

①

$x =$ input Features
 $y =$ labels

First layer w_1, b_1
second layer w_2, b_2

output of layer 1

$$z_1 = w_1 \cdot x + b_1$$

$a_1 = g(z_1)$ g is activation function

Layer 2:

$$z_2 = w_1 \cdot a_1 + b_2$$

$$a_2 = g(z_2)$$

the o/p =
 $\hat{y} = a_2$

task: regression

loss = mse

mean squared error

$$L = \frac{1}{n} \sum (y - \hat{y})^2$$

① start initial guess for w_1, w_2, b_1, b_2

② $w_i = w_i - \alpha \cdot \frac{\partial L}{\partial w_i}$ $b_i = b_i - \alpha \cdot \frac{\partial L}{\partial b_i}$

③ repeat

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update for second layer $\frac{dL}{dw_2}$

$$\frac{dL}{dw_2} = \frac{d}{dw_2} \left[\frac{1}{J} \sum_j (y - \hat{y})^2 \right]$$

$$= 1 \cdot \frac{d}{dw_2} (y - \hat{y})^2$$

$$= 1 \cdot 2 \cdot (y - \hat{y})^{2-1} \cdot \frac{d}{dw_2} [y - \hat{y}]$$

$$= -2 \cdot (y - \hat{y})$$

$$\hat{y} = a_2 = g(z_2)$$

$g = \text{nothing}$

$$\therefore \frac{\partial L}{\partial w_2} = -2 \cdot (y - a_2) \cdot a_1^T$$

$$\therefore \frac{dL}{dw_2} = -2 \cdot (y - a_2)$$

③

$$\frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial a_2} \cdot \frac{da_2}{dz_2} \cdot \frac{dz_2}{da_1} \cdot \frac{da_1}{dz_1} \cdot \frac{dz_1}{dw_1}$$

$$\therefore \frac{\partial L}{\partial w_1} = \frac{\partial L}{\partial a_2} \cdot \frac{da_2}{dz_2} \cdot \frac{dz_2}{da_1} \cdot \frac{da_1}{dz_1} \cdot \frac{dz_1}{dw_1}$$

rewrite using chain rule

$$\frac{\partial L}{\partial w_1} = -2(y - a_2) \frac{dz_2}{da_1} \cdot \frac{da_1}{dz_1} \cdot \frac{dz_1}{dw_1} \quad \left(\frac{dz_2}{da_1} = w_2 \right)$$

$$a_1 = g(z_1) \text{ then } \frac{da_1}{dz_1} = g'(z_1)$$

$$\therefore \frac{\partial L}{\partial w_1} = -2(y - a_2) \cdot w_2 \cdot g'(z_1) \cdot x$$

$$g = \text{sigmoid } \frac{1}{1 + e^{-x}}$$

$$g' = g(z) \cdot (1 - g(z))$$

$$\therefore \frac{\partial L}{\partial w_1} = w_2^T \cdot g'(z_1) \cdot (-2(y - a_2)) \cdot x^T$$

$$\therefore \frac{\partial L}{\partial w_1} = w_2^T \cdot g'(z_1) \cdot (-2(y - a_2))$$

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question for question 1:

the difference between regression
for mean squared error loss + binary classification
logloss, is the loss function it's self,

instead of having logloss in the update
rule you now have mean squared error