Cambridge International AS Level

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary Level

MATHEMATICS **9709/23**
Paper 2 **October/November 2019**
MARK SCHEME
Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.
DM or **DB**	When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(i)	State or imply non-modular inequality $(2x-7)^2 < (2x-9)^2$ or corresponding equation or linear equation (with signs of $2x$ different)	**M1**	
	Obtain critical value 4	**A1**	
	State $x < 4$ only	**A1**	
		3	
1(ii)	Attempt to find *n* from $\ln n$ = *their* critical value from part **(i)**	**M1**	
	Obtain or imply $n < e^4$ and hence 54	**A1**	
		2	

Question	Answer	Marks	Guidance
2	Expand integrand to obtain $4e^{4x} - 4e^{2x} + 1$	**B1**	
	Integrate to obtain at least two terms of form $k_1e^{4x} + k_2e^{2x} + k_3x$	***M1**	
	Obtain correct $e^{4x} - 2e^{2x} + x$	**A1**	
	Apply both limits correctly to their integral	**DM1**	
	Obtain $e^8 - 3e^4 + 2e^2 + 1$	**A1**	
		5	

Question	Answer	Marks	Guidance
3	Use quotient rule (or product rule) to find first derivative	*M1	Must have correct u and v
	Obtain $-\dfrac{1}{x(1+\ln x)^2}$ or (unsimplified) equivalent	A1	
	Use $y=4$ to obtain $\ln x=-\frac{1}{2}$ or exact equivalent for x	B1	
	Substitute for x in their first derivative	DM1	
	Obtain $-4e^{\frac{1}{2}}$ or exact equivalent	A1	Must be simplified to contain a single exponential term
		5	

Question	Answer	Marks	Guidance
4(i)	Substitute $x=2$, equate to zero and attempt solution	M1	
	Obtain $a=4$	A1	
		2	
4(ii)	Divide by $x-2$ at least as far as the x term	M1	By inspection or use of identity
	Obtain $4x^2+12x+9$	A1	
	Conclude $(x-2)(2x+3)^2$	A1	Each factor must be simplified to integer form
		3	

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Question	Answer	Marks	Guidance
4(iii)	Attempt correct process to solve $e^{\sqrt{y}} = k$ where $k > 0$	M1	For $y = (\ln k)^2$
	Obtain 0.48 and no others	A1	AWRT
		2	

Question	Answer	Marks	Guidance
5(i)	Integrate to obtain form $x^3 + k_1 \sin 2x + k_2 \cos x$	*M1	
	Obtain correct $x^3 + 2\sin 2x + \cos x$	A1	
	Apply limits correctly and equate to 2	DM1	
	Confirm given result	A1	AG; necessary detail needed
		4	
5(ii)	Consider sign of $a - \sqrt[3]{3 - 2\sin 2a - \cos a}$ or equivalent for 0.5 and 0.75	M1	
	Obtain –0.26 and 0.10 or equivalents and justify conclusion	A1	AG; necessary detail needed
		2	

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Question	Answer	Marks	Guidance
5(iii)	Use iterative process correctly at least once	M1	Need to see a correct x_3, may be implied by $x_1 = 0.5$ so $x_3 = 0.65256$ or $x_1 = 0.75$ so $x_3 = 0.64897$ OE Must be working with radians
	Obtain final answer 0.651	A1	
	Show sufficient iterations to 5sf to justify answer or show a sign change in the interval $[0.6505,\ 0.6515]$	A1	
		3	

Question	Answer	Marks	Guidance
6(a)	Express equation as $\dfrac{1}{\cos\alpha\sin\alpha} = 7$	B1	OE; May be implied by subsequent work
	Attempt use of identity for $\sin 2\alpha$ or attempt to obtain a quadratic equation in terms of any one of the following: $\sin^2\alpha,\ \cos^2\alpha,\ \cot^2\alpha$ or $\tan^2\alpha$	M1	From equation of form $\sin 2\alpha = k$ where $0 < k < 1$ or from use of correct identities
	Obtain $\sin 2\alpha = \frac{2}{7}$ or a correct 3 term quadratic equation, equated to zero in any one of the following: $\sin^2\alpha,\ \cos^2\alpha,\ \cot^2\alpha$ or $\tan^2\alpha$	A1	
	Attempt correct process to find at least one correct value of α	M1	
	Obtain 8.3 and 81.7 and no others between 0 and 90	A1	
		5	

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Question	Answer	Marks	Guidance
6(b)	Simplify left-hand side to obtain $2\sin\beta\cos 20°$	**B1**	
	Attempt to form equation where $\tan\beta$ is only variable, $\tan\beta \neq 3$	**M1**	
	Obtain $\tan\beta = \dfrac{3}{\cos 20°}$	**A1**	OE
	Obtain $\beta = 72.6$ and no others between 0 and 90	**A1**	
		5	

Question	Answer	Marks	Guidance
7(i)	Obtain $-4y - 4x\frac{dy}{dx}$ from use of the product rule	**B1**	
	Differentiate $-2y^2$ to obtain $-4y\frac{dy}{dx}$	**B1**	
	Obtain $2x$, $\;=0$ with no extra terms	**B1**	
	Rearrange to obtain expression for $\frac{dy}{dx}$ and substitute $x=-1$, $y=2$	**M1**	
	Obtain $\dfrac{dy}{dx} = \dfrac{2x-4y}{4x+4y}$ OE and hence $-\frac{5}{2}$	**A1**	
		5	

Question	Answer	Marks	Guidance
7(ii)	Equate numerator of derivative to zero to produce equation in x and y	**M1**	
	Substitute into equation of curve to produce equation in x or y	**M1**	
	Obtain $-6y^2 = 1$ or $-\frac{3}{2}x^2 = 1$ OE and conclude	**A1**	
		3	
7(iii)	Use denominator of derivative equated to zero with equation of curve to produce equation in x	**M1**	
	Obtain $3x^2 = 1$ and hence $x = \pm\frac{1}{\sqrt{3}}$	**A1**	OE
		2	