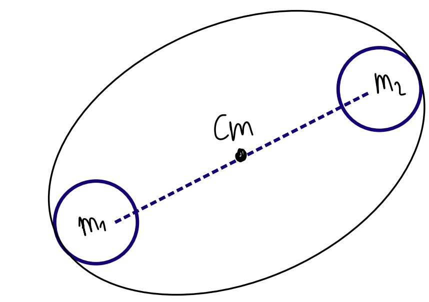
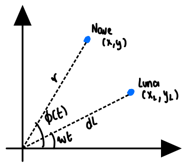
Problema de 2 cuerpos.



Problema 3 cuerpos: Datos:



$x_L(t) = d_L\cos(\omega t)$

$y_L(t) = d_L\sin(\omega t)$

c.) Cuando tengo dos puntos en el espacio, para calcular su distancia en coordenadas polares:

$\Delta^2 = \Delta x^2 + \Delta y^2$

$\Delta^2 = (x-x_L)^2 + (y-y_L)^2 \longrightarrow \Delta^2 = (r\cos\phi - d\cos(\omega t))^2 + (r\sin(\phi) - d\sin(\omega t))^2$

$\longrightarrow \Delta^2 = r^2\cos^2(\phi) - 2r\cos(\phi)d\cos(\omega t) + d^2\cos^2(\omega t) + r^2\sin^2(\phi) - 2r\sin(\phi)d\sin(\omega t) + d^2\sin^2(\omega t)$

$\longrightarrow \Delta^2 = r^2+d^2-2rd(\cos(\phi)+\cos(\omega t)+\sin(\phi)+\sin(\omega t))$

$\longrightarrow \Delta^2 = r^2+d^2-2rd[\cos(\phi-\omega t)] \longrightarrow \boxed{\Delta = \sqrt{r(t)^2+d^2-2r(t)d\cos(\phi-\omega t)}}$

d.) $\hat{H} = T + U$; $\dot q_j = \frac{\partial H}{\partial P_j}$

Energía K. — Hamiltoniano — coordenadas — Momento.

$L = T - U$ (Lagrangiano) ; $H = p_r\dot r + p_\phi\dot\phi - L$

$\longrightarrow$ donde $L_{Nave} = T - V = \underbrace{\frac12 m_{Nave}\dot r^2}_{\text{Traslación}} + \underbrace{\frac12 \overset{mr^2}{I}\dot\phi^2}_{\text{Rotación}} - \left(-\frac{Gm_{Nave}m_T}{r} - \frac{Gm_{Nave}m_L}{\Delta}\right) \longrightarrow$ Para escribir el Lagrangiano en términos de momento:

$P_i = \frac{\partial L}{\partial\dot q_i}$ donde $q_1 = r$ y $q_2=\phi \longrightarrow P_r = \frac{\partial L}{\partial \dot r} = m\dot r$; $P_\phi = \frac{\partial L}{\partial\dot\phi} = mr^2\dot\phi \longrightarrow L = \frac12 P_r\dot r + \frac12 P_\phi\dot\phi + \frac{Gm_{Nave}m_T}{r} + \frac{Gm_{Nave}m_L}{\Delta}$

Entonces: $H = \frac12 P_r\dot r + \frac12 P_\phi\dot\phi - \frac{Gm_Nm_T}{r} - \frac{Gm_Nm_L}{\Delta}$. $\longrightarrow \dot r = \frac{P_r}{m}$; $\dot\phi = \frac{P_\phi}{mr^2} \longrightarrow \boxed{H = \frac{P_r^2}{2m} + \frac{P_\phi^2}{2mr^2} - \frac{Gm_Nm_T}{r} - \frac{Gm_Nm_L}{\Delta}}$

e.) Para llegar a las ecuaciones de movimiento: $\dot q_j = \frac{\partial H}{\partial P_j}$ si $\hat H(r,\phi,P_r,P_\phi)$ ¿?

$\dot r = \frac{\partial H}{\partial P_r} = \frac{2P_r}{2m} + \cancel{\frac{P_\phi^2}{2mr^2}} - \cancel{\frac{Gm_Nm_T}{r}} - \cancel{\frac{Gm_Nm_L}{\Delta}} = \boxed{\frac{P_r}{m}}$

; $\dot\phi = \frac{\partial H}{\partial p_\phi} = \cancel{\frac{P_r^2}{2m}} + \frac{2P_\phi}{2mr^2} - \cancel{\frac{Gm_Nm_T}{r}} - \cancel{\frac{Gm_Nm_L}{\Delta}} = \boxed{\frac{P_\phi}{mr^2}}$

; $\dot P_r = -\frac{\partial H}{\partial r} = \cancel{\frac{P_r^2}{2m}} + \cancel{\frac{P_\phi^2}{2mr^2}} - \cancel{\frac{Gm_Nm_T}{r}} - \frac{\overset{\gamma}{Gm_Nm_L}}{\cancel{\Delta}}$.

Cambia el signo al final

$\frac{-2P_\phi^2r^{-3}}{2m} + Gm_Nm_Tr^{-2} - \gamma(r(t)^2+d^2-2r(t)d\cos(\phi-\omega t))^{-1/2} \longrightarrow -\frac12(r(t)^2+d^2-2r(t)d\cos(\phi-\omega t))^{-3/2}(2r-2d\cos(\phi-\omega t))\cdot\gamma$

$= -\frac{\cancel{2}(r-d\cos(\phi-\omega t))}{\cancel{2}(r(t)^2+d^2-2r(t)d\cos(\phi-\omega t))^{3/2}}\cdot\gamma$

$\longrightarrow \boxed{\frac{P_\phi^2}{mr^3} - \frac{Gm_Nm_t}{r^2} - \frac{Gm_Nm_L[r-d\cos(\phi-\omega t)]}{(r^2+d^2-2rd\cos(\phi-\omega t))^{3/2}}}$

; $\dot p_\phi = -\frac{\partial H}{\partial\phi} = \cancel{\frac{P_r^2}{2m}} + \cancel{\frac{P_\phi^2}{2mr^2}} - \cancel{\frac{Gm_Nm_T}{r}} - \frac{\overset{\gamma}{Gm_Nm_L}}{\sqrt{r(t)^2+d^2-2r(t)d\cos(\phi-\omega t)}} = \boxed{\frac{(Gm_Nm_L)(rd\sin(\phi-\omega t))}{[r^2+d^2-2rd\cos(\phi-\omega t)]^{3/2}}}$

$\gamma[r(t)^2+d^2-2r(t)d\cos(\phi-\omega t)]^{-1/2} \longrightarrow -\frac{1}{\cancel{2}}\gamma[r(t)^2+d^2-2r(t)d\cos(\phi-\omega t)]^{-3/2}[\cancel{2}rd\sin(\phi-\omega t)\cdot 1]$

f.) Se establecen las nuevas variables: $\breve{r} = \frac{r}{d}$; ϕ ; $\breve{p}_r = \frac{p_r}{m_N d}$; $\breve{p}_\phi = \frac{p_\phi}{m d^2}$

* $\breve{r} = \frac{r}{d} \longrightarrow \frac{\partial}{\partial t}\left(\breve{r} = \frac{r}{d}\right) \longrightarrow \dot{\breve{r}} = \frac{\partial}{\partial t}(r/d) = \frac{\dot{r}}{d} \longrightarrow d\dot{\breve{r}} = \dot{r}$

$\breve{p}_r = \frac{p_r}{m_N d} \longrightarrow \breve{p}_r d = \frac{p_r}{m_N} \longrightarrow$ Si la original es $\dot{r} = \frac{p_r}{m_N}$, $d\dot{\breve{r}} = \breve{p}_r d \longrightarrow \dot{\breve{r}} = \breve{p}_r$

* $\dot{\phi} = \frac{p_\phi}{r^2 m}$ Para escribir en términos de las nuevas variables: $\longrightarrow m d^2 \breve{p}_\phi = p_\phi$; $\frac{\breve{p}_\phi d^2}{r^2}$ donde $d = \frac{r}{\breve{r}}$; $\frac{\breve{p}_\phi \frac{\cancel{r^2}}{\breve{r}^2}}{\cancel{r^2}} = \frac{\breve{p}_\phi}{\breve{r}^2}$

* $\dot{\breve{p}}_r = \frac{p_\phi^2}{m r^3} - \frac{G m_N m_t}{r^2} - \frac{G m_N m_L [r - d\cos(\phi - \omega t)]}{(r^2 + d^2 - 2rd\cos(\phi - \omega t))^{3/2}}$. Tenemos que $\breve{p}_r = \frac{p_r}{m_N d}$; para recuperar $\dot{\breve{p}}_r$: $\frac{\partial}{\partial t}\left(\breve{p}_r = \frac{p_r}{m_N d}\right)$

$\longrightarrow \dot{\breve{p}}_r = \frac{\partial}{\partial t}\left(\frac{p_r}{m_N d}\right) = \frac{\dot{p}_r}{m_N d}$