Solution 1:- $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$

Subtract $2R_1$ from R_2

$R_2 = R_2 - 2R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

$R_3 = R_3 - 3R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

$R_4 = R_4 - 6R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

Dividing R_2 by ~~R_2~~ -3

$R_2 = R_2 / -3$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & 1 & -2/3 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_3 = R_3 + 4R_2$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & 1 & -2/3 \\ 0 & -4 & -4 & 1/3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_4 = R_4 + 4R_2$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & 1 & -2/3 \\ 0 & -4 & -4 & 1/3 \\ 0 & -4 & -7 & 7/3 \end{bmatrix}$$

$R_3 = R_3 - R_4$

$R_4 = R_4 - R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & 3 & -6/3 \\ 0 & 0 & -3 & 6/3 \end{bmatrix}$$

$R_4 = R_4 + R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & 3 & -6/3 \\ 0 & 0 & 0 & -4 \end{bmatrix}$$

So, the rank of the matrix is <u>4</u>.

Solution 2: Given :- $T: W \rightarrow P_2$

$$T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b) + (b-c)x + (c-d)x^2$$

Dimension of W:-

Symmetric 2×2 matric has 3 independent values:

The diagonal element and off-diagonal element $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ [b=c] [a] [d]

$$\dim(W) = 3$$

Rank of T :-

T has all polynomials in P_2 that can come by applying T to 2×2 matric (symmetric).

T mapping to all polynomial of degree at 2.

∴, Rank of T = 3.

Using Rank-nullity theorem, we have:-

$$\begin{aligned} \text{(T)nullity} &= \dim(W) - \text{rank}(T) \\ &= 3 - 3 \\ &= 0 \end{aligned}$$

Solution 3: Given: $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

Eigen values of A:

$$(A - \lambda I) = \begin{vmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} = (2-\lambda)^2 - 1 = \lambda^2 - 4\lambda + 3 = 0$$

$$\lambda_1 = 1, \lambda_2 = 3$$

Eigen vectors of A:

$$\lambda_1 = 1$$

$$(A - \lambda_1 I) = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} ; (A - \lambda_1 I) v_1 = 0$$

$$v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Eigen values of A^{-1}:

reciprocals of eigenvalues of A.

$$\lambda_1' = 1/1 = 1$$

$$\lambda_2' = 1/3$$

Eigen vectors of A^{-1} are same as A

Eigen values of $A + 4I$: Add 4 to the diagonal elem

$$A + 4I = \begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix}$$

$$= \begin{vmatrix} 6-\lambda & -1 \\ -1 & 6-\lambda \end{vmatrix} = (6-\lambda)^2 - 1 = \lambda^2 - 12\lambda + 35 = 0$$

$$\lambda_1'' = 5, \lambda_2'' = 7$$

Eigen vectors of $A + 4I$:-

$$(A + 4I) v'' = 0$$

for $\lambda_1'' = 5$:

$$A + 4I - \lambda_1'' I = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$v_1'' = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

For $\lambda_2'' = 7$:

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$$

$(A + 4I - \lambda_2'' I) v_2'' = 0$:-

$$v_2'' = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$A^{-1} = \lambda_1' = 1,\ \lambda_2' = 1/3,\ v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ v_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$A + 4I = \lambda_1'' = 5,\ \lambda_2'' = 7,\ v_1'' = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ v_2'' \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

Solution 4: Given -
$$3x - 0.1y - 0.2z = 7.85$$
$$0.1x + 7y - 0.3z = -19.3$$
$$0.3x - 0.2y + 10z = 71.4$$

$$x = \frac{7.85 + 0.1y + 0.2z}{3}$$

$$y = \frac{-19.3 - 0.1x + 0.3z}{7}$$

$$z = \frac{71.4 - 0.3x + 0.2y}{10}$$

$$x(0) = 0, \quad y(0) = 0, \quad z(0) = 0$$

(1) $$x(1) = \frac{7.85 + 0.1(0) + (0.2)(0)}{3} = \frac{7.85}{3} = 2.61$$

$$y(1) = \frac{(-19.3 - 0.1(2.61) + 0.3(0))}{7} = -2.78$$

$$z(1) = \frac{71.4 - 0.3(2.61) + 0.2(-2.79)}{10} = 7.61$$

(2) $$x(2) = \frac{7.85 + 0.1(-2.78) + (0.2)(7.61)}{3} = 3.000$$

$$y(2) = \frac{-19.3 - 0.1(3.0001) + 0.3(7.61)}{7} = -3.0001$$

$$z(2) = \frac{71.4 - 0.3(3.0001) + 0.2(-3.00013)}{10} = 6.999$$

(3) $$x(3) = \frac{(7.85 + 0.1(-3.00013) + 0.2(6.999))}{3} = 3.00$$

$$y(3) = \frac{(-19.3 - 0.1(3.00) + 0.3(6.999))}{7} = -3.00$$

$$z(3) = \frac{(71.4 - 0.3(3.00) + 0.2(-3.00))}{10} = 7.00$$

Solution-5 Given: $x + 3y + 2z = 0$

$$2x - y + 3z = 0$$
$$3x - 5y + 4z = 0$$
$$x + 17y + 4z = 0$$

$$AX = 0$$

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$[A|0]$ Augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 2 & -1 & 3 & 0 \\ 3 & -5 & 4 & 0 \\ 1 & 17 & 4 & 0 \end{array}\right]$$

Row echelon form A

$R_2 = R_2 - 2R_1$
$R_3 = R_3 - 3R_1$
$R_4 = R_4 - R_1$

$$= \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -14 & -2 \\ 0 & 14 & 2 \end{bmatrix}$$

$$R_3 = R_3 - 2R_2 = \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & 0 & 0 \\ 0 & 14 & 2 \end{bmatrix}$$

$$R_4 = R_4 + 2R_2 = \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$[A|0] = \left[\begin{array}{ccc|c} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\left.\begin{array}{l}\text{Rank of A is 2} \\ \text{Rank of [A|0] is 2}\end{array}\right\} \rightarrow$ the system is consistent

$$x + 3y + 2z = 0 \quad \text{--- (1)}$$
$$-7y - z = 0 \quad \text{--- (2)}$$

$z = -7y$; $x + 3y + 2(-7y) = 0$

$\boxed{z = 0, x = 0, y = 0}$ $x + 3y - 14y = 0$; $x - 11y = 0$

$\boxed{y = \frac{x}{11}}$

Solution 6: Given : $T(a+bx+cx^2) = (a+1)+(b+1)x+(c+1)x^2$

Additivity: $T(u+v) = T(u)+T(v)$ for all u, v in P_2

Homogeneity: $T(cu) = cT(u)$ for all u in P_2 and all scalars in c.

① $u = a+bx+cx^2$ and $v = a'+b'x+c'x^2$ in P_2.

Then $u+v = (a+a')+(b+b')x+(c+c')x^2$

Now, $T(u) = (a+1)+(b+1)x+(c+1)x^2$ and

$T(v) = (a'+1)+(b'+1)x+(c'+1)x^2$

$T(u+v) = (a+a'+1)+(b+b'+1)x+(c+c'+1)x^2$

&

$T(u)+T(v) = (a+1)+(b+1)x+(c+1)x^2+(a'+1)+(b'+1)x+(c'+1)x$

$= (a+a'+1)+(b+b'+1)+(c+c'+1)x^2$

$T(u+v) = T(u)+T(v)$, satisfy additivity.

② $u = a+bx+cx^2 - P_2$, let c be a scalar.

$cu = ca+cbx+ccx^2$

$T(cu) = (ca+1)+(cb+1)x+(cc+1)x^2$

&

$cT(u) = c(a+1)+c(b+1)x+c(c+1)x^2$.

$(ca+1)+(cb+1)x+(cc+1)x^2$

Satisfy Homogeneity

Function T satisfies both additivity and homogeneity it is a linear transformation.

Solution 7:

$$AX = 0 = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 3 & 1 & 9 & 0 \\ -2 & 1 & 3 & 0 \end{array}\right]$$

Row reduced form:

$$R_2 = R_2 - 3R_1 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -9 \\ -2 & 1 & 3 \end{bmatrix}$$

$$R_3 = R_3 + 2R_1 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -9 \\ 0 & 5 & 9 \end{bmatrix}$$

$$R_3 = R_3 + R_2 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -9 \\ 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 2$, The rank is 2 thus it is linearly independent

→ R^3 has a dimension 3,

S is linearly independent → Span R^3

S is linearly independent and consists of 3 vectors in R^3. It forms a basis for R^3.

Solution 8: Given : $3x - 6y + 2z = 23$

$$-4x + y - z = -15$$

$$x - 3y + 7z = 16$$

① $x(0) = 1,\ y(0) = 1,\ z(0) = 1$

$$x(1) = \frac{23 + 6\,y(0) - 2z(0)}{3} = \frac{23 + 6 - 2}{3} = \frac{27}{3} = 9$$

$$y(1) = \frac{-15 + 4*x(0) + z(0)}{1} = -12$$

$$z(1) = \frac{16 - x(0) + 3*y(0)}{7} = \frac{-21}{7} = -3$$

② $$x_2 = \frac{23 + 6*(-12) - 2*(-3)}{3} = \frac{-55}{3}$$

$$y_2 = \frac{(-15 - 4*9 + (-3))}{1} = -54$$

$$z_2 = \frac{(16 - 9 + 3(-3))}{7} = \frac{-2}{7}$$

③ $$x_3 = \frac{23 + 6*(-54) - 2*(-2/7)}{3} = \frac{-17}{21}$$

$$y_3 = \frac{(-15 - 4*(-55/3) + (-2/7))}{1} = \frac{58}{21} = 17.19$$

$$z_3 = \frac{16 - (-55/3) + 3*(-2/7)}{7} = -\frac{137}{147} = -72.71$$

Solution 9: One application of matrix operations in image processing is image transformation, such as rotation, scaling or translation.

For example, let's consider the application of a rotation matrix to rotate an image by a ~~cent~~ certain angle. The rotation of an image can be achieved by multiplying the coordinates of each pixel in the original image by a rotation matrix. The rotation matrix depends on the desired angle of rotation.

The General 2-D rotation matrix:

$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

Let, an image presented as a matrix of pixel values. To rotate this image by 90 degrees counterclockwise, we use the following rotation matrix:

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Multiply each pixel coordinate by this matrix to get new coordinates. Example, the pixel at position (x, y) in the original image would be transformed to position (x', y') in the rotated image using the formula:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

After applying this transformation to every pixel in original image, we obtain the rotated image.

Solution 10:

Linear transformation are fundamental operations in computer vision used to manipulate images, such as rotating a 2D image.

Rotating a 2D image involves applying a linear transform to each pixel in image. The transformation is typically represented by a rotation matrix, which specifies the rotation angle and direction.

For example, the rotation matrix is:

$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

Each pixel in the original image is represented by a coordinate (x, y). To rotate the image, we multiply this coordinate vector by the rotation matrix to ~~adt~~ obtain the new coordinates (x', y') after rotation:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

After this transformation to each pixel in original image, we obtain the rotated image.

Linear transformation offer several advantages in computer vision for rotating images:

1. Efficiency: linear transformations can be applied efficiently to each pixel using matrix multiplication, making them suitable for real-time image processing.
2. Preservation of properties: Linear transformations preserve important properties of images, such as straight lines, angles and distances, ensuring that the rotated image retains its essential characteristics.