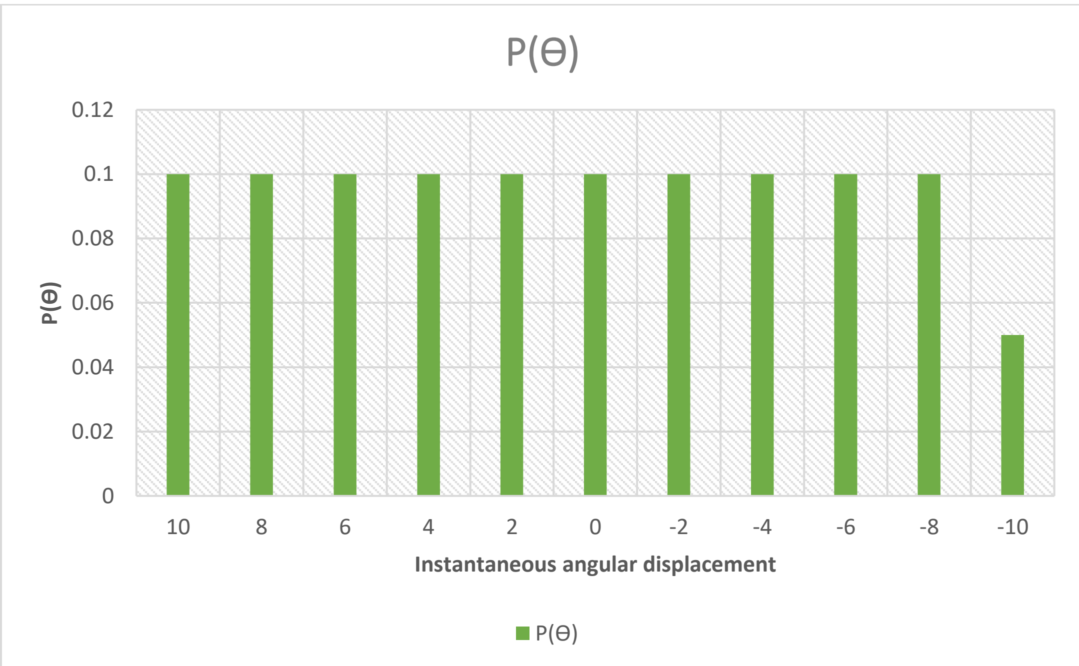
time (in seconds)	**P(t)**
t	
0	0.05
0.1	0.05

0.2	0.05
0.3	0.05
0.4	0.05
0.5	0.05
0.6	0.05
0.7	0.05
0.8	0.05
0.9	0.05
1	0.05
1.1	0.05
1.2	0.05
1.3	0.05
1.4	0.05
1.5	0.05
1.6	0.05
1.7	0.05
1.8	0.05
1.9	0.05
2	0.05



Probability distribution of instantaneous angular displacement [P(Ө)]

Instantaneous angular displacement		P(Ө)
xi	fi	
10	2	0.1
8	2	0.1
6	2	0.1
4	2	0.1
2	2	0.1
0	2	0.1
-2	2	0.1
-4	2	0.1
-6	2	0.1
-8	2	0.1
-10	1	0.05



APPLICATION OF BAYES THEOREM FOR DETERMINING INSTANTANEOUS POSITION OF THE PENDULUM BOB

Bayes Theorem

$$p(H|d) = \frac{p(d|H).p(H)}{p(d)}$$

In the above formula p(H) is the prior probability, p(d|H) is the likelihood function and p(H|d) is the posterior probability which needs to be determined.

Application of bayes theorem-

We are incorporating the bayes theorem for determining the instantaneous position of bob at a particular time t. The prior hypothesis H which is required for the formula can be taken as the value of Θ**.** The value of d would be the time t for determining the position, thus the likelihood function will become p(t|Θ). Thus, the bayes theorem would be:

$$p(\Theta|t) = \frac{p(t|\Theta).p(\Theta)}{p(t)}$$

BAYES FILTER ALGORITHM

Algorithm Bayes_filter($bel(x_{t-1}), u_t, z_t$)**:**
    for all x_t do
        $\overline{bel}(x_t) = \int p(x_t \mid u_t, x_{t-1})\, bel(x_{t-1})\, dx$
        $bel(x_t) = \eta\, p(z_t \mid x_t)\, \overline{bel}(x_t)$
    endfor
    return $bel(x_t)$

Belief Distribution-

For constructing the belief distribution for the instantaneous position of the bob during oscillation, there can be following possibilities. Therefore, the belief distribution would be as follows:

bel (X_o=10)	0.1
bel (X_o=8)	0.1

bel (X_o=6)	0.1
bel (X_o=4)	0.1
bel (X_o=2)	0.1
bel (X_o=0)	0.1
bel (X_o=-2)	0.1
bel (X_o=-4)	0.1
bel (X_o=-6)	0.1
bel (X_o=-8)	0.1
bel (X_o=-10)	0.05

Measurement Probability

bel (X_o=10)	0.1
bel (X_o=8)	0.1

Let us assume one case that at x=0, the pendulum is at the position $\Theta=10^o$. At x=0.2, the next position of bob would be $\Theta=8^o$.

Let us now assume that the sensors are noisy. The noise is characterized by the following conditional probabilities:

P ($Z_{0.2}$=sense_8 | $X_{0.2}$=is_8) = 0.8

P ($Z_{0.2}$=sense_10 | $X_{0.2}$=is_8) = 0.2

Now if we consider the other possible option then it would be as defined as follows:

P ($Z_{0.2}$=sense_8 | $X_{0.2}$=is_10) = 0.2

P ($Z_{0.2}$=sense_10 | $X_{0.2}$=is_10) = 0.8

Finally, let us assume that the action of the actuator would be the effect of gravity. The component of gravity i.e., gsinϴ would be responsible for the simple harmonic motion of the pendulum. The force due to this component is mgsinϴ which would be applicable.

ϴ (in degrees)	**ϴ (in radians)**	**mgsinϴ**
10	0.174603175	1.70243
8	0.13968254	1.364442

Now let us note that if the factor gsinϴ is present, then if the value of Θ_o is 10 then automatically at time=0.2, $\Theta_{0.2}$=8. If Θ_o=10, then at time=0.2, the chance that $\Theta_{0.2}$=10, is 0.8.

State transition probability

P ($X_{0.2}$=is_8 | $U_{0.2}$= mgsin(8^o), X_0= is_8) = 1

P ($X_{0.2}$=is_10 | $U_{0.2}$= mgsin(8^o), X_0= is_8) = 0

P ($X_{0.2}$=is_8 | $U_{0.2}$= mgsin(8^o), X_0= is_10) = 0.8

P ($X_{0.2}$=is_10 | $U_{0.2}$= mgsin(8^o), X_0= is_10) = 0.2

Posterior belief

Let us assume that at time t=0.2s, the control action is $U_{0.2}$=mgsin(ϴ=8^o) and the measurement is taken. The resulting posterior belief can be calculated from the prior belief bel (X_o=10^o).

1. **CONTROL UPDATE STEP**

$\overline{\text{bel}}(x_0) = \int p(xt \mid ut, x0).\, bel(x0)\, dx0$

Since the state space is finite, the integral sign can be turned into summation.

$\overline{\text{bel}}(x_t) = \sum_{xo} p(xt \mid ut, x0).\, bel(x0)$

= p ($x_{0.2}$ | $U_{0.2}$= mgsin(8^o), X_o=is_8).bel(X_o= is_8) + p($x_{0.2}$ | $U_{0.2}$= mgsin(8^o), X_o=is_10).bel(X_o= is_10)

We can now substitute the two possible values for the state variable $X_{0.2}$.

- For the hypothesis $X_{0.2}$=is_8, we obtain

$\overline{\text{bel}}$($X_{0.2}$=is_8) = p($X_{0.2}$=is_8 | $U_{0.2}$= mgsin(8^o), X_o=is_8).bel(X_o= is_8) + p($X_{0.2}$=is_8 | U_t= mgsin(8^o), X_o=is_10). bel (X_o= is_10)

= (1*0.1) + (0.8*0.1)

$\overline{\text{bel}}$($X_{0.2}$=is_8) = 0.18

- Likewise, for $X_{0.2}$=is_10 , we obtain

$\overline{\text{bel}}$($X_{0.2}$=is_10) = p($X_{0.2}$=is_10 | $U_{0.2}$= mgsin(8^o), X_o=is_8).bel(X_o= is_8) + p(X_t=is_10 | $U_{0.2}$= mgsin(8^o), X_o=is_10). bel (X_o= is_10)

= (0*0.1) + (0.2*0.1)

$\overline{\text{bel}}$($X_{0.2}$=is_10) = 0.02

2. **MEASUREMENT UPDATE STEP**

Incorporating the measurement for the bayes filter algorithm:

bel $(X_{0.2}) = \eta.p(Z_{0.2}= sense\_8 \mid X_{0.2}). \overline{bel}(X_{0.2})$

For the two possible cases:

- $X_{0.2}= is\_8$

$bel(X_{0.2}=is\_8) = \eta.p(Z_{0.2}= sense\_8 \mid X_{0.2}=is\_8). \overline{bel}(X_{0.2}=is\_8)$

$= \eta*(0.8) *(0.18)$

$= \eta*(0.144)$

- $X_{0.2}= is\_10$

$bel(X_{0.2}=is\_10) = \eta.p(Z_{0.2}= sense\_8 \mid X_t=is\_10). \overline{bel}(X_t=is\_10)$

$= \eta*(0.2) *(0.02) = \eta*(0.004)$

Now we calculate the normalizer $\eta= (0.114+0.004)^{-1}= 8.47$

Therefore,

$bel(X_{0.2}=is\_8) = 0.97$

$bel(X_{0.2}=is\_10) = 0.03$

At this point at t=0.2s, the probability that pendulum bob would be at 8^o is 0.97 which is certainly very high. This proves that the measurements were correct and the control action of mgsinϴ was responsible for the motion of the pendulum bob from 10^o to 8^o.

Similarly, if consider the posterior belief at time=0.2s as the prior belief for the belief assumed at t=0.4s, then we could automatically form prior and posterior probabilities at various points of time. Thus, in this way we can find the instantaneous position of the bob of simple pendulum at various points of time.

Polar coordinates

We know the instantaneous angular displacement of the bob and this information can be used to determine the polar coordinates of the bob at a particular time period as well. Following are the polar coordinates of the bob at various value of ϴ, for r=L=1m.

Instantaneous angular displacement		**L**	**y=Lsinϴ**	**x=Lcosϴ**
ϴ (in degrees)	**ϴ (in radians)**	**in m**		
10	0.174603175	1	0.173717	0.984796
8	0.13968254	1	0.139229	0.99026
6	0.104761905	1	0.10457	0.994517
4	0.06984127	1	0.069785	0.997562
2	0.034920635	1	0.034914	0.99939
0	0	1	0	1

-2	-0.034920635	1	-0.03491	0.99939
-4	-0.06984127	1	-0.06978	0.997562
-6	-0.104761905	1	-0.10457	0.994517
-8	-0.13968254	1	-0.13923	0.99026
-10	-0.174603175	1	-0.17372	0.984796
-8	-0.13968254	1	-0.13923	0.99026
-6	-0.104761905	1	-0.10457	0.994517
-4	-0.06984127	1	-0.06978	0.997562
-2	-0.034920635	1	-0.03491	0.99939
0	0	1	0	1
2	0.034920635	1	0.034914	0.99939
4	0.06984127	1	0.069785	0.997562
6	0.104761905	1	0.10457	0.994517
8	0.13968254	1	0.139229	0.99026
10	0.174603175	1	0.173717	0.984796

If the instantaneous position of the bob is known then by using the angular displacement, we can also find the distance along the arc of the motion. The distance covered by the bob from the mean position can be presented by a formula where s is the distance covered by the bob, L is the length of the string and ϴ is the angular displacement.

$$s=L\Theta$$

Instantaneous angular displacement		**L**	**s=Lϴ**
ϴ (in degrees)	**ϴ (in radians)**	**in m**	
10	0.174603175	1	0.174603
8	0.13968254	1	0.139683
6	0.104761905	1	0.104762
4	0.06984127	1	0.069841
2	0.034920635	1	0.034921
0	0	1	0
-2	-0.034920635	1	-0.03492
-4	-0.06984127	1	-0.06984
-6	-0.104761905	1	-0.10476
-8	-0.13968254	1	-0.13968
-10	-0.174603175	1	-0.1746
-8	-0.13968254	1	-0.13968
-6	-0.104761905	1	-0.10476
-4	-0.06984127	1	-0.06984
-2	-0.034920635	1	-0.03492

0	0	1	0
2	0.034920635	1	0.034921
4	0.06984127	1	0.069841
6	0.104761905	1	0.104762
8	0.13968254	1	0.139683
10	0.174603175	1	0.174603

CONCLUSION

Using the technique of bayes theorem and bayes filter we were successfully able to determine the instantaneous position of the bob in 1 dimension motion. Bayes filter uses the concept of bayes theorem and apply it for recursive state estimation of a particular robot according to the measurement recorded by sensors and control action of a particular robot. Here, in this particular example the we formed, the probability distribution of angular displacement of the pendulum bob was essential in forming a belief distribution. The belief was defined for various point of time during oscillation. We defined an initial time period and initial angular displacement of the pendulum bob to form a prior condition of motion. Prior probability along with the measurement probability and state transition probability was used to form the posterior at a particular time.

Through recursive state estimation the position of the bob was constantly being updated as the time passed and new belief was formed. In the example that we took to demonstrate our explanation, at initial time 0s, the bel (X_o=10) = 0.1 and bel (X_o=8) = 0.1. But as the time was increased to 0.2s, the bel($X_{0.2}$=is_8) = 0.97 and bel($X_{0.2}$=is_10) = 0.03. At this point at t=0.2s, the probability that pendulum bob would be at 8^o is 0.97 which was certainly very high. This proved that the measurements were correct and the control action of mgsinϴ was responsible for the motion of the pendulum bob from 10^o to 8^o.

Similarly, if consider the posterior belief at time=0.2s as the prior belief for the belief assumed at t=0.4s, then we could automatically form prior and posterior probabilities at various points of time. Thus, in this way we can find the instantaneous position of the bob of simple pendulum at various points of time. This is how we used the concept of recursive state estimation and bayes theorem in determining the instantaneous position during the one-dimension oscillation of the pendulum bob.

Further, if the exact instantaneous angular displacement at a time t is known to us, then using the polar coordinates the exact coordinates of the pendulum bob can be determined at a particular time t.

ACKNOWLEDGEMENTS

This work was completed under the guidance and mentorship of Dr. Sambhunath Nandy, CSIR-CMERI Durgapur. Thanks are due to the Indian Academy of Science for providing student fellows with the opportunity of Summer Research Fellowship Program 2021. Thanks are due to the Central Mechanical Engineering and Research Institute, Durgapur for providing platform for the commencement of the fellowship through online mode, during the COVID-19 pandemic.

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