Cambridge International A Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **May/June 2020**

MARK SCHEME

Maximum Mark: 75

Published

Students did not sit exam papers in the June 2020 series due to the Covid-19 global pandemic.

This mark scheme is published to support teachers and students and should be read together with the question paper. It shows the requirements of the exam. The answer column of the mark scheme shows the proposed basis on which Examiners would award marks for this exam. Where appropriate, this column also provides the most likely acceptable alternative responses expected from students. Examiners usually review the mark scheme after they have seen student responses and update the mark scheme if appropriate. In the June series, Examiners were unable to consider the acceptability of alternative responses, as there were no student responses to consider.

Mark schemes should usually be read together with the Principal Examiner Report for Teachers. However, because students did not sit exam papers, there is no Principal Examiner Report for Teachers for the June 2020 series.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the June 2020 series for most Cambridge IGCSE™ and Cambridge International A & AS Level components, and some Cambridge O Level components.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Mathematics-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

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Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks
1	Use law of the logarithm of a product or power	**M1**
	Obtain a correct linear inequality in any form, e.g. $\ln 2 + (1-2x)\ln 3 < x \ln 5$	**A1**
	Solve for x	**M1**
	Obtain $x > \dfrac{\ln 6}{\ln 45}$	**A1**
		4

Question	Answer	Marks
2(a)	State a correct unsimplified version of the x or x^2 term of the expansion of $(2-3x)^{-2}$ or $\left(1-\dfrac{3}{2}x\right)^{-2}$	**M1**
	State correct first term $\dfrac{1}{4}$	**B1**
	Obtain the next two terms $\dfrac{3}{4}x+\dfrac{27}{16}x^2$	**A1 + A1**
		4
2(b)	State answer $\lvert x\rvert < \dfrac{2}{3}$, or equivalent	**B1**
		1

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Question	Answer	Marks
3	Use tan $(A \pm B)$ formula and obtain an equation in tan θ	**M1**
	Using $\tan 60° = \sqrt{3}$, obtain a horizontal equation in tan θ in any correct form	**A1**
	Reduce the equation to $3\tan^2\theta + 4\tan\theta - 1 = 0$, or equivalent	**A1**
	Solve a 3-term quadratic for tan θ	**M1**
	Obtain a correct answer, e.g. 12.1°	**A1**
	Obtain a second correct answer, e.g. 122.9°, and no others in the given interval	**A1**
		6

Question	Answer	Marks
4(a)	Use product rule	**M1**
	Obtain derivative in any correct form e.g. $2e^{2x}(\sin x + 3\cos x) + e^{2x}(\cos x - 3\sin x)$	**A1**
	Equate derivative to zero and obtain an equation in one trigonometric ratio	**M1**
	Obtain $x = 1.43$ only	**A1**
		4
4(b)	Use a correct method to determine the nature of the stationary point e.g. $x = 1.42,\ y' = 0.06e^{2.84} > 0$ $x = 1.44,\ y' = -0.07e^{2.88} < 0$	**M1**
	Show that it is a maximum point	**A1**
		2

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Question	Answer	Marks
5(a)	Commence division and reach quotient of the form $2x+k$	**M1**
	Obtain quotient $2x-1$	**A1**
	Obtain remainder 6	**A1**
		3
5(b)	Obtain terms x^2-x (**FT** on quotient of the form $2x+k$)	**B1FT**
	Obtain term of the form $a\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$	**M1**
	Obtain term $\frac{6}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$ (**FT** on a constant remainder)	**A1FT**
	Use $x=1$ and $x=3$ as limits in a solution containing a term of the form $a\tan^{-1}(bx)$	**M1**
	Obtain final answer $\frac{1}{\sqrt{3}}\pi+6$, or exact equivalent	**A1**
		5

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Question	Answer	Marks
6(a)	State or imply $AT = r\tan x$ or $BT = r\tan x$	B1
	Use correct area formula and form an equation in r and x	M1
	Rearrange in the given form	A1
		3
6(b)	Calculate the values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.4$	M1
	Complete the argument correctly with correct calculated values	A1
		2
6(c)	Use the iterative formula correctly at least once	M1
	Obtain final answer 1.35	A1
	Show sufficient iterations to 4 d.p. to justify 1.35 to 2 d.p. or show there is a sign change in the interval (1.345, 1.355)	A1
		3

Question	Answer	Marks
7(a)	Use quotient or product rule	M1
	Obtain derivative in any correct form e.g. $\dfrac{-\sin x(1+\sin x)-\cos x(\cos x)}{(1+\sin x)^2}$	A1
	Use Pythagoras to simplify the derivative	M1
	Justify the given statement	A1
		4

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Question	Answer	Marks
7(b)	State integral of the form $a\ln(1+\sin x)$	*M1
	State correct integral $\ln(1+\sin x)$	A1
	Use limits correctly	DM1
	Obtain answer $\ln\frac{4}{3}$	A1
		4
8(a)	State $\frac{dy}{dx}=k\frac{y}{x\sqrt{x}}$, or equivalent	B1
	Separate variables correctly and attempt integration of at least one side	M1
	Obtain term $\ln y$, or equivalent	A1
	Obtain term $-2k\frac{1}{\sqrt{x}}$, or equivalent	A1
	Use given coordinates to find k or a constant of integration c in a solution containing terms of the form $a\ln y$ and $\frac{b}{\sqrt{x}}$, where $ab\neq 0$	M1
	Obtain $k=1$ and $c=2$	A1 + A1
	Obtain final answer $y=\exp\left(-\frac{2}{\sqrt{x}}+2\right)$, or equivalent	A1
		8

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Question	Answer	Marks
8(b)	State that y approaches e^2 (FT *their* c in part **(a)** of the correct form)	**B1FT**
		1
9(a)	State $\overrightarrow{AB}\left(\text{or } \overrightarrow{BA}\right)$ and $\overrightarrow{BC}\left(\text{or } \overrightarrow{CB}\right)$ in vector form	**B1**
	Calculate their scalar product	**M1**
	Show product is zero and confirm angle ABC is a right angle	**A1**
		3
9(b)	Use correct method to calculate the lengths of AB and BC	**M1**
	Show that $AB = BC$ and the triangle is isosceles	**A1**
		2
9(c)	State a correct equation for the line through B and C, e.g. $\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda\,(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ or $\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu\left(-2\mathbf{i} - \mathbf{j} - 2\mathbf{k}\right)$	**B1**
	Taking a general point of BC to be P, form an equation in λ by **either** equating the scalar product of $\overrightarrow{OP}$ and $\overrightarrow{BC}$ to zero, **or** applying Pythagoras to triangle OBP (or OCP), *or* setting the derivative of $\left\lvert\overrightarrow{OP}\right\rvert$ to zero	**M1**
	Solve and obtain $\lambda = -\dfrac{5}{9}$	**A1**
	Obtain answer $\dfrac{1}{3}\sqrt{2}$, or equivalent	**A1**

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Question	Answer	Marks
	Alternative method for question 9(c)	
	Use a scalar product to find the projection *CN* (or *BN*) of *OC* (or *OB*) on *BC*	**M1**
	Obtain answer $CN = \frac{5}{3}\left(\text{or } BN = \frac{14}{3}\right)$	**A1**
	Use Pythagoras to find *ON*	**M1**
	Obtain answer $\frac{1}{3}\sqrt{2}$, or equivalent	**A1**
		4

Question	Answer	Marks
10(a)(i)	Multiply numerator and denominator by $a - 2\mathrm{i}$, or equivalent	**M1**
	Use $\mathrm{i}^2 = -1$ at least once	**A1**
	Obtain answer $\frac{6}{a^2+4} + \frac{3ai}{a^2+4}$	**A1**
		3
10(a)(ii)	Either state that $\arg u = -\frac{1}{3}\pi$ or express u^* in terms of a (FT on u)	**B1**
	Use correct method to form an equation in a	**M1**
	Obtain answer $a = -2\sqrt{3}$	**A1**
		3

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Question	Answer	Marks
10(b)(i)	Show the perpendicular bisector of points representing $2i$ and $1 + i$	**B1**
	Show the point representing $2 + i$	**B1**
	Show a circle with radius 2 and centre $2 + i$ (FT on the position of the point for $2 + i$)	**B1FT**
	Shade the correct region	**B1**
		4
10(b)(ii)	State or imply the critical point $2 + 3i$	**B1**
	Obtain answer 56.3° or 0.983 radians	**B1**
		2

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