Material properties of the element

|  |  |
| --- | --- |
| E | Young's modulus (force/length2) for membrane stiffness of isotropic  Material |
| **n** | Poisson's ratio |
| **r** | Material mass density |
| h | Thickness of the plate |

A drawing of a square with arrows

AI-generated content may be incorrect.

Assumption of Kirchoff’s plate theory

1. The mid plane of the plate is a neutral plane meaning, the mid plane of the plate remains free of in-plane stress/ strain.
2. Plane section remain plane. The line elements lying perpendicular to the middle surface of the plate remain perpendicular to the middle surface during deformation.
3. Line elements lying perpendicular to the mid-surface do not change length during deformation, therefor vertical strain is zero.

Blue lines on a black background

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Based on these assumptions, the displacement in x, y and z directions are

The in-plane strain – displacement relations are

The transverse shear strains are assumed to be constant through the thickness of the plate.

Stresses and the curvatures/ Twist in a linear elastic plate, from Hooke’s law

The inplane stress – strain relations are

The transverse shear stress – strain relations are

Where α is the shear correction factor (constant α = 12/ π2 or 6/5)

For [D] – elasticity matrix.

Stress strain relation is given by,

## Finite Element formulation of the Mindlin plate

Mindlin plate theory accounts for the bending deformation and for transverse shear deformation. Hence, the strain energy U in the plate can be written as

Substituting,

The strain energy U becomes,

Integrating through thickness t in z direction yields

Assuming that w, βx and βy within the i-th plate element are interpolated from the elemental nodal degrees of freedoms [d]i by the following expression.

Where,

Below is the B matrix formulation for elements having n Nodes.

Where K is the stiffness matrix of the element given as follows,

## 3 Node Triangle Element CTRIA3

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Shape functions of 3-Noded triangle element

Kronecker delta property

The shape functions of triangle element at their natural coordinate system ξ & η

Note: Multiplied by -12, (-1 [d][N]) [Ds](-1[d][N])

A point (x,y) inside the triangle with known exterior points (x1,y1) (x2,y2) & (x3,y3), through Local area (natural) co-ordinates is given by

Substituting the shape functions,

Differentiating w.r.t element co-ordinates

Similar to vertex co-ordinates, displacements inside the triangle with known nodal displacements (u1,v1)(u2,v2) & (u3,v3) is given by

Differentiating w.r.t element co-ordinates