Ejercicio 1: Integración.

$$f(x) \backsimeq p_1(x) = \frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b) , \quad \forall x \in [a,b]$$

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→ $$I = \int_a^b f(x)\, dx \cong \int_a^b p_1(x)\, dx = \int_a^b \left[\frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b) \right] dx$$

→ $$I = \int_a^b \frac{x-b}{a-b} f(a)\, dx + \int_a^b \frac{x-a}{b-a} f(b)\, dx \longrightarrow \frac{f(a)}{a-b} \int_a^b (x-b)\, dx + \frac{f(b)}{b-a} \int_a^b (x-a)\, dx$$

→ $$\frac{f(a)}{a-b} \left[\int_a^b x\, dx - \int_a^b b\, dx \right] + \frac{f(b)}{b-a} \left[\int_a^b x\, dx - \int_a^b a\, dx \right]$$

→ $$\frac{f(a)}{a-b} \left[\left(\frac{x^2}{2} \Big|_a^b \right) - \left(bx \Big|_a^b \right) \right] + \frac{f(b)}{b-a} \left[\left(\frac{x^2}{2} \Big|_a^b \right) - \left(ax \Big|_a^b \right) \right]$$

→ $$\frac{f(a)}{a-b} \left[\left(\frac{b^2}{2} - \frac{a^2}{2} \right) - \left(b^2 - ba \right) \right] + \frac{f(b)}{b-a} \left[\left(\frac{b^2}{2} - \frac{a^2}{2} \right) - \left(ab - a^2 \right) \right]$$

→ $$\frac{f(a)}{a-b} \left[-\frac{b^2}{2} - \frac{a^2}{2} + ba \right] + \frac{f(b)}{b-a} \left[\frac{a^2}{2} + \frac{b^2}{2} - ab \right]$$

→ $$-\frac{2f(a)}{a-b} \left(a^2 - 2ba + b^2 \right) + \frac{2f(b)}{b-a} \left(a^2 - 2ab + b^2 \right) \longrightarrow \left(a^2 - 2ab + b^2 \right) \left(\frac{2f(b)}{b-a} - \frac{2f(a)}{a-b} \right)$$

→ $$\left(a^2 - 2ab + b^2 \right) \left(\frac{2f(b) + 2f(a)}{(b-a)} \right) \longrightarrow \left(a^2 - 2ab + b^2 \right) \cdot \frac{2}{(b-a)} (f(a) + f(b))$$

→ $$\frac{(b-a)}{2} (f(a) + f(b)) + \left(a^2 - 2ab + b^2 \right)$$