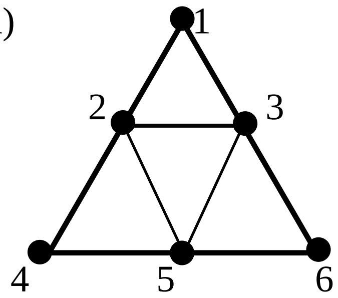
MA238 Quiz 2 10:00 am Tutorial February 12, 2021 20 marks

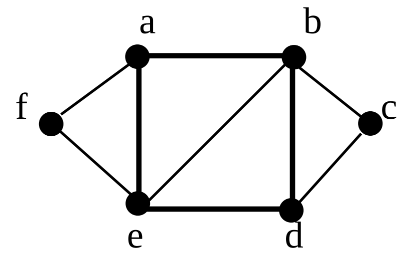
No aids allowed. Insufficient justification will result in a loss of marks.

1. [6 marks] Determine whether the following pairs of graphs are isomorphic.
   If so, give an isomorphism. If not, give a reason.

(a)



G

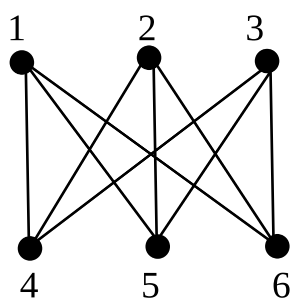


H

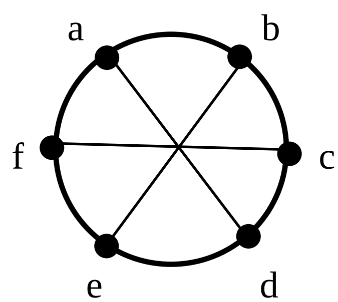
G and H are not isomorphic.
Here are some reasons (one is sufficient and there are others):

- G has 3 vertices of degree 2 whereas H has only 2 vertices of degree 2.
- The degree sequences of the two graphs are for G: (4, 4, 4, 2, 2, 2) and for H: (4, 4, 3, 3, 2, 2).

(b)



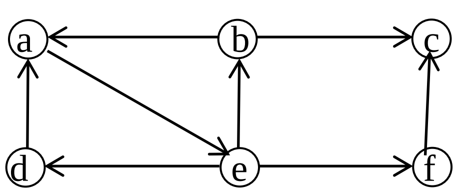
J



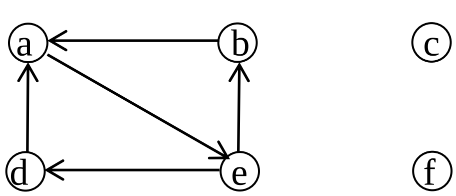
K

J and K are isomorphic.
Here is an isomorphism (there are others):
h: V(J) -> V(K) defined by h(1) = a, h(2) = c, h(3) = e, h(4) = b, h(5) = d, h(6) = f.

2. [3 marks] Draw the strongly connected components of the directed graph below.

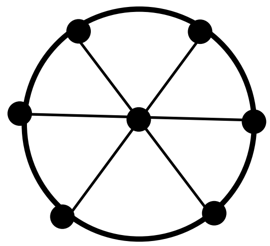


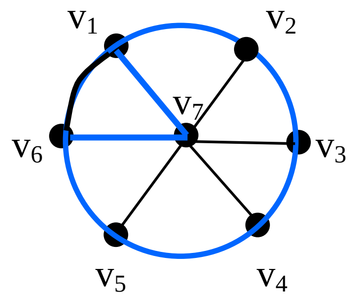
This directed graph has three strongly connected components shown below:



3. [7 marks] Recall that the wheel W_n , $n \geq 3$, consists of the cycle C_n together with a vertex joined to each vertex of C_n.

(a) Draw W_6 twice.
If W_6 has an Euler circuit, mark it on your first picture.
If W_6 has a Hamilton cycle, mark it on your second picture.





W_6 has no Euler circuit because it has odd-degree vertices

W_6 has a Hamilton cycle as shown
$v_1 , v_2 , v_3 , v_4 , v_5 , v_6 , v_7 , v_1$ (there are many)

(b) For what values of n does W_n have an Euler circuit? Explain briefly.

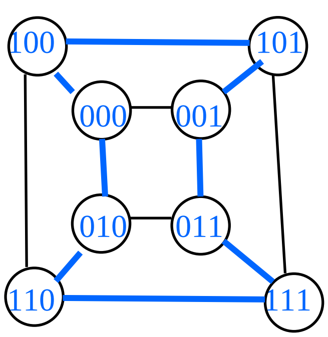
No values of n.
In order for a graph to have an Euler circuit, all vertices must have even degree.
In W_n , where the vertex of degree n is v_{n+1}, all other vertices have degree 3, so there is no Euler circuit.

(c) For what values of n does W_6 have an Hamilton cycle? Explain briefly.

All values of n.
In W_n , where the vertex of degree n is v_{n+1}, the following is a Hamilton cycle (there are others):
$v_1 , v_2 , \ldots , v_n , v_{n+1} , v_1$

4. [4 marks]

(a) Draw the 3-dimensional hypercube Q_3 with its vertices labelled with its vertices labelled appropriately with 01- strings of length 3.
(b) Mark a Hamilton cycle in Q_3 . (One is marked in blue – there are many)
(c) Give a Gray code using your answers to (a) and (b).



Corresponding Gray code: 000 – 100 – 101 – 001 – 011 – 111 – 110 – 010 - 000