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**9231/22**

May/June 2020

**2 hours**

You will need: List of formulae (MF19)

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

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**[Turn over**

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**1** Find the solution of the differential equation

$$\frac{dy}{dx} + 5y = e^{-7x}$$

for which  $y = 0$  when  $x = 0$ . Give your answer in the form  $y = f(x)$ .

[6]

[illegible]

2 It is given that  $y = 2^x$ .

- (a) By differentiating  $\ln y$  with respect to  $x$ , show that  $\frac{dy}{dx} = 2^x \ln 2$ . [3]

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- (b) Write down  $\frac{d^2y}{dx^2}$ . [1]

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- (c) Hence find the first three terms in the Maclaurin's series for  $2^x$ . [3]

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- 3 (a) Find the roots of the equation  $z^3 = -1 - i$ , giving your answers in the form  $re^{i\theta}$ , where  $r > 0$  and  $0 \leq \theta < 2\pi$ . [5]

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Let  $w = z_1^{3k} + z_2^{3k} + z_3^{3k}$ , where  $k$  is a positive integer and  $z_1, z_2, z_3$  are the roots of  $z^3 = -1 - i$ .

- (b) Express  $w$  in the form  $Re^{i\alpha}$ , where  $R > 0$ , giving  $R$  and  $\alpha$  in terms of  $k$ . [3]

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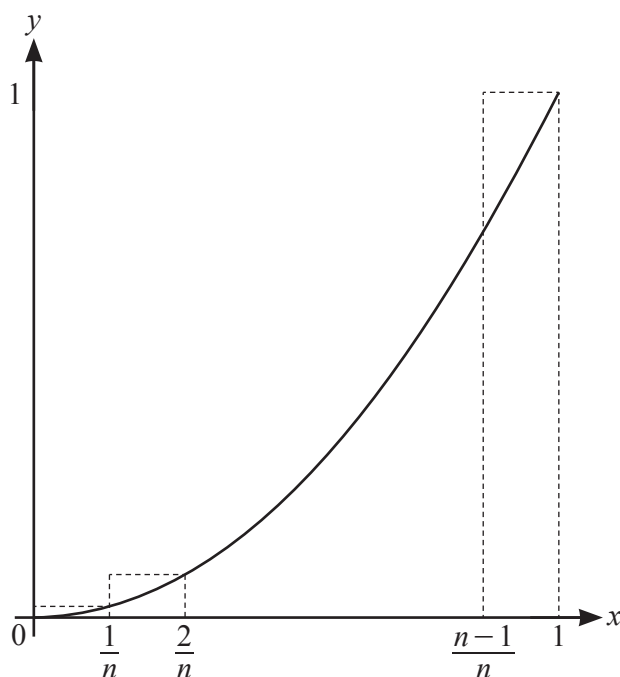
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**(a)** By considering the sum of the areas of these rectangles, show that

$$\int_0^1 x^2 dx < \frac{2n^2 + 3n + 1}{6n^2}. \quad [4]$$

[illegible]

[illegible]

5 The curves  $C_1 : y = \cosh x$  and  $C_2 : y = \sinh 2x$  intersect at the point where  $x = a$ .

(a) Find the exact value of  $a$ , giving your answer in logarithmic form. [4]

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(b) Sketch  $C_1$  and  $C_2$  on the same diagram. [2]



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**6** The integral  $I_n$ , where  $n$  is an integer, is defined by  $I_n = \int_0^{\frac{1}{2}} (1-x^2)^{-\frac{1}{2}n} dx$ .

**(a)** Find the exact value of  $I_1$ .

[2]

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**(b)** By considering  $\frac{d}{dx} \left( x(1-x^2)^{-\frac{1}{2}n} \right)$ , or otherwise, show that

$$nI_{n+2} = 2^{n-1}3^{-\frac{1}{2}n} + (n-1)I_n.$$

[5]

[illegible]

- (c) Find the exact value of  $I_5$  giving the answer in the form  $k\sqrt{3}$ , where  $k$  is a rational number to be determined. [3]

[illegible]

**7** It is given that  $x = t^3y$  and

$$t^3 \frac{d^2 y}{dt^2} + (4t^3 + 6t^2) \frac{dy}{dt} + (13t^3 + 12t^2 + 6t)y = 61e^{\frac{1}{2}t}.$$

(a) Show that

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 13x = 61e^{\frac{1}{2}t}. \quad [4]$$

[illegible]



- 8 (a) Find the values of  $a$  for which the system of equations

$$3x + y + z = 0,$$

$$ax + 6y - z = 0,$$

$$ay - 2z = 0,$$

does not have a unique solution.

[3]

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The matrix  $\mathbf{A}$  is given by

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & 1 \\ 0 & 6 & -1 \\ 0 & 0 & -2 \end{pmatrix}.$$

- (b) Use the characteristic equation of  $\mathbf{A}$  to find the inverse of  $\mathbf{A}^2$ .

[4]

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[illegible]

[illegible]

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