

# Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

# 7 4 3 5 7 7 5 8 8 0

### **FURTHER MATHEMATICS**

9231/11

Paper 1 Further Pure Mathematics 1

May/June 2020

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

### **INSTRUCTIONS**

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### **INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has 16 pages. Blank pages are indicated.

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[Turn over

1 Let *a* be a positive constant.

(a) Sketch the curve with equation  $y = \frac{ax}{x+7}$ . [2]

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(b)	Sketch the curve with equation $y = \left  \frac{1}{x} \right $	$\frac{ax}{+7}$ and	l find the	set of value	es of x for v	which $\left  \frac{ax}{x+7} \right $	
							[4]
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Fin	nd a cubic equation whose roots are $\alpha^2$ , $\beta^2$ , $\gamma^2$ .	]
	is given that $\alpha^2 + \beta^2 + \gamma^2 = 2(\alpha + \beta + \gamma)$	
	is given that $\alpha^2 + \beta^2 + \gamma^2 = 2(\alpha + \beta + \gamma)$ .	
	is given that $\alpha^2 + \beta^2 + \gamma^2 = 2(\alpha + \beta + \gamma)$ .  Find the value of $p$ .	

	•••••			
Find the val	tue of $\alpha^3 + \beta^3 + \beta^3$	$\gamma^3$ .		[2]
		•	•••••	 

(a)	Find the equations of the asymptotes of <i>C</i> .	
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<b>(b)</b>	Find the coordinates of the stationary points on C	
(b)	Find the coordinates of the stationary points on $C$ .	
(b)	Find the coordinates of the stationary points on $C$ .	
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(b)		

(c) Sketch *C*. [3]

4 (a) By first expressing $\frac{1}{r^2-1}$ in partial fractions, show
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$$\sum_{r=2}^{n} \frac{1}{r^2 - 1} = \frac{3}{4} - \frac{an + b}{2n(n+1)},$$

where $a$ and $b$ are integers to be found.	[5]

Deduce the	value of z	$rac{r^2-1}{r^2-1}$	ľ				
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Find the lir	$nit, as n \rightarrow$	$\infty$ , of $\sum_{r=1}^{\infty}$	$\sum_{r=1}^{2n} \frac{n}{r^2 - 1}$	<del>-</del> .			
Find the lin	mit, as $n \rightarrow$	$\infty$ , of $\sum_{r=1}^{\infty}$	$\sum_{n=1}^{2n} \frac{n}{r^2 - 1}$	<u>-</u> . 1	 		
Find the lin	mit, as $n \rightarrow$	$\infty$ , of $\sum_{r=1}^{\infty}$	$\sum_{n+1}^{2n} \frac{n}{r^2 - \dots}$	<u>-</u> . 1	 		
Find the lin	mit, as $n \rightarrow$	$\infty$ , of $\sum_{r=1}^{\infty}$	$\sum_{n+1}^{2n} \frac{n}{r^2 - \dots}$	 1			
Find the lin	mit, as $n \rightarrow$	$\infty$ , of $\sum_{r=1}^{\infty}$	$\sum_{n+1}^{2n} \frac{n}{r^2 - 1}$	<u>-</u> . 1			
Find the lin	mit, as $n \rightarrow$	$\infty$ , of $\sum_{r=1}^{\infty}$	$\sum_{n+1}^{2n} \frac{n}{r^2 - 1}$	<u>-</u> .			
Find the lin	mit, as $n \rightarrow$	$\infty$ , of $\sum_{r=1}^{\infty}$	$\sum_{n+1}^{2n} \frac{n}{r^2 - 1}$	<u>1</u> .			
Find the lin	mit, as $n \rightarrow$	$\infty$ , of $\sum_{r=1}^{\infty}$	$\sum_{n+1}^{2n} \frac{n}{r^2 - \dots}$	<u>1</u> .			

_	Find the shortest distance between $l_1$ and $l_2$ .	
		,

The plane  $\Pi$  contains  $l_1$  and is parallel to the vector  $\mathbf{i} + \mathbf{k}$ . **(b)** Find the equation of  $\Pi$ , giving your answer in the form ax + by + cz = d. [4] (c) Find the acute angle between  $l_2$  and  $\Pi$ . [3]

(a)	$\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}.$ The transformation in the x x plane represented by $\mathbf{A}^{-1}$ transforms a triangle of area $20 \text{ cm}^2$ in
(a)	The transformation in the x-y plane represented by $A^{-1}$ transforms a triangle of area 30 cm <sup>2</sup> in triangle of area $d  \text{cm}^2$ .
	Find the value of $d$ .
(b)	Prove by mathematical induction that, for all positive integers $n$ ,
	$\mathbf{A}^n = \begin{pmatrix} 2^n & 0 \\ 2^n - 1 & 1 \end{pmatrix}.$

$\mathbf{B} = \begin{pmatrix} 1 & 0 \\ 33 & 0 \end{pmatrix}.$	
Find the value of $n$ .	

	17	
The	e curve $C_1$ has polar equation $r = \theta \cos \theta$ , for $0 \le \theta \le \frac{1}{2}\pi$ .	
(a)	The point on $C_1$ furthest from the line $\theta = \frac{1}{2}\pi$ is denoted by $P$ . Show that, at $P$ ,	
	$2\theta \tan \theta - 1 = 0$	
	and verify that this equation has a root between 0.6 and 0.7.	[5
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	e curve $C_2$ has polar equation $r = \theta \sin \theta$ , for $0 \le \theta \le \frac{1}{2}\pi$ . The curves $C_1$ and $C_2$ intersected, denoted by $O$ , and at another point $Q$ .	ct at th
	Find the polar coordinates of $Q$ , giving your answers in exact form.	[2

[3]

(c) Sketch  $C_1$  and  $C_2$  on the same diagram.

d)	Find, in terms of $\pi$ , the area of the region bounded by the arc $OQ$ of $C_1$ and the arc $OQ$ of $C_2$ .

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## **Additional Page**

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.			
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