

Cambridge International AS & A Level

CANDIDATE NAME				
CENTRE NUMBER		CANDIDATE NUMBER		

4 0 0 0 0 0 7 3 6 5 6

FURTHER MATHEMATICS

9231/23

Paper 2 Further Pure Mathematics 2

May/June 2020

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has 16 pages. Blank pages are indicated.

DC (LK/CB) 187469/1 © UCLES 2020

[Turn over

$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2}$	$-8\frac{\mathrm{d}x}{\mathrm{d}t}$	$-9x = 9e^{8t}.$	[6]
•••••	••••••		
 •••••			
 	••••••		
 •••••			
 •••••			
•••••			

	•••••
b) Find the exact value of I_2 .	[3

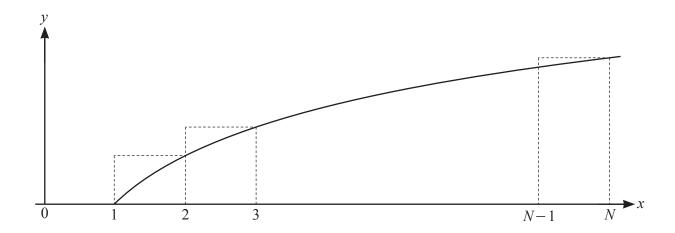
3	The	matrix	Δ	is	given	hv
J	1116	mauix	\boldsymbol{H}	15	given	υy

$$\mathbf{A} = \begin{pmatrix} 5 & -1 & 7 \\ 0 & 6 & 0 \\ 7 & 7 & 5 \end{pmatrix}.$$

(a)	Find the eigenvalues of A .	[4]
		••••
		••••
		••••
		••••
		••••

Use the characteristic equation of A to find A^{-1} .	[4

4



The diagram shows the curve with equation $y = \ln x$ for $x \ge 1$, together with a set of (N-1) rectangles of unit width.

(a) By considering the sum of the areas of these rectangles, show that

$\ln N! > N \ln N - N + 1.$	[5]

5	The curve	C has	parametric	equations
J	THE CUIVE	C Has	parametric	cquations

$x = \frac{1}{2}t^2 - \ln t$, $y = 2t + 1$, for $\frac{1}{2} \le t \le 2$.

Find the exact length of <i>C</i> .	

Find $\frac{d^2y}{dx^2}$ in terms of t, simplifying your answer.	[4]

(a)	Starting from the definitions of tanh and sech in terms of exponentials, prove that						
	$1 - \tanh^2 \theta = \operatorname{sech}^2 \theta.$	[3]					
		••••••					
		•••••					
Γhe	variables x and y are such that $\tanh y = \cos\left(x + \frac{1}{4}\pi\right)$, for $-\frac{1}{4}\pi < x < \frac{3}{4}\pi$.						
(b)	By differentiating the equation $\tanh y = \cos\left(x + \frac{1}{4}\pi\right)$ with respect to x, show that						
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\csc\left(x + \frac{1}{4}\pi\right).$	[4					
	$dx = \cos(x + 4x)$	L'.					
		•••••					
		•••••					
		•••••					

2	en , grving me	onact varaes	or the consta	ants a , b and c .	

_		~ 1							~
7 ((a)	Show	that an	approi	oriate	ıntegra	tıng	factor	tor

$(x^{2}+1)\frac{\mathrm{d}y}{\mathrm{d}x} + y\sqrt{x^{2}+1} = x^{2} - x\sqrt{x^{2}+1}$
is $x + \sqrt{x^2 + 1}$. [4]

	$(x^{2}+1)\frac{dy}{dx} + y\sqrt{x^{2}+1} = x^{2} - x\sqrt{x^{2}+1}$	
for which	$y = \ln 2$ when $x = 0$. Give your answer in the form $y = f(x)$.	[7]
•••••		
•••••		
		••••••
••••••		
•••••		

[t is	given that $\cos^6 \theta = \frac{1}{32} (\cos 6\theta + 6 \cos 4\theta + 1)$	5 cos 2θ + 10).	
	given that $\cos^6 \theta = \frac{1}{32} (\cos 6\theta + 6 \cos 4\theta + 1)$		
	given that $\cos^6 \theta = \frac{1}{32} (\cos 6\theta + 6 \cos 4\theta + 1)$ Find the exact value of $\int_0^{\frac{1}{3}\pi} (\cos^6 (\frac{1}{4}x) + \sin^6 (\frac{1}{4}x)) dx$		

							• • • • • • •
	•••••			•••••	•••••		• • • • • • • •
•••••							• • • • • • • • • • • • • • • • • • • •
Express eac rational nur	ch root of the nber.	he equation	n $16c^6 + 16$	$\left(1-c^2\right)^3-13=$	= 0 in the form	m $\cos k\pi$, when	re k i
•••••				••••••	•••••	••••••	•••••
• • • • • • • • • • • • • • • • • • • •					•••••		•••••
	,						
••••••		••••••		••••••	••••••	•••••	• • • • • • • •
							• • • • • • • • • • • • • • • • • • • •
							• • • • • • • • • • • • • • • • • • • •

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question num must be clearly shown.	ber(s)
	•••••
	•••••
	•••••
	•••••
	•••••

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.