

# Cambridge International AS & A Level

CANDIDATE  
NAME

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CENTRE  
NUMBER

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## MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3

**May/June 2020**

**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

## INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

## INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

- 1 Find the set of values of  $x$  for which  $2(3^{1-2x}) < 5^x$ . Give your answer in a simplified exact form. [4]

This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

- 2 (a) Expand  $(2 - 3x)^{-2}$  in ascending powers of  $x$ , up to and including the term in  $x^2$ , simplifying the coefficients. [4]

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- (b) State the set of values of  $x$  for which the expansion is valid. [1]

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- 3 Express the equation  $\tan(\theta + 60^\circ) = 2 + \tan(60^\circ - \theta)$  as a quadratic equation in  $\tan \theta$ , and hence solve the equation for  $0^\circ \leq \theta \leq 180^\circ$ . [6]

[illegible]

4 The curve with equation  $y = e^{2x}(\sin x + 3 \cos x)$  has a stationary point in the interval  $0 \leq x \leq \pi$ .

(a) Find the  $x$ -coordinate of this point, giving your answer correct to 2 decimal places. [4]

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(b) Determine whether the stationary point is a maximum or a minimum. [2]

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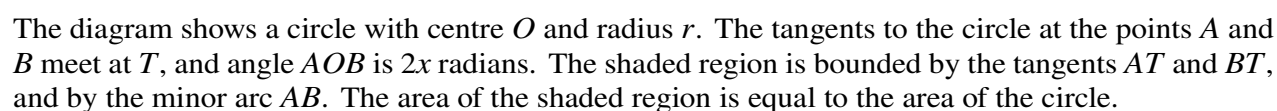
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- 5** (a) Find the quotient and remainder when  $2x^3 - x^2 + 6x + 3$  is divided by  $x^2 + 3$ . [3]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

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- This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



- (b) This equation has one root in the interval  $0 < x < \frac{1}{2}\pi$ . Verify by calculation that this root lies between 1 and 1.4. [2]

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- (c) Use the iterative formula

$$x_{n+1} = \tan^{-1}(\pi + x_n)$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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**7** Let  $f(x) = \frac{\cos x}{1 + \sin x}$ .

(a) Show that  $f'(x) < 0$  for all  $x$  in the interval  $-\frac{1}{2}\pi < x < \frac{3}{2}\pi$ . [4]

[illegible]

(b) Find  $\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} f(x) \, dx$ . Give your answer in a simplified exact form. [4]

This image shows a blank sheet of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting or typing. There are no margins, text, or other markings on the page.

- 8** A certain curve is such that its gradient at a point  $(x, y)$  is proportional to  $\frac{y}{x\sqrt{x}}$ . The curve passes through the points with coordinates  $(1, 1)$  and  $(4, e)$ .

- (a) By setting up and solving a differential equation, find the equation of the curve, expressing  $y$  in terms of  $x$ . [8]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

[1]

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- 9 With respect to the origin  $O$ , the vertices of a triangle  $ABC$  have position vectors

$$\overrightarrow{OA} = 2\mathbf{i} + 5\mathbf{k}, \quad \overrightarrow{OB} = 3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = \mathbf{i} + \mathbf{j} + \mathbf{k}.$$

- (a) Using a scalar product, show that angle  $ABC$  is a right angle. [3]

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- (b) Show that triangle  $ABC$  is isosceles. [2]

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10 (a) The complex number  $u$  is defined by  $u = \frac{3i}{a+2i}$ , where  $a$  is real.

(i) Express  $u$  in the Cartesian form  $x + iy$ , where  $x$  and  $y$  are in terms of  $a$ . [3]

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(ii) Find the exact value of  $a$  for which  $\arg u^* = \frac{1}{3}\pi$ . [3]

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- (b) (i) On a sketch of an Argand diagram, shade the region whose points represent complex numbers  $z$  satisfying the inequalities  $|z - 2i| \leq |z - 1 - i|$  and  $|z - 2 - i| \leq 2$ . [4]

- (ii) Calculate the least value of  $\arg z$  for points in this region. [2]

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[illegible]



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