Tarea 2

Punto 3:

* Solución analítica de la ecuación de Riccati:

$$x^3\frac{dy}{dx} = x^4y^2 - 2x^2y - 1 \longrightarrow dy/dx = \frac{x^4y^2 - 2x^2y - 1}{x^3}.$$

Sabemos que una solución es $y_1 = x^{-2}$.

Consideremos ahora este cambio de variables: $y(x) = y_1(x) + w(x) \longrightarrow y = y_1 + w$.

$$\frac{d}{dx}(y_1 + w) = \frac{x^4(y_1+w)^2 - 2x^2(y_1+w) - 1}{x^3} \longrightarrow \frac{dy_1}{dx} + \frac{dw}{dx} = \frac{x^4(y_1^2 + 2y_1w + w^2) - 2x^2(y_1+w) - 1}{x^3}$$

$$\longrightarrow \frac{dy_1}{dx} + \frac{dw}{dx} = \frac{x^4y_1^2 + 2x^4y_1w + x^4w^2 - 2x^2y_1 - 2x^2w - 1}{x^3} \longrightarrow \frac{dy_1}{dx} + \frac{dw}{dx} = \frac{x^4y_1^2 - 2x^2y_1 - 1}{x^3} + \frac{x^4w^2 - 2x^2w + 2x^4y_1w}{x^3}$$

$$\longrightarrow \frac{dw}{dx} = \frac{x^4w^2 - 2x^2w + 2x^4y_1w}{x^3} = \frac{w(2x^4y_1 - 2x^2) + x^4w^2}{x^3} \longrightarrow \frac{1}{w^2}\frac{dw}{dx} = \frac{\cancel{w}(2x^4y_1 - 2x^2)}{x^3}\cdot\frac{1}{w^{\cancel{2}}} + \frac{x^4\cancel{w^2}}{x^3}\cdot\frac{1}{\cancel{w^2}}$$

$$\longrightarrow \frac{1}{w^2}\frac{dw}{dx} = \frac{2x^4y_1 - 2x^2}{x^3w} + x \quad \boxed{\begin{matrix} u = 1/w \\ \frac{du}{dx} = -\frac{1}{w^2}\frac{dw}{dx}\end{matrix}} \longrightarrow -\frac{du}{dx} = \left(\frac{2x^4y_1 - 2x^2}{x^3}\right)u + x \longrightarrow \frac{du}{dx} + \left(\frac{2x^4y_1 - 2x^2}{x^3}\right)u + x = 0$$

$$\longrightarrow \boxed{\begin{matrix}\text{Si } u = 1/w \text{ y } w = y - y_1: \\ u = 1/y - y_1\end{matrix}} \longrightarrow \frac{du}{dx} + \left(2x\,y_1 - \frac{2}{x}\right)u + x = 0 \quad \text{donde } I(x) = e^{\int^x 2sy_1\,ds - \int^x 2/s\,ds} = e^{y_1x^2 - 2\ln(x)}$$

$$\longrightarrow \frac{du}{dx}\left(e^{y_1x^2 - 2\ln(x)}\right) + u\left(2x\,y_1 - \frac{2}{x}\right)\left(e^{y_1x^2 - 2\ln(x)}\right) + x\left(e^{y_1x^2 - 2\ln(x)}\right) = 0 \longrightarrow \frac{d}{dx}\left(e^{y_1x^2 - 2\ln(x)}u\right) + x\left(e^{y_1x^2 - 2\ln(x)}\right) = 0$$

$$\longrightarrow e^{y_1x^2 - 2\ln(x)}u + \int^x s\left(e^{y_1s^2 - 2\ln(s)}\right)ds = C \longrightarrow u = Ce^{-(y_1x^2 - 2\ln(x))} - e^{-(y_1x^2 - 2\ln(x))}\int^x s\left(e^{y_1s^2 - 2\ln(s)}\right)ds$$

$$\longrightarrow \text{Si } y = y_1 + \frac{1}{u}, \qquad y_1(x) = \frac{1}{\left(Ce^{-(y_1x^2 - 2\ln(x))}\right) - \left[e^{-(y_1x^2 - 2\ln(x))}\int^x s\left(e^{y_1s^2 - 2\ln(s)}\right)ds\right]}$$