Punto 6: Demostración Numerov:

Dada la ecuación diferencial de la forma $\frac{d^2y}{dx^2} = -g(x)y(x) + s(x)$ (1) ; $x \in [a,b]$; siendo $g(x)$ y $s(x)$ funciones continuas en $[a,b]$.

Por series de Taylor: $y(x) = y(x_0) + hy'(x_0) + \frac{h^2}{2!}y''(x_0) + \frac{h^3}{3!}y'''(x_0) + \frac{h^4}{4!}y^{IV}(x_0) + O(h^5)$

$$y_{n+1}(x_n+h) = y_n + hy'_n + \frac{h^2}{2!}y''_n + \frac{h^3}{3!}y'''_n + \frac{h^4}{4!}y^{IV}_n + \dots \quad (2)$$

$$y_{n-1}(x_n-h) = y_n - hy'_n + \frac{h^2}{2!}y''_n - \frac{h^3}{3!}y'''_n + \frac{h^4}{4!}y^{IV}_n + \dots \quad (3)$$

$$(2)+(3) \Longrightarrow \; = 2y_n + h^2 y''_n + \frac{h^4}{12}y^{IV} + O(h^6) \; (4) \longrightarrow \quad y_{n+1} + y_{n-1} - 2y_n = h^2 y''_n + \frac{h^4}{12}y^{IV} \; (5)$$

$$y''_n = g_n y_n + S_n$$

$$y^{IV}_n = \frac{d^2}{dx^2}\left[-g_n y_n + S_n\right] \longrightarrow \text{de } (5): \; h^2 y_n^{IV} = \left[(-g_{n+1})(y_{n+1}) + S_{n+1} + (-g_{n+1})(y_{n-1}) + S_{n-1} - 2(-g_n y_n + S_n)\right] + O(h^4)$$

Ahora, sustituyendo en las ecuaciones:

$$y_{n+1} + y_{n-1} - 2y_n = h^2[-g_n y_n + s_n] + \frac{h^2}{12}\left[(-g_{n+1})(y_{n+1}) + S_{n+1} + (-g_{n-1})(y_{n-1}) + S_{n-1} - 2((-g_n)(y_n) + s_n)\right]$$

$$\longrightarrow y_{n+1}\left[1 + \frac{h^2}{12}g_{n+1}\right] + y_{n-1}\left[1 + \frac{h^2}{12}g_{n-1}\right] - y_n\left[2 - h^2 g_n + \frac{2h^2}{12}g_n\right] = \frac{h^2}{12}\left[S_{n+1} + 10_{S_n} + S_{n-1}\right]$$

$$\boxed{\longrightarrow y_{n+1}\left(1 + \frac{h^2}{12}g_{n+1}\right) - 2y_n\left(1 - \frac{5h^2}{12}g_n\right) + y_{n-1}\left(1 + \frac{h^2}{12}g_{n-1}\right) = \frac{h^2}{12}\left(S_{n+1} + 10S_n + S_{n-1}\right) + O(h^6)}$$