Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

1983241222

FURTHER MATHEMATICS **9231/21**

Paper 2 Further Pure Mathematics 2 **May/June 2020**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.

DC (JP) 197983

BLANK PAGE

1 Find the solution of the differential equation

$$\frac{dy}{dx} + 5y = e^{-7x}$$

for which $y = 0$ when $x = 0$. Give your answer in the form $y = f(x)$. [6]

2 It is given that $y = 2^x$.

(a) By differentiating $\ln y$ with respect to x, show that $\dfrac{dy}{dx} = 2^x \ln 2$. [3]

....

....

....

....

....

....

....

....

....

....

(b) Write down $\dfrac{d^2y}{dx^2}$. [1]

....

....

....

(c) Hence find the first three terms in the Maclaurin's series for 2^x. [3]

....

....

....

....

....

....

....

....

....

....

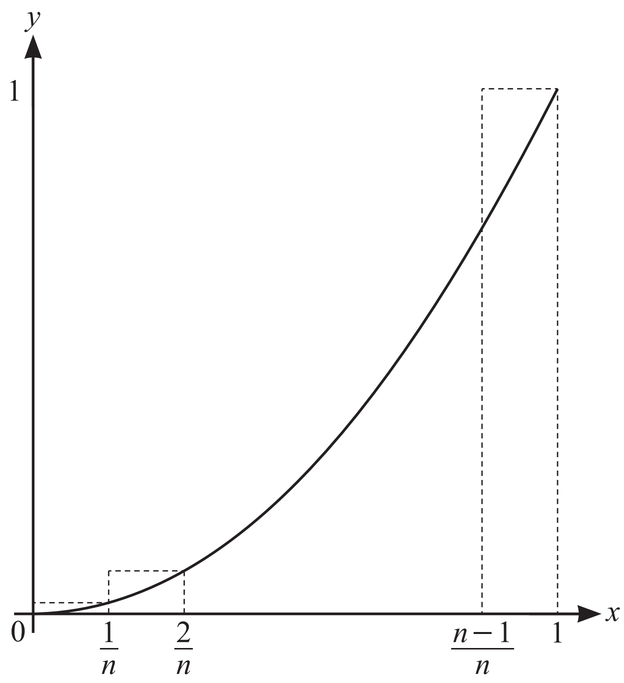
....

3 **(a)** Find the roots of the equation $z^3 = -1 - \mathrm{i}$, giving your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $0 \leqslant \theta < 2\pi$. [5]

Let $w = z_1^{3k} + z_2^{3k} + z_3^{3k}$, where k is a positive integer and z_1, z_2, z_3 are the roots of $z^3 = -1 - \mathrm{i}$.

(b) Express w in the form $R\mathrm{e}^{\mathrm{i}\alpha}$, where $R > 0$, giving R and α in terms of k. [3]

4



The diagram shows the curve with equation $y = x^2$ for $0 \leqslant x \leqslant 1$, together with a set of n rectangles of width $\frac{1}{n}$.

(a) By considering the sum of the areas of these rectangles, show that

$$\int_0^1 x^2 \, dx < \frac{2n^2 + 3n + 1}{6n^2}.$$ [4]

....

....

....

....

....

....

....

....

....

....

....

....

....

(b) Use a similar method to find, in terms of n, a lower bound for $\int_0^1 x^2\,dx$. [4]

5 The curves $C_1 : y = \cosh x$ and $C_2 : y = \sinh 2x$ intersect at the point where $x = a$.

(a) Find the exact value of a, giving your answer in logarithmic form. [4]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

(b) Sketch C_1 and C_2 on the same diagram. [2]

(c) Find the exact value of the length of the arc of C_1 from $x = 0$ to $x = a$. [5]

6 The integral I_n, where n is an integer, is defined by $I_n = \int_0^{\frac{1}{2}} \left(1-x^2\right)^{-\frac{1}{2}n} \mathrm{d}x$.

(a) Find the exact value of I_1. [2]

....

....

....

....

....

(b) By considering $\frac{\mathrm{d}}{\mathrm{d}x}\left(x\left(1-x^2\right)^{-\frac{1}{2}n}\right)$, or otherwise, show that

$$nI_{n+2} = 2^{n-1}3^{-\frac{1}{2}n} + (n-1)I_n.$$ [5]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

(c) Find the exact value of I_5 giving the answer in the form $k\sqrt{3}$, where k is a rational number to be determined. [3]

7 It is given that $x = t^3 y$ and

$$t^3 \frac{d^2y}{dt^2} + \left(4t^3 + 6t^2\right)\frac{dy}{dt} + \left(13t^3 + 12t^2 + 6t\right)y = 61e^{\frac{1}{2}t}.$$

(a) Show that

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 13x = 61e^{\frac{1}{2}t}.$$ [4]

(b) Find the general solution for y in terms of t. [7]

8 **(a)** Find the values of a for which the system of equations

$$\begin{aligned} 3x + y + z &= 0, \\ ax + 6y - z &= 0, \\ ay - 2z &= 0, \end{aligned}$$

does not have a unique solution. [3]

The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & 1 \\ 0 & 6 & -1 \\ 0 & 0 & -2 \end{pmatrix}.$$

(b) Use the characteristic equation of $\mathbf{A}$ to find the inverse of $\mathbf{A}^2$. [4]

(c) Find a matrix **P** and a diagonal matrix **D** such that $\mathbf{A}^5 = \mathbf{PDP}^{-1}$. [7]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....