1 Introduction

Mathematical models play a key role in traffic simulation, by predicting impacts on road networks like identifying bottlenecks and estimating travel time changes due to roadworks. In this report, we focus on the intelligent driver model (IDM) as an example of a time-step based microscopic model for a single-lane circular motorway. By simulating individual vehicle behaviour and interactions, IDM can be used to analyse these interactions with a specific focus on the vehicle in front. Our aim will be to ask two questions, what parameters have the largest effect on the output of the system? And if we introduce a junction, how busy does that junction have to be to merit introducing a traffic light?

2 Modelling the System

2.1 How the Model Works

The IDM is a straightforward car-following model designed to simulate traffic on a single-lane ring motorway. For each vehicle on the motorway, its current position and velocity are initialised. At each time step we determine the acceleration for each vehicle and compute the resulting positions and velocities for each vehicle for the next time step using standard expressions from Newtonian dynamics. The expression for the acceleration is:

$$\dot{v} = a\left[1 - \left(\frac{v}{v_0}\right)^{\delta} - \left(\frac{s^*(v, \Delta v)}{s}\right)^2\right] \tag{2.1.1}$$

where a is the obstacle-free acceleration at rest, v is the velocity of the car, v_0 is the desired velocity, δ is the acceleration exponent which determines the acceleration profile as speed increases, Δv is the approach velocity to the car in front and s is the current bumper-to-bumper distance to the preceding car which is how we will define the headway.

The desired bumper-to-bumper distance to the preceding car, s^*, is given by the following expression:

$$s^* = s_0 + \max\left(0, vT + \frac{v\Delta v}{2\sqrt{ab}}\right) \tag{2.1.2}$$

This expressions contains the standard safety gap $s_0 + vT$, but it also contains a second dynamical term, which takes into account the braking strategy. The parameter b is a braking parameter which alters the rate of deceleration.

These equations leave us with desirable qualities which we would expect in any traffic situation. For example, acceleration and braking changes gradually except in emergency situations and drivers will always try to maintain a minimum gap to the preceding vehicle. The rest of the properties that these equations provide are described in more detail in Traffic Flow Dynamics: Data, Models and Simulation [1].

2.2 Data Colletction

2.2.1 Standard Initial Setup

Before we can start to collect any data we need to determine our parameter setup. We need to consider some of the main characteristics of the model itself:

	Standard Model	Further Investigation
Length of the road, L	1000 m	✗
Desired speed, v_0	70 miles/hour	✓
Time gap, T	1 s	✗
Safety gap, s_0	2 m	✓
Acceleration exponent, δ	4	✓
Maximum acceleration, a	1m/s^2	✓
Comfortable braking, b	$1.5\ \text{m/s}^2$	✓
Time-step, Δt	0.5 s	✗
Number of steps	1000	✗
Length of the car, l	2 m	✗

The values in the 'standard model' column were selected for what we will use as the comparable standard in future analysis. The majority of these values came from Traffic Flow Dynamics: Data, Models and Simulation [1]. The remaining values, that is the length of the road, the number of steps and length of the cars, come from repeat testing at various fixed densities to see what values produced sensible results.

The standard initial setup that we will use for our model is all cars starting at rest in an evenly spaced que that spans a sufficient distance for the amount of cars in it. We found this initial setup to provide more insightful results than simply starting all the cars evenly spaced throughout the loop.

2.2.2 Density, Flow and Average Velocity

The main metrics that we are using to analyse the output behaviour of our basic system are global flow, global density and average velocity. We found that by analysing the relationship between these three metrics we could draw insightful and meaningful conclusions from our model.

Global flow is measured in cars per hour and can be expressed as $\sum_i v_i$ divided by L and where v_i is the i-th car's velocity.

In our initial trials of the simulation we quickly seen that the system would reach an equilibrium state after a certain number of steps. Our goal was to collect one value of flow that accurately described the flow at this equilibrium state. We tried a few ways of doing this, first by selecting a value at a time step close to the end of the simulation, however, we could not know for sure that the system had reached equilibrium and hence, this method was unreliable.

In a later development we searched for the set of 100 values of flow with the lowest standard deviation and took their average, however, this was slow and sometimes identified local plateaus instead of the system's true equilibrium.

Our final method we developed was based on the epsilon definition of convergence. We iterated through every set of 100 values of flow starting at the end of the simulation and stopped when we found a set that had a standard deviation lower than our epsilon range and took the average of this set. This method is much faster and gives a far higher likelihood of identifying the value of flow at the equilibrium of the system. Through repeat testing, we found a reliable value of epsilon to be 0.5.

We used the same method described above to calculate the average velocity of the system at each density. The global density is fixed for each simulation in our basic model. This information allowed us to plot the fundamental diagrams as identified in 'Traffic Flow Dynamics: Data, Models and Simulation' [1] for the standard model.

3 Sensitivity Analysis On Basic Model

We now have the tools ask the question outlined in the introduction; what parameter have the largest effect on the output of the system?. We will use the parameters defined in Sec. 2.2.1 as our standard setup to which all other setups will be compared. The parameters marked with a tick will be the parameters that we will itterate over in this section. Repeat investigation led us to determine that we should iterate each parameter from a third to 3 times their standard values as this gave sensible yet meaningful results that are clearly comparable. Due to the run time of the simulations, we determined that we should select 5 evenly spaced values in this range for each parameter. For each value of the selected parameter we will run the model over 1000 steps for densities from 1 up to 300 cars per kilometre. We found this reliably gave us the results we where looking for in most instances.

The specific metrics that we will use to compare the outcomes of our models are as follows:

- Critical Flow
- Critical Speed
- Critical Density
- Jam Density

where 'critical' refers to the value of the adjoint metric when the system is at the density at which peak flow is achieved and jam density refers to the density at which flow becomes 0.

We can plot the fundamental diagrams with these specific metrics identified as follows.

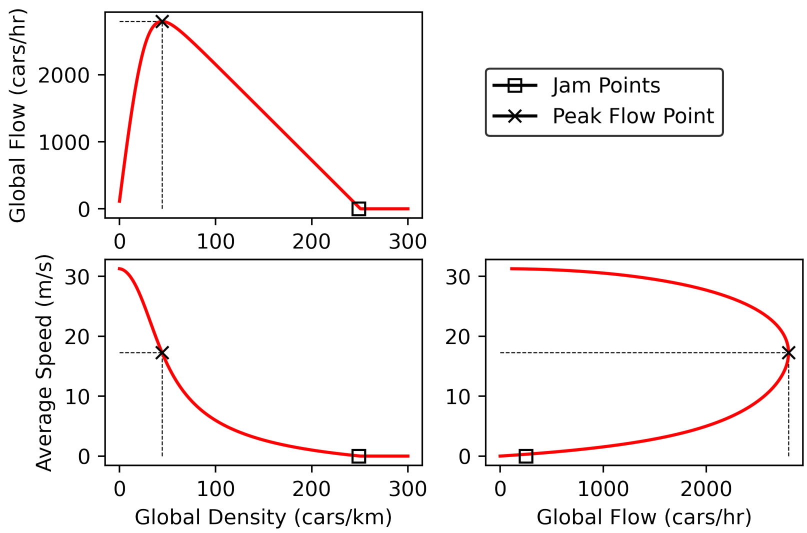


Figure 1: This shows the fundamental diagrams for the standardised model, where the 'x' marks the point of critical flow on each diagram and the square marks the jam point.

3.1 Analysis of Results

We can now take a look at the results of our sensitivity analysis. We will first look at some specific metrics and discuss what effect our model determined they had on the system and we will then compare the sensitivity of each metric against one another.

The first input parameter that we are going to look at altering is the speed limit, v_0. In the standard model we had the speed limit equal to 70 miles per hour or 31.2829 ms^{-1}. Now we are going to see how the specific metrics of the model are affected by increasing and decreasing this input.

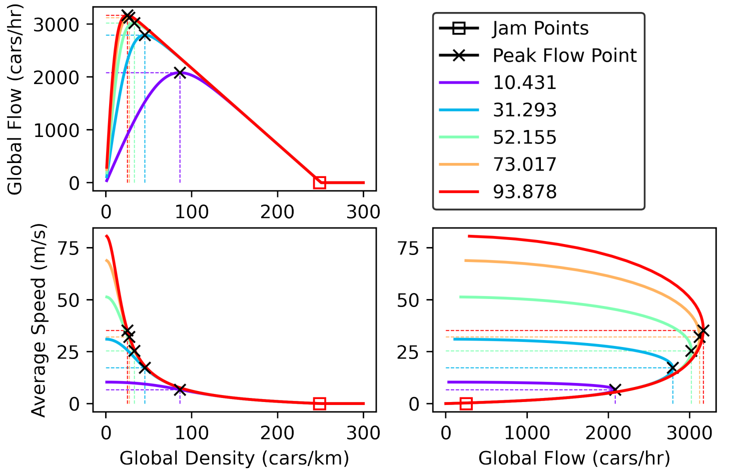


Figure 2: This shows the fundamental diagrams for each iterated value of the maximum velocity, where the 'x' marks the point of critical flow on each diagram and the square marks the jam point. The legend is in ms^{-1}.

When analysing our fundamental diagrams for speed it is useful to think about smart motorways. We can see that at low densities a higher flow can be achieved with a higher speed. However, as the density increases, higher flow is achieved at lower speeds which is the purpose of smart, speed varying motorways. We can note that this parameter had no effect on the crash point.

Another parameter that we decided to iterate over that gave interesting results was the minimum distance between each car.

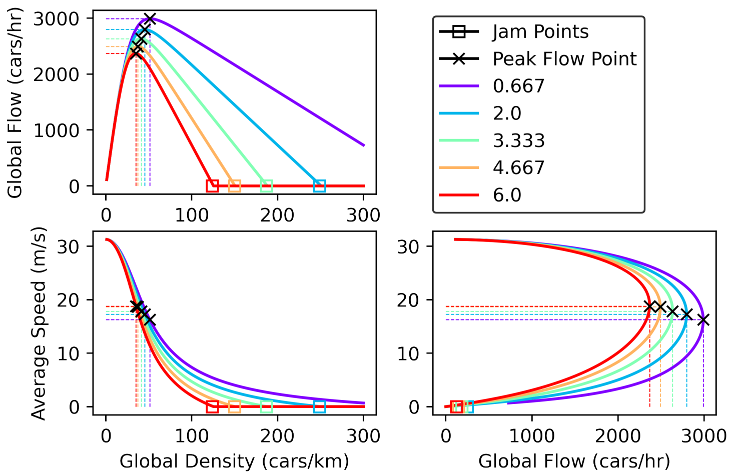


Figure 3: This shows the fundamental diagrams for each iterated value of the minimum safety distance, where the 'x' marks the point of critical flow on each diagram and the square marks the jam point. The legend is in metres.

Here we can see that a decrease in the minimum distance between cars allows for higher flows at higher densities with slightly decreased speeds. This may not be a practical real life solution as decreasing the minimum safety gap may increase the risk of an accident. We can also see a decrease in this parameter causes an increase in the value of the jam point meaning we can have more total cars on the road.

The lowest value of minimum distance gave a jam point that was outside of the range of densities that we tested, but since we wanted to use the extremes of the data in our analysis, we performed a linear regression on the value of the flow from a density of 150 to 300 to predict the jam point for this minimum distance of 0.667 metres. The linear regression predicts that the jam density will be 377 and it returns a standard error value of 8.74×10^{-5} which we deemed acceptably low to use this data in our analysis.

To conduct our final sensitivity analysis we will look at how the high and low extreme values for each parameter we tested affects each of our specific metrics compared to the standard model. The following table presents the percentage of the specific metrics in the standard model produce by the high and low extreme values of each parameter.

Parameter	Critical Flow	Critical Speed	Critical Density	Jam Point
Acceleration Exponent High	108.32678	82.22090	131.7488	100.0
Acceleration Exponent Low	83.58342	120.0015	69.65285	100.0
Braking Constant High	100.0059	100.0	100.0059	100.0
Braking Constant Low	100.0059	100.0	100.0059	100.0
Maximum Acceleration High	100.0096	100.0	100.0096	100.0
Maximum Acceleration Low	95.50748	115.5567	82.66336	100.0
Maximum Velocity High	113.2613	55.55225	203.8829	100.0
Maximum Velocity Low	74.43302	191.1179	38.94751	100.0
Minimum Distance High	84.62973	77.77613	108.8097	49.79920
Minimum Distance Low	106.9111	113.3343	94.33332	151.1130

We can see from this table that in general if the high extreme value has an effect on the standard value then so does the low extreme, except for the maximum acceleration parameter for which it appears that only the low extreme affects any of the specific metrics. This may be because there simply were not enough steps in the model for the system to reach equilibrium with such a low maximum acceleration value. We can also see that changing the braking constant seemed to have minimal effect on the system which is perhaps because its values where not varied enough. These discrepancies could be further investigated in future analysis.

To compare the sensitivity of each specific metric we can take the standard deviation of their percentage change at high and low extremes respectively and then average these two figures. When we do this we see that the standard deviation for the brake constant is negligibly low and that of the maximum acceleration is incomparable due to the issues discussed above. We do find that maximum velocity is by far the most sensitive parameter that we tested with an average standard deviation of 63.62 compared to that of the acceleration exponent (21.13) and the minimum distance (24.36).

4 Extended Model

Now we are going to look at an extension to the basic model we have just analysed. As mentioned in Sec. 1, we are going to look at implementing a junction. We will then follow this up by analysing the impact of introducing a traffic light at the junction. To do this we will need new metrics to analyse our data and we will need to discuss some of the finer points of how the junction itself works in our model.

4.1 Data Collection

For both simulations our initial setup will be the same as the table in Sec. 2.2.1, except the length of the road will now be 2000 metres to allow more cars on the road and the desired speed will now be 40 miles per hour which is a more realistic speed limit for a road with a junction. A new parameter will be the initial density, through repeat testing we have selected this to be 25 cars all starting at rest in a queue in the first 500 metres. We will also now be using 10,000 steps. It is important to run this simulation for much longer to try to lessen the affect of probabilistic inaccuracies since the junctions use a probability function to determine when a car arrives at the junction to better simulate a real life scenario.

Additionally, we determined that a safe distance that the cars need to turn on to the road if they are not protected by a traffic light should be 30 metres and they should initially have a speed of 3 ms^{-1}. We also ensured a sufficient gap of 5 metres was left so that cars have room to turn onto the junction and maintain a safe distance.

We also needed to set up the traffic lights, we found that if we let the traffic lights come on every 1250 steps for 500 steps starting at step 500 our model returned reasonable results.

The new metrics that we will be using to analyse our system are:

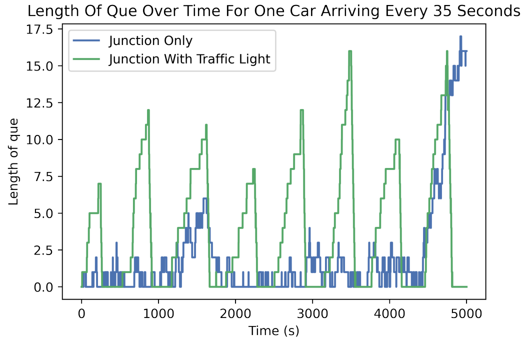
1. The length of the que at the junction at any given time.
2. Three detector points placed 10 metres before, 10 metres after and at the opposite side of the road to the junction that detect the velocity of any car that passes it along with the time at which it passed. These data points can then be extrapolated to give us the number of cars passing each detector per hour (local flow) and then this value can be divided by the average speed of cars passing that point (local density).
3. The number of cars on the road at any given time.

The idea of our simulations is that we will iterate over the 'busyness' of the junction and ask the question, when is installing a traffic light for a road with our given parameters worth it? For our described setups we set the probability that a car will arrive in any given $0.5s$ step. Initially we set this to be 0.05 and iterated over values between 0 and 0.05 but we found the most interesting range to be between 0 and 0.015 so we reran our simulation for values between these figures to achieve a finer data set in this range. We picked 100 values between these points due to simulation time restrictions.

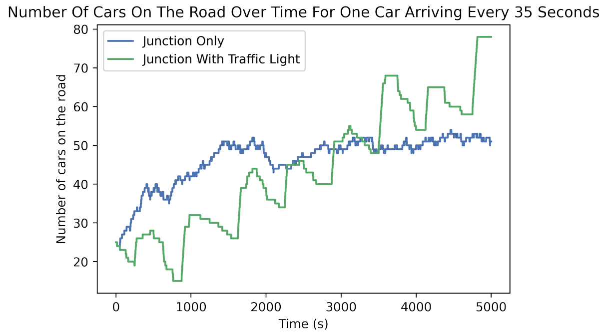
4.2 Analysis of Results

Now we can begin the analysis of our results. In order to answer the question set out in 4.1 we will try to find out how busy the junction has to be for the traffic light setup to consistently out perform just a junction on its own. We will define this as the busyness from which the traffic light setup has a lower average queue than the junction for every busyness measured after that point if indeed this point exists.

Analysing our data set we see that at the point where a car is arriving every 34.237 seconds is where the traffic light consistently out performs the junction only setup. We can take a closer look at this particular busyness to see why this is the case.



(a) Length of queue varying over time



(b) Number of cars on the road varying over time

Figure 4: The figure on the left shows how the length of the queue varies and the figure on the right shows how the number of cars on the road varies over time when a car arrives every 34.327 seconds on average for a system with a junction and a system with a junction with a traffic light.

We can see that there appears to be a fundamental density at which cars cannot turn onto the road in the junction only setup. From our data, the last point at which the queue is zero in this setup is 4427.5 seconds in, if we take the average density from this point forward we get 51 cars per kilometer as our fundamental density.

When a traffic light is added, we can see that initially the average queue is higher in Fig. 4(a) but as the density on the road builds up beyond our fundamental density, the queue is still able to dissipate and more cars can be let onto the road, as seen in Fig. 4(b).

When a junction exhibits this behaviour, that is there exists a point when the road becomes too busy for cars to join, the queue is likely to experience unbounded growth, which can be seen in Fig. 4(a). At this point a traffic light is absolutely crucial to maintain a functional system.

Adding a traffic light has drawbacks as we observe from the data gathered from our detection points.

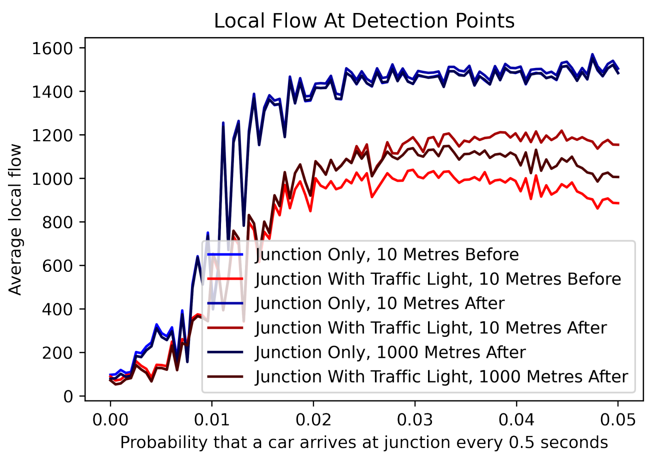


Figure 5: This graph shows local flow at three different detection points. One 10 meters before the junction, one 10 meters after the junction and one 1000 meters after the junction. The blues represent the flow of the junction alone and the reds represent the flow of the junction with a traffic light.

We can see that in general, adding a traffic light reduces the overall flow at all of our detection points, this is the trade off that is made to reduce the overall length of the queue and increase the cars on the road.

It is worth noting that our model only simulates for around 1 hour and 25 minutes and does not take into account differences in busyness at different times of day. It also does not account for pedestrians, likelihood of an accident, difficulty of installing traffic lights or many other factors. However, our analysis forms the basis for the idea of how you would determine how busy a junction would have to be to merit adding additional traffic signalling measures.

References

[1] M. Treiber and A. Kesting, Traffic Flow Dynamics: Data, Models and Simulation https://edisciplinas.usp.br/pluginfile.php/5564382/mod_resource/content/0/Martin_Treiber___Arne_Kesting__auth._Traffic_FBookZZ.org.pdf

[2] Chapter 04: Fundamental diagrams
https://ocw.tudelft.nl/course-readings/chapter-4-fundamental-diagrams/
(accessed 5/3/24)