

# CS 412 - Introduction to Machine Learning

## Assignment 10

Q1a) Slope of line 1 ( $L_1$ ) : Marginal Hyperplane

$$= \left( \frac{5-2}{11-7} \right) = \left( \frac{3}{4} \right)$$

Slope of line 2 ( $L_2$ ) : Marginal Hyperplane

$$= \left( \frac{9-6}{8-4} \right) = \left( \frac{3}{4} \right)$$

$\therefore$  The slope of the hyperplane ( $L_3$ ) will also be  $3/4$ .

Distance between  $L_1$  and  $L_2$  = perpendicular distance  
between the two points on  
the lines.

$$\therefore \text{dist.} = \sqrt{(7-4)^2 + (2-6)^2}$$

$$= 5$$

Distance between the 2 points = 5

$$\therefore \text{Margin} (l) = \left( \frac{5}{2} \right) = 2.5$$



Q1 b) Choose three support vectors, write out system of equations for those support vector data points  
 $y_i (w_0 + w_1 x_{i,1} + w_2 x_{i,2}) = 1$

A: 3 support vectors: (7, 2) (11, 5) (4, 6)

class labels : 1, 1, -1

∴ We get the following set of equations:

$$7w_1 + 2w_2 + w_0 = 1 \quad \dots (i)$$

$$11w_1 + 5w_2 + w_0 = 1 \quad \dots (ii)$$

$$4w_1 + 6w_2 + w_0 = -1 \quad \dots (iii)$$

Solving equations (i) & (ii)

$$4w_1 + 3w_2 = 0 \quad \dots (iv)$$

Solving eq. (ii) & (iii)

$$7w_1 - w_2 = 2 \quad \dots (v)$$

Solving eq. (v) & (iv)

$$25w_1 = 6$$

$$w_1 = \left( \frac{6}{25} \right) = 0.24$$

Substituting  $w_1$  in (iv)

$$4 \left( \frac{6}{25} \right) + 3w_2 = 0$$

$$w_2 = -8/25 = -0.32$$



Solving eq. (1)

$$7 \left( \frac{6}{25} \right) + 2 \left( \frac{-8}{25} \right) + w_0 = 1$$

$$\left( \frac{42}{25} \right) + \left( \frac{-16}{25} \right) + w_0 = 1$$

$$\therefore w_0 = \frac{-1}{25} = -0.04$$

$\therefore w_1 = 0.24$
$w_2 = -0.32$
$w_0 = -0.04$



Q1. c) Removals are independent

i) No

As we have 4 points on the marginal hyperplanes & we only need 3 points as support vectors to solve for  $w_0, w_1, w_2$ . The fourth point is redundant.

ii) No

It is not a support vector.

iii) No

$(10, 0)$  is not a support vector.

iv) Yes

As we are deleting 2 support vectors on one side. It will change the decision boundary.

v) Yes

Both are support vector. The decision boundary will shift.



Q1. d)

i) No

It does not lie between two marginal hyperplanes.

ii) Yes

Since it is within the two marginal hyperplanes.

iii) ~~Yes~~ No

Since it ~~is~~ within the two marginal hyperplanes.  
does not lie

iv) No

Because both the points do not lie between the two marginal hyperplanes.

v) Yes

Since both points are within the two marginal hyperplanes.



Q2.

subplot	1	2	3	4	5	6
model	d	f	b	a	c	e

- a) Linear SVM meaning a linear boundary.  
 small value of  $C (0.1)$ ,  $\therefore$  more errors are allowed  
 As  $C$  focuses on loss (how much it violates the margin)  
4 fig - The line does not separate properly.

- b) Linear SVM meaning a linear boundary.  
 large value of  $C (10)$ ,  $\therefore$  less errors  
3 fig - The line separates properly.

- c) Decision function of quadratic kernel  

$$f(x) = \sum_i \alpha_i (x_i \cdot x + (x_i \cdot x)^2) + b$$

$$\therefore f(x) = 0$$

$\therefore x$  can either be an ellipse or a hyperbolic curve  
 $\therefore$  fig. 5.

- d) Decision function

$$f(x) = \sum_i \alpha_i \exp(-\gamma \|x_i - x\|^2)$$

if  $\gamma$  is large, kernel value is small and vice versa.  
 less number of support vectors  
 more

$\gamma = 1/4$  for fig. 1, classification is difficult if kernel value is small.



Q2.

e)  $\gamma$  is large, more number of support vectors based on Q2.d)

$\gamma = 4$

fig. 6

Q2

f)

None of the above - fig. 2.