Punto 3

Utilizando el operador Laplaciano es posible escribir un método explícito de solución para la ecuación de onda en coordenadas cilíndricas.

Definimos: $\lambda := \frac{\Delta\rho}{\Delta\phi}$ y $V := \frac{\alpha\Delta t}{\Delta\rho}$

Operador de Laplace

$$\nabla^2 u(\rho,\phi) = \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\Delta\rho^2} + \frac{1}{\rho[i]}\left(\frac{u_{i,j} - u_{i-1,j}}{\Delta\rho}\right)$$
$$+ \frac{1}{\rho[i]^2}\left(\frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{\Delta\phi^2}\right)$$

$\partial_{tt} u(\rho,\phi) = \alpha^2 \nabla^2 u(\rho,\phi) \rightarrow$ Ec onda

$$\rightarrow \frac{u^{l+1}_{i,j} - 2u^{l}_{i,j} + u^{l-1}_{i,j}}{\Delta t^2} = \alpha^2\left(\frac{u^{l}_{i+1,j} - 2u^{l}_{i,j} + u^{l}_{i-1,j}}{\Delta\rho^2} + \frac{1}{\rho[i]}\cdots\right.$$
$$\left.\left(\frac{u^{l}_{i,j} - u^{l}_{i-1,j}}{\Delta\rho}\right) + \frac{1}{\rho[i]^2}\left(\frac{u^{l}_{i,j+1} - 2u^{l}_{i,j} + u^{l}_{i,j-1}}{\Delta\phi^2}\right)\right)$$

Luego se despeja $u^{l+1}_{i,j}$, se multiplica el $\Delta\rho^2$ y usamos el $\lambda = \frac{\Delta\rho}{\Delta\phi}$ y queda

$$\frac{\partial^2 u}{\partial^2 t} = \Delta\rho^2 = \alpha^2 \nabla^2 u \,\Delta\rho^2$$

$$u_{i,j}^{l+1} = \frac{(\Delta t)^2 \alpha^2}{\Delta \rho^2}\left(u_{i+1,j}^{l} - 2u_{i,j}^{l} + \frac{\Delta \rho}{\rho[i]}\left(u_{i,j}^{l} - u_{i-1,j}^{l}\right) + \frac{\Delta \rho^2}{\rho[i]^2 \Delta \phi^2}\left(u_{i,j+1}^{l} - 2u_{i,j}^{l} + u_{i,j-1}^{l}\right)\right)$$

$$+ 2u_{i,j}^{l} - u_{i,j}^{l-1}$$

Ahora se usa el $r = \frac{\alpha \Delta t}{\Delta \rho}$

$$u_{i,j}^{l+1} = r^2\left(u_{i+1,j}^{l} - 2u_{i,j}^{l} + u_{i-1,j}^{l} + \frac{\Delta \rho}{\rho[i]}\left(u_{i,j}^{l} - u_{i-1,j}^{l}\right) + \frac{\lambda^2}{\rho[i]^2}\left(u_{i,j+1}^{l} - 2u_{i,j}^{l} + u_{i,j-1}^{l}\right)\right) + 2u_{i,j}^{l} - u_{i,j}^{l-1}$$