Ejercicios

① Mostrar el n-ésimo +1 término de las siguientes relaciones recursivas:

* $X_{n+1} = 4X_n - X_n^2$, $X_0 = 4\sin^2(\theta)$ conduce a $X_{n+1} = 4\sin^2(2^{n+1}\theta)$, $\theta \in [0, \pi/2]$

Solución:

$$X_0 = 4\sin^2(\theta)$$

$$X_1 = 4X_0 - X_0^2 = 4(4\sin^2(\theta)) - (4\sin^2(\theta))^2 = 16\sin^2(\theta) - 16\sin^4(\theta) = 16\sin^2(\theta)[1 - \sin^2(\theta)] = 16\sin^2(\theta)\cos^2(\theta)$$

$$16\sin^2(\theta)\cos^2(\theta) = 16\left[(\sin(\theta)\cos(\theta))^2\right] = 16\left[\left(\frac{\sin(2\theta)}{2}\right)^2\right] = \frac{16}{4}\sin^2(2\theta) = 4\sin^2(2\theta) = 4\sin^2(2^{n+1}\theta)$$

$$X_2 = 4(4\sin^2(2\theta)) - (4\sin^2(2\theta))^2 = 16\sin^2(2\theta) - 16\sin^4(2\theta) = 16\sin^2(2\theta)[1 - \sin^2(2\theta)]$$

$$= 16\sin^2(2\theta)\cos^2(2\theta) = 16[(\sin(2\theta)\cos(2\theta))^2] = 16\left[\left(\frac{\sin(4\theta)}{2}\right)^2\right] = \frac{16}{4}\sin^2(4\theta) = 4\sin^2(4\theta)$$
$$= 4\sin^2(2^2\theta)$$

$$X_{i+1} = 4(4\sin^2(2^{i+1}\theta)) - (4\sin^2(2^{i+1}\theta))^2 = 16\sin^2(2^{i+1}\theta) - 16\sin^4(2^{i+1}\theta) = 16\sin^2(2^{i+1}\theta)[1 - \sin^2(2^{i+1}\theta)]$$

$$= 16\sin^2(2^{i+1}\theta)\cos^2(2^{i+1}\theta) = 16[(\sin(2^{i+1}\theta)\cos(2^{i+1}\theta))^2] = 16\left[\left(\frac{\sin(2^{i+1}\theta)}{2}\right)^2\right] = 4\sin^2(2^{i+1}\theta)$$

* $X_{n+1} = 4X_n - 4X_n^2$, $X_0 = \sin^2(\theta)$ conduce a $X_{n+1} = \sin^2(2^{n+1}\theta)$, $\theta \in [0, \pi/2]$

$$X_0 = \sin^2(\theta)$$

$$X_1 = 4(\sin^2(\theta)) - 4(\sin^2(\theta))^2 = 4\sin^2(\theta) - 16\sin^4(\theta) = 4\sin^2(\theta)[1 - 4\sin^2(\theta)] = \frac{4\sin^2(\theta)}{4}[4 - \sin^2(\theta)]$$

$$= \sin^2(\theta)[4 - \sin^2(\theta)] = 4\sin^2(\theta) - \sin^4(\theta) = 4\sin^2(\theta) - \sin^4(\theta)$$

$$2\sin^2(\theta)\left[2 - \frac{\sin^2(\theta)}{2}\right] = 2\sin^2(\theta)\,2\cos^2(\theta) = 4[(\sin(\theta)\cos(\theta))^2] = 4\left[\left(\frac{\sin(2\theta)}{2}\right)^2\right] = \sin^2(2\theta)$$

$$X_2 = 4(\sin^2(2\theta)) - 4(\sin^2(2\theta))^2 = 4\sin^2(2\theta) - 16\sin^4(2\theta) = 4\sin^2(2\theta)[1 - 4\sin^4(2\theta)] = \sin^2(2\theta)[4 - \sin^2(2\theta)]$$

$$= 4\sin^2(2\theta) - \sin^4(2\theta) = 2\sin^2(2\theta)\left[2 - \frac{\sin^2(2\theta)}{2}\right] = 2\sin^2(2\theta)2\cos^2(2\theta) = \frac{4}{4}\sin^2(4\theta) = \sin^2(4\theta) = \sin^2(2^2\theta))$$

$$X_{i+1} = 4(\sin^2(2^{i+1}\theta)) - 4(\sin^2(2^{i+1}\theta))^2$$

$$= 4\sin^2(2^{i+1}\theta) - 16\sin^4(2^{i+1}\theta) = 4\sin^2(2^{i+1}\theta)[1 - 4\sin^4(2^{i+1}\theta)] = \sin^2(2^{i+1}\theta)[4 - \sin^2(2^{i+1}\theta)]$$

$$= 4\sin^2(2^{i+1}\theta) - \sin^4(2^{i+1}\theta) = 2\sin^2(2^{i+1}\theta)\left[2 - \frac{\sin^2(2^{i+1}\theta)}{2}\right] = 2\sin^2(2^{i+1}\theta)2\cos^2(2^{i+1}\theta) = \frac{4}{4}\sin^2(2^{i+1}\theta) = \sin^2(2^{i+1}\theta) = \sin^2(2^{i+1}\theta))$$