Integrador de Adams-Boshforth

Tres puntos:

$$P(x) = f(x_i) + \binom{s}{1}\Delta f_{i-1} + \binom{s+1}{2}\Delta^2 f_{i-2}$$

$$\hookrightarrow \binom{s}{m} = \frac{s!}{m!(s-n)!} \rightarrow \binom{s}{1} = \frac{s!}{1!(s-1)!}$$

$$y_{i+1} - y_i = \int_0^1 P(x)\, h\, ds$$

menos

$$= \int_0^1 \left(f(t_i) + \frac{s\Delta f_{i-1}}{1!} + \frac{(s+1)\, s\Delta^2 f_{i-2}}{2!} \right) h\, ds$$

$$= h\left[\int_0^1 f(t_i)\, ds + \int_0^1 \frac{s\Delta f_{i-1}}{1!}\, ds + \int_0^1 (s^2+s)\frac{\Delta^2 f_{i-2}}{2!} \right]$$

$$= h\left[f(t_i)\, s\Big|_0^1 + \frac{s^2}{2}\Big|_0^1 \Delta f_{i-1} + \left(\frac{s^3}{3} + \frac{s^2}{2}\right)\Big|_0^1 \frac{\Delta^2 f_{i-2}}{2!} \right]$$

$$= h\left[f(t_i) + \frac{1}{2}\Delta f_{i-1} + \left(\frac{1}{3} + \frac{1}{2}\right)\frac{\Delta f_{i-2}}{2} \right]$$

$$y_{i+1} - y_i = h\left[f_i + \frac{1}{2}\Delta f_{i-1} + \frac{5}{12}\Delta^2 f_{i-2} \right]$$

$$y_{i+1} = y_i + h\left[f_i + \frac{1}{2}\Delta f_{i-1} + \frac{5}{12}\Delta^2 f_{i-2} \right]$$

El minimo común multiplo entre 2 y 12 es 12, así que lo multiplicamos en cada termino.

$$y_{i+1} = y_i + h\left[\frac{12f_i + 6\Delta f_{i-1} + 5\Delta^2 f_{i-2}}{12}\right]$$

$$y_{i+1} = y_i + \frac{h}{12}\left[12f_i + 6\Delta f_{i-1} + 5\Delta^2 f_{i-2}\right]$$

$$\Delta f_{i-1} = f_i - f_{i-1}$$

$$\Delta^2 f_{i-1} = f_i - 2f_{i-1} + f_{i-2}$$ $\longrightarrow$ Reemplazamos

$$y_{i+1} = y_i + \frac{h}{12}\left[12f_i + 6(f_i - f_{i-1}) + 5(f_i - 2f_{i-1} + f_{i-2})\right]$$

$$= y_i + \frac{h}{12}\left[12f_i + 6f_i + 5f_i - 6f_{i-1} - 10f_{i-1} + 5f_{i-2}\right]$$

$$\boxed{= y_i + \frac{h}{12}\left[23f_i - 16f_{i-1} + 5f_{i-2}\right]}$$

Cuatro puntos:

$$P(x) = f(x_i) + \binom{s}{1}\Delta f_{i-1} + \binom{s+1}{2}\Delta^2 f_{i-2} + \binom{s+2}{3}\Delta^3 f_{i-3}$$

$$\hookrightarrow \binom{s}{m} = \frac{s!}{m!(s-n)!} \rightarrow \binom{s}{1} = \frac{s!}{1!(s-1)!}$$

$$y_{i+1} - y_i = \int_0^1 P(x)\, h\, ds$$

$$= \int_0^1 \left[f(t_i) + \frac{s\Delta f_{i-1}}{1!} + \frac{(s-1)s\,\Delta^2 f_{i-2}}{2!} + \frac{(s+2)(s+1)s}{3!}\Delta^3 f_{i-3}\right] h\, ds$$

$$= h\left[\int_0^1 f(t_i)\, ds + \int_0^1 \frac{s\,\Delta f_{i-1}}{1!}\, ds + \int_0^1 \frac{(s^2-s)\,\Delta^2 f_{i-2}}{2!}\, ds + \int_0^1 \frac{(s^3+3s^2+2s)\,\Delta^3 f_{i-3}}{3!}\, ds\right]$$

$$= h\left[f(t_i)\, s\Big|_0^1 + \frac{s^2}{2}\Big|_0^1 \Delta f_{i-1} + \left(\frac{s^3}{3} + \frac{s^2}{2}\right)\Big|_0^1 \frac{\Delta f_{i-2}}{2!} + \frac{1}{3!}\left(\frac{s^4}{4} + \frac{3s^3}{3} + \frac{2s^2}{2}\right)\Big|_0^1 \Delta^3 f_{i-3}\right.$$

$$= h\left[f(t_i) + \frac{1}{2}\Delta f_{i-1} + \left(\frac{1}{3} + \frac{1}{2}\right)\frac{\Delta f_{i-2}}{2} + \frac{1}{6}\left(\frac{1}{4} + 1 + 1\right)\Delta^3 f_{i-3}\right.$$

$$y_{i+1} = y_i + h\left[f_i + \frac{1}{2}\Delta f_{i-1} + \frac{5}{12}\Delta^2 f_{i-2} + \frac{3}{8}\Delta^3 f_{i-3}\right]$$

El minimo comun multiplo entre 2, 12 y 8 es 24, así que multiplicamos en cada termino

$$y_{i+1} = y_i + h\left[\frac{24 f_i + 12\Delta f_{i-1} + 10\Delta^2 f_{i-2} + 9\Delta^3 f_{i-3}}{24}\right]$$

$$= y_i + \frac{h}{24}\left[24 f_i + 12\Delta f_{i-1} + 10\Delta^2 f_{i-2} + 9\Delta^3 f_{i-3}\right]$$

$$\Delta f_{i-1} = f_i - f_{i-1}$$

$$\Delta^2 f_{i-2} = f_i - 2f_{i-1} + f_{i-2}$$ → Reemplazamos

$$\Delta^3 f_{i-3} = f_i - 3f_{i-1} + 3f_{i-2} - f_{i-3}$$

$$= y_i + \frac{h}{24}\left[24 f_i + 12(f_i - f_{i-1}) + 10(f_i - 2f_{i-1} + f_{i-2}) + 9(f_i - 3f_{i-1} + 3f_{i-2} - f_{i-3})\right]$$

$$= y_i + \frac{h}{24}\left[24 f_i + 12 f_i + 10 f_i + 9 f_i - 12 f_{i-1} - 20 f_{i-1} - 27 f_{i-1} + 10 f_{i-2} + 27 f_{i-2} - 9 f_{i-3}\right]$$

$$\boxed{= y_i + \frac{h}{24}\left[55 f_i - 59 f_{i-1} + 37 f_{i-2} - 9 f_{i-3}\right]}$$