Cambridge International A Level

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MATHEMATICS **9709/31**

Paper 3 **October/November 2019**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	State $1+e^{2y}=e^{x}$	B1	
	Make y the subject	M1	Rearrange to $e^{2y}=\ldots$ and use logs
	Obtain answer $y=\frac{1}{2}\ln\left(e^{x}-1\right)$	A1	OE
		3	

Question	Answer	Marks	Guidance
2	State or imply non-modular inequality $(2x-3)^2 > 4^2(x+1)^2$, or corresponding quadratic equation, or pair of linear equations $(2x-3) = \pm 4(x+1)$	B1	$12x^2 + 44x + 7 < 0$
	Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	M1	Correct method seen, or implied by correct answers
	Obtain critical values $x = -\frac{7}{2}$ and $x = -\frac{1}{6}$	A1	
	State final answer $-\frac{7}{2} < x < -\frac{1}{6}$	A1	
	Alternative method for question 2		
	Obtain critical value $x = -\frac{7}{2}$ from a graphical method, or by inspection, or by solving a linear equation or an inequality	B1	
	Obtain critical value $x = -\frac{1}{6}$ similarly	B2	
	State final answer $-\frac{7}{2} < x < -\frac{1}{6}$	B1	
		4	

Question	Answer	Marks	Guidance
3	State $\frac{dx}{dt} = 2 + 2\cos 2t$	B1	
	Use the chain rule to find the derivative of y	M1	
	Obtain $\frac{dy}{dt} = \frac{2\sin 2t}{1-\cos 2t}$	A1	OE
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
	Obtain $\frac{dy}{dx} = \operatorname{cosec} 2t$ correctly	A1	AG
		5	

Question	Answer	Marks	Guidance
4(i)	State $\frac{dN}{dt} = ke^{-0.02t}N$ and show $k = -0.01$	B1	OE $(-10 = k \times 1 \times 1000)$
		1	
4(ii)	Separate variables correctly and integrate at least one side	B1	$\int \frac{1}{N} dN = \int -0.01e^{-0.02t} dt$
	Obtain term $\ln N$	B1	OE
	Obtain term $0.5e^{-0.02t}$	B1	OE
	Use $N = 1000$, $t = 0$ to evaluate a constant, or as limits, in a solution with terms $a \ln N$ and $be^{-0.02t}$, where $ab \neq 0$	M1	
	Obtain correct solution in any form e.g. $\ln N - \ln 1000 = 0.5\left(e^{-0.02t} - 1\right)$	A1	$\ln 1000 - \frac{1}{2} = 6.41$
	Substitute $N = 800$ and obtain $t = 29.6$	A1	
		6	
4(iii)	State that N approaches $\frac{1000}{\sqrt{e}}$	B1	Accept 606 or 607 or 606.5
		1	

Question	Answer	Marks	Guidance
5(i)	Use correct product rule	M1	
	Obtain correct derivative in any form $\frac{\mathrm{d}y}{\mathrm{d}x} = -2\mathrm{e}^{-2x}\ln(x-1) + \frac{\mathrm{e}^{-2x}}{x-1}$	A1	
	Equate derivative to zero and derive $x = 1 + e^{\frac{1}{2(x-1)}}$ or $p = 1 + ^{\frac{1}{2(p-1)}}$	A1	AG
		3	
5(ii)	Calculate values of a relevant expression or pair of relevant expressions at x = 2.2 and x = 2.6 $\mathrm{f}(x) = \ln(x-1) - \frac{1}{2(x-1)} \Rightarrow \mathrm{f}(2.2) = -0.234, \mathrm{f}(2.6) = 0.317$ $\mathrm{f}(x) = 2\mathrm{e}^{-2x}\ln(x-1) + \frac{\mathrm{e}^{-2x}}{x-1} \Rightarrow \mathrm{f}(2.2) = 0.005\ldots, \mathrm{f}(2.6) = -0.0017\ldots$	M1	
	Complete the argument correctly with correct calculated values	A1	
		2	

Question	Answer	Marks	Guidance
5(iii)	Use the iterative process $p_{n+1} = 1 + \exp\left(\dfrac{1}{2(p_n - 1)}\right)$ correctly at least once	**M1**	
	Obtain final answer 2.42	**A1**	
	Show sufficient iterations to 4 d.p. to justify 2.42 to 2 d.p., or show there is a sign change in the interval (2.415, 2.425)	**A1**	
		3	

Question	Answer	Marks	Guidance
6(i)	Use correct quotient rule	M1	
	Obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = -\operatorname{cosec}^2 x$ correctly	A1	AG
		2	
6(ii)	Integrate by parts and reach $ax\cot x + b\int \cot x\,\mathrm{d}x$	*M1	
	Obtain $-x\cot x + \int \cot x\,\mathrm{d}x$	A1	OE
	State $\pm\ln\sin x$ as integral of $\cot x$	M1	
	Obtain complete integral $-x\cot x + \ln\sin x$	A1	OE
	Use correct limits correctly	DM1	$0 + 0 + \frac{\pi}{4} - \ln\frac{1}{\sqrt{2}}$
	Obtain $\frac{1}{4}(\pi + \ln 4)$ following full and exact working	A1	AG
		6	

Question	Answer	Marks	Guidance
7(i)	Express general point of l or m in component form e.g. $(a + \lambda, 2 - 2\lambda, 3 + 3\lambda)$ or $(2 + 2\mu, 1 - \mu, 2 + \mu)$	**B1**	
	Equate at least two pairs of corresponding components and solve for λ or for μ	**M1**	
	Obtain **either** $\lambda = -2$ or $\mu = -5$ **or** $\lambda = \frac{1}{3}a$ or $\mu = \frac{2}{3}a - 1$ **or** $\lambda = \frac{1}{5}(a-4)$ or $\mu = \frac{1}{5}(3a-7)$	**A1**	
	Obtain $a = -6$	**A1**	
		4	
7(ii)	Use scalar product to obtain a relevant equation in a, b and c, e.g. $a - 2b + 3c = 0$	**B1**	
	Obtain a second equation, e.g. $2a - b + c = 0$ and solve for one ratio	**M1**	
	Obtain $a : b : c = 1 : 5 : 3$	**A1**	OE
	Substitute a relevant point and values of a, b, c in general equation and find d	**M1**	
	Obtain correct answer $x + 5y + 3z = 13$	**A1FT**	OE. The FT is on a from part (**i**), if used
	Alternative method for question 7(ii)		
	Attempt to calculate vector product of relevant vectors,	**M1**	e.g. $(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}).(2\mathbf{i} - \mathbf{j} + \mathbf{k})$
	Obtain two correct components	**A1**	
	Obtain correct answer, e.g. $\mathbf{i} + 5\mathbf{j} + 3\mathbf{k}$	**A1**	
	Substitute a relevant point and find d	**M1**	
	Obtain correct answer $x + 5y + 3z = 13$	**A1FT**	OE. The FT is on a from part (**i**), if used

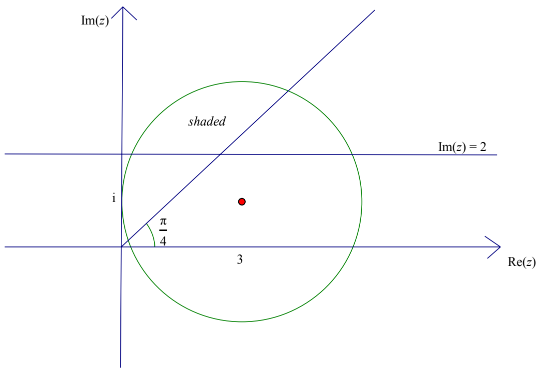
Question	Answer	Marks	Guidance
7(ii)	**Alternative method for question 7(ii)**		
	Using a relevant point and relevant vectors, form a 2-parameter equation for the plane	**M1**	
	State a correct equation, e.g. $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) + \mu(2\mathbf{i} - \mathbf{j} + \mathbf{k})$	**A1FT**	
	State three correct equations in x, y, z, λ and μ	**A1FT**	
	Eliminate λ and μ	**M1**	
	Obtain correct answer $x + 5y + 3z = 13$	**A1FT**	OE. The FT is on a from part (**i**), if used
		5	

Question	Answer	Marks	Guidance
8(i)	State or imply the form $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2}$	**B1**	
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A = -1$, $B = 3$, $C = 2$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	Allow in the form $\frac{Ax+B}{x^2} + \frac{C}{x+2}$
		5	

Question	Answer	Marks	Guidance
8(ii)	Integrate and obtain terms $\ln x - \frac{3}{x} + 2\ln(x+2)$	B1FT + B1FT + B1FT	The FT is on *A*, *B*, *C*; or on *A*, *D*, *E*.
	Substitute limits correctly in an integral with terms $a\ln x$, $\frac{b}{x}$ and $c\ln(x+2)$, where $abc \neq 0$	M1	$-\ln 4 - \frac{3}{4} + 2\ln 6(+\ln 1) + 3 - 2\ln 3$
	Obtain $\frac{9}{4}$ following full and exact working	A1	AG – work to combine or simplify logs is required
		5	

Question	Answer	Marks	Guidance
9(i)	Use $\cos(A + B)$ formula to express $\cos 3x$ in terms of trig functions of $2x$ and x	**M1**	
	Use double angle formulae and Pythagoras to obtain an expression in terms of $\cos x$ only	**M1**	
	Obtain a correct expression in terms of $\cos x$ in any form	**A1**	
	Obtain $\cos 3x \equiv 4\cos^3 x - 3\cos x$	**A1**	AG
		4	
9(ii)	Use identity and solve cubic $4\cos^3 x = -1$ for x	**M1**	$\cos x = -0.6299...$
	Obtain answer 2.25 and no other in the interval	**A1**	Accept 0.717π M1A0 for $129.0°$
		2	

Question	Answer	Marks	Guidance
9(iii)	Obtain indefinite integral $\frac{1}{12}\sin 3x + \frac{3}{4}\sin x$	B1 + B1	
	Substitute limits in an indefinite integral of the form $a\sin 3x + b\sin x$, where $ab \neq 0$	M1	$\frac{1}{4}\left[\frac{1}{3}\sin \pi + 3\sin\frac{\pi}{3} - \frac{1}{3}\sin\frac{\pi}{2} - 3\sin\frac{\pi}{6}\right]$
	Obtain answer $\frac{1}{24}\left(9\sqrt{3}-11\right)$, or exact equivalent	A1	
	Alternative method for question 9(iii)		
	$\int \cos x\left(1-\sin^2 x\right)\mathrm{d}x = \sin x - \frac{1}{3}\sin^3 x (+C)$	B1 + B1	
	Substitute limits in an indefinite integral of the form $a\sin x + b\sin^3 x$ where $ab \neq 0$	M1	$\left(\frac{\sqrt{3}}{2} - \frac{1}{2} - \frac{1}{4}\frac{\sqrt{3}}{2} + \frac{1}{24}\right)$
	Obtain answer $\frac{1}{24}\left(9\sqrt{3}-11\right)$, or exact equivalent	A1	
		4	

Question	Answer	Marks	Guidance
10(a)	Square $a + ib$ and equate real and imaginary parts to – 3 and $-2\sqrt{10}$ respectively	*M1	
	Obtain $a^2 - b^2 = -3$ and $2ab = -2\sqrt{10}$	A1	
	Eliminate one unknown and find an equation in the other	DM1	
	Obtain $a^4 + 3a^2 - 10 = 0$, or $b^4 - 3b^2 - 10 = 0$, or horizontal 3-term equivalent	A1	
	Obtain answers $\pm\left(\sqrt{2} - \sqrt{5}\mathrm{i}\right)$, or exact equivalent	A1	
		5	
10(b)	Show point representing 3 + i in relatively correct position	B1	
	Show a circle with radius 3 and centre not at the origin	B1	
	Show correct half line from the origin at $\frac{1}{4}\pi$ to the real axis	B1	
	Show horizontal line $y = 2$	B1	
	Shade the correct region	B1	
		5	