Understanding Drivers in a Car-lite Singapore

40.316 Game Theory Final Presentation

Group 5

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Content

- Introduction
- Problem Formulation
- Collection of Data
- Analysis
- Conclusion

Introduction

Motivation

- Singapore aims to:

Reduce traffic congestion

Reduce carbon emissions

- Increased uptake in cycling

Alternative transport mode

Increased bicycle lanes

Objectives

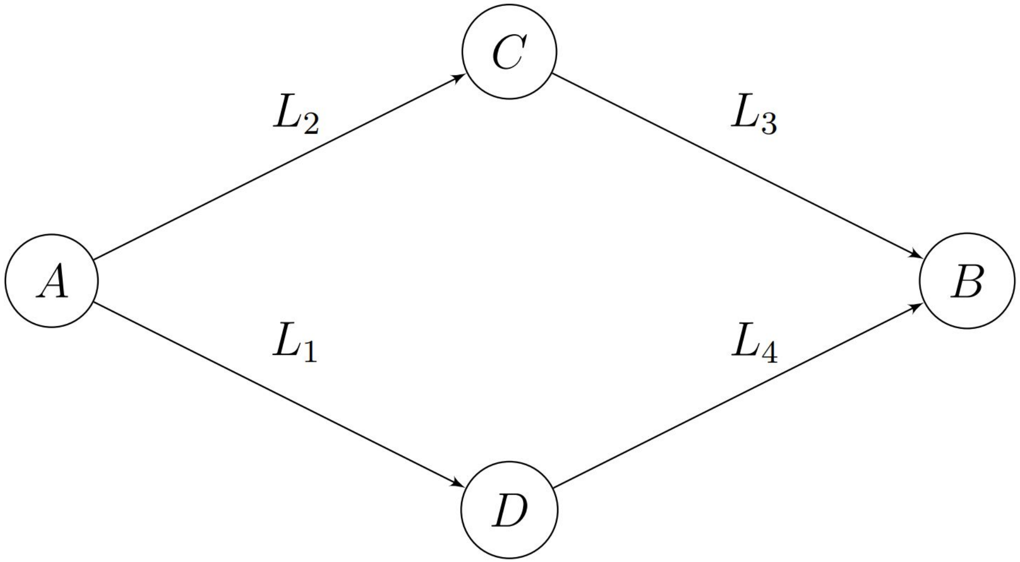
3 main objectives:

1. Develop a traffic network model that accounts for:
    a. Proportion of users travelling a certain path
    b. Average number of lanes
    c. Current traffic congestion
        i. Peak vs Non-peak hours
2. Analyze if Braess' Paradox exists
3. Examine if cycling and bicycle-lanes, can help to ease traffic conditions

Problem Formulation

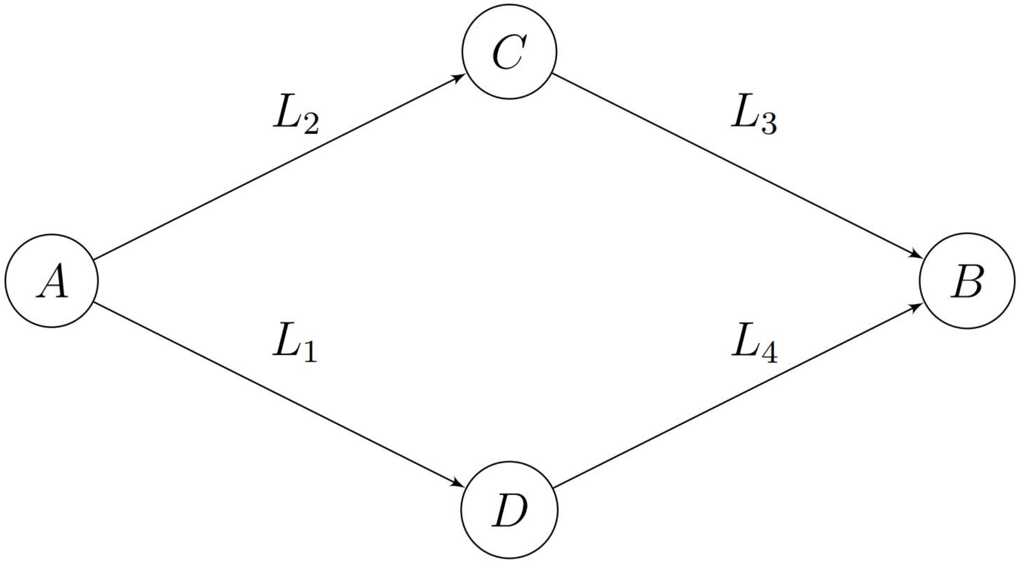
Directed Graph Representation

- Edges: Roads with latency function L_i
- Nodes: Checkpoints along the roads



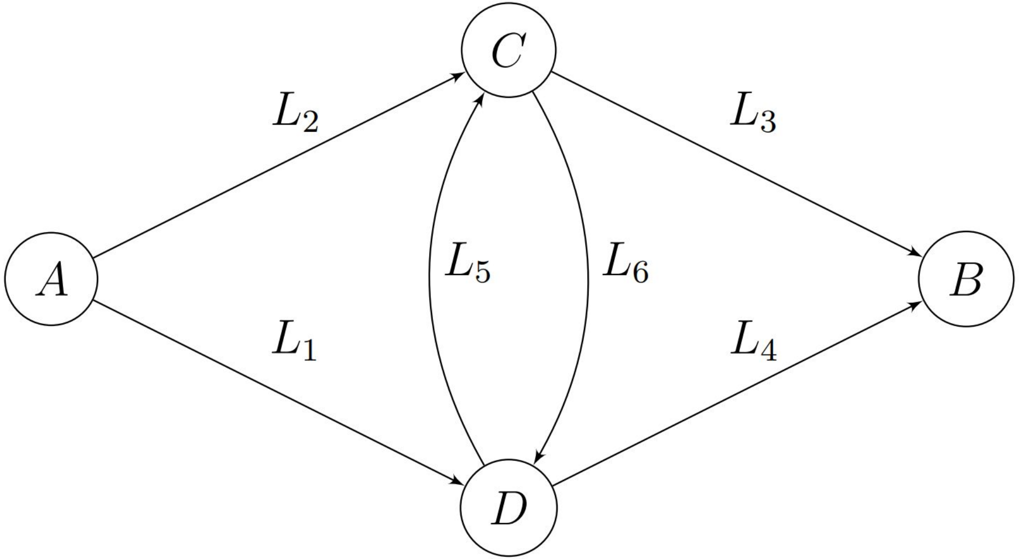
Our Networks

- Basic Traffic Network



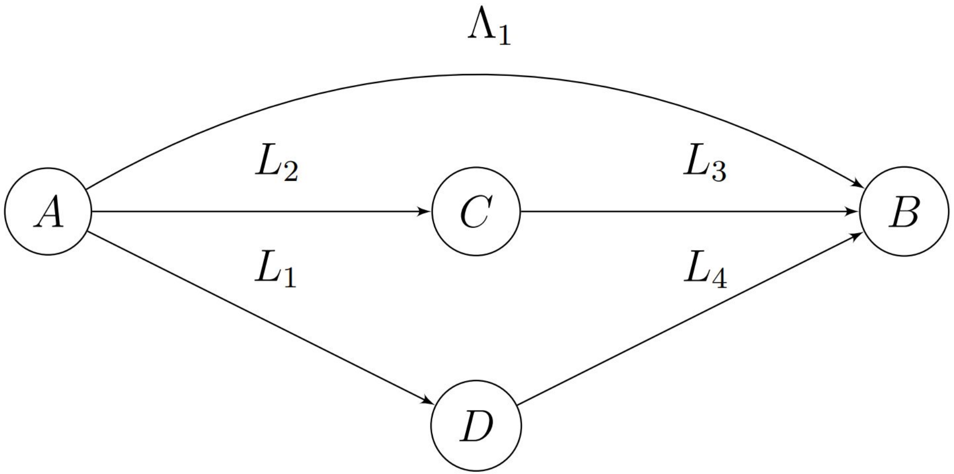
Our Networks

- Basic Traffic Network
- Augmented Traffic Network



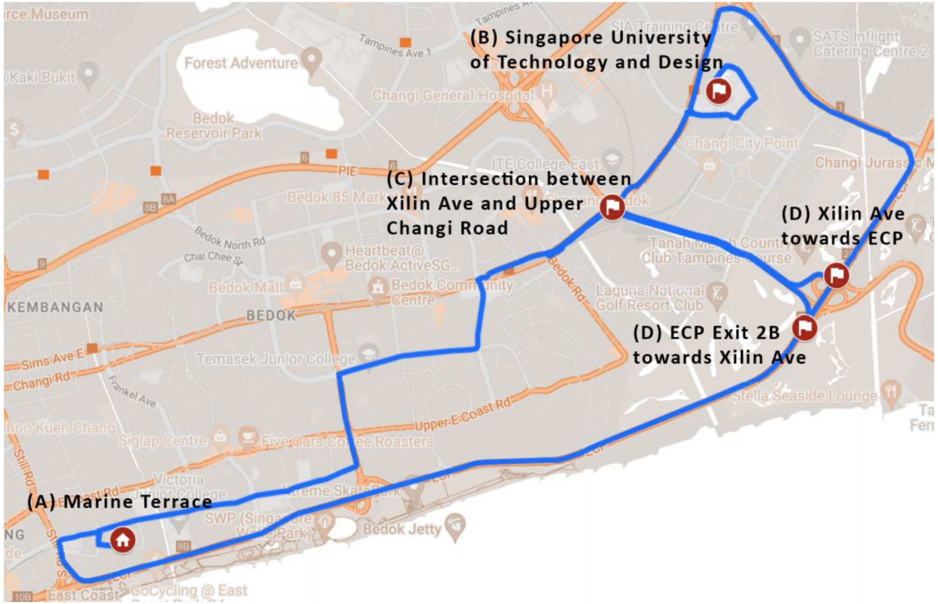
Our Networks

- Basic Traffic Network
- Augmented Traffic Network
- Modified Traffic Network (with cycling)



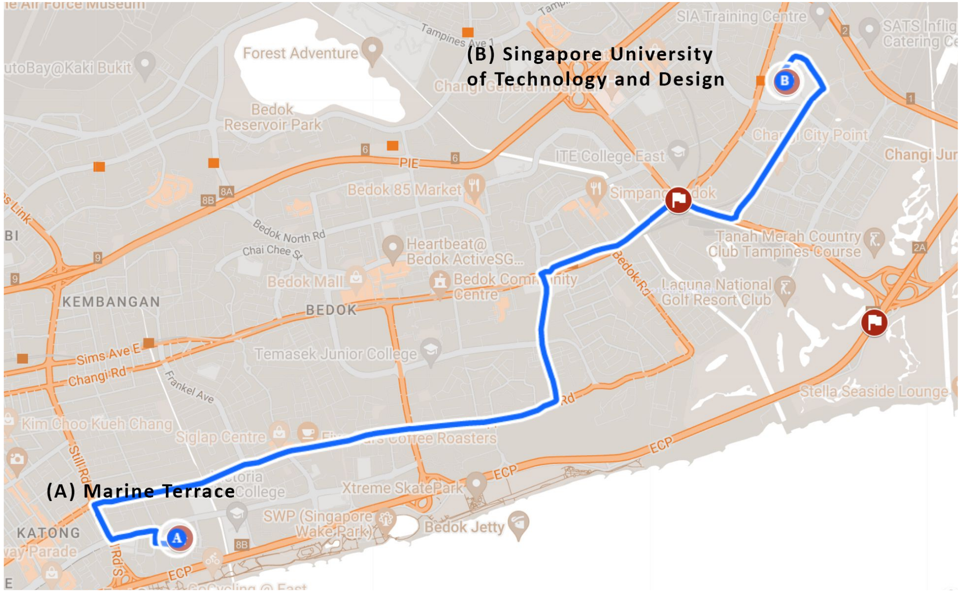
Traffic Networks on Google Maps

- Choice of two arbitrary start & end location:
  - Marine Terrace
  - SUTD



Traffic Networks on Google Maps

- Cycling route



Latency Function: Variables

Based off our 1st objective:

$$L_i(x_i, \Gamma_i, \Theta_i)$$

measures the delay in the i-th edge where:

1. Proportion of users travelling, x_i
2. Average number of lanes per metre, Γ_i
3. Current traffic congestion per metre, Θ_i

1) Proportion of users x_i

- Assumption: Total proportion of users of the network adds to 1

$$\sum_{i=1}^{n} x_i = 1$$

$$0 \leq x_i \leq 1$$

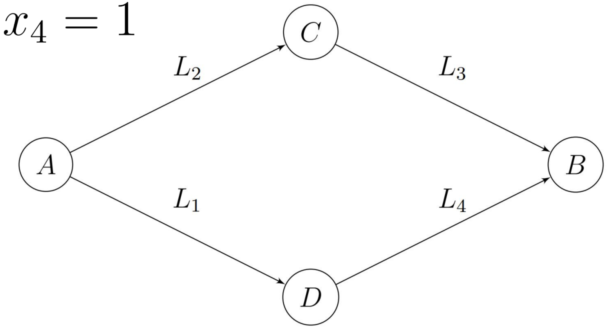
1) Proportion of users x_i

- Assumption: Total proportion of users of the network adds to 1

$$\sum_{i=1}^{n} x_i = 1$$

$$0 \leq x_i \leq 1$$

- E.g. $x_1 + x_2 = 1, x_3 + x_4 = 1$



2) Average Number of Lanes per metre Γ_i

- Y_i : total path length, j : number of sections
- γ_{ij} : number of lanes per section, y_{ij}: road section length

$$\Gamma_i = \frac{1}{Y_i} \sum_{j=1}^{J} (\gamma_{ij} \cdot y_{ij}) \text{ where } Y_i = \sum_{j=1}^{J} (y_{ij})$$

2) Average Number of Lanes per metre Γ_i

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Number of lanes: 3 2 4

Distance, m: 200 150 100

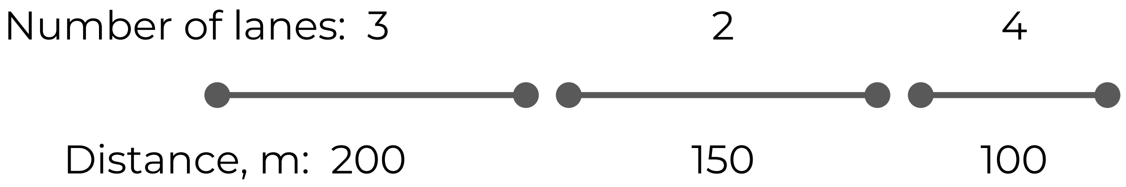
2) Average Number of Lanes per metre Γ_i

- Y_i : total path length, j : number of sections
- γ_{ij} : number of lanes per section, y_{ij}: road section length

$$\Gamma_i = \frac{1}{Y_i} \sum_{j=1}^{J} (\gamma_{ij} \cdot y_{ij}) \text{ where } Y_i = \sum_{j=1}^{J} (y_{ij})$$

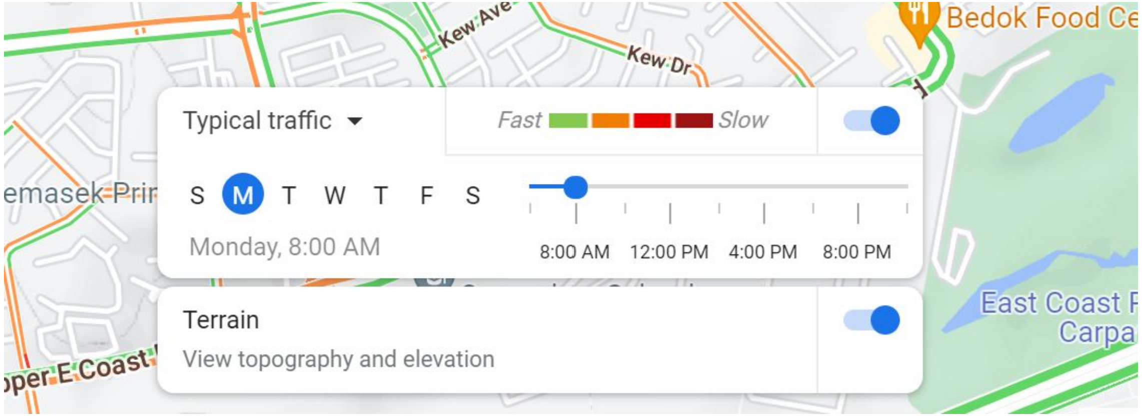
- E.g. Avg number of lanes per m

  = (3 x 200 + 2 x 150 + 4 x 100)/(200 + 150 + 100) = 2.89



Traffic State

- θ_{ik} : traffic state per section
- Based on Google Maps, 4 states assigned a discrete value between 0 & 1



- {Green, Orange, Red, Brown} = {0.125, 0.375, 0.625, 0.875}

3) Traffic Congestion per metre Θ_i

- Y_i : total path length, j : number of sections
- θ_{ik} : traffic state per section, y_{ik} : road section length

$$\Theta_i = \frac{1}{Y_i} \sum_{k=1}^{K} (\theta_{ik} \cdot y_{ik}) \text{ where } Y_i = \sum_{k=1}^{K} (y_{ik})$$

3) Traffic Congestion per metre Θ_i

- Y_i : total path length, j : number of sections
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Traffic State: 0.125 0.375 0.125

Distance, m: 200 150 100

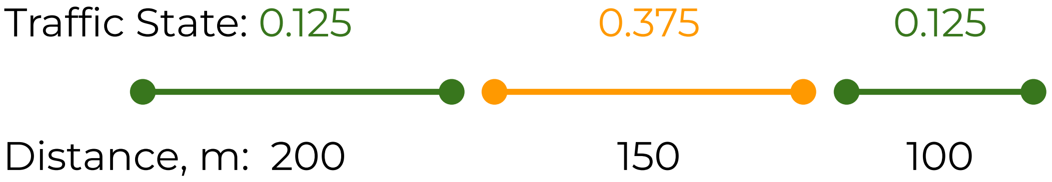
3) Traffic Congestion per metre Θ_i

- Y_i : total path length, j : number of sections
- θ_{ik} : traffic state per section, y_{ik} : road section length

$$\Theta_i = \frac{1}{Y_i}\sum_{k=1}^{K}(\theta_{ik} \cdot y_{ik}) \text{ where } Y_i = \sum_{k=1}^{K}(y_{ik})$$

- E.g. Traffic congestion per metre

  = (0.125 x 200 + 0.375 x 150 + 0.125 x 100)/(200 + 150 + 100) = 0.208



Motivation: Bureau of Public Roads (BPR)

- BPR Function: Positive relationship between traffic volume and delay experienced by road users

$$t = t_0\left(1 + \beta\left(\frac{v_a}{c_a}\right)^{\alpha}\right)$$

$t_0 =$ travel time given 0 traffic volume, minimum time to travel
$v_a =$ vehicle flow rate of the road (passenger car unit (pcu)/hour)
$c_a =$ capacity of the road (pcu/h)
$\alpha, \beta =$ coefficients

Latency Function: Modeling

$$L_i(x_i, \Gamma_i, \Theta_i) = t_i(1 + x_i \cdot \sigma(\frac{1}{\Gamma_i \cdot \Theta_i}))$$

measures the delay in the i-th edge where:

1. Proportion of users travelling, x_i
2. Average number of lanes per metre, Γ_i
3. Current traffic congestion pre metre, Θ_i
4. Travel time given 0 traffic (minimum travel time), t_i
5. Sigmoid function, σ

Latency Function: Interpretation

- Induced demand relationship:
    - Additional travels is induced by lower cost resulting from capacity expansion [2]

$$L_i(x_i, \Gamma_i, \Theta_i) = t_i(1 + x_i \cdot \sigma(\frac{1}{\Gamma_i \cdot \Theta_i}))$$

Latency Function: Interpretation

- Induced demand relationship:
  - Additional travels is induced by lower cost resulting from capacity expansion [2]

$$L_i(x_i, \Gamma_i, \Theta_i) = t_i(1 + x_i \cdot \sigma(\frac{1}{\Gamma_i \cdot \Theta_i}))$$

Average number of congested lanes per square metre

Inverse to account for the induced demand relationship

Latency Function: Interpretation

$$L_i(x_i, \Gamma_i, \Theta_i) = t_i(1 + x_i \cdot \sigma(\frac{1}{\Gamma_i \cdot \Theta_i}))$$

Sigmoid activation function

To squeeze the induced travel effect to between 0 and 1

Latency Function: Interpretation

$$L_i(x_i, \Gamma_i, \Theta_i) = t_i(1 + \underbrace{x_i \cdot \sigma(\frac{1}{\Gamma_i \cdot \Theta_i})}_{})$$

Proportion of users affected

by induced travel effect

Latency Function: Interpretation

$$L_i(x_i, \Gamma_i, \Theta_i) = t_i(1 + x_i \cdot \sigma(\frac{1}{\Gamma_i \cdot \Theta_i}))$$

Extra delay (min) as a fraction of the minimum time to travel

Latency Function: Interpretation

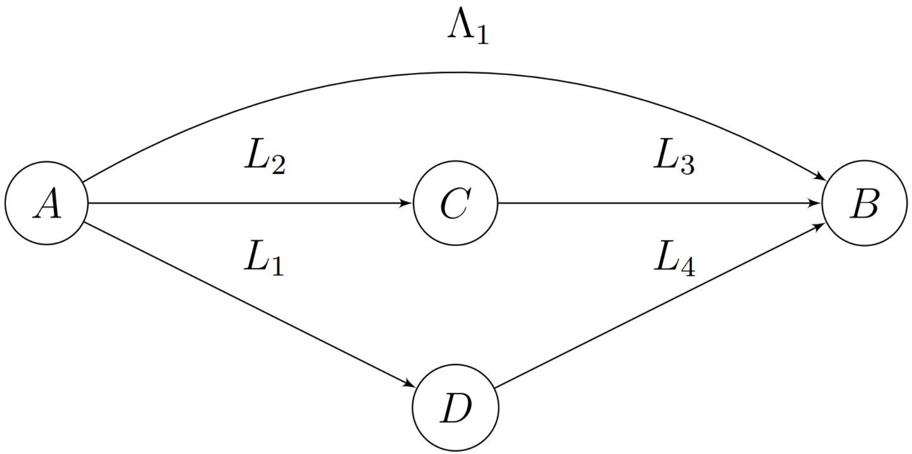
$$L_i(x_i, \Gamma_i, \Theta_i) = \overline{t_i(1} + \underline{x_i \cdot \sigma(\frac{1}{\Gamma_i \cdot \Theta_i}}))$$

Summation of the **minimum travel time** and the **extra delay**

Latency Function: Cycling

- Assumed to have no congestion
- Just the time taken to cycle from start to end location

$$\Lambda_1 = t_{\text{cycle}}$$



Peak vs Non-peak Hours

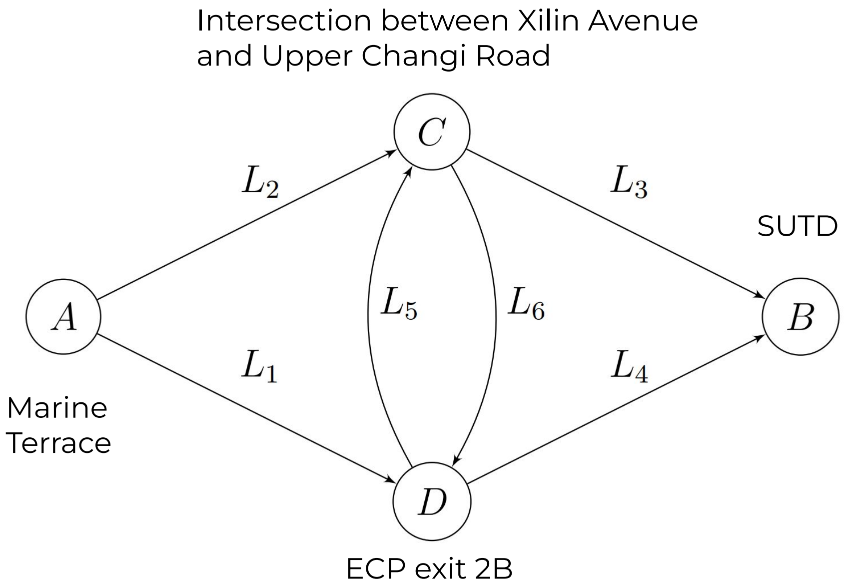
- According to LTA:
  - AM Peak is from 0800hrs to 0900hrs
  - PM Peak is from 1800hrs to 1900hrs
- To keep our analysis shorter and more concise
  - 0830hrs (AM peak hour)
  - 1330hrs (PM non-peak hour)
  - More common timings to head to school for classes

Collection of Data

Travel Times t_i

- Using Google Maps, at 3.00am where the traffic congestion is very low
- Assumed to be the min time to travel

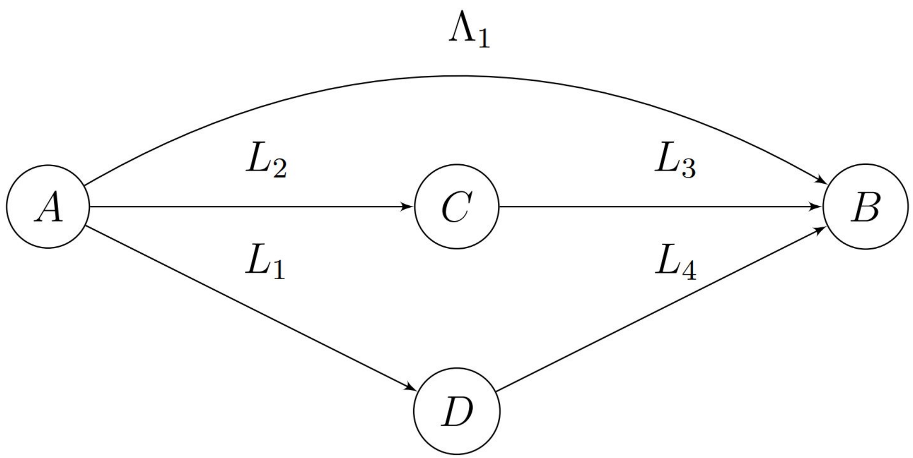
	t_i **/min**
From A to D	7
From A to C	10
From C to B	5
From D to B	7
From C to D	7
From D to C	3



Travel Times (Cycling)

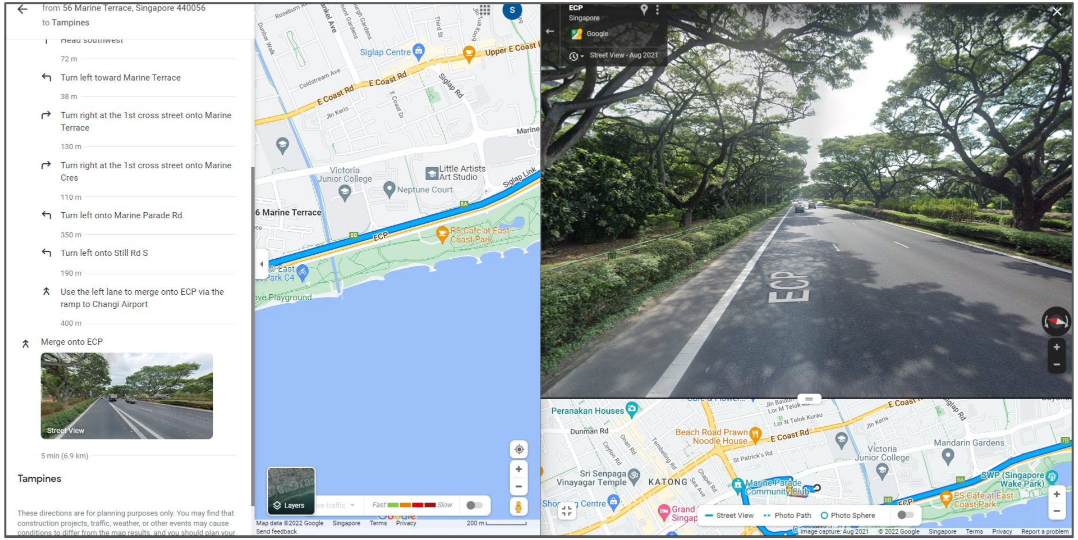
- Using Google Maps, at 3.00am where the traffic congestion is very low
- Assumed to be the min time to travel

	Λ_1/min
From A to B	34



Average number of lanes per metre Γ_i

- Using Google Maps, manually checked the number of lanes using satellite imagery and distance measure



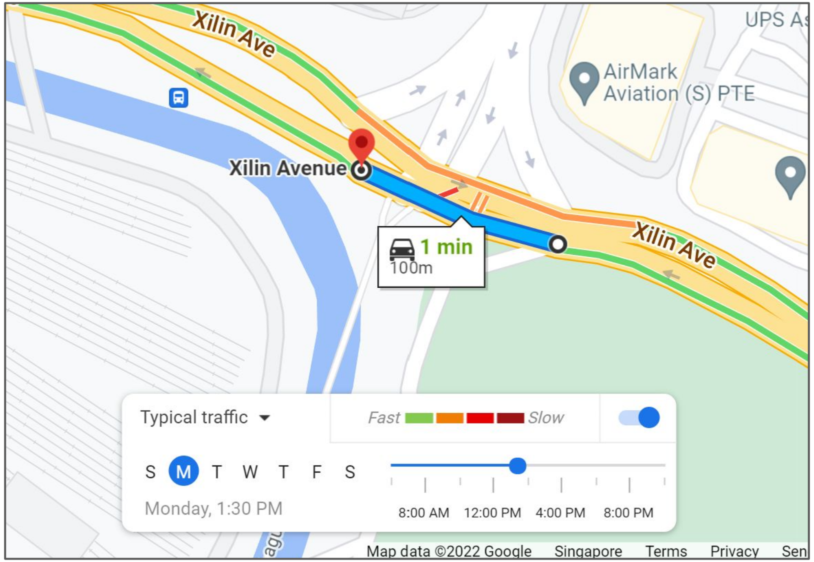
Average number of lanes per metre Γ_i

- Using Google Maps, manually checked the number of lanes using satellite imagery and distance measure

	Avg number of lanes per m
From A to D	2.85
From A to C	2.63
From C to B	2.43
From D to B	2.57
From C to D / D to C	3

Average Traffic Congestion per metre Θ_i

- Using Google Maps, manually checked the typical traffic states at 0830hrs & 1330hrs and using the distance measure



Average Traffic Congestion per metre Θ_i

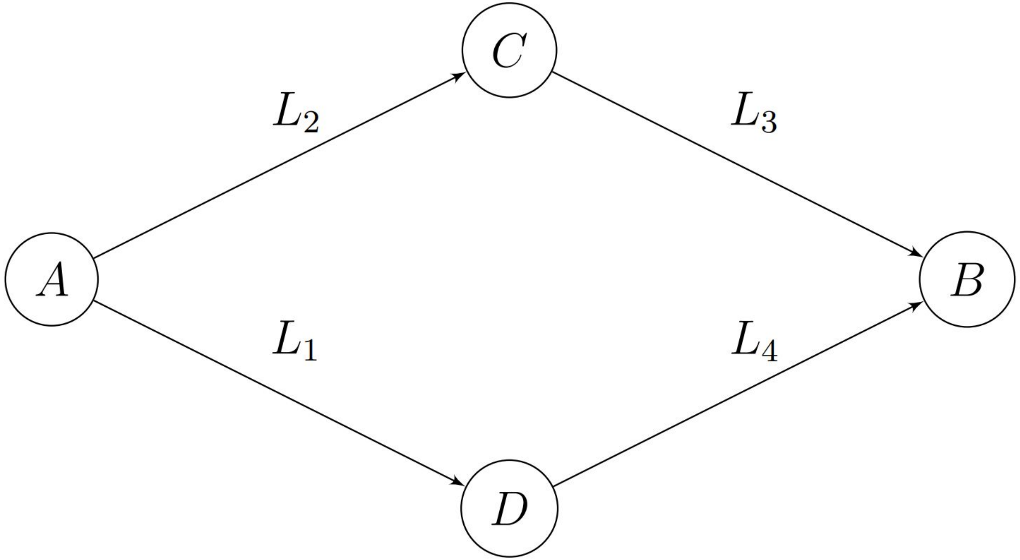
- Using Google Maps, manually checked the typical traffic states at 0830hrs & 1330hrs and using the distance measure

	AM peak	**PM non-peak**
From A to D	0.160	0.153
From A to C	0.308	0.249
From C to B	0.241	0.203
From D to B	0.176	0.137
From D to C	0.216	0.142
From C to D	0.222	0.136

Analysis

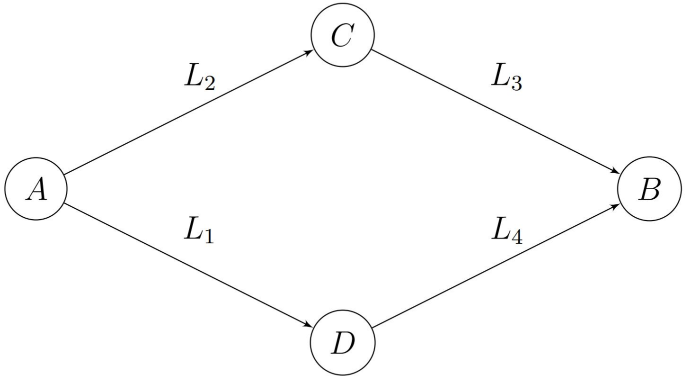
Nash Equilibrium

- Recall our Basic Traffic Network:



Nash Equilibrium

- Recall our Basic Traffic Network:
- For AM peak, 0830hrs:



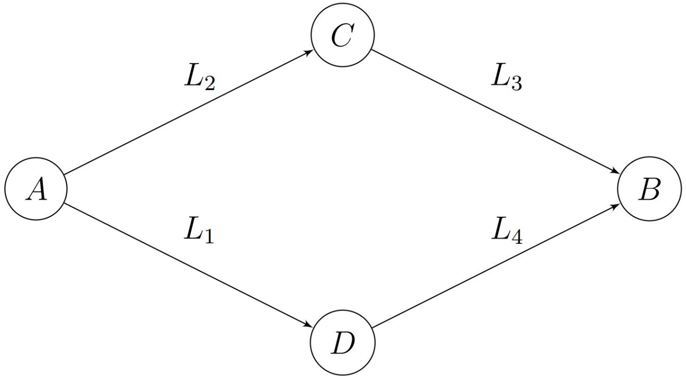
$$\text{Total Delay top path} = x \cdot (L_2 + L_3)$$

$$\text{Total Delay bottom path} = x \cdot (L_1 + L_4)$$

$$\text{where } x = 1 \text{ is all users travelling that direction}$$

Nash Equilibrium

- Recall our Basic Traffic Network:
- For AM peak, 0830hrs:



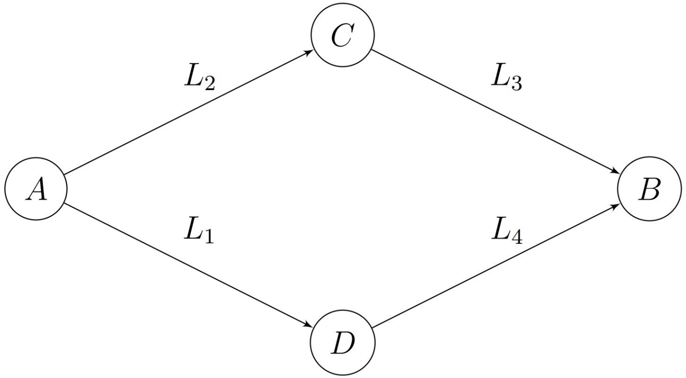
$$
\begin{aligned}
\text{Total Delay top path} &= x \cdot (L_2 + L_3) \\
&= 10(1+\sigma(1/0.81004)) + 5(1+\sigma(1/0.58563)) = 29.98
\end{aligned}
$$

$$
\begin{aligned}
\text{Total Delay bottom path} &= x \cdot (L_1 + L_4) \\
&= 7(1+\sigma(1/0.456)) + 5(1+\sigma(1/0.45232)) = 26.61
\end{aligned}
$$

where $x = 1$ is all users travelling that direction

Nash Equilibrium

- Recall our Basic Traffic Network:
- For AM peak, 0830hrs:



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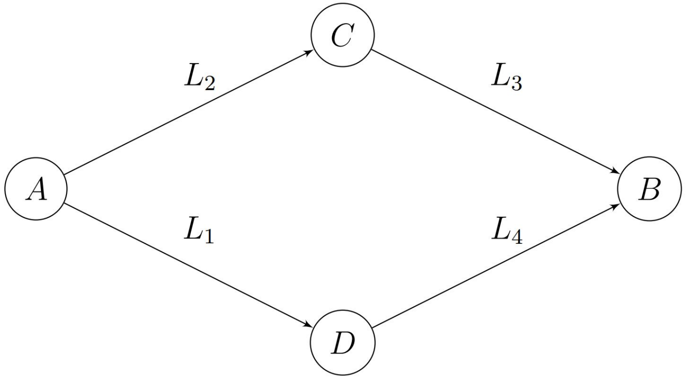
Nash Equilibrium



- Recall our Basic Traffic Network:
- For PM non-peak, 1330hrs:

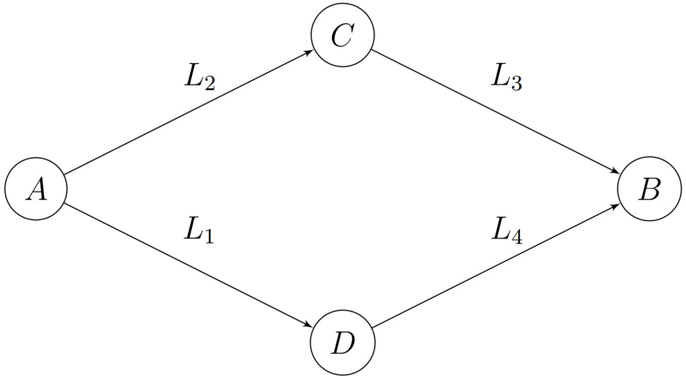
$$\text{Total Delay top path} = x \cdot (L_2 + L_3)$$

$$\text{Total Delay bottom path} = x \cdot (L_1 + L_4)$$

$$\text{where } x = 1 \text{ is all users travelling that direction}$$

Nash Equilibrium

- Recall our Basic Traffic Network:
- For PM non-peak, 1330hrs:



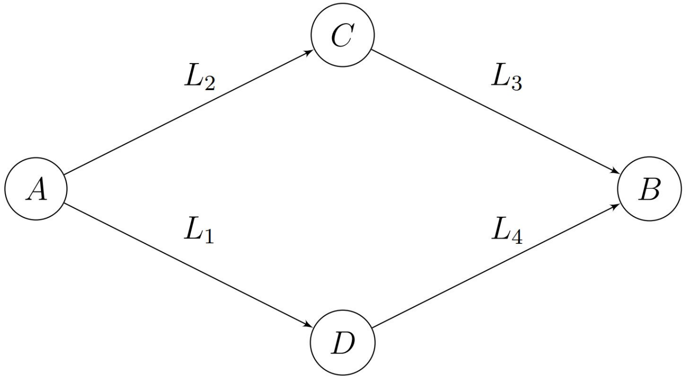
$$\text{Total Delay top path} = x \cdot (L_2 + L_3)$$
$$= 10(1+\sigma(1/0.65487)) + 5(1+\sigma(1/0.49329)) = 27.63$$

$$\text{Total Delay bottom path} = x \cdot (L_1 + L_4)$$
$$= 7(1+\sigma(1/0.43605)) + 5(1+\sigma(1/0.35209)) = 26.97$$

$$\text{where } x = 1 \text{ is all users travelling that direction}$$

Nash Equilibrium

- Recall our Basic Traffic Network:
- For PM non-peak, 1330hrs:



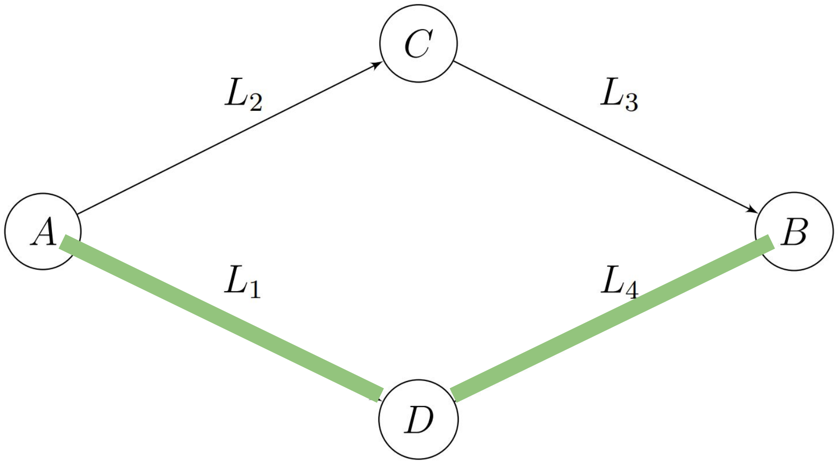
$$\begin{aligned}\text{Total Delay top path} &= x \cdot (L_2 + L_3)\\ &= 10(1+\sigma(1/0.65487)) + 5(1+\sigma(1/0.49329)) = 27.63\end{aligned}$$

$$\begin{aligned}\text{Total Delay bottom path} &= x \cdot (L_1 + L_4)\\ &= 7(1+\sigma(1/0.43605)) + 5(1+\sigma(1/0.35209)) = 26.97\end{aligned}$$

where $x = 1$ is all users travelling that direction

Nash Equilibrium

- Recall our Basic Traffic Network:
- **Bottom path chosen regardless of timing**



Social Welfare

- To minimize total delay for both AM peak & PM non-peak:
- The optimization problem is

$$\min \sum_{i=1}^{4} (L_i x_i) \text{ where}$$

$$L_i = t_i(1 + x_i \cdot \sigma(\frac{1}{\Gamma_i \cdot \Theta_i})) \forall i = 1, 2, 3, 4$$

$$x_1 + x_2 = 1$$

$$x_2 = x_3$$

$$x_1 = x_4$$

$$0 \leq x_i \leq 1$$

Social Welfare

- Solving the optimization using **cvxpy** in Python:
    - AM peak 0830hrs:

Proportion of users travelling top path	Proportion of users travelling bottom path	Total delay/ min
0.492	0.508	20.645

- PM non-peak, 1330hrs:

Proportion of users travelling top path	Proportion of users travelling bottom path	Total delay/ min
0.487	0.513	20.897

Price of Anarchy (PoA)

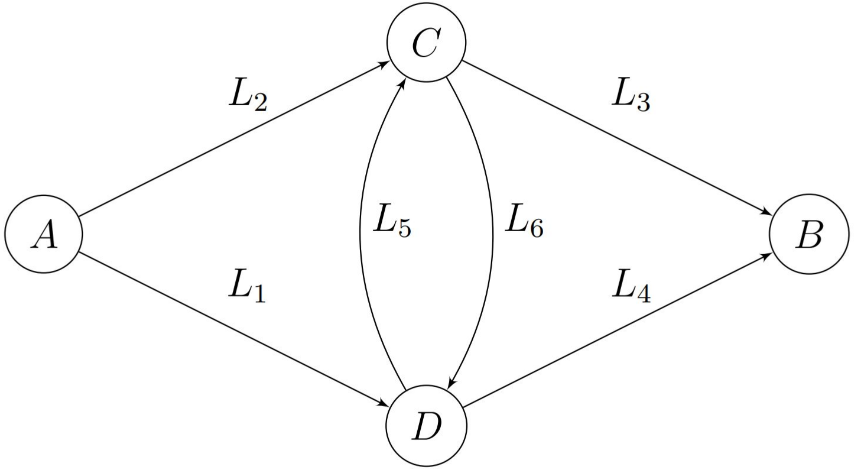
- PoA: Ratio of social welfare between social optimum & the worst NE
- Measures how a system efficiency degrades due to selfish player behavior

$$\mathrm{PoA}_{\text{peak}} = (1/20.645)/(1/26.61) = 1.29$$
$$\mathrm{PoA}_{\text{non-peak}} = (1/20.897)/(1/26.97) = 1.29$$

- Traffic network performs 29% worse due to selfish behaviour

Braess Paradox

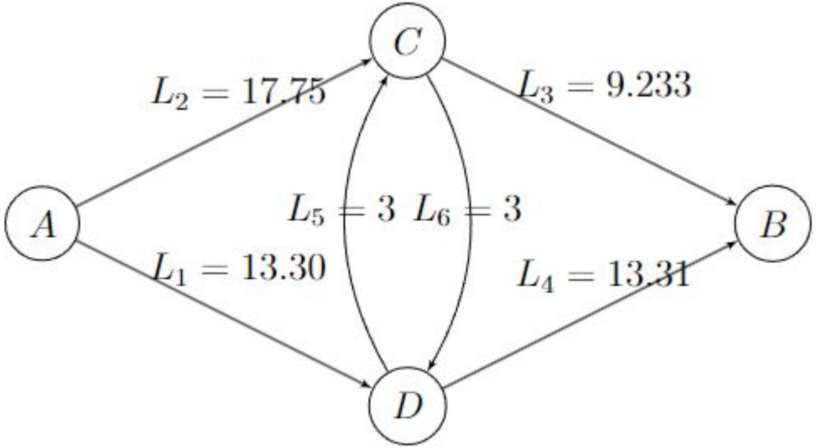
- Recall our augmented traffic network:
- Calculating the latency for every edge, assuming all selfish users goes along it



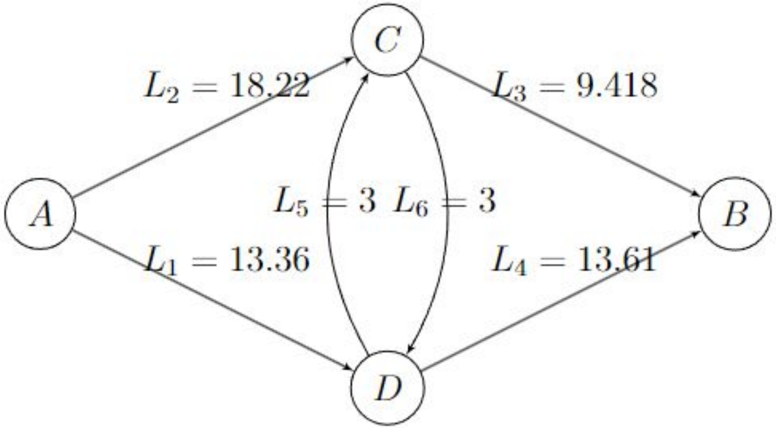
Braess Paradox

- Augmented traffic network with labelled latency function values

For AM peak, 0830hrs:



For PM non-peak, 1330hrs:

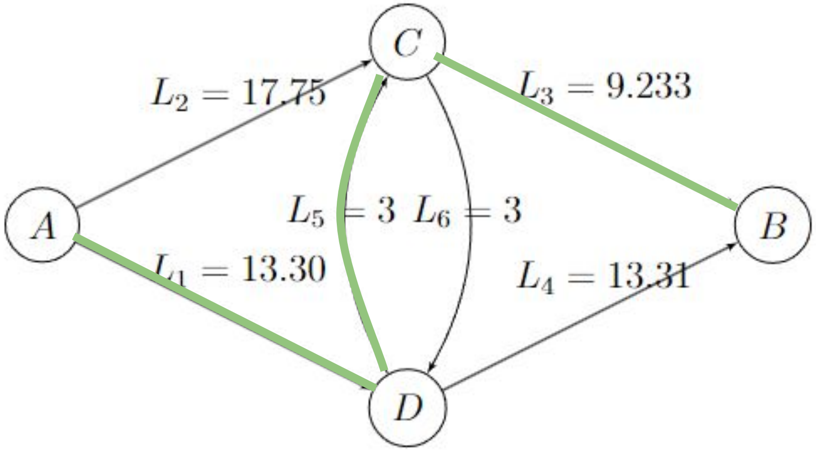


Even though we calculated the effect of congestion for the augmented paths 5 & 6,
We use the min travel timing under the assumption of a congestion-free augmented path

Braess Paradox

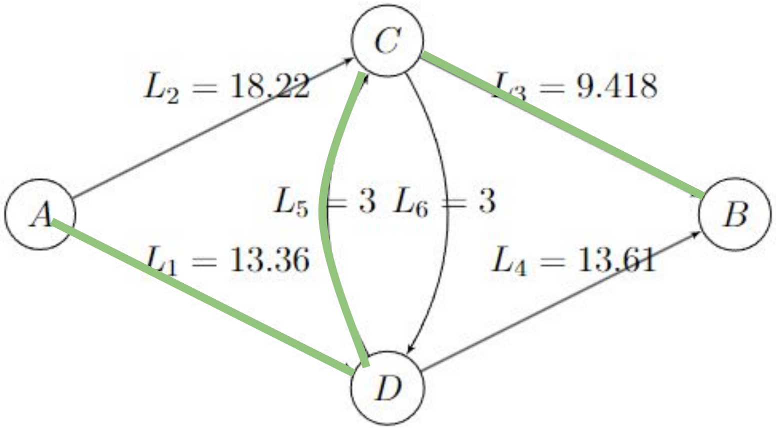
- At NE, selfish users would travel along the green path

For AM peak, 0830hrs:



Total Delay: 13.30+3+9.233 = **25.533**

For PM non-peak, 1330hrs:



13.36+3+9.418 = **25.778**

Braess Paradox

- Augmented network actually improves the traffic network

	AM peak, 0830hrs, Total delay	**PM non-peak, 1330hrs, Total delay**
NE basic network	26.61	26.97
NE augmented network	25.533	25.778

Braess Paradox

- Still performs worse than social optimum of basic network

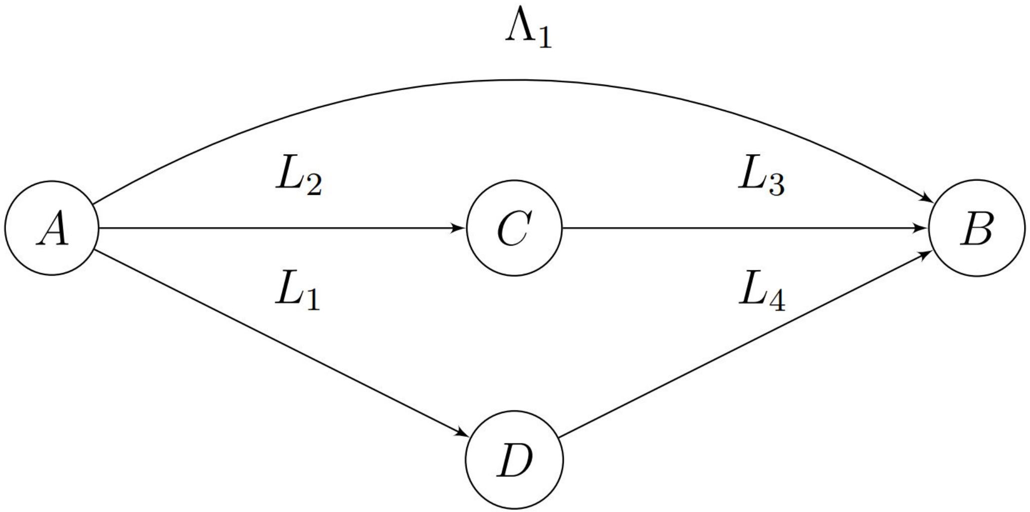
	AM peak, 0830hrs, Total delay	PM non-peak, 1330hrs, Total delay
Social optimum basic network	20.645	20.897
NE augmented network	25.533	25.778

Braess Paradox

- Paradox requires the right combinations of factors to happen in real-life examples and does not happen all the time [4]

Cycling Route

- Recall our modified traffic network:



Cycling Route: Set-up

Assumptions:

- Travel times do not affect proportion of users switching to cycling
- Cycling route congestion-free

Cycling Route: Set-up

Assumptions:

- Travel times do not affect proportion of users switching to cycling
- Cycling route congestion-free

Modifications to problem:

- Proportion of users switching to cycling, c
- According to LTA:
    - Normal lane width = 3.2m
    - Cycling lane width = 2m

Cycling Route: Set-up

Assumptions:

- Travel times do not affect proportion of users switching to cycling
- Cycling route congestion-free

Modifications to problem:

- Proportion of users switching to cycling, $\boldsymbol{c}$
- According to LTA:
  - Normal lane width = 3.2m
  - Cycling lane width = 2m

Γ_i decreased by 2 / 3.2 = 0.625

Cycling Route: Model

- To investigate the effect of adding cycling routes at social optimum
- Optimized at various values of c from 0.1 to 0.9
- The optimization problem is:

$$\min \sum_{i=1}^{4} (L_i x_i) \text{ where}$$

$$L_i = t_i(1 + x_i \cdot \sigma(\frac{1}{\Gamma_i \cdot \Theta_i}))\forall i = 1, 2, 3, 4$$

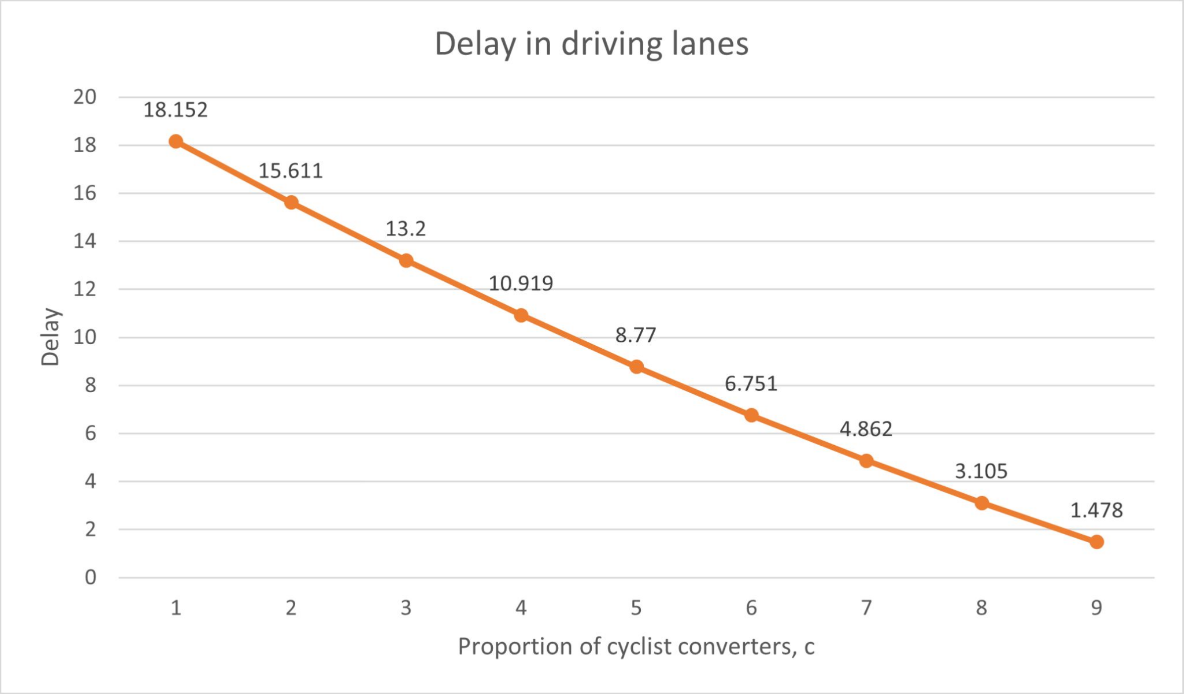
$$x_1 + x_2 = 1 - c$$

$$x_2 = x_3$$

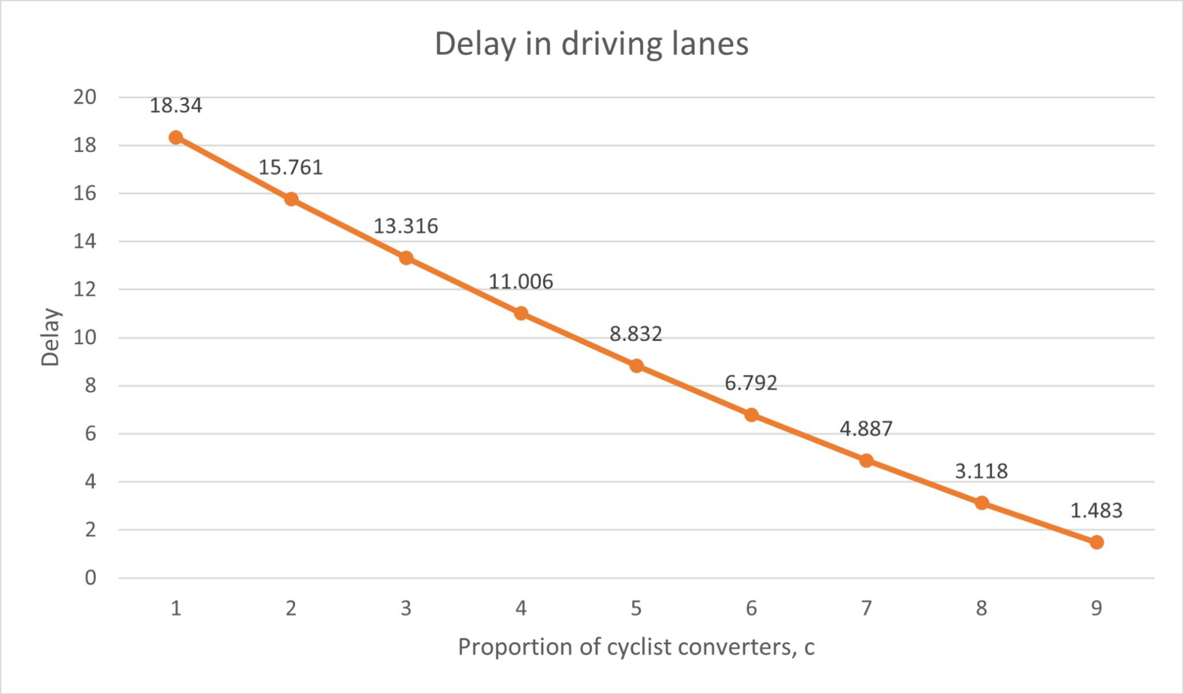
$$x_1 = x_4$$

$$0 \leq x_i \leq 1$$

Cycling Route: Results (AM Peak)



Cycling Route: Results (PM Non-peak)



Cycling Route: Results

- Largest delay, *discounting delay from cycling* (at $\mathcal{C}$ = 0.1) already an improvement

	AM peak, 0830hrs, Total delay	**PM non-peak, 1330hrs, Total delay**
Basic Network (NE)	26.61	26.97
Basic Network (Social Eqm)	20.645	20.645
Cycling Network	18.152 (without cycling delay)	18.34 (without cycling delay)

Cycling Route: Results

- Not *discounting the delay from cycling* (at $\mathcal{C}$ = 0.1),
  only slightly worse than the social optimum of the basic network

	AM peak, 0830hrs, Total delay	**PM non-peak, 1330hrs, Total delay**
Basic Network (NE)	26.61	26.97
Basic Network (Social Eqm)	20.645	20.645
Cycling Network	21.552 (with cycling delay)	21.74 (with cycling delay)

Conclusion

Limitations

- Our model
    - Overestimates the time taken (total delay) to travel between nodes
    - Estimates delay during peak hour to be lower than non-peak hour

Possible improvement:

- Incorporate coefficients (similar to BPR function)
    - Adjust the delays to Google Map timings during those timings
    - Requires more data (possibly from LTA)

Summary of Findings

- Selfish players result in worse delays than social optimum levels
- Braess’ Paradox only holds provided that certain conditions are met
    - Such conditions are ideal but not realistic.
- Cycling lanes reduces the delay from driving (as expected)
    - Still worse than social optimum
- Total delay increases as proportion of users cycling increases
    - Due to long cycling timings

Thank You!
Q&A

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