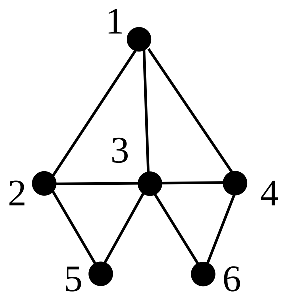
MA238 Quiz 2 11:30 am Tutorial February 12, 2021 20 marks

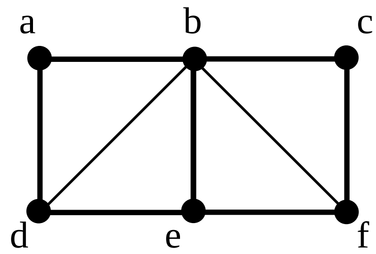
No aids allowed. Insufficient justification will result in a loss of marks.

1. [6 marks] Determine whether the following pairs of graphs are isomorphic. If so, give an isomorphism. If not, give a reason.

(a)



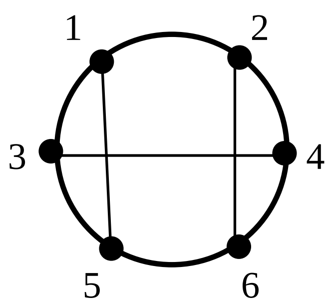
G



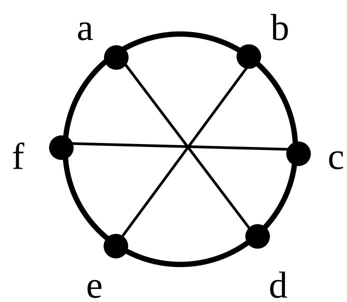
H

G and H are isomorphic.
Here is an isomorphism (there are others):
h: V(G) -> V(H) defined by h(1) = e, h(2) = f, h(3) = b, h(4) = c, h(5) = a, h(6) = c.

(b)



J



K

J and K are not isomorphic.
Here are some reasons (one is sufficient and there are others):

- J has a 3-cycle but K does not.
- K is bipartite but J is not since J contains a 3-cycle

2. [2 marks] Consider the graph called G in problem 1.

(a) Does G have a Hamilton cycle? If so, what is the Hamilton cycle? If not, give a reason.
G contains a Hamilton cycle: 1, 2, 5, 3, 6, 4, 1

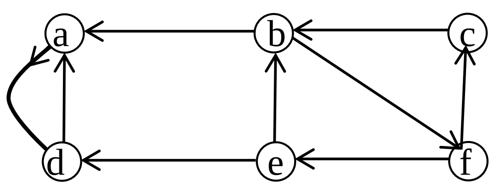
(b) Does Dirac's Theorem tell us that G has a hamilton cycle?
No. In order for Dirac's Theorem to apply, every vertex of G would have to have degree at least n/2, where n is the number of vertices.
Here, the number of vertices is 6, so every vertex would need degree at least 3, but vertices 5 and 6 have degree 2.

3. [6 marks]
(a) What is the degree of each vertex in K_n ? n-1
(b) For what values of n does K_n have an Euler circuit? Explain briefly.
K_n has an Euler circuit for odd values of n.
The Euler circuit theorem says that a graph has an Euler circuit if and only if it is connected and every vertex has even degree.
K_n is always connected and all vertices have even degree exactly when n is odd.

(c) For what values of n does K_n have an Euler trail but no Euler circuit? Explain briefly.
n = 2.
A graph has an Euler trail but no Euler circuit if and only if it is connected and has exactly two vertices of odd degree.
K_n is always connected and has exactly two vertices of odd degree precisely when it has 2 vertices.

4. [3 marks] For a simple graph with adjacency matrix A, state how to use matrix multiplication to determine the number of walks of length 12 from vertex c to vertex a.
The number of walks of length 12 from vertex c to vertex a is the (c,a) entry in A^{12}.
(Note that the (c,a) entry is the entry in the row for vertex c and column for vertex a.
A^{12} is calculated by multiplying together 12 copies of A.)

5. [3 marks] Draw the strongly connected components of the directed graph below.



This directed graph has 2 strongly connected components: