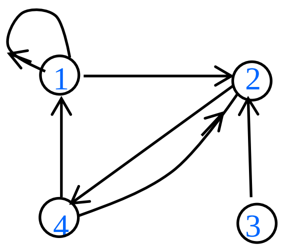
MA238 Quiz 1 10 am Tutorial January 22, 2021 [20 marks]

No aids allowed. Insufficient justification will result in a loss of marks.

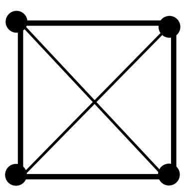
1. [5 marks]
   (a) Draw a directed graph on the vertex-set {1,2,3,4} with edge-set
   { (1,1), (1,2), (2,4), (3,2), (4,1), (4,2) }
   (b) Write the adjacency matrix of the graph, labelling the rows and columns.



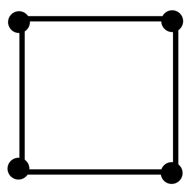
$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

[2.5 marks each, -0.5 for each error, -0.5 if circles or dots not used for vertices, -0.5 if labels not given for matrix.]

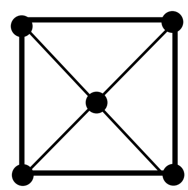
2. [4 marks – 1 mark for each graph and 1 for (b)]
   (a) Draw the complete graph K_4, the cycle C_4, and the wheel W_4.
   (Say which graph is which.)



K_4



C_4



W_4

(b)Which of the above graphs are bipartite? Only C_4

3. [2 marks] Can (5, 4, 4, 3, 3, 3, 2, 2, 2, 2, 1) be the degree sequence of a graph?
   If your answer is yes, draw such a graph. If your answer is no, give a reason.
   No [0.5], every graph has an even number of odd-degree vertices, [1]
   But this sequence has 5 odd numbers [0.5]
   And thus cannot be the degree sequence of a graph

4. [2 marks] In a friendship graph, the vertices correspond to people and two vertices are joined by an edge if the corresponding people are friends. Consider the vertex s corresponding to Sam.
   (a) What does the degree of vertex s represent?
   The degree of vertex s is the **number** of friends Sam has (in the group) [1]
   (b) What does the neighbour-set of vertex s represent?
   The neighbour-set of s represents the set of Sam's friends (in the group) [1]

5. [7 marks]
   (a) For any graph, how are the number of edges and the degrees related?
   The sum of the degrees is the twice the number of edges. [1]
   (b) How many edges does the complete graph on n vertices have? Show your reasoning.
   The complete graph has n vertices and the degree of each vertex is n-1. [1]
   By (a), $2|E| = n(n-1)$, and thus $|E| = n(n-1)/2$ [1 for correct answer, 1 for reasoning]

(c) Recall that a graph is k-regular if every vertex has degree k.
How many edges does a k-regular graph on n vertices have? Show your reasoning.
Since the degree of each vertex is k, by (a), $2|E| = nk$, and thus $|E| = nk/2$
[1 for correct answer, 1 for reasoning]

(d) Suppose G is a k-regular graph on n vertices. How many edges does the complement $\bar{G}$ of G have?
Solution 1:
An edge is in $\bar{G}$ if and only if it is not in G.
Thus number of edges of $\bar{G}$ = number of edges of K_n – number of edges of G
$= n(n-1)/2 - kn/2$
[1 for correct answer, 1 for reasoning]

Solution 2:
Since each vertex of G has degree k, the degree of each vertex of $\bar{G}$ is $(n-1)-k$.
By (c), the number of edges of $\bar{G}$ is $n[(n-1)-k]/2$