Demostrar estabilidad de la función de onda para $\lambda < 1$.

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0 \longrightarrow \frac{\partial u}{\partial t} - c\frac{\partial u}{\partial x} \longrightarrow \frac{u_j^{p+1} - u_j^p}{\Delta x} = c\left(\frac{u_{j+1}^p - u_j^p}{\Delta x}\right) \longrightarrow u_j^{p+1} = u_j^p - \left(\frac{c\Delta t}{\Delta x}\right)\left(u_{j+1}^p - u_j^p\right)$$

Considerando el error de la forma:

$$\varepsilon_j^p = e^{\gamma p\Delta t} e^{i\beta_m j\Delta x} \longrightarrow u_N = u_E + u_\varepsilon \longrightarrow N = E + \epsilon \longrightarrow \varepsilon_j^{p+1} = \varepsilon_j^p - \left(\frac{c\Delta t}{\Delta x}\right)\left(\varepsilon_{j+1}^p - \varepsilon_j^p\right)$$

$$\longrightarrow \varepsilon_j^{p+1} = e^{\gamma(p+1)\Delta t} e^{i\beta_m j\Delta x} \longrightarrow \varepsilon_{j-1}^p = e^{\gamma p\Delta t} e^{i\beta_m (j+1)\Delta x} = e^{\gamma p\Delta t} e^{i\beta_m j\Delta x} \longrightarrow \varepsilon_{j-1}^p = e^{\gamma p\Delta t} e^{i\beta_m (j-1)\Delta x} = e^{\gamma p\Delta t} e^{i\beta_m j\Delta x} e^{-i\beta_m\Delta x}$$

$$\longrightarrow e^{\gamma\Delta t} e^{\gamma p\Delta t} e^{i\beta_m j\Delta x} = e^{\gamma p\Delta t} e^{i\beta_m j\Delta x}$$

$$\longrightarrow -\left(\frac{c\Delta t}{\Delta x}\right)\left(e^{\gamma p\Delta t} e^{i\beta_m j\Delta x} e^{i\beta_m\Delta x} - e^{\gamma p\Delta t} e^{i\beta_m j\Delta x}\right)$$

$$\longrightarrow e^{\gamma\Delta t} = 1 - \left(\frac{c\Delta t}{\Delta x}\right)\left(e^{i\beta_m\Delta x} - 1\right) \qquad C \longrightarrow e^{\gamma\Delta t} = 1 - C\left(e^{i\beta_m\Delta x} - 1\right)$$

$$G = \frac{\varepsilon_j^{p+1}}{\varepsilon_j^p} = \frac{e^{\gamma\Delta t} e^{\gamma p\Delta t} e^{i\beta_m j\Delta x}}{e^{\gamma p\Delta t} e^{i\beta_m j\Delta x}} = e^{\gamma\Delta t} = 1 - C\left(e^{i\beta_m\Delta x} - 1\right) = \left|1 + C - Ce^{i\beta_m\Delta x}\right|$$

Para la estabilidad: $G \leq 1$ o $\left|\frac{\varepsilon_j^{p+1}}{\varepsilon_j^p}\right| \leq 1 \longrightarrow \left|1 + C - Ce^{i\beta_m\Delta x}\right| \leq 1$