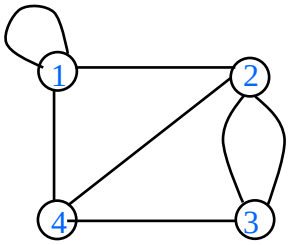
MA238 Quiz 1 11:30 am Tutorial January 22, 2021 20 marks

No aids allowed. Insufficient justification will result in a loss of marks.

1. [4 marks] For each course at a university, there may be one or more other courses that are its prerequisites.
   (a) How can a graph or directed graph be used to model the courses and which courses are prerequisites for which courses? Describe the vertices and when there is an edge in your model. Is your graph directed or undirected?
   I would construct a directed graph [0.5] as follows.
   There is a vertex for each course. [1]
   There is an edge (a,b) [directed from vertex a to vertex b] exactly when course a is a prerequisite for course b. [1.5]
   (b) What does the degree or indegree of a vertex in your model represent?
   The indegree of a vertex represents the number of prerequisites for the course corresponding to that vertex. [1]

2. [5 marks] (a) Draw an undirected graph with vertex-set V = {1,2,3,4} and edge-set E ={ {1,1}, {1,2}, {2,3}, {3,2}, {3,4}, {4,1}, {4,2} }.

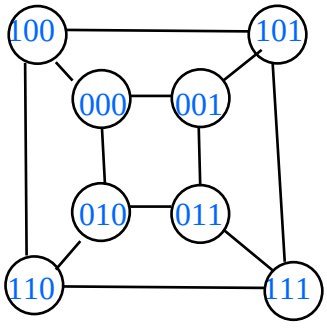
   (b) Write adjacency matrix of the graph, labelling the rows and columns.



	1	2	3	4
1	1	1	0	1
2	1	0	2	1
3	0	2	0	1
4	1	1	1	0

[2.5 marks each, -0.5 for each error, -0.5 if circles or dots not used for vertices, -0.5 if labels not given for matrix, -1 if the only errors in the adjacency matrix are that it is not symmetric.]

3. [3 marks] Draw the 3-dimensional hypercube with vertices labelled appropriately.



4. [8 marks] (a) For any graph G = (V, E), the sum of the degrees of vertices of G equals $2|E|$ (twice the number of edges). [1]
   (b) What is the degree of each vertex in the complete graph K_n? $n-1$ [1]
   (c) How many edges does K_n have? $n(n-1)/2$ [1]

(d) **For this question, show the calculations you used to get your answer. Do not just draw a specific graph. (a) – (c) should help.**

Suppose a simple graph G has degree sequence (4, 3, 3, 2, 1, 1).

(i) How many edges does G have?

From (a), number of edges = (½) sum of the degrees = $\frac{4+ 3+ 3+ 2+ 1+ 1}{2} = 7$

Showing work [0.5], answer [0.5]

(ii) How many edges does the complement, $\bar{G}$, of G have?

G has 6 vertices. K_6 has $6 \cdot 5/2 = 15$ edges. Thus $\bar{G}$ has 8 edges. Showing work [1], answer [1]

(iii) What is the degree sequence of complement, $\bar{G}$, of G?

Each vertex of K_6 has degree 5. Thus if the degree of a vertex of G is t, its degree in $\bar{G}$ is 5-t.
Thus the degrees of vertices of $\bar{G}$ are 5-4=1, 5- 3=2, 5-3=2, 5-2=3, 5-1=4, 5-1=4
And writing these in non-decreasing order, we get the degree sequence of G which is (4, 4, 3, 2, 2, 1)
Showing work [1], answer [1]