NEW SOUTH WALES
HIGHER SCHOOL CERTIFICATE
Mathematics Extension 2

Exercise 40/67

by James Coroneos*

Find the following integrals.

1. $\int \frac{x\ dx}{x^2+4}$ **2.** $\int \frac{x\ dx}{\sqrt{x^2+4}}$ **3.** $\int \frac{5x+2}{x^2-4}\ dx$ **4.** $\int \sin x \cos^3 x\ dx$ **5.** $\int \sin x \sec^3 x\ dx$
6. $\int \cos^2 \frac{x}{2}\ dx$ **7.** $\int x \sin x\ dx$ **8.** $\int x \sec^2 2x\ dx$ **9.** $\int \tan^{-1} 2x\ dx$ **10.** $\int \frac{x^3\ dx}{x^2+1}$
11. $\int \frac{x\ dx}{(x+2)(x+4)}$ **12.** $\int \frac{(x-1)(x+1)\ dx}{(x-2)(x-3)}$ **13.** $\int \frac{(2x-1)\ dx}{x^2+2x+3}$ **14.** $\int \frac{x^3\ dx}{2x-1}$ **15.** $\int \frac{(1+x)\ dx}{\sqrt{1-x-x^2}}$
16. $\int \frac{dx}{x^2(1-x^2)^{\frac{1}{2}}}$ **17.** $\int \frac{dx}{x\sqrt{a^2+x^2}}$ **18.** $\int \frac{dx}{x\sqrt{a^2-x^2}}$ **19.** $\int \frac{dx}{x\sqrt{x^2-a^2}}$ **20.** $\int \frac{x\ dx}{\sqrt{x}+1}$
21. $\int \frac{\cos^{-1} x}{\sqrt{1-x^2}}\ dx$ **22.** $\int \sqrt{\frac{x+1}{x-1}}\ dx$ **23.** $\int \frac{dx}{x(\log x)^3}$ **24.** $\int \sec^4 3x\ dx$ **25.** $\int \frac{dx}{x^2(1-x)}$
26. $\int \frac{dx}{x^2(1+x^2)}$ **27.** $\int \frac{dx}{(1+x^2)^2}$ **28.** $\int \tan^3 x\ dx$ **29.** $\int \frac{dx}{5+3\cos x}$ **30.** $\int \frac{dx}{3+5\cos x}$
31. $\int \frac{\sin x\ dx}{5+3\cos x}$ **32.** $\int \frac{dx}{1+\cos^2 x}$ **33.** $\int \frac{dx}{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}$ **34.** $\int x^2 \sin x\ dx$
35. $\int \frac{x^2\ dx}{(x-1)(x-2)(x-3)}$ **36.** $\int \frac{e^x\ dx}{e^x-1}$ **37.** $\int \frac{dx}{3\sin^2 x+5\cos^2 x}$ **38.** $\int x^3 e^{5x^4-7}\ dx$
39. $\int x^5 \log x\ dx$ **40.** $\int \frac{(3x+2)\ dx}{x(x+1)^3}$ **41.** $\int \log x^3\ dx$ **42.** $\int \frac{dx}{e^x+e^{-x}}$
43. $\int (5x^3+7x-1)^{\frac{3}{2}}.(15x^2+7)\ dx$ **44.** $\int \frac{dx}{(x^2+1)(x^2+4)}$ **45.** $\int (x^2+x-+1)^{-1}\ dx$
46. $\int e^x \sin 2x\ dx$ **47.** $\int (x^2+x-1)^{-1}\ dx$ **48.** $\int (x^2-x)^{-\frac{1}{2}}\ dx$ **49.** $\int \frac{1-2x}{3+x}\ dx$
50. $\int x^3(4+x^2)^{-\frac{1}{2}}\ dx$ **51.** $\int \frac{\sin 2x\ dx}{3\cos^2 x+4\sin^2 x}$ **52.** $\int \frac{x^2\ dx}{1-x^4}$ **53.** $\int \frac{dx}{\sin x \cos x}$
54. $\int \log \sqrt{x-1}\ dx$ **55.** $\int \frac{dx}{e^x-1}$ **56.** $\int \frac{\sec^2 x\ dx}{\tan^2 x-3\tan x+2}$ **57.** $\int \frac{(x+1)\ dx}{(x^2-3x+2)^{\frac{1}{2}}}$
58. $\int \sin 2x \cos x\ dx$ **59.** $\int \frac{x\ dx}{1+x^3}$ **60.** $\int x \tan^{-1} x\ dx$ **61.** $\int (1+3x+2x^2)^{-1}\ dx$
62. $\int (9-x^2)^{\frac{1}{2}}\ dx$ **63.** $\int (9+x^2)^{\frac{1}{2}}\ dx$ **64.** $\int x(9+x^2)^{\frac{1}{2}}\ dx$ **65.** $\int \sec^2 x \tan^3 x\ dx$
66. $\int x^2 e^{-x}\ dx$ **67.** $\int x e^{x^2}\ dx$ **68.** $\int \sin x \tan x\ dx$ **69.** $\int \sin^4 x \cos^3 x\ dx$
70. $\int \frac{(x^3+1)\ dx}{x^3-x}$ **71.** $\int \log(x+\sqrt{x^2-1})\ dx$ **72.** $\int \frac{dx}{(x+1)^{\frac{1}{2}}+(x+1)}$

Evaluate the following definite integrals, leaving results in irrational form.

73. $\int_0^4 \frac{x\ dx}{\sqrt{x+4}}$ **74.** $\int_1^2 \frac{dx}{x(1+x^2)}$ **75.** $\int_1^2 \frac{\log x}{x}\ dx$ **76.** $\int_0^1 \cos^{-1} x\ dx$ **77.** $\int_1^2 \frac{(x+1)\ dx}{\sqrt{-2+3x-x^2}}$
78. $\int_0^{\frac{\pi}{2}} \frac{dx}{\cos^2 x+2\sin^2 x}$ **79.** $\int_0^1 x\sqrt{1-x^2}\ dx$ **80.** $\int_2^4 x \log x\ dx$ **81.** $\int_1^2 \frac{dx}{x^2+5x+4}$
82. $\int_0^{\frac{\pi}{2}} (1+\frac{1}{2}\sin x)^{-1}\ dx$ **83.** $\int_0^1 x^2 e^{-x}\ dx$ **84.** $\int_0^1 \frac{(7+x)\ dx}{1+x+x^2+x^3}$ **85.** $\int_0^1 \frac{e^{-2x}\ dx}{e^{-x}+1}$

*Other resources by James Coroneos are available. Write to P.O. Box 25, Rose Bay, NSW, 2029, Australia, for a catalogue.

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TeX

86. $\int_0^{\frac{a}{2}} \frac{y}{a-y}\ dy$ **87.** $\int_0^a \frac{(a-x)^2\ dx}{a^2+x^2}$ **88.** $\int_0^1 \frac{(x+3)\ dx}{(x+2)(x+1)^2}$ **89.** $\int_0^1 \frac{x^2\ dx}{x^6+1}$

90. $\int_0^{\pi} \cos^2 mx\ dx,\ m$ integral **91.** $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} x \sin 2x\ dx$ **92.** $\int_0^{\frac{a}{2}} x^2\sqrt{a^2-x^2}\ dx$

93. $\int_0^{\frac{\pi}{4}} \sec^2 x \tan x\ dx$ **94.** $\int_0^1 (x+2)(x^2+4x+5)^{\frac{1}{2}}\ dx$ **95.** $\int_1^2 x(\log x)^2\ dx$

96. $\int_3^4 \frac{x^2+4}{x^2-1}\ dx$ **97.** $\int_1^4 \frac{x^2+4}{x(x+2)}\ dx$ **98.** $\int_0^{\frac{\pi}{2}} \frac{\cos x\ dx}{5-3\sin x}$ **99.** $\int_0^1 \frac{dx}{(4-x^2)^{\frac{3}{2}}}$

100. $\int_0^{\frac{\pi}{2}} 2\sin\theta\cos\theta(3\sin\theta - 4\sin^3\theta)\ d\theta$

SET 4J (page 140)

1. $\frac{1}{2}\log(x^2+4)$

2. $\sqrt{x^2+4}$

3. $3\log(x-2) + 2\log(x+2)$

4. $-\frac{1}{4}\cos^4 x$

5. $\frac{1}{2}\sec^2 x$

6. $\frac{1}{2}[x + \sin x]$

7. $-x\cos x + \sin x$

8. $\frac{1}{2}x\tan 2x + \frac{1}{4}\log\cos 2x$

9. $x\tan^{-1}2x - \frac{1}{4}\log(1+4x^2)$

10. $\frac{1}{2}x^2 - \frac{1}{2}\log(1+x^2)$

11. $2\log(x+4) - \log(x+2)$

12. $x - 3\log(x-2) + 8\log(x-3)$

13. $\log(x^2+2x+3) - \frac{3}{\sqrt{2}}\tan^{-1}(\frac{x+1}{\sqrt{2}})$

14. $\frac{1}{6}x^3 + \frac{1}{8}x^2 + \frac{1}{8}x + \frac{1}{16}\log(2x-1)$

15. $\frac{1}{2}\sin^{-1}(\frac{2x+1}{\sqrt{5}}) - \sqrt{1-x-x^2}$

16. $-\frac{\sqrt{1-x^2}}{x}$

17. $-\frac{1}{a}\log\left[\frac{\sqrt{a^2+x^2}+a}{x}\right]$ or $-\frac{1}{a}\log\left[\frac{x}{\sqrt{a^2+x^2}-a}\right]$

18. $-\frac{1}{a}\log\left[\frac{a+\sqrt{a^2-x^2}}{x}\right]$ or $-\frac{1}{a}\log\left[\frac{x}{a-\sqrt{a^2-x^2}}\right]$

19. $\frac{1}{a}\sec^{-1}\frac{x}{a}$

20. $\frac{2}{3}x^{3/2} - x + 2x^{1/2} - 2\log(1+x^{1/2})$

21. $-\frac{1}{2}(\cos^{-1}x)^2$

22. $\sqrt{x^2-1} + \log[x+\sqrt{x^2-1}]$

23. $\frac{-1}{2(\log x)^2}$

24. $\frac{1}{3}\tan 3x + \frac{1}{9}\tan^3 3x$

25. $\log x - \frac{1}{x} - \log(1-x)$

26. $-\frac{1}{x} - \tan^{-1}x$

27. $\frac{1}{2}\tan^{-1}x + \frac{x}{2(1+x^2)}$

28. $\frac{1}{2}\tan^2 x + \log\cos x$

29. $\frac{1}{2}\tan^{-1}(\frac{\tan x/2}{2})$

30. $\frac{1}{4}\log(\frac{2+\tan x/2}{2-\tan x/2})$

31. $-\frac{1}{3}\log(5+3\cos x)$

32. $\frac{1}{\sqrt{2}}\tan^{-1}(\frac{\tan x}{\sqrt{2}})$

33. $\log(\sec x + \tan x) = \log\tan(\frac{x}{2} + \frac{\pi}{4})$

34. $-x^2\cos x + 2x\sin x + 2\cos x$

35. $\frac{1}{2}\log(x-1) - 4\log(x-2) + \frac{9}{2}\log(x-3)$

36. $\log(e^x-1)$

37. $\frac{1}{\sqrt{15}}\tan^{-1}\left(\sqrt{\frac{3}{5}}\tan x\right)$

38. $\frac{1}{20}e^{5x^4-7}$

39. $\frac{x^6}{6}\log x - \frac{x^6}{36}$

40. $2\log x - 2\log(x+1) + \frac{2}{x+1} - \frac{1}{2(x+1)^2}$

41. $3[x\log x - x]$

42. $\tan^{-1}(e^x)$

43. $\frac{2}{5}(5x^3+7x-1)^{5/2}$

44. $\frac{1}{3}[\tan^{-1}x - \frac{1}{2}\tan^{-1}\frac{x}{2}]$

45. $\frac{2}{\sqrt{3}}\tan^{-1}(\frac{2x+1}{\sqrt{3}})$

46. $\frac{e^x}{5}(\sin 2x - 2\cos 2x)$

47. $\frac{1}{\sqrt{5}}\log(\frac{2x+1-\sqrt{5}}{2x+1+\sqrt{5}})$

48. $\log[(x-\frac{1}{2}) + \sqrt{x^2-x}]$

49. $-2x+7\log(3+x)$

50. $\frac{1}{3}(x^2-8)\sqrt{4+x^2}$

51. $\log(3+\sin^2 x)$

52. $\frac{1}{4}\log(1+x) - \frac{1}{4}\log(1-x) - \frac{1}{2}\tan^{-1}x$

53. $\log\tan x$ or $-\log(\operatorname{cosec} 2x + \cot 2x)$

54. $\frac{1}{2}(x-1)\log(x-1) - \frac{1}{2}x$

55. $\log(e^x-1) - x$

56. $\log(\frac{\tan x - 2}{\tan x - 1})$

57. $\sqrt{x^2-3x+2} + \frac{5}{2}\log[x - \frac{3}{2} + \sqrt{x^2-3x+2}]$

58. $-\frac{2}{3}\cos^3 x$

59. $\frac{1}{6}\log(1-x+x^2) - \frac{1}{3}\log(1+x) + \frac{1}{\sqrt{3}}\tan^{-1}(\frac{2x-1}{\sqrt{3}})$

60. $\frac{1}{2}[x^2\tan^{-1}x + \tan^{-1}x - x]$

61. $\log\frac{1+2x}{1+x}$

62. $\frac{1}{2}[x\sqrt{9-x^2} + 9\sin^{-1}\frac{x}{3}]$

63. $\frac{1}{2}\{x\sqrt{9+x^2} + 9\log[x+\sqrt{9+x^2}]\}$

64. $\frac{1}{3}(9+x^2)^{3/2}$

65. $\frac{1}{4}\tan^4 x$

66. $-e^{-x}(x^2+2x+2)$

67. $\frac{1}{2}e^{x^2}$

68. $\log(\sec x + \tan x) - \sin x$

69. $\frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x$

70. $x+\log(x-1)-\log x$

71. $x\log(x+\sqrt{x^2-1})-\sqrt{x^2-1}$

72. $2\log[1+\sqrt{x+1}]$

73. $\frac{16}{3}(2-\sqrt{2})$

74. $\frac{1}{2}\log(\frac{8}{5})$

75. $\frac{1}{2}(\log 2)^2$

76. 1

77. $\frac{5\pi}{2}$

78. $\frac{\pi\sqrt{2}}{4}$

79. $\frac{1}{3}$

80. $14\log 2 - 3$

81. $\frac{1}{3}\log(\frac{5}{4})$

82. $\frac{2\pi}{3\sqrt{3}}$

83. $2 - \frac{5}{e}$

84. $\frac{3}{2}\log 2 + \pi$

85. $\log(\frac{e+1}{2e}) - \frac{1}{e} + 1$

86. $\frac{a}{2}(\log 4 - 1)$

87. $a(1 - \log 2)$

88. $1 + \log(\frac{3}{4})$

ANSWERS

89. $\frac{\pi}{12}$ 90. $\frac{\pi}{2}$ 91. $\frac{1}{4}(\pi-1)$

92. $\frac{(4\pi-3\sqrt{3})a^4}{192}$ 93. $\frac{1}{2}$ 94. $\frac{5\sqrt{5}}{3}(2\sqrt{2}-1)$

95. $2(\log 2)^2 - 2 \log 2 + \frac{3}{4}$ 96. $1 + \frac{5}{2} \log \frac{6}{5}$

97. 3 98. $\frac{1}{3} \log(\frac{5}{2})$

99. $\frac{1}{4\sqrt{3}}$ 100. $\frac{2}{5}$

EXERCISES SET 4J

1 $\int \frac{xdx}{x^2+4} = \frac{1}{2}\int \frac{2xdx}{x^2+4} = \frac{1}{2}\ln(x^2+4) + C$

2 $\int \frac{xdx}{\sqrt{x^2+4}} = \frac{1}{2}\int \frac{2xdx}{\sqrt{x^2+4}} = \sqrt{x^2+4} + C$

3 **Let** $\frac{5x+2}{x^2-4} = \frac{a}{x+2} + \frac{b}{x-2}$

Then $5x+2 = a(x-2) + b(x+2)$
$= (a+b)x - 2a + 2b$

By equating coefficients

$x : a+b = 5 \quad \ldots\ldots(1)$

$x^0 : -2a + 2b = 2 \quad \ldots\ldots(2)$

Equation $(2) - 2(1)$

$-4a = -8, \therefore a = 2$

Substitute $a = 2$ **into equation** (1)

$2 + b = 5,\ b = 3$

Thus $\frac{5x+2}{x^2-4} = \frac{2}{x+2} + \frac{3}{x-2}$

$$\int \frac{5x+2}{x^2-4}dx = \int (\frac{2}{x+2} + \frac{3}{x-2})dx$$

$$= 2\int \frac{dx}{x+2} + 3\int \frac{dx}{x-2}$$

$$= 2\ln(x+2) + 3\ln(x-2) + C$$

4 **Let** $u = \cos x, du = -\sin x dx$

$$\int \sin x \cos^3 x dx = -\int \overbrace{-\sin x dx}^{du} \cos^3 x$$

$$= -\int u^3 du = -\frac{u^4}{4} + C$$

$$= -\frac{\cos^4 x}{4} + C$$

5 **Let** $u = \cos x, du = -\sin x dx$

$$\int \sin x \sec^3 x dx = \int \frac{\sin x dx}{\cos^3 x} = -\int \frac{\overbrace{-\sin x dx}^{du}}{\cos^3 x}$$

$$= -\int \frac{du}{u^3}$$

$$= -\frac{u^{-2}}{-2} + C = \frac{1}{2u^2} + C$$

$$= \frac{1}{2\cos^2 x} + C$$

$$= \frac{1}{2}\sec^2 x + C$$

6 $\int \frac{\cos^2 x}{2}dx = \int \frac{1+\cos 2x}{2}dx$

$$= \frac{1}{2}\int dx + \frac{1}{2}\int \cos x dx$$

$$= \frac{1}{2}x + \frac{1}{2}\sin x + C$$

$$= \frac{1}{2}(x + \sin x) + C$$

7 Let $u = x, du = dx$ **and** $dv = \sin x dx,$
$v = -\cos x$

$$\int x \sin x dx = uv - \int v du = x \times -\cos x - \int -\cos x dx$$

$$= -x\cos x + \int \cos x dx$$

$$= -x\cos x + \sin x + C$$

8 Let $u = x, du = dx$; $dv = \sec^2 2x dx,$

$$v = \frac{1}{2}\tan 2x$$

$$\int x \sec^2 2x dx = uv - \int v du = x \times \frac{1}{2}\tan 2x - \int \frac{1}{2}\tan 2x \times dx$$

$$= \frac{1}{2}x\tan 2x - \frac{1}{2}\int \tan 2x dx$$

$$= \frac{1}{2}x\tan 2x - \frac{1}{2}\int \frac{\sin 2x}{\cos 2x}dx$$

$$= \frac{1}{2}x\tan 2x + \frac{1}{4}\int \frac{-2\sin 2x}{\cos 2x}dx$$

$$= \frac{1}{2}x\tan 2x + \frac{1}{4}\ln\cos 2x + C$$

9 Let $u = \tan^{-1} 2x, du = \dfrac{2}{1+4x^2}dx$; $dv = 1dx,$
$v = x$

$$\int \tan^{-1} 2x dx = uv - \int v du = \tan^{-1} 2x \times x - \int x \times \frac{2}{1+4x^2}dx$$

$$= x\tan^{-1} 2x - \int \frac{2x}{1+4x^2}dx$$

$$= x\tan^{-1} 2x - \frac{1}{4}\int \frac{8x}{1+4x^2}dx$$

$$= x\tan^{-1} 2x - \frac{1}{4}\ln(1+4x^2) + C$$

10 Let $\dfrac{x^3}{x^2+1} = mx + d + \dfrac{ax+b}{x^2+1}$

Then $x^3 = (mx+d)(x^2+1) + ax + b$
$= mx^3 + dx^2 + (a+m)x + b + d$
By equating coefficients
$x^3 : m = 1$
$x^2 : d = 0$
$x^1 : a + m = 0$ **or** $a + 1 = 0, \therefore \quad a = -1$
$x^0 : b + d = 0$ **or** $b + 0 = 0, b = 0$

Thus $\dfrac{x^3}{x^2+1} = x - \dfrac{x}{x^2+1}$

$$\int \frac{x^3}{x^2+1}dx = \int (x - \frac{x}{x^2+1})dx$$

$$= \int x dx - \int \frac{x}{x^2+1}dx$$

$$= \int x dx - \frac{1}{2}\int \frac{2x dx}{x^2+1}$$

$$= \frac{x^2}{2} - \frac{1}{2}\ln(x^2+1) = C$$

11 Let $\dfrac{x}{(x+2)(x+4)} = \dfrac{a}{x+2} + \dfrac{b}{x+4}$

Thus $x = a(x+4) + b(x+2)$
$= (a+b)x + 4a + 2b$
By equating coefficients
$x : a + b = 1 \quad \ldots\ldots\ldots(1)$
$x^0 : 4a + 2b = 0 \quad \ldots\ldots\ldots(2)$

Equation $(2) - 2(1) : 2a = -2, \therefore a = -1$
Substitute $a = -1$ **into equation** (1)

$-1 + b = 1, \therefore b = 2$

$$\therefore \frac{x}{(x+2)(x+4)} = -\frac{1}{x+2} + \frac{2}{x+4}$$

$$\int \frac{x}{(x+2)(x+4)}dx = \int (-\frac{1}{x+2} + \frac{2}{x+4})dx$$

$$= -\int \frac{dx}{x+2} + \int \frac{2dx}{x+4}$$

$$= -\ln(x+2) + 2\ln(x+4) + C$$

12 Let $\dfrac{x^2-1}{x^2-5x+6} = m + \dfrac{a}{x-2} + \dfrac{b}{x-3}$

Then $x^2 - 1 = m(x^2 - 5x + 6) + a(x-3) + b(x-2)$
$= mx^2 + (a + b - 5m)x + (-3a - 2b + 6m)x^0$
By equating coefficients
$x^2 : m = 1$

$x^1 : a+b-5=0$ **or**

$a+b=5 \quad \ldots\ldots\ldots(1)$

$x^0 : -3a-2b+6=-1$ **or**

$-3a-2b=-7 \quad \ldots\ldots\ldots(2)$

Equation $(2)+2(1)$

$-a=3, \therefore a=-3$

Substitute $a=-3$ **into equation** (1)

$-3+b=5, \therefore b=8$

Thus $\dfrac{x^2-1}{x^2-5x+6} = 1 - \dfrac{3}{x-2} + \dfrac{8}{x-3}$

$$\int \frac{x^2-1}{x^2-5x+6}dx = \int (1 - \frac{3}{x-2} + \frac{8}{x-3})dx$$

$$= \int dx - \int \frac{3dx}{x-2} + \int \frac{8dx}{x-3}$$

$$= x - 3\ln(x-2) + 8\ln(x-3) + C$$

13 $\displaystyle\int \frac{(2x-1)dx}{x^2+2x+3} = \int \frac{(2x-1)dx}{\underbrace{x^2+2x+1}_{(x+1)^2} + \underbrace{2}_{(\sqrt{2})^2}}$

$$= \int \frac{(2x-1)dx}{(x+1)^2+(\sqrt{2})^2}$$

$$= \int \frac{2xdx}{(x+1)^2+(\sqrt{2})^2} - \int \frac{dx}{(x+1)^2+(\sqrt{2})^2}$$

$$= \int \frac{(2x+2-2)dx}{(x+1)^2+(\sqrt{2})^2} - \int \frac{dx}{(x+1)^2+(\sqrt{2})^2}$$

$$= \int \frac{(2x+2)dx}{(x+1)^2+(\sqrt{2})^2} - \int \frac{3dx}{(x+1)^2+(\sqrt{2})^2}$$

$$= \int \frac{(2x+2)dx}{x^2+2x+3} - \int \frac{3dx}{(x+1)^2+(\sqrt{2})^2}$$

$$= \ln(x^2+2x+3) - \frac{3}{\sqrt{2}}\tan^{-1}(\frac{x+1}{\sqrt{2}}) + C$$

14 $\displaystyle\int \frac{x^3dx}{2x-1} = \frac{1}{2}\int \frac{x^3dx}{(x-1/2)}$

$$= \frac{1}{2}\int \frac{(x^3-1/8+1/8)dx}{(x-1/2)}$$

$$= \frac{1}{2}\int \frac{(x^3-1/8)dx}{(x-1/2)} + \frac{1}{16}\int \frac{dx}{(x-1/2)}$$

$$= \frac{1}{2}\int \frac{(x-1/2)(x^2+x/2+1/4)dx}{(x-1/2)} + \frac{1}{16}\int \frac{dx}{(x-1/2)}$$

$$= \frac{1}{2}\int (x^2+x/2+1/4)dx + \frac{1}{16}\int \frac{dx}{(x-1/2)}$$

$$= \frac{1}{2}(\frac{x^3}{3} + \frac{x^2}{4} + \frac{x}{4}) + \frac{1}{16}\ln(x-1/2) + C$$

$$= \ln(x^2+2x+3) - \frac{3}{\sqrt{2}}\tan^{-1}(\frac{x+1}{\sqrt{2}}) + C$$

15 $\dfrac{d}{dx}(1-x-x^2) = -1-2x$

Also

$1-x-x^2 = -(x^2+x-1)$

$$= -(x^2+x+\frac{1}{4}-1-\frac{1}{4})$$

$$= -[(x+\frac{1}{2})^2 - (\frac{\sqrt{5}}{2})^2]$$

$$= (\frac{\sqrt{5}}{2})^2 - (x+\frac{1}{2})^2$$

Then $\displaystyle\int \frac{(1+x)dx}{\sqrt{1-x-x^2}} = -\frac{1}{2}\int \frac{(-2-2x)dx}{\sqrt{1-x-x^2}}$

$$= -\frac{1}{2}\int \frac{[-1+(-1-2x)]dx}{\sqrt{1-x-x^2}}$$

$$= \frac{1}{2}\int \frac{dx}{\sqrt{1-x-x^2}} - \frac{1}{2}\int \frac{(-1-2x)dx}{\sqrt{1-x-x^2}}$$

$$= \frac{1}{2}\int \frac{dx}{\sqrt{(\frac{\sqrt{5}}{2})^2 - (x+\frac{1}{2})^2}} - \frac{1}{2}\int \frac{(-1-2x)dx}{\sqrt{1-x-x^2}}$$

$$= \frac{1}{2}\sin^{-1}\frac{(x+1/2)}{\frac{\sqrt{5}}{2}} - \sqrt{1-x-x^2} + C$$

$$= \frac{1}{2}\sin^{-1}\frac{(2x+1)}{\sqrt{5}} - \sqrt{1-x-x^2} + C$$

16 Let $x = \dfrac{1}{u}$**, then** $dx = -\dfrac{1}{u^2}du$

Also $u^2-1=z$, **thus** $2udu=dz$

$$\text{i.e. } \int \frac{dx}{x^2(1-x^2)^{1/2}} = \int \frac{-\frac{1}{u^2}du}{\frac{1}{u^2}(1-\frac{1}{u^2})^{1/2}}$$

$$= -\int \frac{udu}{(u^21)^{1/2}} = -\int \frac{udu}{\sqrt{u^2-1}}$$

$$= -\frac{1}{2}\int \frac{\overbrace{2udu}^{\mathbf{dz}}}{\sqrt{\underbrace{u^2-1}_{\mathbf{z}}}} = -\frac{1}{2}\int \frac{dz}{\sqrt{z}}$$

$$= -\sqrt{z}+C = -\sqrt{u^2-1}+C$$

$$= -\sqrt{\frac{1}{x^2}-1}+C = -\sqrt{\frac{1-x^2}{x^2}}+C$$

$$= -\frac{\sqrt{\mathbf{1-x}^2}}{\mathbf{x}}+C$$

17 **Let** $x=a\tan\theta, dx=a\sec^2\theta d\theta$

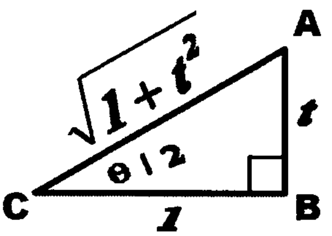
$a^2+x^2=a^2+a^2\tan^2\theta=a^2(1+\tan^2\theta)$
$=a^2\sec^2\theta$

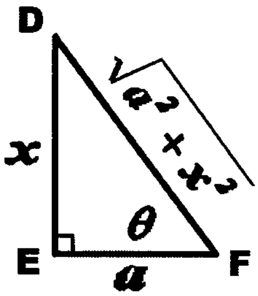
$$\text{i.e. } \int \frac{dx}{x\sqrt{a^2+x^2}} = \int \frac{a\sec^2\theta d\theta}{a\tan\theta\sqrt{a^2\sec^2\theta}}$$

$$= \frac{1}{a}\int \frac{\sec\theta d\theta}{\tan\theta} = \frac{1}{a}\int \frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}d\theta$$

$$= \frac{1}{a}\int \frac{d\theta}{\sin\theta}$$

Now, let $t=\tan\frac{\theta}{2}$





$$\tan\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \sqrt{\frac{1-\frac{a}{\sqrt{a^2+x^2}}}{1+\frac{a}{\sqrt{a^2+x^2}}}}$$

$$= \sqrt{\frac{\frac{\sqrt{a^2+x^2}-a}{\sqrt{a^2+x^2}}}{\frac{\sqrt{a^2+x^2}+a}{\sqrt{a^2+x^2}}}}$$

$$= \sqrt{\frac{\sqrt{a^2+x^2}-a}{\sqrt{a^2+x^2}+a}}$$

$$= \sqrt{\frac{\sqrt{a^2+x^2}-a}{\sqrt{a^2+x^2}+a}\times\frac{\sqrt{a^2+x^2}-a}{\sqrt{a^2+x^2}-a}}$$

$$= \sqrt{\frac{(\sqrt{a^2+x^2}-a)^2}{a^2+x^2-a^2}}$$

$$= \frac{\sqrt{a^2+x^2}-a}{x}$$

$$\text{i.e. } \frac{1}{a}\int\frac{d\theta}{\sin\theta} = \frac{1}{a}\int\frac{\frac{2dt}{1+t^2}}{\frac{2t}{1+t^2}} = \frac{1}{a}\int\frac{dt}{t}$$

$$= \frac{1}{a}\ln t + C = \frac{1}{a}\ln(\tan\frac{\theta}{2})+C$$

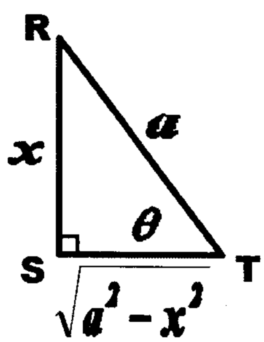
$$= \frac{1}{a}\ln(\frac{\sqrt{a^2+x^2}-a}{x})+C \text{ or}$$

$$-\frac{1}{a}\ln(\frac{x}{\sqrt{a^2+x^2}-a})+C$$

18 **Let** $x=a\sin\theta, dx=a\cos\theta d\theta$

$a^2-x^2=a^2-a^2\tan^2\theta=a^2(1-\sin^2\theta)$
$=a^2\cos^2\theta$

Now, let $t=\tan\frac{\theta}{2}$



$$\tan\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \sqrt{\frac{1-\frac{\sqrt{a^2-x^2}}{a}}{1+\frac{\sqrt{a^2-x^2}}{a}}}$$

$$= \sqrt{\frac{\frac{a - \sqrt{a^2 - x^2}}{a}}{\frac{a + \sqrt{a^2 - x^2}}{a}}}$$

$$= \sqrt{\frac{a - \sqrt{a^2 - x^2}}{a + \sqrt{a^2 - x^2}}}$$

$$= \sqrt{\frac{a - \sqrt{a^2 - x^2}}{a + \sqrt{a^2 - x^2}} \times \frac{a + \sqrt{a^2 - x^2}}{a + \sqrt{a^2 - x^2}}}$$

$$= \sqrt{\frac{a^2 - (\sqrt{a^2 - x^2})^2}{(a + \sqrt{a^2 - x^2})^2}}$$

$$= \frac{x}{a + \sqrt{a^2 - x^2}}$$

i.e. $\int \frac{dx}{x\sqrt{a^2 - x^2}} = \int \frac{a\cos\theta d\theta}{a\sin\theta\sqrt{a^2 - (a\sin\theta)^2}}$

$$= \int \frac{a\cos\theta d\theta}{a\sin\theta a\cos\theta} = \frac{1}{a}\int \frac{d\theta}{\sin\theta}$$

$$= \frac{1}{a}\int \frac{\frac{2dt}{1+t^2}}{\frac{2t}{1+t^2}} = \frac{1}{a}\int \frac{dt}{t} = \frac{1}{a}\ln t + C$$

i.e. $\frac{1}{a}\ln(\tan\frac{\theta}{2}) + C$

$$= \frac{1}{a}\ln(\frac{x}{a + \sqrt{a^2 - x^2}}) + C \text{ or}$$

$$= -\frac{1}{a}\ln(\frac{\sqrt{a^2 + x^2} + a}{x}) + C$$

19 **Let** $x = a\sec\theta, dx = a\sec\theta\tan\theta d\theta$
Also $x^2 - a^2 = a^2\sec^2\theta - a^2$

$$= a^2(\sec^2\theta - 1)$$
$$= a^2\tan^2\theta$$

i.e. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \int \frac{a\sec\theta\tan\theta d\theta}{a\sec\theta\sqrt{a^2\tan^2\theta}}$

$$= \int \frac{d\theta}{a}$$

$$= \frac{1}{a}\theta + C$$

$$= \frac{1}{a}\sec^{-1}\frac{x}{a} + C$$

20 $u = \sqrt{x} = x^{1/2}, du = \frac{1}{2\sqrt{x}}dx$ **or**

$$dx = 2\sqrt{x}du = 2udu$$

i.e. $\int \frac{xdx}{\sqrt{x} + 1} = \int \frac{u^2 \times 2udu}{u+1} = 2\int \frac{u^3du}{u+1}$

$$= 2\int \frac{(u^3 + 1 - 1)du}{u+1} = 2[\int \frac{(u^3+1)du}{u+1} - \int \frac{du}{u+1}]$$

$$= 2[\int \frac{(u+1)(u^2 - u + 1du}{u+1} - \ln(u+1)]$$

$$= 2[\int (u^2 - u + 1)du - \ln(u+1)]$$

$$= 2[\frac{u^3}{3} - \frac{u^2}{2} + u - \ln(u+1)] + C$$

$$= \frac{2x^{3/2}}{3} - x + 2x^{1/2} - 2\ln(x^{1/2} + 1)] + C$$

21 $u = \cos^{-1}x, du = -\frac{1}{\sqrt{x^2 - 1}}dx$

i.e. $\int \frac{\cos^{-1}xdx}{\sqrt{1 - x^2}} = -\int \frac{-\cos^{-1}xdx}{\sqrt{1 - x^2}} = -\int \boldsymbol{udu}$

$$= -\frac{u^2}{2} + C = -\frac{(\cos^{-1}x)^2}{2} + C$$

22 $\int \sqrt{\frac{x+1}{x-1}}dx = \int \frac{\sqrt{x+1}}{\sqrt{x-1}} \times \frac{\sqrt{x+1}}{\sqrt{x+1}}dx$

$$= \int \frac{x+1}{\sqrt{x^2 - 1}}dx = \int \frac{xdx}{\sqrt{x^2 - 1}} + \int \frac{dx}{\sqrt{x^2 - 1}}$$

$$= \frac{1}{2}\int \frac{2xdx}{\sqrt{x^2 - 1}} + \int \frac{dx}{\sqrt{x^2 - 1}}$$

$$= \sqrt{x^2 - 1} + \ln(x + \sqrt{x^2 - 1}) + C$$

23 $u = \log x, du = \frac{1}{x}dx$

$\therefore$

i.e. $\int \frac{dx}{x(\log x)^3} = \int \frac{du}{u^3} = -\frac{1}{2u^2} + C$

$$= -\frac{1}{2(\log x)^2} + C$$

24 **Let** $u = \sec^2 3x, du = 2\sec 3x \times 3\sec 3x\tan 3x$

$$= 6\sec^2 3x \tan 3x dx$$

$$dv = \sec^2 3x dx,\ v = \frac{1}{3}\tan 3x$$

i.e. $\int \sec^4 3x dx = \int \sec^2 3x \times \sec^2 3x dx$

i.e. $uv - \int v du = \sec^2 3x \times \frac{1}{3}\tan 3x -$

$$\int \frac{1}{3}\tan 3x \times 6\sec^2 3x \tan 3x dx$$

i.e. $\frac{1}{3}\sec^2 3x \tan 3x - 2 \underbrace{\int \sec^2 3x \tan^2 3x dx}_{\frac{1}{9}\tan^3 3x}$

Now, let $u' = \tan^2 3x, du' = 2\tan 3x \times$
$\sec^2 3x \times 3$
$= 6\tan 3x \sec^2 3x$

$$dv' = \sec^2 3x dx, v' = \frac{1}{3}\tan 3x$$

$$\int \sec^2 3x \tan^2 3x dx = u'v' - \int v' du'$$
$$= \tan^2 3x \times \frac{1}{3}\tan 3x -$$
$$\int \frac{1}{3}\tan 3x \times 6\tan 3x \sec^2 3x dx$$

$$= \frac{1}{3}\tan^3 3x -$$
$2\int \sec^2 3x \tan^2 3x dx$ **or**

$3\int \sec^2 3x \tan^2 3x dx = \frac{1}{3}\tan^3 3x$ **or**

$$\int \sec^2 3x \tan^2 3x dx = \frac{1}{9}\tan^3 3x$$

i.e. $\int \sec^4 3x dx = \frac{1}{3}\sec^2 3x \tan 3x -$

$$2\left(\frac{1}{9}\tan^3 3x\right) + C$$

$$= \frac{1}{3}(1 + \tan^2 3x)\tan 3x -$$

$$\frac{2}{9}\tan^3 3x + C$$

$$= \frac{1}{3}\tan 3x + \frac{1}{3}\tan^3 3x -$$

$$\frac{2}{9}\tan^3 3x) + C$$

$$= \frac{1}{3}\tan 3x + \frac{1}{9}\tan^3 3x + C$$

25 **Let** $\frac{1}{x^2(1-x)} = \frac{a}{x^2} + \frac{b}{x} + \frac{c}{1-x}$

Then $1 = a(1-x) + bx(1-x) + cx^2$
$= a + (-a+b)x + (c-b)x^2$

By equating coefficients

$x^0 : a = 1$

$x^1 : -a + b = 0$ **or**
$-1 + b = 0, \therefore b = 1$

$x^2 : c - b = 0$ **or** $c - 1 = 0, \therefore c = 1$

Thus $\frac{1}{x^2(1-x)} = \frac{1}{x^2} + \frac{1}{x} + \frac{1}{1-x}$

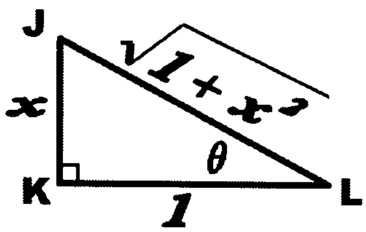
$$\int \frac{1}{x^2(1-x)}dx = \int\left(\frac{1}{x^2} + \frac{1}{x} + \frac{1}{1-x}\right)dx$$

$$= \int \frac{dx}{x^2} + \int \frac{dx}{x} + \int \frac{dx}{1-x}$$

$$= \frac{x^{-1}}{-1} + \ln x - \ln(1-x) + C$$

$$= -\frac{1}{x} + \ln x - \ln(1-x) + C$$

26 **Let** $x = \tan\theta, dx = \sec^2\theta d\theta$
$1 + x^2 = 1 + \tan^2\theta = \sec^2\theta$



i.e. $\int \frac{dx}{x^2(1+x^2)} = \int \frac{\sec^2\theta d\theta}{\tan^2\theta(\sec^2\theta)}$

i.e. $\int \frac{d\theta}{\tan^2\theta} = \int \frac{d\theta}{\frac{\sin^2\theta}{\cos^2\theta}} = \int \frac{\cos^2\theta d\theta}{\sin^2\theta}$

i.e. $\int \frac{(1-\sin^2\theta)d\theta}{\sin^2\theta} = \int(\operatorname{cosec}^2\theta - 1)d\theta$

i.e. $-\cot\theta - \theta + C = -\frac{1}{x} - \tan^{-1}x + C$

27 **Let** $x = \tan\theta, dx = \sec^2\theta d\theta$
$1 + x^2 = 1 + \tan^2\theta = \sec^2\theta$

i.e. $\int \frac{dx}{(1+x^2)^2} = \int \frac{\sec^2\theta d\theta}{(\sec^2\theta)^2}$

i.e. $\int \frac{d\theta}{\sec^2\theta} = \int \cos^2\theta d\theta = \int \frac{(1+\cos 2\theta)d\theta}{2}$

i.e. $\frac{1}{2}(\theta + \frac{1}{2}\sin 2\theta) + C$

i.e. $= \frac{1}{2}(\theta + \sin\theta\cos\theta) + C$

i.e. $= \frac{1}{2}\tan^{-1}x + \frac{1}{2} \times \frac{x}{\sqrt{1+x^2}} \times$

$$\frac{1}{\sqrt{1+x^2}} + C$$

i.e. $= \frac{1}{2}\tan^{-1}x + \frac{1}{2}\frac{x}{1+x^2} + C$

28 $\int \tan^3 x dx = \int \tan x \times \tan^2 x dx$

i.e. $\int \tan x(\sec^2 x - 1)dx = \int \tan x \sec^2 x dx -$

$$\int \tan x dx$$

i.e. $\int \tan x \sec^2 x dx - \int \frac{\sin x}{\cos x} dx$

i.e. $\int \tan x \underbrace{\sec^2 x}_{\frac{d(\tan x)}{dx}} dx - \int \frac{\sin x}{\cos x} dx$

i.e. $\frac{\tan^2 x}{2} - \int \frac{\sin x}{\cos x} dx = \frac{\tan^2 x}{2} +$

$$\int \frac{\overbrace{-\sin x}^{\frac{d(\cos x)}{dx}}}{\cos x} dx$$

i.e. $\frac{\tan^2 x}{2} dx + \ln(\cos x) + C$

29 **Let** $t = \tan\frac{x}{2}$, $dt = \frac{1}{2}\sec^2\frac{x}{2}dx$ **or**

$$dx = \frac{2dt}{\sec^2\frac{x}{2}} = 2\cos^2\frac{x}{2}dt$$

$$= 2(\frac{1}{\sqrt{1+t^2}})^2 dt$$

$$= \frac{2dt}{1+t^2}$$

Also $\cos x = 1 - 2\sin^2\frac{x}{2}$

$$= 1 - 2(\frac{t}{\sqrt{1+t^2}})^2$$

$$= 1 - \frac{2t^2}{1+t^2} = \frac{1-t^2}{1+t^2}$$

i.e. $\int \frac{dx}{5 + 3\cos x} = \int \frac{\frac{2dt}{1+t^2}}{5 + 3(\frac{1-t^2}{1+t^2})}$

i.e. $\int \frac{\frac{2dt}{1+t^2}}{\frac{5+5t^2+3-3t^2}{1+t^2}} = \int \frac{2dt}{8+2t^2}$

i.e. $\int \frac{dt}{4+t^2} = \frac{1}{2}\tan^{-1}\frac{t}{2} + C$

$$= \frac{1}{2}\tan^{-1}\frac{\tan\frac{x}{2}}{2} + C$$

30 **Let** $t = \tan\frac{x}{2}$, $dt = \frac{1}{2}\sec^2\frac{x}{2}dx$ **or**

$$dx = \frac{2dt}{\sec^2\frac{x}{2}} = 2\cos^2\frac{x}{2}dt$$

$$= 2(\frac{1}{\sqrt{1+t^2}})^2 dt$$

$$= \frac{2dt}{1+t^2}$$

Also $\cos x = 1 - 2\sin^2\frac{x}{2}$

$$= 1 - 2(\frac{t}{\sqrt{1+t^2}})^2$$

$$= 1 - \frac{2t^2}{1+t^2} = \frac{1-t^2}{1+t^2}$$

i.e. $\int \frac{dx}{3 + 5\cos x} = \int \frac{\frac{2dt}{1+t^2}}{3 + 5(\frac{1-t^2}{1+t^2})}$

i.e. $\int \frac{\frac{2dt}{1+t^2}}{\frac{3+3t^2+5-5t^2}{1+t^2}} = \int \frac{2dt}{8-2t^2}$

i.e. $\int \frac{dt}{4-t^2} = \frac{1}{2(2)}\log_e\frac{2+t}{2-t} + C$

i.e. $\frac{1}{4}\log_e \dfrac{2+\tan\frac{x}{2}}{2-\tan\frac{x}{2}} + C$

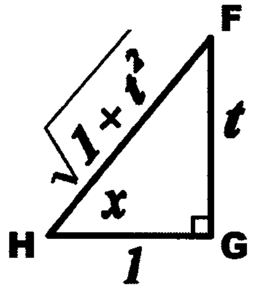
31 **Let** $u = 5 + 3\cos x, du = -3\sin x dx$

i.e. $\int \dfrac{\sin x dx}{5+3\cos x} = -\frac{1}{3}\int \dfrac{-3\sin x dx}{5+3\cos x}$

$$= -\frac{1}{3}\int \frac{du}{u}$$

i.e. $-\frac{1}{3}\ln u + C = -\frac{1}{3}\ln(5+3\cos x) + C$

32 **Let** $t = \tan x, dt = \sec^2 x dx$ **or** $dx = \cos^2 x dt$



i.e. $\int \dfrac{dx}{1+\cos^2 x} = \int \dfrac{\frac{1}{1+t^2}dt}{1+\frac{1}{1+t^2}} = \int \dfrac{dt}{t^2+2}$

i.e. $\frac{1}{\sqrt{2}}\tan^{-1}\frac{t}{\sqrt{2}} + C$

i.e. $\frac{1}{\sqrt{2}}\tan^{-1}\frac{\tan x}{\sqrt{2}} + C$

33 **Let** $t = \tan\frac{x}{2}$

i.e. $\int \dfrac{dx}{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}} = \int \dfrac{dx}{\cos x}$

i.e. $\int \dfrac{\frac{2dt}{1+t^2}}{\frac{1-t^2}{1+t^2}} = \int \dfrac{2dt}{1-t^2} = 2\int \dfrac{dt}{1-t^2}$

i.e. $2 \times \frac{1}{2(1)}\log_e \frac{1+t}{1-t} + C$

i.e. $\log_e \dfrac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}} + C$

Now $\dfrac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}} = \dfrac{\tan\frac{\pi}{4}+\tan\frac{x}{2}}{1-\tan\frac{\pi}{4}\tan\frac{x}{2}}$

$= \tan(\frac{\pi}{4}+\frac{x}{2})$ **or**

$$\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}} = \frac{1+\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}{1-\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}} = \frac{\sin\frac{x}{2}+\cos\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}$$

$$\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}} \times \frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}$$

$$\frac{(\cos\frac{x}{2}+\sin\frac{x}{2})^2}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}} = \frac{1+2\sin\frac{x}{2}\cos\frac{x}{2}}{\cos x}$$

$$\frac{1+\sin x}{\cos x} = \frac{1}{\cos x} + \frac{\sin x}{\cos x} = \sec x + \tan x$$

i.e. $\log_e \dfrac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}} + C = \log_e(\sec x + \tan x) + C$

$= \log_e(\tan(\frac{\pi}{4} + \frac{x}{2}) + C$

34 **Let** $u = x^2, du = 2x dx$ **and** $dv = \sin x dx,\ v = -\cos x$

i.e. $\int x^2 \sin x dx = uv - \int v du$

i.e. $x^2 \times -\cos x - \int -\cos x \times 2x dx$

i.e. $-x^2\cos x + 2\int x\cos x dx$

Now, let $u' = x, du' = dx$**and** $dv' = \cos x dx, v' = \sin x$

$\int x\cos x dx = u'v' - \int v' du'$

$$= x \times \sin x - \int \sin x \times dx$$

$$= x\sin x - \int \sin x dx$$

$$= x\sin x + \cos x + C$$

i.e. $-x^2\cos x + 2\int x\cos x dx = -x^2\cos x + 2(x\sin x + \cos x)$

i.e. $-x^2\cos x + 2x\sin x + 2\cos x + C$

35 Let $\dfrac{x^2}{(x-1)(x-2)(x-3)} = \dfrac{a}{x-1} + \dfrac{b}{x-2} + \dfrac{c}{x-3}$

Then $x^2 = a(x-2)(x-3) + b(x-1)(x-3) + c(x-1)(x-2)$
$= a(x^2-5x+6) + b(x^2-4x+3) + c(x^2-3x+2)$
$= (a+b+c)x^2 + (-5a-4b-3c)x + 6a+3b+2c$

By equating coefficients

$x^2 : a+b+c = 1 \quad \ldots\ldots(1)$
$x^1 : -5a-4b-3c = 0 \quad \ldots\ldots(2)$
$x^0 : 6a+3b+2c = 0 \quad \ldots\ldots(3)$
Equations $3(1)+(2) : -2a-b = 3 \quad \ldots\ldots(4)$
Equations $2(1)-(3) : -4a-b = 2 \quad \ldots\ldots(5)$
Equations $(4)-(5) : 2a = 1, \therefore a = 1/2$

Substitute $a = 1/2$ into equation (4)
$-2(1/2) - b = 3, \therefore b = -4$
Substitute a and b into equation (1)
$1/2 - 4 + c = 1, \therefore c = 9/2$

Thus $\dfrac{x^2}{(x-1)(x-2)(x-3)} = \dfrac{1}{2(x-1)} - \dfrac{4}{x-2} + \dfrac{9}{2(x-3)}$

$$\int \frac{x^2}{(x-1)(x-2)(x-3)}dx = \int\left[\frac{1}{2(x-1)} - \frac{4}{x-2} + \frac{9}{2(x-3)}\right]dx$$

$$= \frac{1}{2}\ln(x-1) - 4\ln(x-2) + \frac{9}{2}\ln(x-3) + C$$

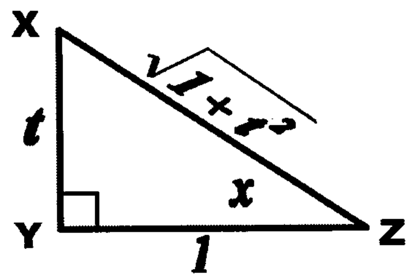
36 Let $u = e^x - 1, du = e^x dx$

i.e. $\int \dfrac{e^x dx}{e^x - 1} = \int \dfrac{du}{u} = \ln u + C = \ln(e^x - 1) + C$

37 Let $t = \tan x, dt = \sec^2 x dx$ or dx

$$= \cos^2 x dt = \frac{1}{1+t^2}dt$$

Also $3\sin^2 x + 5\cos^2 x = 3\sin^2 x + 5(1-\sin^2 x)$
$= 5 - 2\sin^2 x$



i.e. $\int \dfrac{dx}{3\sin^2 x + 5\cos^2 x} = \int \dfrac{dx}{5 - 2\sin^2 x}$

i.e. $\int \dfrac{\frac{1}{1+t^2}dt}{5 - 2(\frac{t^2}{1+t^2})} = \int \dfrac{\frac{dt}{1+t^2}}{\frac{5+5t^2-2t^2}{1+t^2}}$

i.e. $\int \dfrac{dt}{5+3t^2} = \dfrac{1}{3}\int \dfrac{dt}{t^2+5/3}$

i.e. $\dfrac{1}{3} \times \dfrac{1}{\sqrt{5/3}}\tan^{-1}(\dfrac{t}{\sqrt{5/3}}) + C$

i.e. $\dfrac{1}{\sqrt{15}}\tan^{-1}(\sqrt{3/5}t) + C$

i.e. $\dfrac{1}{\sqrt{15}}\tan^{-1}(\sqrt{3/5}\tan x) + C$

38 $\dfrac{d(5x^4-7)}{dx} = 20x^3$

i.e. $\int x^3 e^{5x^4-7}dx = \dfrac{1}{20}\int 20x^3 e^{5x^4-7}dx$

i.e. $\dfrac{1}{20}e^{5x^4-7} + C$

39 Let $u = \log x, du = \dfrac{1}{x}dx$ and $dv = x^5 dx$

$$v = \frac{x^6}{6}$$

i.e. $\int x^5 \log x dx = uv - \int v du$

i.e. $\log x \times \dfrac{x^6}{6} - \int \dfrac{x^6}{6} \times \dfrac{1}{x}dx$

i.e. $\dfrac{x^6}{6}\log x - \dfrac{1}{6}\int x^5 dx = \dfrac{x^6}{6}\log x - \dfrac{x^6}{36} + C$

40 Let $\dfrac{3x+2}{x(x+1)^3} = \dfrac{a}{x} + \dfrac{b}{(x+1)^3} +$

$$\frac{c}{(x+1)^2} + \frac{d}{x+1}$$

Then $3x+2 = a(x+1)^3 + bx + cx(x+1) +$

$$\begin{aligned} & dx(x+1)^2 \\ &= a(x^3+3x^2+3x+1) + bx + \\ & cx^2 + cx + dx(x^2+2x+1) \\ &= (a+d)x^3 + (3a+c+2d)x^2 + \\ & (3a+b+c+d)x + a \end{aligned}$$

By equating coefficients

$x^0 : a = 2$ ......(1)

$x^1 : 3a+b+c+d = 3$ ......(2)

$x^2 : 3a+c+2d = 0$ ......(3)

$x^3 : a+d = 0$ ......(4)

Substitute $a=2$ **into equation** (4)

$2+d=0, \therefore d=-2$

Substitute a **and** d **into equation** (3)

$3(2)+c+2(-2)=0, \therefore c=-2$

Substitute a, c **and** d **into equation** (2)

$3(2)+b-2-2=3, \therefore b=1$

Thus $\dfrac{3x+2}{x(x+1)^3} = \dfrac{2}{x} + \dfrac{1}{(x+1)^3} -$

$$\frac{2}{(x+1)^2} - \frac{2}{x+1}$$

$$\int \frac{3x+2}{x(x+1)^3}dx = \int \left(\frac{2}{x} + \frac{1}{(x+1)^3} - \frac{2}{(x+1)^2} - \frac{2}{x+1}\right)dx$$

$$= 2\int \frac{dx}{x} + \int \frac{dx}{(x+1)^3} - \int \frac{2dx}{(x+1)^2} - \int \frac{2dx}{x+1}$$

$$= 2\ln x + \frac{(x+1)^{-2}}{-2} - \frac{2(x+1)^{-1}}{-1} - 2\ln(x+1) + C$$

$$= 2\ln x - \frac{1}{2(x+1)^2} + \frac{2}{x+1} - 2\ln(x+1) + C$$

41 Let $u = \log x^3 = 3\log x, du = 3\dfrac{1}{x}dx = \dfrac{3dx}{x}$

$dv = dx,\ v = x$

i.e. $\int \log x^3 dx = uv - \int vdu$

i.e. $3\log x \times x - \int x \times \dfrac{3dx}{x} = 3x\log x - 3\int dx$

i.e. $= 3x\log x - 3x + C = 3(x\log x) + C$

42 $u = e^x, du = e^x dx$

i.e. $\displaystyle\int \frac{dx}{e^x + e^{-x}} = \int \frac{dx}{e^x + \frac{1}{e^x}} = \int \frac{e^x dx}{e^{2x}+1}$

i.e. $\displaystyle\int \frac{du}{u^2+1} = \tan^{-1} u + C = \tan^{-1} e^x + C$

43 If $u = 5x^3 + 7x - 1, du = (15x^2+7)dx$

i.e. $\int (5x^3+7x-1)^{3/2}(15x^2+7)dx$

i.e. $\displaystyle\int u^{3/2}du = \frac{u^{5/2}}{5/2} + C = \frac{2}{5}u^{5/2} + C$

i.e. $\dfrac{2}{5}(5x^3+7x-1)^{5/2} + C$

44 Let $\dfrac{1}{(x^2+1)(x^2+4)} = \dfrac{ax+b}{x^2+1} + \dfrac{cx+d}{x^2+4}$

Then $1 = (ax+b)(x^2+4) + (cx+d)(x^2+1)$

$$\begin{aligned} &= (a+c)x^3 + (b+d)x^2 + \\ & (4a+c)x + 4b + d \end{aligned}$$

By equating coefficients

$x^0 : 4b+d = 1$ ......(1)

$x^1 : 4a+c = 0$ ......(2)

$x^2 : b+d = 0$ ......(3)

$x^3 : a+c = 0$ ......(4)

Equations $(1)-(3)$

$3b = 1, \therefore b = 1/3$

Equations $(2)-(4)$

$3a = 0, \therefore a = 0$

Substitute $a=0$ **into equation** (2)

$4(0)+c=0, \therefore c=0$

Substitute $b=1/3$ **into equation** (3)

$1/2 + d = 0, \therefore d = -1/3$

Thus $\dfrac{1}{(x^2+1)(x^2+4)} = \dfrac{1/3}{x^2+1} + \dfrac{-1/3}{x^2+4}$

$$\int \frac{1}{(x^2+1)(x^2+4)}dx = \int \left(\frac{1/3}{x^2+1} + \frac{-1/3}{x^2+4}\right)dx$$

$$= \frac{1}{3}\int \frac{dx}{x^2+1} -$$

$$\frac{1}{3}\int \frac{dx}{x^2+4}$$

$$= \frac{1}{3}\tan^{-1} x -$$

$$\frac{1}{3} \times \frac{1}{2}\tan^{-1}\frac{x}{2} + C$$

$$= \frac{1}{3}(\tan^{-1} x -$$

$$\frac{1}{2}\tan^{-1}\frac{x}{2}) + C$$

45 If $x^2 + x + 1 = x^2 + x + \frac{1}{4} + 1 - \frac{1}{4}$

$$= (x+\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2$$

i.e. $\int (x^2+x+1)^{-1}dx = \int \frac{dx}{x^2+x+1}$

i.e. $\int \frac{dx}{(x+\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2}$

i.e. $\frac{1}{\frac{\sqrt{3}}{2}}\tan^{-1}(\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}}) + C$

i.e. $\frac{2}{\sqrt{3}}\tan^{-1}(\frac{2x+1}{\sqrt{3}}) + C$

46 Let $u = e^x, du = e^x dx$; $dv = \sin 2x dx$

$$v = -\frac{1}{2}\cos 2x$$

i.e. $\int e^x \sin 2x dx = uv - \int v du$

i.e. $e^x \times -\frac{1}{2}\cos 2x - \int -\frac{1}{2}\cos 2x \times e^x dx$

i.e. $-\frac{1}{2}e^x \cos 2x + \frac{1}{2}\int e^x \cos 2x dx$

Now, let $u' = e^x, du' = e^x dx; dv' = \cos 2x dx$

$$v' = \frac{1}{2}\sin 2x$$

$\therefore \int e^x \cos 2x dx = u'v' - \int v' du'$

$$= e^x \times \frac{1}{2}\sin 2x - \int \frac{1}{2}\sin 2x \times e^x dx$$

$$= \frac{1}{2}e^x \sin 2x - \frac{1}{2}\int e^x \sin 2x dx$$

Then $\int e^x \sin 2x dx = -\frac{1}{2}e^x \cos 2x +$

$$\frac{1}{2}(\frac{1}{2}e^x \sin 2x - \frac{1}{2}\int e^x \sin 2x dx)$$

$$= -\frac{1}{2}e^x \cos 2x +$$

$$\frac{1}{4}e^x \sin 2x - \frac{1}{4}\int e^x \sin 2x dx$$

i.e. $\frac{5}{4}\int e^x \sin 2x dx = -\frac{1}{2}e^x \cos 2x +$

$$\frac{1}{4}e^x \sin 2x$$

i.e. $\int e^x \sin 2x dx = \frac{4}{5}(-\frac{1}{2}e^x \cos 2x +$

$$\frac{1}{4}e^x \sin 2x) + C$$

i.e. $\int e^x \sin 2x dx = \frac{e^x}{5}(\sin 2x - 2\cos 2x) + C$

47 If $x^2 + x - 1 = x^2 + x + \frac{1}{4} - 1 - \frac{1}{4}$

$$= (x+\frac{1}{2})^2 - \frac{5}{4}$$

$$= (x+\frac{1}{2})^2 - (\frac{\sqrt{5}}{2})^2$$

i.e. $\int (x^2+x-1)^{-1}dx = \int \frac{dx}{x^2+x-1}$

i.e. $\int \frac{dx}{(x+\frac{1}{2})^2 - (\frac{\sqrt{5}}{2})^2}$

i.e. $\frac{1}{2(\frac{\sqrt{5}}{2})}\log_e(\frac{x+\frac{1}{2}-\frac{\sqrt{5}}{2}}{x+\frac{1}{2}+\frac{\sqrt{5}}{2}}) + C$

i.e. $\frac{1}{\sqrt{5}}\log_e(\frac{2x+1-\sqrt{5}}{2x+1+\sqrt{5}}) + C$

48 If $x^2 - x = x^2 - x + \frac{1}{4} - \frac{1}{4}$

$$= (x-\frac{1}{2})^2 - (\frac{1}{2})^2$$

i.e. $\int(x^2-x)^{-1/2}dx = \int \frac{dx}{\sqrt{x^2-x}}$

i.e. $\int \frac{dx}{\sqrt{(x-\frac{1}{2})^2-(\frac{1}{2})^2}}$

i.e. $\ln(x-\frac{1}{2}+\sqrt{x^2-x})+C$

49 $\int(\frac{1-2x}{3+x})dx = \int \frac{dx}{3+x} - 2\int \frac{xdx}{3+x}$

i.e. $\ln(3+x) - 2\int \frac{(x+3-3)dx}{3+x}$

i.e. $\ln(3+x) - 2\int dx + \int \frac{6dx}{3+x}$

i.e. $\ln(3+x) - 2x + 6\ln(3+x) + C$
i.e. $7\ln(3+x) - 2x + C$

50 Let $u = x^2+4, du = 2xdx$

Also $x^3dx = \frac{1}{2}(\underbrace{2xdx}_{\text{du}} \times \overbrace{x^2}^{\text{u}-4})$

$= \frac{1}{2}[du(u-4)]$

i.e. $\int x^3(4+x^2)^{-1/2}dx = \int \frac{x^3dx}{\sqrt{x^2+4}}$

i.e. $\int \frac{\frac{1}{2}[du(u-4)]}{\sqrt{u}} = \frac{1}{2}\int \frac{(u-4)du}{\sqrt{u}}$

i.e. $\frac{1}{2}\int(\sqrt{u}-\frac{4}{\sqrt{u}})du = \frac{1}{2}[\frac{u^{3/2}}{3/2} -$

$\frac{4u^{1/2}}{1/2}] + C$

i.e. $\frac{1}{2}[\frac{2u^{3/2}}{3} - 8u^{1/2}] + C = \frac{u^{3/2}}{3} -$

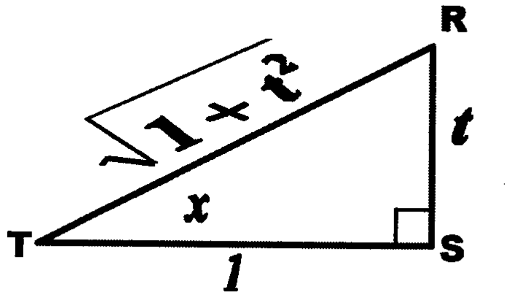
$4u^{1/2} + C$

i.e. $u^{1/2}(\frac{u}{3}-4)+C$

i.e. $\sqrt{x^2+4}(\frac{x^2+4}{3}-4)+C$

i.e. $\sqrt{x^2+4}(\frac{x^2-8}{3})+C$

51 let $t = \tan x, dt = \sec^2 xdx$ or $dx = \cos^2 xdt$



Let $\frac{2t}{(4t^2+3)(1+t^2)} = \frac{at+b}{4t^2+3} + \frac{ct+d}{1+t^2}$

Then $2t = (at+b)(1+t^2) + (ct+d)(4t^2+3)$
$= (a+4c)t^3 + (b+4d)t^2 + (a+3c)t + b + 3d$

By equating coefficients
$t^0 : b+3d = 0$ ......(1)
$t^1 : a+3c = 2$ ......(2)

$t^2 : b+4d = 0$ ......(3)
$t^3 : a+4c = 0$ ......(4)
Equations $(4)-(2) : c = -2$
Equations $(3)-(1) : d = 0$
Substitute $c = -2$ **into equation** (2)
$a + 3(-2) = 2, \therefore a = 8$
Substitute $d = 0$ **into equation** (3)
$b + 4(0) = 0, \therefore b = 0$

i.e. $\int \frac{\sin 2xdx}{3\cos^2 x + 4\sin^2 x}$

i.e. $\int \frac{2\sin x\cos x(\cos^2 xdt)}{3\cos^2 x + 4(1-\cos^2 x)}$

i.e. $\int \frac{2\sin x\cos x(\cos^2 xdt)}{4-\cos^2 x}$

i.e. $\int \frac{(\frac{2t}{1+t^2})(\frac{1}{1+t^2})dt}{4-\frac{1}{1+t^2}}$

i.e. $\int \frac{2tdt}{(4t^2+3)(1+t^2)} = \int(\frac{8tdt}{4t^2+3} - \int \frac{2tdt}{1+t^2}$

$= \ln(4t^2+3) -$
$\ln(1+t^2) + C$

i.e. $\ln\frac{4t^2+3}{1+t^2} + C = \ln\frac{\frac{4\sin^2 x}{\cos^2 x}+3}{1+\frac{\sin^2 x}{\cos^2 x}} + C$

$$\text{i.e. } \ln \frac{\frac{4\sin^2 x + 3\cos^2 x}{\cos^2 x}}{\frac{\cos^2 x + \sin^2 x}{\cos^2 x}} + C$$

$$\text{i.e. } \ln \frac{4\sin^2 x + 3(1 - \sin^2 x)}{\cos^2 x + \sin^2 x} + C$$

i.e. $\ln(3+\sin^2 x)+C$

52 Let $\frac{x^2}{1-x^4} = \frac{ax+b}{1+x^2} + \frac{cx+d}{1-x^2}$

Then $x^2 = (ax+b)(1-x^2) + (cx+d)(1+x^2)$
$= (-a+c)x^3 + (-b+d)x^2 + (a+c)x+b+d$

By equating coefficients

$x^0 : b + d = 0 \quad \ldots\ldots(1)$
$x^1 : a + c = 0 \quad \ldots\ldots(2)$
$x^2 : -b + d = 1 \quad \ldots\ldots(3)$
$x^3 : -a + c = 0 \quad \ldots\ldots(4)$

Equations $(1) + (3) :\ 2d = 1, \therefore d = 1/2$
Equations $(2) + (4) :\ 2c = 0,\ c = 0$
Substitute $d = 1/2$ **into equation** (1)
$b + 1/2 = 0, \therefore b = -1/2$
Substitute $c = 0$ **into equation** (2)
$a + 0 = 0, \therefore a = 0$

Thus $\frac{x^2}{1-x^4} = \frac{-1/2}{1+x^2} + \frac{1/2}{1-x^2}$

$$\text{i.e.} \int \frac{x^2}{1-x^4} = \int \left(\frac{-1/2}{1+x^2} + \frac{1/2}{1-x^2}\right)dx$$

$$\text{i.e. } -\frac{1}{2}\int \frac{dx}{1+x^2} + \frac{1}{2}\int \frac{dx}{1+x^2}$$

$$\text{i.e.} -\frac{1}{2}\tan^{-1} x + \frac{1}{2} \times \frac{1}{2(1)} \ln\left(\frac{1+x}{1-x}\right) + C$$

$$\text{i.e.} -\frac{1}{2}\tan^{-1} x + \frac{1}{4}\ln\left(\frac{1+x}{1-x}\right) + C$$

53 $\int \frac{dx}{\sin x \cos x} = \int \frac{2dx}{\underbrace{2\sin x \cos x}_{\sin 2x}} = \int \frac{dx}{\sin 2x}$

$$\text{i.e. } \int \cos ec2x dx = \int \cos ecex \times \frac{(\cos ec2x + \cot 2x)}{(\cos ec2x + \cot 2x)}$$

i.e. $-\ln(\cos ec2x + \cot 2x) + C$

54 Let $u = \log\sqrt{x-1}, du = \frac{1}{\sqrt{x-1}} \times \frac{1}{2\sqrt{x-1}}dx$ and

$dv = 1dx, v = x$

i.e. $\int \log\sqrt{x-1}dx = uv - \int vdu$

$$\text{i.e. } \log\sqrt{x-1} \times x - \int x \times \frac{1}{2(x-1)}dx$$

$$\text{i.e. } x\log\sqrt{x-1} - \frac{1}{2}\int \frac{xdx}{x-1}$$

$$\text{i.e. } x\log\sqrt{x-1} - \frac{1}{2}\int \frac{(x-1+1)dx}{x-1}$$

$$\text{i.e. } x\log\sqrt{x-1} - \frac{1}{2}\int dx - \frac{1}{2}\int \frac{dx}{x-1}$$

$$\text{i.e. } x\log\sqrt{x-1} - \frac{1}{2}x - \frac{1}{2}\ln(x-1) + C$$

55 Let $\frac{1}{u(u-1)} = \frac{a}{u} + \frac{b}{u-1}$

Then $1 = a(u-1) + bu$
$= (a+b)u - a$

By equating coefficients

$u^0 : -a = 1$ **or** $a = -1 \quad \ldots\ldots(1)$
$u^1 : a + b = 0 \quad \ldots\ldots(2)$

From equation (1) **Substitute** $a = -1$ **into equation** (2)
$-1 + b = 0, \therefore b = 1$
Thus

$$\frac{1}{u(u-1)} = -\frac{1}{u} + \frac{1}{u-1}$$

If $u = e^x, du = e^x dx$ **or** $dx = \frac{du}{e^x} = \frac{du}{u}$

$$\text{i.e. } \int \frac{dx}{e^x - 1} = \int \frac{\frac{du}{u}}{u-1} = \int \frac{du}{u(u-1)}$$

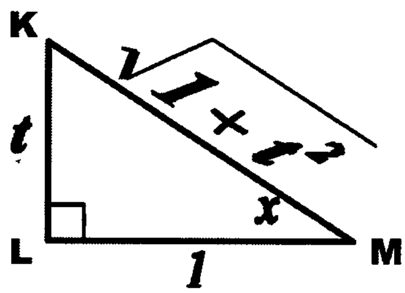
$$\text{i.e.} \int \frac{1du}{u(u-1)} = \int \left(-\frac{1}{u} + \frac{1}{u-1}\right)du$$

i.e. $-\ln u + \ln(u-1) + C$

i.e. $-\ln e^x + \ln(e^x - 1) + C$

i.e. $\ln(e^x-1)-x+C$

56 **If** $t = \tan x$, $dt = \sec^2 x dx$ **or** $dx = \cos^2 x dt$



Let $\dfrac{1}{t^2 - 3t + 2} = \dfrac{a}{t-1} + \dfrac{b}{t-2}$

Then $1 = a(t-2) + b(t-1)$
$= (a+b)t - 2a - b$
By equating coefficients
$t^0 : -2a - b = 1 \quad \ldots\ldots(1)$
$t^1 : \quad a + b = 0 \quad \ldots\ldots(2)$
Equations $(1) + (2)$
$-a = 1, \therefore \quad a = -1$
Substitute $a = -1$ **into equation** (2)
$-1 + b = 0, \therefore b = 1$

Thus $\dfrac{1}{t^2 - 3t + 2} = -\dfrac{1}{t-1} + \dfrac{1}{t-2}$

i.e. $\displaystyle\int \frac{\sec^2 x dx}{\tan^2 x - 3\tan x + 2}$

i.e. $\displaystyle\int \frac{\frac{1}{\cos^2 x} dx}{\frac{\sin^2 x}{\cos^2 x} - 3(\frac{\sin x}{\cos x}) + 2}$

i.e. $\displaystyle\int \frac{\frac{1}{\cos^2 x} dx}{\frac{1-\cos^2 x}{\cos^2 x} - 3(\frac{\sin x}{\cos x}) + 2}$

i.e. $\displaystyle\int \frac{dx}{1 - \cos^2 x - 3(\sin x \cos x) + 2\cos^2 x}$

i.e. $\displaystyle\int \frac{dx}{1 + \cos^2 x - 3\sin x \cos x}$

i.e. $\displaystyle\int \frac{\cos^2 x dt}{1 + \frac{1}{1+t^2} - 3 \times \frac{t}{\sqrt{1+t^2}} \times \frac{1}{\sqrt{1+t^2}}}$

i.e. $\displaystyle\int \frac{\frac{1}{1+t^2} dt}{1 + \frac{1}{1+t^2} - \frac{3t}{1+t^2}}$

i.e. $\displaystyle\int \frac{dt}{1 + t^2 + 1 - 3t}$

i.e. $\displaystyle\int \frac{dt}{t^2 - 3t + 2} = \int(-\frac{1}{t-1} + \frac{1}{t-2})dt$

i.e. $\displaystyle -\int \frac{dt}{t-1} + \int \frac{dt}{t-2} = \ln(t-2) -$

$\ln(t-1) + C$

i.e. $\ln(\tan x - 2) - \ln(\tan x - 1) + C$

i.e. $\ln(\dfrac{\tan x - 2}{\tan x - 1}) + C$

57 **If** $u = x^2 - 3x + 2$, $\dfrac{d(x^2 - 3x + 2)}{dx} = 2x - 3$

i.e. $\displaystyle\int \frac{(x+1)dx}{(x^2 - 3x + 2)^{1/2}} = \frac{1}{2}\int \frac{2(x+1)dx}{(x^2 - 3x + 2)^{1/2}}$

i.e. $\displaystyle\frac{1}{2}\int \frac{(2x+2)dx}{(x^2 - 3x + 2)^{1/2}}$

i.e. $\displaystyle\frac{1}{2}\int \frac{(2x - 3 + 5)dx}{(x^2 - 3x + 2)^{1/2}}$

i.e. $\displaystyle\frac{1}{2}\int \frac{(2x-3)dx}{(x^2 - 3x + 2)^{1/2}} +$

$\displaystyle\frac{5}{2}\int \frac{dx}{(x^2 - 3x + 2)^{1/2}}$

i.e. $\sqrt{x^2 - 3x + 2} +$

$\dfrac{5}{2}\ln(x - \dfrac{3}{2} + \sqrt{x^2 - 3x + 2})$

Since $x^2 - 3x + 2 = \underbrace{x^2 - 3x + \frac{9}{4}}_{\mathbf{(x-3/2)^2}} \underbrace{- \frac{9}{4} + 2}_{\mathbf{(1/2)^2}}$

58 **If** $u = \cos x, du = -\sin x dx$

i.e. $\int \sin 2x \cos x dx = \int 2\sin x \cos x \cos x dx$

i.e. $\int 2\sin x \cos^2 x dx = -2\int -\sin x \underbrace{\cos^2 x}_{\mathbf{u^2}} dx$

i.e. $-2\int u^2 du = -2\dfrac{u^3}{3} + C$

i.e. $-\dfrac{2}{3}\cos^3 x + C$

59 Let $\dfrac{x}{1+x^3} = \dfrac{a}{1+x} + \dfrac{bx+c}{x^2-x+1}$

Then $x = a(x^2-x+1)+(bx+c)(1+x)$
$= (a+b)x^2 + (-a+b+c)x + a + c$

By equating coefficients

$x^0 : a+c=0 \quad \ldots\ldots(1)$

$x^1 : -a+b+c=1 \quad \ldots\ldots(2)$

$x^2 : a+b=0 \quad \ldots\ldots(3)$

Equations $(2)-(3):\ -2a+c=1 \quad \ldots\ldots(4)$

Equations $(4)-(1):\ -3a=1, \therefore a=-1/3$

Substitute $a=-1/3$ **into equation** (1)

$-1/3+c=0 \quad , \therefore c=1/3$

Substitute a **and** b **into equation** (2)

$1/3+b+1/3=1, \therefore b=1/3$

Thus $\dfrac{x}{1+x^3} = -\dfrac{1/3}{1+x} + \dfrac{x/3+1/3}{x^2-x+1}$

Also $x^2-x+1 = \underbrace{x^2-x+\frac{1}{4}}_{(x-1/2)^2}\underbrace{-\frac{1}{4}+1}_{(\sqrt{3}/2)^2}$

i.e. $\displaystyle\int \frac{xdx}{1+x^3} = \int\left(-\frac{1/3}{1+x} + \frac{x/3+1/3}{x^2-x+1}\right)dx$

i.e. $\displaystyle -\frac{1}{3}\ln(x+1) + \frac{1}{3}\int \frac{(x+1)dx}{x^2-x+1}$

i.e. $\displaystyle -\frac{1}{3}\ln(x+1) + \frac{1}{6}\int \frac{(2x+2)dx}{x^2-x+1}$

i.e. $\displaystyle -\frac{1}{3}\ln(x+1) + \frac{1}{6}\int \frac{(2x-1+3)dx}{x^2-x+1}$

i.e. $\displaystyle -\frac{1}{3}\ln(x+1) + \frac{1}{6}\int \frac{(2x-1)dx}{x^2-x+1} + \frac{1}{6}\int \frac{3dx}{x^2-x+1}$

i.e. $\displaystyle -\frac{1}{3}\ln(x+1) + \frac{1}{6}\ln(x^2-x+1) + \frac{1}{6}\int \frac{3dx}{(x-1/2)^2}$

i.e. $\displaystyle -\frac{1}{3}\ln(x+1) + \frac{1}{6}\ln(x^2-x+1) + \frac{1}{2}\times\frac{1}{\sqrt{3}/2}\tan^{-1}\frac{x-1/2}{\frac{\sqrt{3}}{2}} + C$

i.e. $\displaystyle -\frac{1}{3}\ln(x+1) + \frac{1}{6}\ln(x^2-x+1) + \frac{1}{\sqrt{3}}\tan^{-1}\frac{2x-1}{\sqrt{3}} + C$

60 Let $u = \tan^{-1}x, du = \dfrac{1}{1+x^2}dx$ **and**

$dv = xdx,\ v = \dfrac{x^2}{2}$

i.e. $\int x\tan^{-1}xdx = uv - \int vdu$

i.e. $\displaystyle \tan^{-1}x \times \frac{x^2}{2} - \int \frac{x^2}{2}\times\frac{1}{1+x^2}dx$

i.e. $\displaystyle \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int \frac{x^2dx}{1+x^2}$

i.e. $\displaystyle \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int \frac{(x^2+1-1)dx}{1+x^2}$

i.e. $\displaystyle \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int dx + \frac{1}{2}\int \frac{dx}{1+x^2}$

i.e. $\displaystyle \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}x + \frac{1}{2}\tan^{-1}x + C$

i.e. $\displaystyle \frac{1}{2}(x^2\tan^{-1}x + \tan^{-1}x - x) + C$

61 Let $2x^2+3x+1$

$= 2(x^2 + \frac{3}{2}x + \frac{1}{2})$

$= 2(\underbrace{x^2+\frac{3}{2}x+\frac{9}{16}}_{(x+3/4)^2}\underbrace{-\frac{9}{16}+\frac{1}{2}}_{(1/4)^2})$

$= 2[(x+\frac{3}{4})^2 - (\frac{1}{4})^2]$

i.e. $\displaystyle \int(1+3x+2x^2)^{-1}dx = \int \frac{dx}{x^2+\frac{3}{2}x+\frac{1}{2}}$

i.e. $\displaystyle \int \frac{dx}{2[(x+\frac{3}{4})^2-(\frac{1}{4})^2]}$

i.e. $\displaystyle \frac{1}{2}\int \frac{dx}{(x+\frac{3}{4})^2-(\frac{1}{4})^2}$

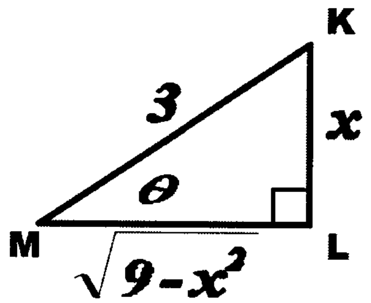
i.e. $\frac{1}{2} \times \frac{1}{2(\frac{1}{4})} \log_e(\frac{x+\frac{3}{4}-\frac{1}{4}}{x+\frac{3}{4}+\frac{1}{4}}) + C_1$

i.e. $\log_e(\frac{x+\frac{1}{2}}{x+1}) + C_1 = \log_e \frac{2x+1}{2(x+1)} + C_1$

i.e. $\log_e(2x+1) - \log_2 2 - \log_e(x+1) + C_1$

i.e. $\log_e(1+2x) - \log_e(x+1) + C$

62 If $x = 3\sin\theta, dx = 3\cos\theta d\theta$



i.e. $\int(9-x^2)^{1/2}dx = \int(9-9\sin^2\theta)^{1/2}\times 3\cos\theta d\theta$

i.e. $9\int(1-\sin^2\theta)^{1/2}\cos\theta d\theta = 9\int\cos\theta\cos\theta d\theta$

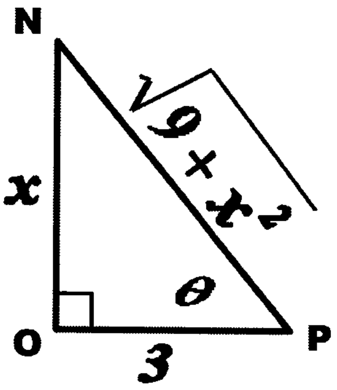
i.e. $\frac{9}{2}\int(1+\cos 2\theta)d\theta = \frac{9}{2}(\int d\theta + \int\cos 2\theta d\theta)$

i.e. $\frac{9}{2}(\theta + \frac{\sin 2\theta}{2}) + C = \frac{9}{2}(\theta + \frac{2\sin\theta\cos\theta}{2}) + C$

i.e. $\frac{9}{2}(\theta + \sin\theta\cos\theta) + C = \frac{9}{2}(\sin^{-1}\frac{x}{3} + \frac{x}{3}\times\frac{\sqrt{9-x^2}}{3}) + C$

i.e. $\frac{1}{2}(9\sin^{-1}\frac{x}{3} + x\sqrt{9-x^2}) + C$

63 If $x = 3\tan\theta, dx = 3\sec^2\theta d\theta$
Then $9 + x^2 = 9 + 9\tan^2\theta = 9(1+\tan^2\theta)$
$= 9\sec^2\theta$



i.e. $\int(9+x^2)^{1/2}dx = \int(9\sec^2\theta)^{1/2}3\sec^2\theta d\theta$

i.e. $\int 3\sec\theta\times 3\sec^2\theta d\theta = 9\int\sec\theta\sec^2\theta d\theta$

$= 9\int\sec^3\theta d\theta$

Let $u = \sec\theta, du = \sec\theta\tan\theta d\theta$ **and**
$dv = \sec^2\theta d\theta, v = \tan\theta$

$\therefore \int\sec\theta\sec^2\theta d\theta = uv - \int vdu$

$\int\sec^3\theta d\theta = \sec\theta\times\tan\theta - \int\tan\theta\times\sec\theta\tan\theta d\theta$

$\int\sec^3\theta d\theta = \sec\theta\tan\theta - \int\sec\theta\tan^2\theta d\theta$

$\int\sec^3\theta d\theta = \sec\theta\tan\theta - \int\sec\theta(\sec^2\theta - 1)d\theta$ **or**

$2\int\sec^3\theta d\theta = \sec\theta\tan\theta + \int\sec\theta d\theta$

$= \sec\theta\tan\theta + \int\sec\theta(\frac{\sec\theta+\tan\theta}{\sec\theta+\tan\theta})d\theta$

$= \sec\theta\tan\theta + \ln(\sec\theta+\tan\theta)$

$\therefore \int\sec^3\theta d\theta = \frac{\sec\theta\tan\theta}{2} + \frac{1}{2}\ln(\sec\theta+\tan\theta)$

i.e. $9\int\sec^3\theta d\theta = 9[\frac{\sec\theta\tan\theta}{2} + \frac{1}{2}\ln(\sec\theta+\tan\theta)]$

i.e. $\frac{9}{2}\sec\theta\tan\theta + \frac{9}{2}\ln(\sec\theta+\tan\theta) + C_1$

i.e. $\frac{9}{2}(\frac{\sqrt{9+x^2}}{3})(\frac{x}{3}) +$

$$\frac{9}{2}\ln(\frac{\sqrt{9+x^2}}{3}+\frac{x}{3})+C_1$$

i.e. $\frac{1}{2}x\sqrt{9+x^2}+\frac{9}{2}\ln(\frac{x+\sqrt{9+x^2}}{3})+C_1$

i.e. $\frac{1}{2}x\sqrt{9+x^2}+$

$$\frac{9}{2}[\ln(x+\sqrt{9+x^2})-\ln 3]+C_1$$

i.e. $\frac{1}{2}x\sqrt{9+x^2}+\frac{9}{2}\ln(x+\sqrt{9+x^2})+C$

i.e. $\frac{1}{2}(x\sqrt{9+x^2}+9\ln(x+\sqrt{9+x^2}))+C$

64 **If** $u=9+x^2, du=2xdx$

i.e. $\int x(9+x^2)^{1/2}dx=\frac{1}{2}\int 2x(9+x^2)^{1/2}dx$

i.e. $\frac{1}{2}\int u^{1/2}du=\frac{1}{2}[\frac{u^{3/2}}{3/2}]+C$

i.e. $\frac{1}{3}[u^{3/2}]+C$

i.e. $\frac{1}{3}[(9+x^2)^{3/2}]+C$

65 **If** $u=\tan^3 x, du=3\tan^2 x\sec^2 xdx$ **and** $dv=\sec^2 xdx,\ v=\tan x$

i.e. $\int \sec^2 x\tan^3 xdx=uv-\int vdu$

i.e. $\tan^3 x\times\tan x-\int\tan x\times 3\tan^2 x\sec^2 xdx$

i.e. $\tan^4 x-3\int\sec^2 x\tan^3 xdx$

i.e. $\int\sec^2 x\tan^3 xdx=\tan^4 x-$

$$3\int\sec^2 x\tan^3 xdx$$

i.e. $4\int\sec^2 x\tan^3 xdx=\tan^4 x$

i.e. $\int\sec^2 x\tan^3 xdx=\frac{\tan^4 x}{4}+C$

66 **If** $u=x^2, du=2xdx$ **and** $dv=e^{-x}dx,\ v=-e^{-x}$

i.e. $\int x^2e^{-x}dx=uv-\int vdu$

i.e. $x^2\times -e^{-x}-\int -e^{-x}\times 2xdx$

i.e. $-x^2e^{-x}+2\int xe^{-x}dx$

Let $u'=x, du'=dx$ **and**

$dv'=e^{-x}dx,\ v'=-e^{-x}$

$\therefore \int xe^{-x}dx=u'v'-\int v'du'$

$$=x\times -e^{-x}-\int -e^{-x}\times dx$$

$$=-xe^{-x}+\int e^{-x}dx$$

i.e. $-x^2e^{-x}+2\int xe^{-x}dx=-x^2e^{-x}+$

$$2(-xe^{-x}+\int e^{-x}dx)$$

i.e. $-x^2e^{-x}+2\int xe^{-x}dx=-x^2e^{-x}-$

$$2xe^{-x}+2\int e^{-x}dx$$

i.e. $-x^2e^{-x}+2\int xe^{-x}dx=-x^2e^{-x}-$

$$2xe^{-x}-2e^{-x}+C$$

i.e. $-x^2e^{-x}+2\int xe^{-x}dx=-e^{-x}(x^2+$

$$2x+2)+C$$

67 **Let** $u=x^2, du=2xdx$

i.e. $\int xe^{x^2}dx=\frac{1}{2}\int 2xe^{x^2}dx=\frac{1}{2}\int e^u du$

i.e. $\frac{1}{2}e^u+C$

i.e. $\frac{1}{2}e^{x^2}du+C$

68 $\int\sin x\tan xdx=\int\sin x\times\frac{\sin x}{\cos x}dx$

i.e. $\int\frac{\sin^2 x}{\cos x}dx=\int\frac{(1-\cos^2 x)}{\cos x}dx$

i.e. $\int\frac{dx}{\cos x}-\int\frac{\cos^2 x}{\cos x}dx$

i.e. $\int\sec xdx-\int\cos xdx$

i.e.$\int\sec x(\frac{\sec x+\tan x}{\sec x+\tan x})dx-\int\cos xdx$

i.e.$\ln(\sec x+\tan x)dx-\sin x+C$

69 **Let** $u=\sin x, du=\cos xdx$

i.e. $\int\sin^4 x\cos^3 xdx=\int\sin^4 x\cos x\times\cos^2 xdx$

i.e. $\int \sin^4 x \cos x \times (1-\sin^2 x)dx$

i.e. $\int u^4(1-u^2)du = \int (u^5-u^6)du$

i.e. $\frac{u^6}{6} - \frac{u^7}{7} + C$

70 **Let** $\frac{x^2-x+1}{x^3-x} = m + \frac{a}{x} + \frac{b}{x-1}$

Then $x^2-x+1 = mx(x-1)+a(x-1)+bx$
$= mx^2+(a+b-m)x-a$

By equating coefficients

$x^0 : -a = 1$ ......(1)
$x^1 : a+b-m = -1$ ......(2)
$x^2 : m = 1$ ......(3)

Substitute $a=-1$ **and** $m=1$ **into equation** (2)

$-1+b-1=-1, \therefore b=1$

Thus $\frac{x^2-x+1}{x^3-x} = 1 - \frac{1}{x} + \frac{1}{x-1}$

i.e. $\int \frac{x^2-x+1}{x^3-x}dx = \int (1-\frac{1}{x}+\frac{1}{x-1})dx$

i.e. $\int (1-\frac{1}{x}+\frac{1}{x-1})dx = \int dx -$

$\int \frac{dx}{x} + \int \frac{dx}{x-1}$

i.e. $x - \ln x + \ln(x-1) + C$

71 **Let** $u = \log(x+\sqrt{x^2-1})$

$du = \frac{1}{x+\sqrt{x^2-1}} \times$

$(1+\frac{1}{2\sqrt{x^2-1}} \times 2x)dx$

$= \frac{1}{x+\sqrt{x^2-1}} \times (1+\frac{x}{\sqrt{x^2-1}})dx$

$= \frac{1}{x+\sqrt{x^2-1}} \times (\frac{\sqrt{x^2-1}+x}{\sqrt{x^2-1}})dx$

$= \frac{dx}{\sqrt{x^2-1}}$ **and** $dv = 1dx$, $v = x$

i.e. $\int \log(x+\sqrt{x^2-1})dx = uv - \int vdu$

i.e. $\log(x+\sqrt{x^2-1}) \times x - \int x \times \frac{dx}{\sqrt{x^2-1}}$

i.e. $x\log(x+\sqrt{x^2-1}) - \frac{1}{2}\int \frac{2xdx}{\sqrt{x^2-1}}$

i.e. $x\log(x+\sqrt{x^2-1}) - \sqrt{x^2-1} + C$

72 **If** $u = \sqrt{x+1}, du = \frac{1}{2\sqrt{x+1}}dx$ **or**

$dx = 2\sqrt{x+1}du = 2udu$

i.e. $\int \frac{dx}{(x+1)^{1/2}+(x+1)} = \int \frac{2udu}{u+u^2}$

$= \int \frac{2du}{1+u}$

i.e. $2\ln(1+u)+C = \mathbf{2\ln(1+\sqrt{x+1})+C}$

73 $\int_0^4 \frac{xdx}{\sqrt{x+4}} = \int_0^4 \frac{(x+4-4)dx}{\sqrt{x+4}}$

i.e. $\int_0^4 (\sqrt{x+4} - \frac{4}{\sqrt{x+4}})dx$

i.e. $[\frac{(x+4)^{3/2}}{3/2} - \frac{4(x+4)^{1/2}}{1/2}]_0^4$

i.e. $[\frac{2(x+4)^{3/2}}{3} - 8(x+4)^{1/2}]_0^4$

i.e. $2(x+4)^{1/2}[\frac{(x+4)}{3} - 4]_0^4$

i.e. $2(x+4)^{1/2}[\frac{(x-8)}{3}]_0^4$

i.e. $\frac{2}{3}(x+4)^{1/2}[(x-8)]_0^4$

i.e. $\{\frac{2}{3}(2^3)^{1/2}[-2^2]\} - \{\frac{2}{3}(2^2)^{1/2}[(-8)]\}$

i.e. $\frac{16}{3}(2-\sqrt{2})$

74 **Let** $\frac{1}{x(1+x^2)} = \frac{a}{x} + \frac{bx+c}{1+x^2}$

Then $1 = a(1+x^2)+(bx+c)x$
$= (a+b)x^2+cx+a$

By equating coefficients

$x^0 : a = 1$ ......(1)
$x^1 : c = 0$ ......(2)
$x^2 : a+b = 0$ ......(3)

From (1) **substitute** $a=1$ **into equation** (3)

$1+b=0, \therefore b=-1$

Thus $\frac{1}{x(1+x^2)} = \frac{1}{x} - \frac{x}{1+x^2}$

i.e. $\int_1^2 \frac{1}{x(1+x^2)}dx = \int_1^2(\frac{1}{x} - \frac{x}{1+x^2})dx$

i.e. $\int_1^2(\frac{dx}{x} - \frac{1}{2}\int \frac{2xdx}{1+x^2} = \ln x-$

$\frac{1}{2}\ln(1+x^2)]_1^2$

i.e. $\ln x - \ln\sqrt{(1+x^2)}]_1^2 = \ln\frac{x}{\sqrt{(1+x^2)}}]_1^2$

i.e. $[(\ln\frac{2}{\sqrt{5}}) - (\ln\frac{1}{\sqrt{2}})] = \ln\frac{\frac{2}{\sqrt{5}}}{\frac{1}{\sqrt{2}}})$

i.e. $\ln\frac{2\sqrt{2}}{\sqrt{5}} = \ln\frac{\sqrt{8}}{\sqrt{5}} = \ln\sqrt{\frac{8}{5}}$

i.e. $\frac{1}{2}\ln\frac{8}{5}$

75 **Let** $u = \log x, du = \frac{1}{x}dx$

If $x = 1, u = 0$; **if** $x = 2, u = \log 2$

i.e. $\int_1^2 \frac{\log x}{x}dx = \int_0^{\log 2} udu = [\frac{u^2}{2}]_0^{\log 2}$

i.e. $\frac{1}{2}[u^2]_0^{\log 2} = \frac{1}{2}[(\log 2)^2 - 0^2]$

i.e. $\frac{1}{2}(\log 2)^2$

76 **Let** $u = \cos^{-1}x, du = -\frac{1}{\sqrt{1-x^2}}dx$ **and** $dv = 1dx,\ v = x$

i.e. $\int_0^1 \cos^2 xdx = uv - \int vdu$

i.e. $\cos^{-1}x \times x]_0^1 - \int_0^1 x \times -\frac{1}{\sqrt{1-x^2}}dx$

i.e. $x\cos^{-1}x]_0^1 + \int_0^1 \frac{xdx}{\sqrt{1-x^2}}$

i.e. $x\cos^{-1}x]_0^1 + \frac{1}{2}\int_0^1 \frac{2xdx}{\sqrt{1-x^2}}$

i.e. $x\cos^{-1}x]_0^1 - \frac{1}{2}\int_0^1 \frac{-2xdx}{\sqrt{1-x^2}}$

i.e. $[x\cos^{-1}x]_0^1 - \sqrt{1-x^2}]_0^1$

i.e. $[(1\underbrace{\cos^{-1}1}_{0} - \sqrt{0}) -$

$(0\underbrace{\cos^{-1}0}_{\pi/2} - \sqrt{1-0^2})]$

i.e. 1

77 **If** $\frac{d(-2+3x-x^2)}{dx} = -2x+3$

Also
$-2+3x-x^2 = -(x^2-3x+2)$

$= -(\underbrace{x^2 - 3x + \frac{9}{4}}_{(x-3/2)^2} \underbrace{-\frac{9}{4} + 2)}_{(1/2)^2}$

$= (\frac{1}{2})^2 - (x - \frac{3}{2})^2$

i.e. $\int_1^2 \frac{(x+1)dx}{\sqrt{-2+3x-x^2}}$

i.e. $\frac{1}{2}\int_1^2 \frac{2(x+1)dx}{\sqrt{-2+3x-x^2}}$

i.e. $-\frac{1}{2}\int_1^2 \frac{(-2x-2)dx}{\sqrt{-2+3x-x^2}}$

i.e. $-\frac{1}{2}\int_1^2 \frac{(-2x+3-5)dx}{\sqrt{-2+3x-x^2}}$

i.e. $-\frac{1}{2}\int_1^2 \frac{(-2x+3)dx}{\sqrt{-2+3x-x^2}} +$

$\frac{5}{2}\int_1^2 \frac{dx}{\sqrt{-2+3x-x^2}}$

i.e. $-\sqrt{-2+3x-x^2}]_1^2 +$

$\frac{5}{2}\int_1^2 \frac{dx}{\sqrt{(\frac{1}{2})^2 - (x-\frac{3}{2})^2}}$

i.e. $-\sqrt{-2+3x-x^2}]_1^2 + \frac{5}{2}\sin^{-1}\frac{x-\frac{3}{2}}{\frac{1}{2}}]_1^2$

i.e. $[(-\sqrt{-2+6-4}+\frac{5}{2}\sin^{-1}\frac{2-\frac{3}{2}}{\frac{1}{2}})-$

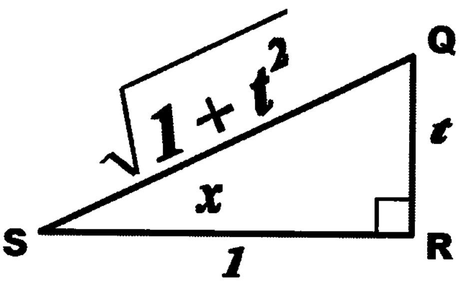
$$(-\sqrt{-2+3-1}+\frac{5}{2}\sin^{-1}\frac{1-\frac{3}{2}}{\frac{1}{2}})]$$

i.e. $[(0+\frac{5}{2}\sin^{-1}1)-(0+\frac{5}{2}\sin^{-1}(-1))]$

i.e. $[(0+\frac{5}{2}\underbrace{\sin^{-1}1}_{\pi/2})-(0+\frac{5}{2}\underbrace{\sin^{-1}(-1)}_{-\pi/2})]$

i.e. $\frac{5\pi}{4}+\frac{5\pi}{4}=\mathbf{\frac{5\pi}{2}}$

78 **Let** $t=\tan x$
If $x=0, t=0$; **if** $x=\pi/2, t=\infty$



i.e. $\int_0^{\pi/2}\frac{dx}{\cos^2 x+2\sin^2 x}$

i.e. $\int_0^{\pi/2}\frac{dx}{1-\sin^2 x+2\sin^2 x}$

i.e. $\int_0^{\pi/2}\frac{dx}{1+\sin^2 x}=\int_0^{\infty}\frac{\frac{dt}{1+t^2}}{1+\frac{t^2}{1+t^2}}$

$$=\int_0^{\infty}\frac{dt}{2t^2+1}$$

i.e. $\frac{1}{2}\int_0^{\infty}\frac{2dt}{t^2+(\frac{1}{\sqrt{2}})^2}=\frac{1}{2}\times$

$$\frac{1}{\frac{1}{\sqrt{2}}}\tan^{-1}\frac{t}{\frac{1}{\sqrt{2}}}]_0^{\infty}$$

i.e. $\frac{1}{\sqrt{2}}\tan^{-1}\sqrt{2}(\tan x)]_0^{\infty}$

i.e. $\frac{1}{\sqrt{2}}[\underbrace{\tan^{-1}\sqrt{2}(\tan x)}_{\pi/2}-\underbrace{\tan^{-1}\sqrt{2}(\tan 0)}_{0}]$

i.e. $\frac{1}{\sqrt{2}}[\frac{\pi}{2}-0]=\frac{\pi}{2\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=\mathbf{\frac{\pi\sqrt{2}}{4}}$

79 **Let** $u=1-x^2, du=-2xdx$
If $x=0, u=1$; **if** $x=1, u=0$

i.e. $\int_0^1 x\sqrt{1-x^2}dx=-\frac{1}{2}\int_{0}^{1} 2x\sqrt{1-x^2}dx$

i.e. $-\frac{1}{2}\int_1^0\sqrt{u}du=-\frac{1}{2}\frac{u^{3/2}}{3/2}]_1^0=-\frac{1}{3}[u^{3/2}]_1^0$

i.e. $-\frac{1}{3}[0^{3/2}-1^{3/2}]=\mathbf{\frac{1}{3}}$

80 **Let** $u=\log x, du=\frac{1}{x}dx$ **and**

$dv=xdx,\ v=\frac{x^2}{2}$

i.e. $\int_2^4 x\log xdx=uv-\int vdu$

i.e. $\log x\times\frac{x^2}{2}]_2^4-\int_2^4\frac{x^2}{2}\times\frac{1}{x}dx$

i.e. $[\frac{x^2}{2}\log x-\frac{1}{2}\int_2^4 x]_2^4=[\frac{x^2}{2}\log x-\frac{x^2}{4}]_2^4$

i.e. $[(8\log 2^2-4)-(2\log 2-1)]$

i.e. $14\log 2-3$

81 **Let** $\frac{1}{x^2+5x+4}=\frac{a}{x+1}+\frac{b}{x+4}$

Then $1=a(x+4)+b(x+1)$
$=(a+b)x+4a+b$
By equating coefficients
$x^0: 4a+b=1 \quad \ldots\ldots(1)$
$x^1: a+b=0 \quad \ldots\ldots(2)$
Equations (1)−(2)
$3a=1, \therefore a=1/3$
Substitute $a=1/3$ **into equation** (2)
$1/3+b=0, b=-1/3$

Thus $\frac{1}{x^2+5x+4}=\frac{1/3}{x+1}-\frac{1/3}{x+4}$

i.e. $\int_1^2\frac{1}{x^2+5x+4}dx=\int_1^2(\frac{1/3}{x+1}-\frac{1/3}{x+4})dx$

i.e. $\int_1^2(\frac{1/3}{x+1}-\frac{1/3}{x+4})dx=\frac{1}{3}\ln(x+1)-$

$$\frac{1}{3}\ln(x+4)]_1^2$$

i.e. $\frac{1}{3}[\ln(x+1)-\ln(x+4)]_1^2 = \frac{1}{3}[\ln(\frac{x+1}{x+4})]_1^2$

i.e. $\frac{1}{3}[(\ln\frac{3}{6})-(\ln\frac{2}{5})] = \frac{1}{3}[(\ln\frac{\frac{1}{2}}{\frac{2}{5}})]$

i.e. $\frac{1}{3}\ln\frac{5}{4}$

82 Let $t=\tan\frac{x}{2}, dt=\frac{1}{2}\sec^2\frac{x}{2}dx$ **or**

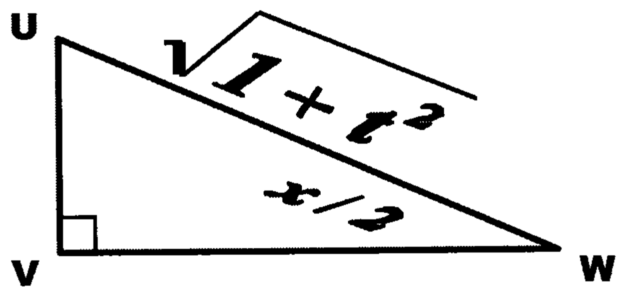
$$dx = 2\cos^2\frac{x}{2}dt = 2(\frac{1}{1+t^2})dt$$

$$= \frac{2dt}{1+t^2}$$

If $x=0, t=0$; **if** $x=\pi/2, t=1$

Also $t^2+t+1 = t^2+t+\frac{1}{4}-\frac{1}{4}+1$

$$= (t+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2$$



i.e. $\int_0^{\pi/2}(1+\frac{1}{2}\sin x)^{-1}dx = \int_0^{\pi/2}\frac{dx}{1+\frac{1}{2}\sin x}$

i.e. $\int_0^1 \frac{\frac{2dt}{1+t^2}}{1+\frac{1}{2}\times\frac{2t}{1+t^2}} = \int_0^1\frac{2dt}{t^2+t+1}$

i.e. $\int_0^1 \frac{2dt}{(t+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2} = 2\times$

$$\frac{1}{\frac{\sqrt{3}}{2}}\tan^{-1}(\frac{t+\frac{1}{2}}{\frac{\sqrt{3}}{2}})]_0^1$$

i.e. $\frac{4}{\sqrt{3}}\tan^{-1}(\frac{2t+1}{\sqrt{3}})]_0^1$

i.e. $\frac{4}{\sqrt{3}}[\tan^{-1}(\sqrt{3})-\tan^{-1}(\frac{1}{\sqrt{3}})]$

i.e. $\frac{4}{\sqrt{3}}[\frac{\pi}{3}-\frac{\pi}{6}] = \frac{4}{\sqrt{3}}[\frac{\pi}{6}] = \frac{2\pi}{3\sqrt{3}}$

i.e. $\mathbf{\frac{2\pi\sqrt{3}}{9}}$

83 Let $u=x^2, du=2xdx$ **and**
$dv=e^{-x}dx, v=-e^{-x}$

i.e. $\int x^2e^{-x}dx = uv-\int vdu$

i.e. $x^2\times -e^{-x}-\int -e^{-x}\times 2xdx$

i.e. $-x^2e^{-x}+\int 2xe^{-x}dx$

If $u'=x, du'=dx$ **and**
$dv'=e^{-x}dx, v'=-e^{-x}$

$\therefore \int xe^{-x}dx = u'v'-\int v'du'$

$$= x\times -e^{-x}-\int -e^{-x}\times dx$$

$$= -xe^{-x}+\int e^{-x}dx = -xe^{-x}-e^{-x}$$

i.e. $-x^2e^{-x}+\int_0^1 2xe^{-x}dx = -x^2e^{-x}+$

$$2(-xe^{-x}-e^{-x})]_0^1$$

i.e. $-e^{-x}(x^2+2x+2)]_0^1 = \{-e^{-1}(1^2+$

$$2\times 1+2)\}-\{-e^0(0^2+2\times 0+2)\}$$

i.e. $\{-e^{-1}(5)\}-\{-1(2)\} = 2-5e^{-1}$

i.e. $2-\frac{5}{e}$

84 If $1+x+x^2+x^3 = (1+x)+x^2(1+x)$
$= (x^2+1)(x+1)$

Let $\frac{7+x}{(x^2+1)(x+1)} = \frac{a}{x+1}+\frac{bx+c}{x^2+1}$

Then $7+x = a(x^2+1)+(bx+c)(x+1)$
$= (a+b)x^2+(b+c)x+a+c$

By equating coefficients

$x^0: a+c=7$ ......(1)

$x^1: b+c=1$ ......(2)

$x^2 : a+b=0 \qquad \ldots\ldots(3)$

Equations $(1)+(2)+(3)$

$2(a+\underbrace{b+c}_{1}) = 8$ **or** $a+1=4, \therefore a=3$

Substitute $a=3$ **into equations** (3) **and** (1)

$3+b=0, \therefore b=-3$

Also $3+c=7, \therefore c=4$

Thus $\dfrac{7+x}{(x^2+1)(x+1)} = \dfrac{3}{x+1} + \dfrac{-3x+4}{x^2+1}$

i.e. $\int_0^1 \dfrac{7+x}{(x^2+1)(x+1)}dx = \int_0^1 (\dfrac{3}{x+1} + \dfrac{-3x+4}{x^2+1})dx$

i.e. $3\ln(x+1)]_0^1 - \int_0^1 \dfrac{3xdx}{x^2+1} + \int_0^1 \dfrac{4dx}{x^2+1}$

i.e. $3\ln(x+1)]_0^1 - \dfrac{3}{2}\int_0^1 \dfrac{2xdx}{x^2+1} + \int_0^1 \dfrac{4dx}{x^2+1}$

i.e. $3\ln(x+1)]_0^1 - \dfrac{3}{2}\ln(x^2+1) + 4\tan^{-1}x]_0^1$

i.e. $[3\ln(x+1) - \dfrac{3}{2}\ln(x^2+1) + 4\tan^{-1}x]_0^1$

i.e. $[(3\ln 2 - \dfrac{3}{2}\ln 2 + 4\underbrace{\tan^{-1}1}_{\pi/4}) - (3\underbrace{\ln 1}_{0} - \dfrac{3}{2}\underbrace{\ln 1}_{0} + 4\underbrace{\tan^{-1}0}_{0})]$

i.e. $[(\dfrac{3}{2}\ln 2 + \pi) - (0-0+0)]$

i.e. $\dfrac{3}{2}\ln 2 + \pi$

i.e. $x - \ln x + \ln(x-1) + C$

85 **Let** $u = e^{-x}, du = -e^{-x}dx$ **or** $dx = -\dfrac{du}{u}$

If $x=0, u=1$; **If** $x=1, u=e^{-1}$

i.e. $\int_0^1 \dfrac{e^{-2x}dx}{e^{-x}+1} = \int_1^{e^{-1}} \dfrac{u^2(-\frac{du}{u})}{u+1}$

$= -\int_1^{e^{-1}} \dfrac{udu}{u+1}$

i.e. $-\int_1^{e^{-1}} \dfrac{(u+1-1)du}{u+1} = -\int_1^{e^{-1}} du + \int_1^{e^{-1}} \dfrac{du}{u+1}$

i.e. $[-u + \ln(u+1)]_1^{e^{-1}}$

i.e. $[\{-e^{-1} + \ln(e^{-1}+1)\} - \{-1 + \ln(2)\}]$

i.e. $1 + \dfrac{1}{e} + \ln(\dfrac{1}{e}+1) - \ln 2$

i.e. $1 + \dfrac{1}{e} + \ln(\dfrac{1+e}{e}) - \ln 2$

i.e. $1 + \dfrac{1}{e} + \ln\dfrac{1+e}{2e}$

86 $\int_0^{a/2} \dfrac{ydy}{a-y} = -\int_0^{a/2} \dfrac{-ydy}{a-y}$

$= -\int_0^{a/2} \dfrac{(a-y-a)dy}{a-y}$

i.e. $-\int_0^{a/2}(1 - \dfrac{a}{a-y})dy = -y]_0^{a/2} + a\int_0^{a/2} \dfrac{dy}{a-y}$

i.e. $[-y - a\ln(a-y)]_0^{a/2}$

i.e. $[\{-a/2 - a\ln(a-a/2)\} - \{0 - a\ln(a-0\}]$

i.e. $-a/2 - a\ln(a/2)\} + a\ln a$

i.e. $-a/2 - a\ln(a/2) + a\ln a = -a/2 - a\ln a + a\ln 2 + a\ln a$

i.e. $-a/2 - a\ln 2 + a\ln 2 + a\ln a$

i.e. $-a/2 + a\ln a = a(\ln a - 1/2)$

i.e. $\dfrac{a}{2}(2\ln a - 1) = \dfrac{a}{2}(\ln 4 - 1)$

i.e. $= \dfrac{a}{2}(2\ln 2 - 1)$

87 $\int_0^a \dfrac{(a-x)^2dx}{a^2+x^2} = \int_0^a \dfrac{(a^2+x^2-2ax)dx}{a^2+x^2}$

i.e. $\int_0^a dx - \int_0^a \dfrac{2axdx}{a^2+x^2} = x - a\ln(a^2+x^2)]_0^a$

i.e. $[(a - a\ln(a^2+a^2)) - (0 - a\ln(a^2+0^2))]$

i.e. $a - a\ln 2a^2 + a\ln a^2 = a - a\ln 2-$

$a\ln a^2 + a\ln a^2$

i.e. $a - a\ln 2 = a(1-\ln 2)$

88 Let $\dfrac{x+3}{(x+2)(x+1)^2} = \dfrac{a}{x+2} + \dfrac{b}{(x+1)^2} + \dfrac{c}{x+1}$

Then $x+3 = a(x+1)^2 + b(x+2) + c(x+1)(x+2)$
$= a(x^2+2x+1) + b(x+2) + c(x^2+3x+2)$
$= (a+c)x^2 + (2a+b+3c)x + a+2b+2c$

By equating coefficients

$x^0 : a+2b+2c = 3 \quad \ldots\ldots(1)$
$x^1 : 2a+b+3c = 1 \quad \ldots\ldots(2)$
$x^2 : a+c = 0 \quad \ldots\ldots(3)$

From equation (3) **substitute** $a=-c$ **into equations** (1) **and** (2)

$-c+2b+2c = 3$ **or**
$2b+c = 3 \quad \ldots\ldots(4)$

Also $2(-c)+b+3c = 1$ **or**
$b+c = 1 \quad \ldots\ldots(5)$

Equations $(4)-(5) : b=2$

Substitute $b=2$ **into equation** (5)

$2+c = 1, \therefore c=-1$

Substitute $c=-1$ **into equation** (3)

$a-1=0, \therefore a=1$

Thus $\dfrac{x+3}{(x+2)(x+1)^2} = \dfrac{1}{x+2} + \dfrac{2}{(x+1)^2} - \dfrac{1}{x+1}$

i.e. $\int_0^{31} \dfrac{x+3}{(x+2)(x+1)^2}dx = \int_0^1 (\dfrac{1}{x+2} + \dfrac{2}{(x+1)^2} - \dfrac{1}{x+1})dx$

i.e. $[\ln(x+2) + \dfrac{2(x+1)^{-1}}{-1} - \ln(x+1)]_0^1$

i.e. $[\ln\dfrac{x+2}{x+1} - \dfrac{2}{x+1}]_0^1$

i.e. $[\ln\dfrac{3}{2} - 1 - (\ln 2 - 2)] = [\ln\dfrac{\frac{3}{2}}{2} + 1]$

i.e. $\ln\dfrac{3}{4} + 1$

89 Let $u = x^3, du = 3x^2dx$

If $x=0, u=0$; **if** $x=1, u=1$

i.e. $\int_0^1 \dfrac{x^2dx}{x^6+1} = \dfrac{1}{3}\int_0^1 \dfrac{3x^2dx}{(x^3)^2+1}$

$= \dfrac{1}{3}\int_0^1 \dfrac{du}{u^2+1}$

i.e. $\dfrac{1}{3}\tan^{-1} u]_0^1 = \dfrac{1}{3}[\tan^{-1} 1 - \tan^{-1} 0]$

i.e. $\dfrac{1}{3}[\dfrac{\pi}{4} - 0] = \mathbf{\dfrac{\pi}{12}}$

90 $\int_0^\pi \cos^2 mx dx = \int_0^\pi (\dfrac{1+\cos 2mx}{2})dx$

i.e. $\dfrac{1}{2}\int_0^\pi dx + \dfrac{1}{2}\int_0^\pi (\cos 2mx)dx$

i.e. $\dfrac{1}{2}x + \dfrac{1}{2} \times \dfrac{\sin 2mx}{2}]_0^\pi = [\dfrac{1}{2}x + \dfrac{1}{4}\sin 2mx]_0^\pi$

i.e. $[(\dfrac{1}{2}\pi + \dfrac{1}{4}\underbrace{\sin 2m\pi}_{0}) - (\dfrac{1}{2}\times 0 + \dfrac{1}{4}\sin 2m \times 0)]$

i.e. $\mathbf{\dfrac{\pi}{2}}$

91 Let $u = x, du = dx$ **and**
$dv = \sin 2x dx, v = -\dfrac{\cos 2x}{2}$

i.e. $\int_{\pi/4}^{\pi/2} x\sin 2x dx = uv - \int v du$

i.e. $x \times -\dfrac{\cos 2x}{2}]_{\pi/4}^{\pi/2} - \int_{\pi/4}^{\pi/2} -\dfrac{\cos 2x}{2} \times dx$

i.e. $-\dfrac{x\cos 2x}{2}]_{\pi/4}^{\pi/2} + \dfrac{1}{2}\int_{\pi/4}^{\pi/2} \cos 2x dx$

i.e. $-\dfrac{x\cos 2x}{2}]_{\pi/4}^{\pi/2} + \dfrac{1}{2} \times \dfrac{\sin 2x}{2}]_{\pi/4}^{\pi/2}$

i.e. $[-\dfrac{x\cos 2x}{2} + \dfrac{1}{4}\sin 2x]_{\pi/4}^{\pi/2}$

i.e. $[(-\dfrac{\pi/2\cos 2\times\pi/2}{2} + \dfrac{1}{4}\sin 2\times\pi/2) - (-\dfrac{\pi/4\cos 2\times\pi/4}{2} + \dfrac{1}{4}\sin 2\times\pi/4)]$

i.e. $[(\frac{\pi}{4}+0)-(-0+\frac{1}{4})]$

i.e. $\frac{1}{4}(\pi-1)$

92 Let $x=a\sin\theta . dx=a\cos\theta d\theta$
If $x=0,\theta=0$; if $x=a/2,\theta=\pi/6$

i.e. $\int_0^{a/2} x^2\sqrt{a^2-x^2}dx$

i.e. $\int_0^{\pi/6} a^2\sin^2\theta\sqrt{a^2-a^2\sin^2\theta}a\cos\theta d\theta$

i.e. $\int_0^{\pi/6} a^2\sin^2\theta a\cos\theta a\cos\theta d\theta$

i.e. $\int_0^{\pi/6} a^4\sin^2\theta\cos^2\theta d\theta$

i.e. $a^4\int_0^{\pi/6}(\frac{\sin 2\theta}{2})^2\theta d\theta$

i.e. $\frac{a^4}{4}\int_0^{\pi/6}\sin^2 2\theta d\theta$

i.e. $\frac{a^4}{4}\int_0^{\pi/6}(\frac{1-\cos 4\theta}{2})d\theta$

i.e. $\frac{a^4}{8}\int_0^{\pi/6}(1-\cos 4\theta)d\theta$

i.e. $\frac{a^4}{8}[\theta-\frac{\sin 4\theta}{4}]_0^{\pi/6}$

i.e. $\frac{a^4}{8}[(\pi/6-\frac{\sin 4\times\pi/6}{4})-$

$(0-\frac{\sin 4\times 0}{4})]$

i.e. $\frac{a^4}{8}[(\pi/6-\frac{\sqrt{3}/2}{4})-(0-0)]$

i.e. $\frac{a^4}{8}[(\frac{4\pi-3\sqrt{3}}{24})=\frac{a^4}{182}(4\pi-3\sqrt{3})$

93 Let $u=\tan x, du=\sec^2 xdx$ and
$dv=\sec^2 xdx, v=\tan x$

i.e. $\int_0^{\pi/4}\sec^2 x\tan xdx=uv-\int vdu$

i.e. $\tan x\times\tan x]_0^{\pi/4}-\int_0^{\pi/4}\tan x\times\sec^2 xdx$

i.e. $\tan^2 x]_0^{\pi/4}-\int_0^{\pi/4}\tan x\sec^2 xdx$

i.e. $\tan^2 x]_0^{\pi/4}-\frac{\tan^2 x}{2}]_0^{\pi/4}=[\tan^2 x-$

$\frac{\tan^2 x}{2}]_0^{\pi/4}$

i.e. $[\frac{\tan^2 x}{2}]_0^{\pi/4}=\frac{1}{2}[\tan^2 x]_0^{\pi/4}$

i.e. $\frac{1}{2}[\tan^2\frac{\pi}{4}-\tan^2 0]=\frac{1}{2}[1-0]$

i.e. $\frac{1}{2}$

94 Let $u=x^2+4x+5, du=(2x+4)dx$
$=2(x+2)dx$
If $x=0,u=5$; if $x=1,u=10$

i.e. $\int_0^1(x+2)(x^2+4x+5)^{1/2}dx$

i.e. $\int_0^1\frac{1}{2}\times 2(x+2)(x^2+4x+5)^{1/2}dx$

i.e. $\frac{1}{2}\times\frac{(x^2+4x+5)^{3/2}}{3/2}]_0^1=\frac{1}{3}u^{3/2}]_5^{10}$

i.e. $\frac{1}{3}[10^{3/2}-5^{3/2}]=\frac{1}{3}[(2\times 5)^{3/2}-5^{3/2}]$

i.e. $\frac{1}{3}\times 5^{3/2}[2^{3/2}-1]$

95 Let $u=(\log x)^2, du=2\log x\times\frac{1}{x}dx$

$=\frac{2}{x}\log xdx$

Also $dv=xdx, v=\frac{x^2}{2}$

i.e. $\int_1^2 x(\log x)^2dx=uv-\int vdu$

i.e. $(\log x)^2\times\frac{x^2}{2}]_1^2-\int_1^2\frac{x^2}{2}\times\frac{2}{x}\log xdx$

i.e. $\frac{x^2}{2}\ (\log x)^2]_1^2-\int_1^2 x\log xdx$

Now, let $u'=\log x, du'=\frac{1}{x}dx$;

$dv'=xdx,\ v'=\frac{x^2}{2}$

$\therefore\int_1^2 x\log xdx=u'v'-\int v'du'$

$=\log x\times\frac{x^2}{2}-\int\frac{x^2}{2}\times\frac{1}{x}dx$

$$= \frac{x^2}{2}\log x - \frac{1}{2}\int x dx$$

$$= \frac{x^2}{2}\log x - \frac{1}{2}\times\frac{x^2}{2}]_1^2$$

$$= \frac{x^2}{2}\log x - \frac{x^2}{4}]_1^2$$

i.e. $\frac{x^2}{2}\ (\log x)^2]_1^2 - \int_1^2 x\log x dx =$

i.e. $\frac{x^2}{2}\ (\log x)^2]_1^2 - (\frac{x^2}{2}\log x - \frac{x^2}{4})]_1^2$

i.e. $\frac{x^2}{2}\ (\log x)^2 - \frac{x^2}{2}\log x + \frac{x^2}{4}]_1^2$

i.e. $[\frac{x^2}{2}\ (\log x)^2 - \frac{x^2}{2}\log x + \frac{x^2}{4}]_1^2$

i.e. $[(2\ (\log 2)^2 - 2\log 2 + 1) -$

$$(\frac{1}{2}\ (\log 1)^2 - \frac{1^2}{2}\log 1 + \frac{1^2}{4})]$$

i.e. $[(2\ (\log 2)^2 - 2\log 2 + 1) -$

$$(\frac{1}{2}\ \underbrace{(\log 1)^2}_{0} - \frac{1^2}{2} + \frac{1^2}{4})]$$

i.e. $\mathbf{2\ (\log 2)^2 - 2\log 2 + \frac{3}{4}}$

96 $\int_3^4 (\frac{x^2+4}{x^2-1})dx = \int_3^4 (\frac{x^2-1+5}{x^2-1})dx$

i.e. $\int_3^4 (1+\frac{5}{x^2-1})dx = \int_3^4 dx + \int_3^4 \frac{5dx}{x^2-1}$

i.e. $\int_3^4 dx + \frac{5}{2}\int_3^4 \frac{2dx}{x^2-1} = [x + \frac{5}{2}\ln\frac{x-1}{x+1}]_3^4$

i.e. $[(4+\frac{5}{2}\ln\frac{3}{5}) - (3+\frac{5}{2}\ln\frac{1}{2})]$

i.e. $1 + \frac{5}{2}(\ln\frac{3}{5} - \ln\frac{1}{2}) = 1 + \frac{5}{2}\ln\frac{\frac{3}{5}}{\frac{1}{2}}$

i.e. $\mathbf{1 + \frac{5}{2}\ln\frac{6}{5}}$

97 **Let** $\frac{x^2+4}{x(x-2)} = m + \frac{a}{x} + \frac{b}{x+2}$

Then $x^2+4 = mx(x+2) + a(x+2) + bx$
$= mx^2 + (a+b+2m)x + 2a$

By equating coefficients

$x^0 : 2a = 4$ ......(1)

$x^1 : a + b + 2m = 0$ ......(2)

$x^2 : m = 1$ ......(3)

From equation (1) $a = 2$

Substitute $a = 2$ **and** $m = 1$ **into equation** (2)

$2 + b + 2\times 1 = 0, \therefore b = -4$

Thus $\frac{x^2+4}{x(x-2)} = 1 + \frac{2}{x} - \frac{4}{x+2}$

i.e. $\int_1^4 \frac{x^2+4}{x(x-2)}dx = \int_1^4 (1+\frac{2}{x} - \frac{4}{x+2})dx$

i.e. $[x + 2\ln x - 4\ln(x+2)]_1^4$

i.e. $[(4 + 2\ln 4 - 4\ln 6) - (1 + 2\underbrace{\ln 1}_{0} - 4\ln 3)]$

i.e. $[(4 + 2\underbrace{\ln 4}_{2\ln 2} - 4\underbrace{\ln 6}_{\ln 2 + \ln 3}) -$

$$(1 + 2\underbrace{\ln 1}_{0} - 4\ln 3)]$$

i.e. $[(4 + 4\ln 2 - 4\ln 2 - 4\ln 3) -$

$$(1 + 2\underbrace{\ln 1}_{0} - 4\ln 3)]$$

i.e. $\mathbf{3}$

98 **Let** $u = 5 - 3\sin x, du = -3\cos x dx$

If $x = 0, u = 5$; **if** $x = \pi/2, u = 2$

i.e. $\int_0^{\pi/2} \frac{\cos x dx}{5 - 3\sin x} = -\frac{1}{3}\int_0^{\pi/2} \frac{-3\cos x dx}{5 - 3\sin x}$

i.e. $-\frac{1}{3}\int_2^5 \frac{du}{u} = -\frac{1}{3}\ln u]_2^5 = -\frac{1}{3}[\ln u]_2^5$

i.e. $-\frac{1}{3}[\ln 5 - \ln 2]$

i.e. $\mathbf{-\frac{1}{3}\ln\frac{5}{2}}$

99 **Let** $x = 2\sin\theta, dx = 2\cos\theta d\theta$

If $x = 0, \theta = 0$; **if** $x = 1, u = \pi/6$

i.e. $\int_0^1 \frac{dx}{(4-x^2)^{3/2}} = \int_0^{\pi/6} \frac{2\cos\theta d\theta}{(4 - 4\sin^2\theta)^{3/2}}$

i.e. $\int_0^{\pi/6} \frac{2\cos\theta d\theta}{(2^2 - 2^2\sin^2\theta)^{3/2}}$

i.e. $\frac{1}{4}\int_0^{\pi/6}\frac{\cos\theta d\theta}{(\underbrace{1-\sin^2\theta}_{\cos^2\theta})^{3/2}}$

i.e. $\frac{1}{4}\int_0^{\pi/6}\frac{\cos\theta d\theta}{\cos^3\theta}=\frac{1}{4}\int_0^{\pi/6}\sec\theta d\theta$

i.e. $\frac{1}{4}\tan\theta]_0^{\pi/6}=\frac{1}{4}[\tan\frac{\pi}{6}-\tan 0]$

i.e. $\frac{1}{4}[\frac{\sqrt{3}}{3}-0]$

i.e. $\mathbf{\frac{\sqrt{3}}{12}}$

$\boxed{100}$ **Let** $u=\sin\theta, du=\cos\theta d\theta$
If $\theta=0, u=0$; **if** $x=\pi/2, u=1$

i.e. $\int_0^{\pi/2}2\sin\theta\cos\theta(3\sin\theta-4\sin^3\theta)d\theta$

i.e. $\int_0^1 2u(3u-4u^3)du=\int_0^1(6u^2-8u^4)du$

i.e. $[\frac{6u^3}{3}-\frac{8u^5}{5}]_0^1=[2u^3-\frac{8u^5}{5}]_0^1$

i.e.$[(2-\frac{8}{5})-(2\times 0^3-\frac{8\times 0^5}{5})]$

i.e. $\mathbf{\frac{2}{5}}$