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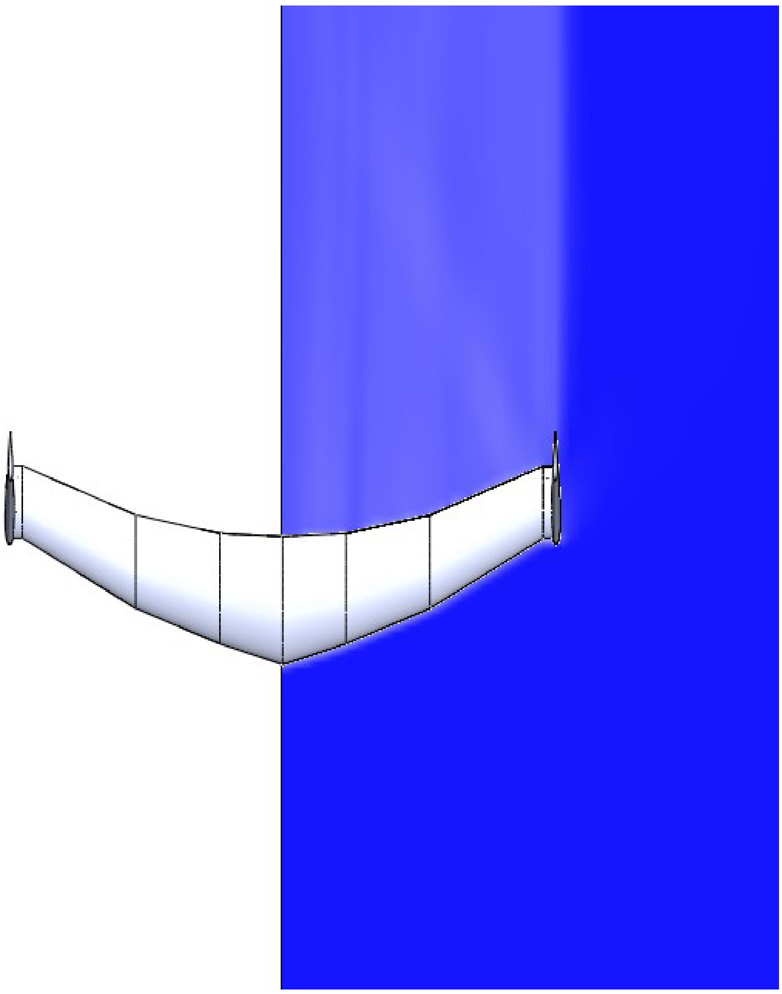
MAE 635 - Comp Fluid Dynamics II

Validation Simulation Supersonic Linearized Theory

Reference papers : *NASA Technical Paper 1406, US Standard Atmosphere*



Abstract

This paper presents a computational study aimed at verifying the results obtained in a NASA Technical Paper 1406. Through careful implementation of numerical methods and computational fluid dynamics (CFD) techniques, we reproduce and validate the the shock angle previously reported. The verification process includes detailed comparisons of flow characteristics and numerical solutions, providing insights into the reliability and accuracy of the original research.

Contents

1 Theory

1.1 Governing equations

The full compressible potential equation, which is a second-order partial differential equation, can be expressed in terms of the velocity potential ϕ as follows:

$$(1 - M^2\phi_x^2)\phi_{xx} - 2M^2\phi_x\phi_y\phi_{xy} + (1 - M^2\phi_y^2)\phi_{yy} + \phi_{zz} = 0, \tag{1}$$

where M is the Mach number, and ϕ_x, ϕ_y, and ϕ_z are the partial derivatives of the velocity potential with respect to the spatial coordinates x, y, and z, respectively. The subscripts denote partial differentiation.

1.2 Hypothesis

- Inviscid, adiabatic, perfect-gas flow, steady

$$P = \rho RT \quad ; \quad \gamma = const \quad ; \quad div(v) = 0 \quad ; \quad \frac{\partial}{\partial t} = 0$$

- Small perturbations of the flow field

$$v = v_0 + v' \quad ; \quad P = P_0 + P' \quad ; \quad T = T_0 + T'$$

- Disturbance velocities

$$v' = \epsilon v_1 + \epsilon^2 v_2 + \epsilon^3 v_3 + ... \quad ; \quad \epsilon << 1$$

1.3 Linearized equations

The linearized equations are obtained by substituting the perturbation velocities into the governing equations and neglecting higher-order terms. The resulting equations can be expressed as follows:

$$(1 - M_\infty^2)\phi_{xx} + \phi_{yy} + \phi_{zz} = 0. \tag{2}$$

This limited development neglects all terms of order $O(\phi_x^2)$ and above, as well as shock-shock interactions, thereby restricting valid Mach and deflection ranges.

1.4 Shock angle

The exact oblique-shock relation for a wedge half-angle θ is

$$tan\theta = 2\cot\beta \frac{M_\infty^2 \sin^2\beta - 1}{M_\infty^2(\gamma + \cos 2\beta) + 2}. \tag{3}$$

In the limit $\theta \to 0$ with attached flow, the critical condition arises when the numerator vanishes:

$$M_\infty^2 \sin^2\beta_d - 1 = 0 \quad \Longrightarrow \quad \beta_d = \sin^{-1}(1/M_\infty).$$

So,

$$\beta_d \approx \sin^{-1}\Big(\frac{1}{M_\infty}\Big) \tag{4}$$

offers a first-order shock-detachment estimate. Note the distinction from the Mach angle $\mu = \sin^{-1}(1/M_\infty)$, which applies to infinitesimal disturbances rather than finite shocks.

1.5 Applicaiton of the theory