**Part 1**

*Method*

To create a process that will approximate a sine wave with minimal SFDR at an arbitrary, I first consider the phase values which are input to the sinusoidal function over the first 50 periods of the signal. This value is expressed by , where is the fractional frequency. If we let so that the phase value begins at 0, we can iterate over the rest of the phases and find the point at which the phase best approximates 0 again. This will tell us the point at which the signal can be ‘re-read’ with very little distortion because the transition from ‘0’ back to the beginning of the digital readout (technically, back to the second sample, after zero) will be the most seamless. I make the assumption that a minimized distortion of phase over will result in a minimal distortion of SFDR.

*Time Domain Results*

Figure 1 shows the result using this method to find the best index at which to repeat for the case of fractional frequency

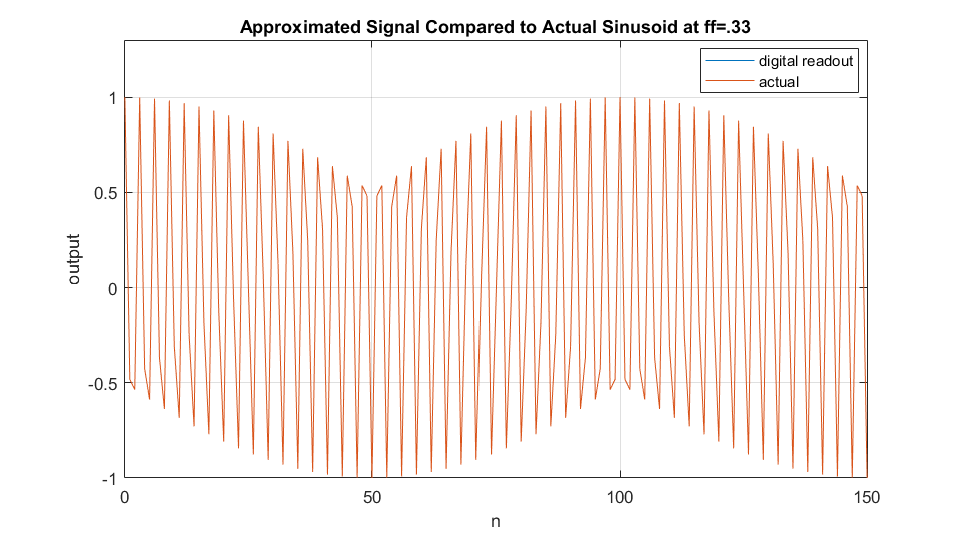


Figure 1 - Time-domain readout compared to the sinusoid it approximates, ff=.33

From Figure 1, we see that the algorithm has successfully selected a repetition point which succeeds in imitating a true sinusoidal signal at least for a short duration of 150 samples. Figure 2 shows the error of the signal compared to a true sinusoid for the first 500 periods of that sinusoid.

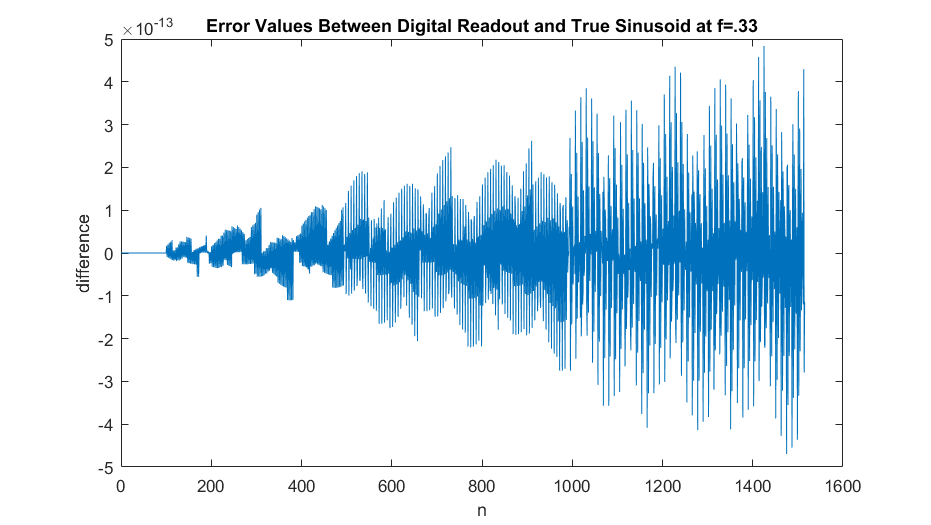


Figure 2 - Digital readout error

Figure 2 shows that the error of the digital readout system is quite small in magnitude, although it accumulates over time. This makes sense considering that the phase of the generated signal gradually drifts from that of the target signal every time the readout is repeated.

*Frequency Domain Results*

Figure 3 presents the resulting signal in the time domain using both Hamming and Blackman windows.

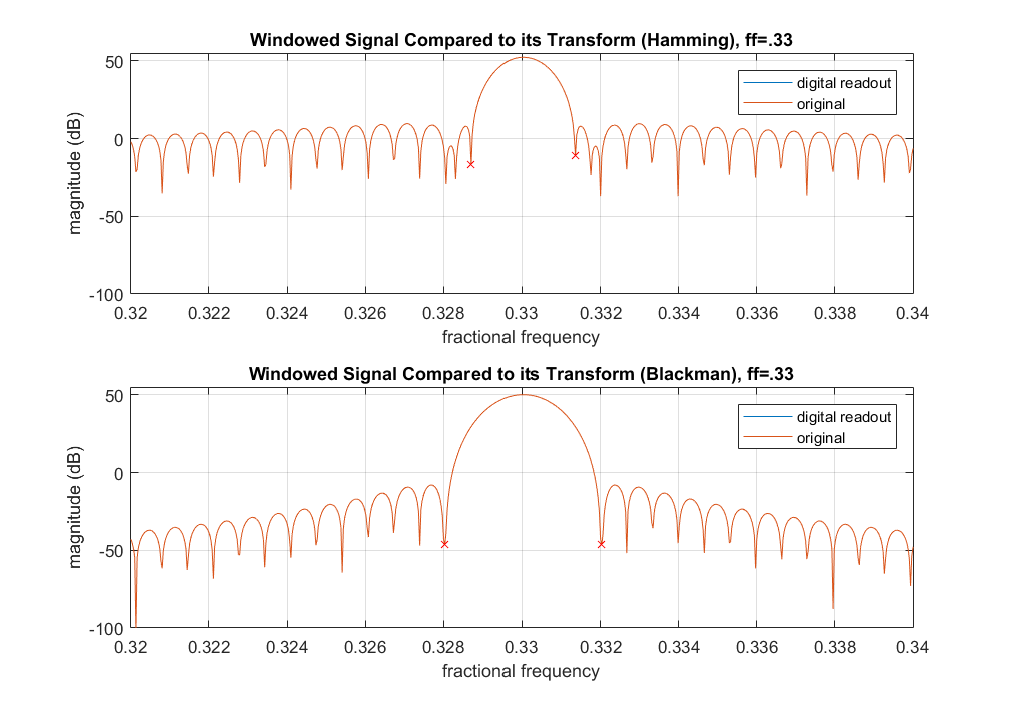


Figure 3 - Comparing the frequency domains using different windows

First, we can make some observations about the window shape themselves. We know that the plots shown in Figure 3 are just scaled and shifted versions of the windows represented in the frequency-domain. This is because we multiplied the windows by a sinusoidal signal and then took the DFT of the result. This operation corresponds to convolving the transform of the window against the transform of a sinusoid which is a scaled and shifted pair of delta functions. The result of this convolution is a pair of scaled and shifted windows. Figure 3 focuses on the frequencies around , so the other transform of the window at is not visible. We can see that the Hamming window has a very narrow main lobe compared to the Blackman window, while the Blackman window drops off much faster and farther than the Hamming window does once outside of the main lobe.

*Determining SFDR*

It is worth noting that, for , the readout signal so closely approximates the original that the effects of the windows are more prominent than the signal’s distortion. If we were to find the SFDR of this signal naively by comparing the main lobe against the sidelobes, we would be essentially measuring a property that is inherent to the windowing function and not the signal of interest.

Instead, we can first calculate the distortion power that is a result of the digital readout approximation by taking the magnitude of the differences between the window frequency representation and the signal frequency representation.

The two red ‘X’ marks on each subplot of Figure 3 denote the boundaries of the main lobe as determined by locate\_mainlobelims, a custom function I wrote (see accompanying files). This utility will assist in determining the SFDR by enabling me to make a distinction between the main lobe and the sidelobes.

My method for determining the SFDR is simply to take the maximum value within the main lobe and compare it against the maximum value among all frequencies of the distortion signal. The distortion signal calculated using both window types is shown below in Figure 4.

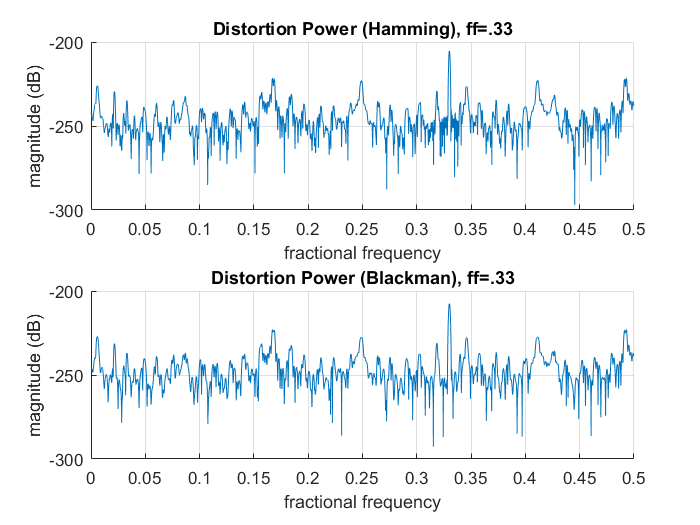


Figure - The frequency content of the distortion signal

Interestingly, the fractional frequency of the sinusoid is frequency at which the distortion power is highest.

The output of my code provides the calculated SFDR of the generated signal for both the Hamming and Blackman windows.

SFDR\_HAMMING =

257.5824

SFDR\_BLACKMAN =

257.5978

The consistent results of this calculation appear to show that this method of calculating SFDR is impacted very little by the windowing method used. This would suggest that the calculated SFDR of roughly 257.6 dB is inherent to the signal.

**Part 2**