El polinomio interpolador está definido en el conjunto $\Omega = \{(a, f(a)), (x_m, f(x_m)), (b, f(b))\}$ donde $x_m = \frac{a+b}{2}$ es el punto medio del intervalo.

Partimos del hecho de que $f(x) \approx p_2(x) = \frac{(x-b)(x-x_m)}{(a-b)(a-x_m)} f(a) + \frac{(x-a)(x-b)}{(x_m-a)(x_m-b)} f(x_m) + \frac{(x-a)(x-x_m)}{(b-a)(b-x_m)} f(b)$

Entonces $\int_a^b f(x)\,dx = \int_a^b \left[\frac{(x-b)(x-x_m)}{(a-b)(a-x_m)} f(a) + \frac{(x-a)(x-b)}{(x_m-a)(x_m-b)} f(x_m) + \frac{(x-a)(x-x_m)}{(b-a)(b-x_m)} f(b)\right] dx$

$$= \int_a^b \left[\frac{(x-b)(x-x_m)}{(a-b)(a-x_m)} f(a)\right] dx + \int_a^b \left[\frac{(x-a)(x-b)}{(x_m-a)(x_m-b)} f(x_m)\right] dx + \int_a^b \left[\frac{(x-a)(x-x_m)}{(b-a)(b-x_m)} f(b)\right] dx$$

$$\frac{f(a)}{(a-b)} \int_a^b \left[\frac{(x-b)(x-x_m)}{(a-x_m)}\right] dx + f(a+b/2) \int_a^b \left[\frac{(x-a)(x-b)}{(x_m-a)(x_m-b)}\right] dx + \frac{f(b)}{(b-a)} \int_a^b \left[\frac{(x-a)(x-x_m)}{(b-x_m)}\right] dx$$

$$\frac{f(a)}{(a-b)} \int_a^b \left[\frac{x^2 - xx_m - bx + bx_m}{(a-x_m)}\right] dx + f(a+b/2) \int_a^b \left[\frac{x^2 - bx - ax + ab}{(x_m^2 - bx_m - ax_m + ab)}\right] dx + \frac{f(b)}{(b-a)} \int_a^b \left[\frac{x^2 - xx_m - ax + ax_m}{(b-x_m)}\right] dx$$

$$\frac{f(a)}{(a-b)} \int_a^b \left[\frac{x^2 - x\left(\frac{a}{2}+\frac{b}{2}\right) - bx + b\left(\frac{a}{2}+\frac{b}{2}\right)}{\left(a - \left(\frac{a}{2}+\frac{b}{2}\right)\right)}\right] dx + f\left(\frac{a}{2}+\frac{b}{2}\right) \int_a^b \left[\frac{x^2 - x(a+b) + ab}{\left(\frac{a}{2}+\frac{b}{2}\right)^2 - b\left(\frac{a}{2}+\frac{b}{2}\right) - a\left(\frac{a}{2}+\frac{b}{2}\right) + ab}\right] dx$$

$$+ \frac{f(b)}{(b-a)} \int_a^b \left[\frac{x^2 - x\left(\frac{a}{2}+\frac{b}{2}\right) - ax + a\left(\frac{a}{2}+\frac{b}{2}\right)}{\left(b - \left(\frac{a}{2}+\frac{b}{2}\right)\right)}\right] dx$$

$$\frac{f(a)}{(a-b)} \left[\left(\int_a^b \frac{2x^2}{a-b} dx\right) - \left(\int_a^b \frac{x(a+b)}{a-b} dx\right) - \left(\int_a^b \frac{2bx}{a-b} dx\right) + \left(\int_a^b \frac{b(a+b)}{(a-b)} dx\right)\right]$$

$$\frac{f(a)}{(a-b)} \left[\frac{2}{a-b}\left(\int_a^b x^2 dx\right) - \frac{(a+b)}{(a-b)}\left(\int_a^b x\, dx\right) - \frac{2b}{(a-b)}\left(\int_a^b x\, dx\right) + \left(-\frac{b(a+b)\cancel{(b-a)}}{\cancel{b-a}}\right)\right]$$

$$\frac{f(a)}{(a-b)} \left[\frac{2}{a-b}\left(\frac{b^3}{3} - \frac{a^3}{3}\right) - \frac{(a+b)}{(a-b)}\left(\frac{b^2}{2} - \frac{a^2}{2}\right) - \frac{2b}{(a-b)}\left(\frac{b^2}{2} - \frac{a^2}{2}\right) - b(a+b)\right]$$

$$\frac{f(a)}{(a-b)} \left[\frac{2}{3}\left(\frac{b^3 - a^3}{a-b}\right) - \frac{b^2 - a^2}{2}\left(\frac{a+3b}{a-b}\right) - b(a+b)\right]$$

$$\rightarrow f\left(\frac{a}{2}+\frac{b}{2}\right)\int_a^b\left[\frac{x^2-x(a+b)+ab}{\left(\frac{a}{2}+\frac{b}{2}\right)^2-b\left(\frac{a}{2}+\frac{b}{2}\right)-a\left(\frac{a}{2}+\frac{b}{2}\right)+ab}\right]dx \rightarrow f\left(\frac{a+b}{2}\right)\int_a^b\left[\frac{x^2-x(a+b)+ab}{\frac{(a+b)^2}{4}-\frac{(a+b)^2}{2}+ab}\right]dx$$

$$\rightarrow f\left(\frac{a}{2}+\frac{b}{2}\right)\int_a^b\left[\frac{x^2-x(a+b)+ab}{\frac{-a^2-2ab-b^2+4ab}{4}}\right]dx \rightarrow f\left(\frac{a}{2}+\frac{b}{2}\right)\int_a^b\left[\frac{4(x^2-x(a+b)+ab)}{-a^2+2ab-b^2}\right]dx$$

$$\rightarrow \frac{4f\left(\frac{a+b}{2}\right)}{-(a-b)^2}\left[\left(\int_a^b x^2\,dx\right)-(a+b)\left(\int_a^b x\,dx\right)+ab(b-a)\right] \rightarrow \frac{4f\left(\frac{a+b}{2}\right)}{-(a-b)^2}\left[\left(\frac{b^3-a^3}{3}\right)-(a+b)\left(\frac{b^2-a^2}{2}\right)+ab(b-a)\right]$$

$$\rightarrow \frac{f(b)}{(b-a)}\int_a^b\left[\frac{x^2-x\left(\frac{a}{2}+\frac{b}{2}\right)-ax+a\left(\frac{a}{2}+\frac{b}{2}\right)}{\left(b-\left(\frac{a}{2}+\frac{b}{2}\right)\right)}\right]dx$$

$$\rightarrow \frac{f(b)}{(b-a)}\left[\left(\int_a^b\frac{2x^2}{b-a}\,dx\right)-\left(\int_a^b\frac{x(a+b)}{b-a}\,dx\right)-\left(\int_a^b\frac{2ax}{b-a}\,dx\right)+\left(\int_a^b\frac{a(a+b)}{(b-a)}\,dx\right)\right]$$

$$\rightarrow \frac{f(b)}{(b-a)}\left[\frac{2}{b-a}\left(\frac{b^3-a^3}{3}\right)-\frac{(a+b)}{(b-a)}\left(\frac{b^2-a^2}{2}\right)-\frac{2a}{b-a}\left(\frac{b^2-a^2}{2}\right)+\frac{a(a+b)}{\cancel{(b-a)}}\cancel{(b-a)}\right]$$

$$\rightarrow \frac{f(b)}{(b-a)}\left[\frac{2}{3}\left(\frac{b^3-a^3}{b-a}\right)+\left(\frac{b^2-a^2}{2}\right)\left(\frac{3a+b}{b-a}\right)+a(a+b)\right]$$

$$\rightarrow \frac{f(a)}{(a-b)}\left[\frac{2}{3}\left(\frac{b^3-a^3}{a-b}\right)-\frac{b^2-a^2}{2}\left(\frac{a+3b}{a-b}\right)-b(a+b)\right]+\frac{4f\left(\frac{a+b}{2}\right)}{-(a-b)^2}\left[\left(\frac{b^3-a^3}{3}\right)-(a+b)\left(\frac{b^2-a^2}{2}\right)+ab(b-a)\right]$$

$$-\frac{f(b)}{(a-b)}\left[\frac{2}{3}\left(\frac{b^3-a^3}{b-a}\right)+\left(\frac{b^2-a^2}{2}\right)\left(\frac{3a+b}{b-a}\right)+a(a+b)\right]$$

se cancelan

$$\rightarrow \frac{1}{a-b}\left[f(a)\left[\frac{2}{3}\left(\frac{b^3-a^3}{a-b}\right)-\frac{b^2-a^2}{2}\left(\frac{a+3b}{a-b}\right)-b(a+b)\right]+f(b)\left[-\frac{2}{3}\left(\frac{b^3-a^3}{a-b}\right)+\left(\frac{b^2-a^2}{2}\right)\left(\frac{3a+b}{a-b}\right)+a(a+b)\right]\right.$$

$$\left.-\frac{4f(x_m)}{a-b}\left[\left(\frac{b^3-a^3}{3}\right)-(a-b)\left(\frac{b^2-a^2}{2}\right)+ab(a-b)\right]\right] \rightarrow \frac{1}{a-b}\left[f(a)+f(b)-4f(x_m)\left(\frac{b^3-a^3}{3(a-b)}-\left(\frac{b^2-a^2}{2}\right)+ab\right)\right]$$

$$\rightarrow \frac{h^3}{3}\left(f(a)+4f(x_m)+f(b)\right)$$