Material properties of the element

|  |  |
| --- | --- |
| E | Young's modulus (force/length2) for membrane stiffness of isotropic  Material |
| **n** | Poisson's ratio |
| **r** | Material mass density |

A drawing of a square with arrows

AI-generated content may be incorrect.

Assumption of Kirchoff’s plate theory

1. The mid plane of the plate is a neutral plane meaning, the mid plane of the plate remains free of in-plane stress/ strain.
2. Plane section remain plane. The line elements lying perpendicular to the middle surface of the plate remain perpendicular to the middle surface during deformation.
3. Line elements lying perpendicular to the mid-surface do not change length during deformation, therefor vertical strain is zero.

Based on these assumptions, the displacement in x and y directions are

The in-plane strain – displacement relations are

The transverse shear strains are assumed to be constant through the thickness of the plate.

Stresses and the curvatures/ Twist in a linear elastic plate, from Hooke’s law

The inplane stress – strain relations are

The transverse shear stress – strain relations are

Where α is the shear correction factor (constant α = 1.2)

For [D] – elasticity matrix.

Stress strain relation is given by,

## Finite Element formulation of the Mindlin plate

Mindlin plate theory accounts for the bending deformation and for transverse shear deformation. Hence, the strain energy U in the plate can be written as

Integrating through thickness h in z direction yields

Assuming that w, βx and βy within the i-th plate element are interpolated from the elemental nodal degrees of freedoms [d]i by the following expression.

Where,

Where K is the stiffness matrix of the element given as follows,