

1 Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} - 8\frac{dx}{dt} - 9x = 9e^{8t}. \quad [6]$$

[illegible]

2 Let $I_n = \int_0^1 (1+3x)^n e^{-3x} dx$, where n is an integer.

(a) Show that $3I_n = 1 - 4^n e^{-3} + 3nI_{n-1}$. [3]

[illegible]

(b) Find the exact value of I_2 . [3]

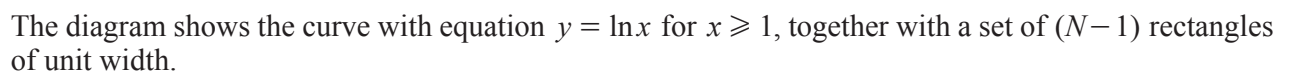
[illegible]

(a) Find the eigenvalues of \mathbf{A} .

[4]

This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dotted lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

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$$\ln N! > N \ln N - N + 1. \quad [5]$$
[illegible]

- (b)** Use a similar method to find, in terms of N , an upper bound for $\ln N!$. [3]

[illegible]

5 The curve C has parametric equations

$$x = \frac{1}{2}t^2 - \ln t, \quad y = 2t + 1, \quad \text{for } \frac{1}{2} \leq t \leq 2.$$

(a) Find the exact length of C .

[5]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(b) Find $\frac{d^2y}{dx^2}$ in terms of t , simplifying your answer.

[4]

[illegible]

- 6 (a)** Starting from the definitions of \tanh and sech in terms of exponentials, prove that

$$1 - \tanh^2 \theta = \operatorname{sech}^2 \theta. \quad [3]$$

[illegible]

The variables x and y are such that $\tanh y = \cos\left(x + \frac{1}{4}\pi\right)$, for $-\frac{1}{4}\pi < x < \frac{3}{4}\pi$.

- (b)** By differentiating the equation $\tanh y = \cos\left(x + \frac{1}{4}\pi\right)$ with respect to x , show that

$$\frac{dy}{dx} = -\operatorname{cosec}\left(x + \frac{1}{4}\pi\right). \quad [4]$$

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (c) Hence find the first three terms in the Maclaurin's series for $\tanh^{-1}\left(\cos\left(x+\frac{1}{4}\pi\right)\right)$ in the form $\frac{1}{2}\ln a + bx + cx^2$, giving the exact values of the constants a , b and c . [5]

[illegible]

7 (a) Show that an appropriate integrating factor for

$$(x^2 + 1) \frac{dy}{dx} + y\sqrt{x^2 + 1} = x^2 - x\sqrt{x^2 + 1}$$

is $x + \sqrt{x^2 + 1}$.

[4]

[illegible]

(b) Hence find the solution of the differential equation

$$(x^2 + 1) \frac{dy}{dx} + y\sqrt{x^2 + 1} = x^2 - x\sqrt{x^2 + 1}$$

for which $y = \ln 2$ when $x = 0$. Give your answer in the form $y = f(x)$. [7]

[illegible]

- 8 (a)** Use de Moivre's theorem to show that $\sin^6 \theta = -\frac{1}{32}(\cos 6\theta - 6 \cos 4\theta + 15 \cos 2\theta - 10)$. [6]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

It is given that $\cos^6 \theta = \frac{1}{32}(\cos 6\theta + 6 \cos 4\theta + 15 \cos 2\theta + 10)$.

- (b)** Find the exact value of $\int_0^{\frac{1}{3}\pi} \left(\cos^6\left(\frac{1}{4}x\right) + \sin^6\left(\frac{1}{4}x\right) \right) dx$. [4]

[illegible]

- (c) Express each root of the equation $16c^6 + 16(1 - c^2)^3 - 13 = 0$ in the form $\cos k\pi$, where k is a rational number. [5]

[illegible]

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