

Cambridge International AS & A Level

MATHEMATICS

Paper 1 Pure Mathematics 1

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always whole marks (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded positively:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

© UCLES 2021 Page 2 of 21

Math	Mathematics Specific Marking Principles				
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.				
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.				
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.				
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).				
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.				
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.				

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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- **B** Mark for a correct result or statement independent of method marks.
- DM or DB When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
 - FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

© UCLES 2021 Page 4 of 21

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Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

WWW Without Wrong Working

AWRT Answer Which Rounds To

© UCLES 2021 Page 5 of 21

Question	Answer	Marks	Guidance
1	$2\cos^2\theta - 7\cos\theta + 3[=0]$	M1	Forming a 3-term quadratic expression with all terms on the same side or correctly set up prior to completing the square. Allow \pm sign errors.
	$(2\cos\theta - 1)(\cos\theta - 3) = 0$	DM1	Solving <i>their</i> 3-term quadratic using factorisation, formula or completing the square.
	$[\cos \theta = \frac{1}{2} \text{ or } \cos \theta = 3 \text{ leading to}] \theta = -60^{\circ} \text{ or } \theta = 60^{\circ}$	A1	
	θ = -60° and θ = 60°	A1 FT	FT for \pm same answer between 0° and 90° or 0 and $\frac{\pi}{2}$. $\pm \frac{\pi}{3}$ or ± 1.05 AWRT scores maximum M1M1A0A1FT. Special case: If M1 DM0 scored then SC B1 for $\theta = -60^{\circ}$ or $\theta = 60^{\circ}$, and SC B1 FT can be awarded for $\pm (their \ 60^{\circ})$.
		4	

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Question	Answer	Marks	Guidance
2(a)	Stretch with [scale factor] either ± 2 or $\pm \frac{1}{2}$	B1	
	Scale factor $\frac{1}{2}$ in the x-direction	B1	
	Translation $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$ or translation of 3 units in negative <i>y</i> -direction	B1	
		3	
2(b)	(10,9)	B1 B1	B1 for each correct co-ordinate.
		2	

Question	Answer	Marks	Guidance
3(a)	$f(5) = \begin{bmatrix} 2 \end{bmatrix} \text{ and } f(their 2) = \begin{bmatrix} 5 \end{bmatrix} \text{ OR } ff(5) = \begin{bmatrix} \frac{2+3}{2-1} \end{bmatrix}$ $OR \frac{\frac{x+3}{x-1} + 3}{\frac{x+3}{x-1} - 1} \text{ and an attempt to substitute } x = 5.$	M1	Clear evidence of applying f twice with $x = 5$.
	5	A1	
		2	

© UCLES 2021 Page 7 of 21

Question	Answer	Marks	Guidance
3(b)	$\frac{x+3}{x-1} = y \Rightarrow x+3 = xy - y \text{ OR } \frac{y+3}{y-1} = x \Rightarrow y+3 = xy - x$	*M1	Setting $f(x) = y$ or swapping x and y , clearing of fractions and expanding brackets. Allow \pm sign errors.
	$xy - x = y + 3 \Rightarrow x = \frac{y+3}{y-1}$ OE OR $y+3 = xy - x \Rightarrow y = \left[\frac{x+3}{x-1}\right]$ OE	DM1	Finding x or $y = $. Allow \pm sign errors.
	$[f^{-1}(x) \text{ or } y] = \frac{x+3}{x-1}$	A1	OE e.g. $1 + \frac{4}{x-1}$ etc. Must be a function of x, cannot be $x = $.
		3	

Question	Answer	Marks	Guidance
4	$\frac{8}{3}$ $\left[\cdot , \cdot \right]$	*B1	For $(3x+2)^{-1}$
	$y = -\frac{3}{(3x+2)}[+c]$	DB1	For $-\frac{8}{3}$
	$5\frac{2}{3} = -\frac{\frac{8}{3}}{(3 \times 2 + 2)} + c$	M1	Substituting $\left(2, 5\frac{2}{3}\right)$ into <i>their</i> integrated expression – defined by power = -1, or dividing by their power. + c needed
	$y = -\frac{8}{3(3x+2)} + 6$	A1	OE e.g. $y = -\frac{8}{3}(3x+2)^{-1} + 6$
		4	

© UCLES 2021 Page 8 of 21

Question	Answer	Marks	Guidance
5(a)	$[(3^{\text{rd}} \text{ term} - 1^{\text{st}} \text{ term}) = (5^{\text{th}} \text{ term} - 3^{\text{rd}} \text{ term}) \text{ leading to}]$ $-6\sqrt{3} \sin x - 2\cos x = 10\cos x + 6\sqrt{3}\sin x$ $[\text{leading to } -12\sqrt{3}\sin x = 12\cos x]$ OR $[(1^{\text{st}} \text{ term} + 5^{\text{th}} \text{ term}) = 2 \times 3^{\text{rd}} \text{ term leading to}] 12\cos x = -12\sqrt{3}\sin x$	*M1	OE. From the given terms, obtain 2 expressions relating to the common difference of the arithmetic progression, attempt to solve them simultaneously and achieve an equation just involving sinx and cosx.
	Elimination of sinx and cosx to give an expression in tanx $\left[\tan x = -\frac{1}{\sqrt{3}}\right]$	DM1	For use of $\frac{\sin x}{\cos x} = \tan x$
	$[x=]\frac{5\pi}{6} \text{ only}$	A1	CAO. Must be exact.
		3	
5(b)	$d = 2\cos x$ or $d = 2\cos(their x)$	B1 FT	Or an equivalent expression involving $\sin x$ and $\cos x$ e.g. $-3\sqrt{3}\sin(their\ x) - \cos(their\ x) \left[= -\sqrt{3} \right]$ FT for their x from (a) only. If not $\pm \sqrt{3}$, must see unevaluated form.
	$S_{25} = \frac{25}{2} \left(2 \times \left(2\cos\left(theirx\right) \right) + \left(25 - 1 \right) \times \left(theird\right) \right)$	M1	Using the correct sum formula with $\frac{25}{2}$, $(25-1)$ and with
	$S_{25} = \frac{25}{2} \left(2 \times \left(2\cos(their x) \right) + (25 - 1) \times (their d) \right)$ $\left[= 12.5 \left(2 \times \left(-\sqrt{3} \right) + 24 \left(-\sqrt{3} \right) \right) \right]$		a replaced by either $2(\cos(their x))$ or $\pm \sqrt{3}$ and d replaced by either $2(\cos(their x))$ or $\pm \sqrt{3}$.
	$-325\sqrt{3}$	A1	Must be exact.
		3	

© UCLES 2021 Page 9 of 21

Question	Answer	Marks	Guidance
6	$ar = 54$ and $\frac{a \text{ or } their a}{1-r} = 243$	B1	SOI
	$\frac{54}{r} = 243(1-r) \text{ leading to } 243r^2 - 243r + 54[=0] [9r^2 - 9r + 2 = 0]$ OR $a^2 - 243a + 13122[=0]$	*M1	Forming a 3-term quadratic expression in r or a using their 2nd term and S_{∞} . Allow \pm sign errors.
	k(3r-2)(3r-1)[=0] OR $(a-81)(a-162)[=0]$	DM1	Solving <i>their</i> 3-term quadratic using factorisation, formula or completing the square. If factorising, factors must expand to give $\pm their$ coefficient of r^2 .
	$54 \div \left(their \frac{2}{3}\right) = a \text{ OR } 54 \div \left(their 81\right) = r$	DM1	May be implied by final answer.
	Tenth term = $\frac{512}{243} \left[\text{OR } 81 \times \left(\frac{2}{3}\right)^9 \text{OR } 54 \times \left(\frac{2}{3}\right)^8 \right]$	A1	OE. Must be exact. Special case: If B1M1DM0DM1 scored then SC B1 can be awarded for the correct final answer.
		5	

© UCLES 2021 Page 10 of 21

Question	Answer	Marks	Guidance
7(a)	EITHER By using trigonometry: $B\hat{A}C = 0.6435$ and $A\hat{B}C = \frac{\pi - 0.6435}{2}$ OR By Pythagoras: $AP = 12 \Rightarrow BP = 3$ so $\tan A\hat{B}C = \frac{9}{3}$ OR Using ΔPBC and either the sine or cosine rule $\sin A\hat{B}C = \frac{3}{\sqrt{10}}$ or $\cos A\hat{B}C = \frac{\sqrt{10}}{10}$	M1	$\frac{3}{\sqrt{10}} = 0.9486 \frac{\sqrt{10}}{10} = 0.3162$
	$A\hat{B}C = \frac{\pi - 0.6435}{2} \text{ or } \tan^{-1} \frac{9}{3} \text{ or } \sin^{-1} \frac{3}{\sqrt{10}} \text{ or } \cos^{-1} \frac{\sqrt{10}}{10} \text{ or}$ $1.249(04) \text{ or} 71.56^{\circ} = 1.25 \text{ radians } (3 \text{ sf})$	A1	AG. Final answer must be 1.25, more accurate value 1.24904 with no rounding to 3sf seen as the final answer gets M1A0. If decimals are used all values must be given to at least 4sf for A1.
		2	
7(b)	$BC = \sqrt{(their 3)^2 + 9^2}$ or $\frac{9}{\sin 1.25} = \sqrt{90}$, $3\sqrt{10}$ or 9.48697	M1	Using correct method(s) to find BC.
	Area of sector = $\frac{1}{2} \times (their BC)^2 \times tan^{-1} 3 = 56.207 \text{ or } 56.25$	M1	Using tan ⁻¹ 3 or 1.25 and <i>their BC</i> , but not 9 or 15, in correct area of sector formula.
	Area of triangle $PBC = 13.4$ to 13.6 or $\frac{1}{2} \times 9 \times 3$	B1	
	[Area = (56.207 or 56.25) – their 13.5 =] 42.7 or 42.8	A1	AWRT
		4	

© UCLES 2021 Page 11 of 21

Question	Answer	Marks	Guidance
8(a)	Terms required for x^2 : $-5 \times 2^4 \times ax + 10 \times 2^3 \times a^2 x^2 = -80ax + 80a^2 x^2$	B1	Can be seen as part of an expansion or in correct products.
	$2 \times (\pm their \text{ coefficient of } x) + 4 \times (\pm their \text{ coefficient of } x^2)$	*M1	
	x^{2} coefficient is $320a^{2} - 160a = -15$ $\Rightarrow 64a^{2} - 32a + 3 \Rightarrow (8a - 3)(8a - 1)$	DM1	Forming a 3-term quadratic in a , with all terms on the same side or correctly setting up prior to completing the square and solving using factorisation, formula or completing the square. If factorising, factors must expand to give <i>their</i> coefficient of a^2 .
	$a = \frac{1}{8} \text{ or } a = \frac{3}{8}$	A1	OE. Special case: If DM0 for solving quadratic, SC B1 can be awarded for correct final answers.
		4	

© UCLES 2021 Page 12 of 21

Question	Answer	Marks	Guidance
8(b)	$320a^2 - 160a = k \implies 320a^2 - 160a - k[=0]$	M1	Forming a 3-term quadratic in a with all terms on the same side. Allow \pm sign errors.
	Their $b^2 - 4ac$ [= 0], [$160^2 - 4 \times 320 \times (-k) = 0$]	M1	Any use of discriminant on a 3-term quadratic.
	k = -20	A1	
	$a = \frac{1}{4}$	B1	Condone $a = \frac{1}{4}$ from $k = 20$.
	Alternative method for question 8(b)		
	$320a^2 - 160a = k \text{ and divide by } 320 \left[a^2 - \frac{a}{2} = \frac{k}{320} \right]$	M1	Allow ± sign errors.
	Attempt to complete the square $\left[\left(a - \frac{1}{4} \right)^2 - \frac{1}{16} = \frac{k}{320} \right]$	M1	Must have $\left(a - \frac{1}{4}\right)^2$
	$a=\frac{1}{4}$	A1	
	k = -20	B1	

© UCLES 2021 Page 13 of 21

Question	Answer	Marks	Guidance	
8(b) cont'd	ont'd Alternative method for question 8(b)			
	$320a^2 - 160a = k$ and attempt to differentiate LHS [$640a - 160$]	M1	Allow \pm sign errors.	
	Setting <i>their</i> $(640a-160)=0$ and attempt to solve.	M1		
	$a = \frac{1}{4}$	A1		
	k = -20	B1		
		4		

© UCLES 2021 Page 14 of 21

Question	Answer	Marks	Guidance
9(a)	$\left[\frac{\mathrm{d}V}{\mathrm{d}r} = \right] \frac{9}{2} \left(r - \frac{1}{2}\right)^2$	B1	OE. Accept unsimplified.
	$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{\mathrm{d}r}{\mathrm{d}V} \times \frac{\mathrm{d}V}{\mathrm{d}t} = \frac{1.5}{their} \frac{\mathrm{d}V}{\mathrm{d}r} \left[= \frac{1.5}{\frac{9}{2} \left(5.5 - \frac{1}{2}\right)^2} = \frac{1.5}{112.5} \right]$	M1	Correct use of chain rule with 1.5, their differentiated expression for $\frac{dV}{dr}$ and using $r = 5.5$.
	0.0133 or $\frac{3}{225}$ or $\frac{1}{75}$ [metres per second]	A1	
		3	
9(b)	$\frac{\mathrm{d}V}{\mathrm{d}r} \text{ or } their \frac{\mathrm{d}V}{\mathrm{d}r} = \frac{1.5}{0.1} \text{ or } 15 \text{ OR } 0.1 = \frac{1.5}{their} \frac{\mathrm{d}V}{\mathrm{d}r} = \frac{2 \times 1.5}{9 \left(r - \frac{1}{2}\right)^2} \text{ OE}$	B1 FT	Correct statement involving $\frac{dV}{dr}$ or their $\frac{dV}{dr}$, 1.5 and 0.1.
1	$\left[\frac{9}{2}\left(r - \frac{1}{2}\right)^2 = 15 \Longrightarrow\right] r = \frac{1}{2} + \sqrt{\frac{10}{3}}$	B1	OE e.g. AWRT 2.3 Can be implied by correct volume.
I	[Volume =] 8.13 AWRT	<u>B1</u>	OE e.g. $\frac{-3 + 5\sqrt{30}}{3}$. CAO.
		3	

© UCLES 2021 Page 15 of 21

Question	Answer	Marks	Guidance
10(a)	$[f'(x) =] 2x - \frac{k}{x^2}$	B1	
	$f'(2) = 0 \left[2 \times 2 - \frac{k}{2^2} = 0 \right] \Rightarrow k = \dots$	M1	Setting <i>their</i> 2-term $f'(2) = 0$, at least one term correct and attempting to solve as far as $k = .$
	<i>k</i> = 16	A1	
		3	
10(b)	$f''(2) = e.g. 2 + \frac{2k}{2^3}$	M1	Evaluate a two term f"(2) with at least one term correct. Or other valid method.
	$\left[2 + \frac{2k}{2^3}\right] > 0 \Rightarrow \text{minimum or} = 6 \Rightarrow \text{minimum}$	A1 FT	WWW. FT on positive <i>k</i> value.
		2	
10(c)	When $x = 2$, $f(x) = 14$	B1	SOI
	[Range is or y or $f(x)$] \geqslant their $f(2)$	B1 FT	Not $x \ge their f(2)$
		2	

© UCLES 2021 Page 16 of 21

Question	Answer	Marks	Guidance
11(a)	$\frac{dy}{dx} = \frac{1}{2} + \frac{1}{3(x-2)^{\frac{4}{3}}}$	B1	OE. Allow unsimplified.
	Attempt at evaluating their $\frac{dy}{dx}$ at $x = 3 \left[\frac{1}{2} + \frac{1}{3(3-2)^{\frac{4}{3}}} = \frac{5}{6} \right]$	*M1	Substituting $x = 3$ into <i>their</i> differentiated expression – defined by one of 3 original terms with correct power of x .
	Gradient of normal = $\frac{-1}{their} \frac{dy}{dx} \left[= -\frac{6}{5} \right]$	*DM1	Negative reciprocal of <i>their</i> evaluated $\frac{dy}{dx}$.
	Equation of normal $y - \frac{6}{5} = (their \text{ normal gradient})(x-3)$ $\left[y = -\frac{6}{5}x + 4.8 \Rightarrow 5y = -6x + 24 \right]$	DM1	Using <i>their</i> normal gradient and <i>A</i> in the equation of a straight line. Dependent on *M1 and *DM1.
	[When $y = 0$,] $x = 4$	A1	or (4, 0)
		5	

© UCLES 2021 Page 17 of 21

Question	Answer	Marks	Guidance
11(b)	Area under curve = $\int \left(\frac{1}{2} x + \frac{7}{10} - \frac{1}{(x-2)^{\frac{1}{3}}} \right) [dx]$	M1	For intention to integrate the curve (no need for limits). Condone inclusion of π for this mark.
	$\frac{1}{4}x^2 + \frac{7}{10}x - \frac{3(x-2)^{\frac{2}{3}}}{2}$	A1	For correct integral. Allow unsimplified. Condone inclusion of π for this mark.
	$\left(\frac{9}{4} + 2.1 - \frac{3}{2}\right) - \left(\frac{6.25}{4} + 1.75 - \frac{3 \times 0.5^{\frac{2}{3}}}{2}\right)$	M1	Clear substitution of 3 and 2.5 into <i>their</i> integrated expression (with at least one correct term) and subtracting.
	0.48[24]	A1	If M1A1M0 scored then SC B1 can be awarded for correct answer.
	[Area of triangle =] 0.6	B1	OE
	[Total area =] 1.08	A1	Dependent on the first M1 and WWW.
		6	

© UCLES 2021 Page 18 of 21

Question	Answer	Marks	Guidance		
12(a)	Centre is $(3, -2)$	B1			
	Gradient of radius = $\frac{(their - 2) - 4}{(their 3) - 5} [= 3]$	*M1	Finding gradient using <i>their</i> centre (not $(0, 0)$) and $P(5,4)$.		
	Equation of tangent $y-4=-\frac{1}{3}(x-5)$	DM1	Using P and the negative reciprocal of <i>their</i> gradient to find the equation of AB .		
	Sight of $[x =]17$ and $[y =]\frac{17}{3}$	A1			
	$\left[\text{Area} = \frac{1}{2} \times \frac{17}{3} \times 17 = \right] \frac{289}{6}$	A1	Or $48\frac{1}{6}$ or AWRT 48.2.		
	Alternative method for question 12(a)				
	$2x + 2y\frac{\mathrm{d}y}{\mathrm{d}x} - 6 + 4\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	B1			
	At $P: 10 + 8\frac{dy}{dx} - 6 + 4\frac{dy}{dx} = 0 \left[\Rightarrow \frac{dy}{dx} = -\frac{1}{3} \right]$	*M1	Find the gradient using $P(5,4)$ in <i>their</i> implicit differential (with at least one correctly differentiated y term).		
	Equation of tangent $y-4=-\frac{1}{3}(x-5)$	DM1	Using P and <i>their</i> value for the gradient to find the equation of AB .		
	Sight of $[x =]17$ and $[y =]\frac{17}{3}$	A1			
	$\left[\text{Area} = \frac{1}{2} \times \frac{17}{3} \times 17 = \right] \frac{289}{6}$	A1	Or $48\frac{1}{6}$ or AWRT 48.2.		

© UCLES 2021 Page 19 of 21

Question	Answer	Marks	Guidance
12(a)	Alternative method for question 12(a)		
cont'd		B1	OE. Correct differentiation of rearranged equation.
	$\frac{dy}{dx} = (3-5)(31+6(5)-(5)^2)^{-\frac{1}{2}} \left[\Rightarrow \frac{dy}{dx} = -\frac{1}{3} \right]$	*M1	Find the gradient using $x = 5$ in <i>their</i> differential (with clear use of chain rule).
	Equation of tangent $y-4=-\frac{1}{3}(x-5)$	DM1	Using P and their value for the gradient to find the equation of AB .
	Sight of $[x =]17$ and $[y =]\frac{17}{3}$	A1	
	$\left[\text{Area} = \frac{1}{2} \times \frac{17}{3} \times 17 = \right] \frac{289}{6}$	A1	Or $48\frac{1}{6}$ or AWRT 48.2.
		5	

© UCLES 2021 Page 20 of 21

Question	Answer	Marks	Guidance
12(b)	Radius of circle = $\sqrt{40}$,	B1	Or $2\sqrt{10}$ or 6.32 AWRT or $r^2 = 40$.
	Area of $\triangle CRQ = \frac{1}{2} \times (their r)^2 \sin 120 \left[= \frac{1}{2} \times 40 \times \frac{\sqrt{3}}{2} \right]$ OR Area of $\triangle CQX = \frac{1}{2} \times \sqrt{40}\cos 30 \times \sqrt{40}\cos 60$ OE $\left[= \frac{1}{2} \times \sqrt{30} \times \sqrt{10} \right]$ OR Area of circle $-3 \times$ Area of segment $= 40\pi - 3 \times (40\frac{\pi}{3} - 10\sqrt{3})$ OR $QR = \sqrt{120} \text{ or } 2\sqrt{30} \text{ and area} = \frac{1}{2}QR^2 \sin 60$	M1	Using $\frac{1}{2}r^2\sin\theta$ with their r and 120 or 60 [×3] Using $\frac{1}{2}$ ×base×height in a correct right-angled triangle [×6].
	$QR = \sqrt{120} \text{ or } 2\sqrt{30} \text{ and area} = \frac{-QR}{2} \sin 60$		Use of cosine rule and area of large triangle
	$30\sqrt{3}$	A1	AWRT 52[.0] implies B1M1A0.
		3	See diagram for points stated in 'Answer' column.

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