Integrador de Adams- Moulton

Tres puntos:

$$P(x) = f(t_{i+1}) + \binom{s}{1}\Delta f_i + \binom{s+1}{2}\Delta^2 f_{i-1}$$

$$\int_{y_i}^{y_{i+1}} dy = \int_{t_i}^{t_{i+1}} \left[f(t_{i+1}) + \binom{s}{1}\Delta f_i + \binom{s+1}{2}\Delta^2 f_{i-1} \right] dt$$

$$= \int_{-1}^{0} \left[f(t_{i+1}) + s\Delta f_i + \frac{s^2+s}{2}\Delta^2 f_{i-1} \right] h\,ds$$

$$= \int_{-1}^{0} f(t_{i+1})\,h\,ds + \int_{-1}^{0} s\Delta f_i\,h\,ds + \int_{-1}^{0} \frac{s^2+s}{2}\Delta^2 f_{i-1}\,h\,ds$$

$$= h f(t_{i+1})\, s\Big|_{-1}^{0} + \frac{s^2}{2}\Delta f_i h\Big|_{-1}^{0} + \left(\frac{s^3}{3}+\frac{s^2}{2}\right)\frac{1}{2}\Delta^2 f_{i-1} h$$

$$= h\left[f t_{i+1} - \frac{1}{2}\Delta f_i - \frac{1}{12}\Delta^2 f_{i-1} \right]$$

$\Delta f = f_{i+1} - f_i$

$\longrightarrow$ Reemplazamos

$\Delta^2 f = f_{i+1} - 2f_i + f_{i-1}$

$$= f_{i+1} - \frac{1}{2}(f_{i+1} - f_i) - \frac{1}{12}(f_{i+1} - 2f_i + f_{i-1})$$

Minimo comun multiplo $\rightarrow$ 12

$$= \frac{1}{12}\left[12 f_{i+1} - 6 f_{i+1} + 6 f_i - f_{i+1} + 2 f_i - f_{i-1}\right] h$$

$$\boxed{y_{i+1} = y_i + \frac{h}{12}\left[5 f_{i+1} + 8 f_i - f_{i-1}\right]}$$

Cuatro puntos:

$$P(x) = f(t_{i+1}) + \binom{s}{1}\Delta f_i + \binom{s+1}{2}\Delta^2 f_{i-1} + \binom{s+2}{3}\Delta^3 f_{i-2}$$

$$\int_{y_i}^{y_{i+1}} dy = \int_{t_i}^{t_{i+1}} \left[f(t_{i+1}) + \binom{s}{1}\Delta f_i + \binom{s+1}{2}\Delta^2 f_{i-1} + \binom{s+2}{3}\Delta^3 f_{i-2} \right] dt$$

$$= \int_{-1}^{0} \left[f(t_{i+1}) + s\Delta f_i + \frac{s^2+s}{2}\Delta^2 f_{i-1} + \frac{s^3+3s^2+2s}{6}\Delta^3 f_{i-2} \right] h\,ds$$

$$= \int_{-1}^{0} f(t_{i+1}) h\,ds + \int_{-1}^{0} s\Delta f_i h\,ds + \int_{-1}^{0} \frac{s^2+s}{2}\Delta^2 f_{i-1} h\,ds + \int \frac{s^3+3s^2+2s}{6}\Delta^3 f_{i-2} h\,ds$$

$$= h f(t_{i+1}) s \Big|_{-1}^{0} + \frac{s^2}{2}\Delta f_i h \Big|_{-1}^{0} + \left(\frac{s^3}{3} + \frac{s^2}{2}\right)\frac{1}{2}\Delta^2 f_{i-1} h \Big|_{-1}^{0} - \frac{1}{24}\Delta^3 f_{i-2} h$$

$$= h\left[f_{i+1} - \frac{1}{2}\Delta f_i - \frac{1}{12}\Delta^2 f_{i-1} - \frac{1}{24}\Delta^3 f_{i-2} \right]$$

$$\Delta f = f_{i+1} - f_i$$

$$\Delta^2 f = f_{i+1} - 2f_i + f_{i-1} \longrightarrow \text{Reemplazamos}$$

$$\Delta^3 f = f_{i+1} - 3f_i + 3f_{i-1} - f_{i-2}$$

$$= f_{i+1} - \frac{1}{2}(f_{i+1} - f_i) - \frac{1}{12}(f_{i+1} - 2f_i + f_{i-1}) - \frac{1}{24}(f_{i+1} - 3f_i + 3f_{i-1} - f_{i-2})$$

Minimo comun multiplo $\rightarrow$ 24

$$= \frac{1}{24}\left[24f_{i+1} - 12f_{i+1} + 12f_i - 2f_{i+1} + 4f_i - 2f_{i-1} - f_{i+1} + 3f_i - 3f_{i-1} + f_{i-2}\right]$$

$$\boxed{y_{i+1} = y_i + \frac{h}{24}\left[9f_{i+1} + 19f_i - 5f_{i-1} + f_{i-2}\right]}$$