The Relationship Between Commodity Prices and the Canada-US Exchange Rate: An Empirical Investigation II

Introduction:

The relationship between exchange rates and commodity prices has long been a subject of economic inquiry, particularly in commodity-exporting economies like Canada. Theoretical and empirical literature suggest that exchange rates and commodity prices may exhibit long-run equilibrium relationships, alongside short-run fluctuations driven by macroeconomic fundamentals (Chen et al., 2010). Given Canada's reliance on commodity exports, fluctuations in commodity prices can influence the exchange rate through changes in trade balances, capital flows, and monetary policy adjustments. This study extends previous empirical analyses by employing the Present-Value (PV) Approach to assess whether exchange rate movements are systematically linked to commodity prices and whether additional macroeconomic fundamentals, such as interest rate differentials, influence this relationship.

The PV approach, based on asset price theory, implies that exchange rates must equal the discounted present value of future expected fundamentals, such as commodity prices and interest rate differentials (Devereux & Smith, 2021). When the model is valid, it implies a cointegrating relation between exchange rates and commodity prices, i.e., they have a common stochastic trend over time. In addition, the PV framework forecasts Granger causality to run from commodity prices to exchange rates when market participants use changes in commodity prices to forecast exchange rate changes. However, empirical studies have yielded mixed results, whereby some of them detected reverse causality or bidirectional relations (Chen et al., 2010; Devereux & Smith, 2021).

To empirically assess these relationships, this paper follows a two-part structure. In Part I, we apply unit root tests, Johansen cointegration tests, and Granger causality tests to examine the Canada-US exchange rate and the total commodity price index, deflated by the US price index. Since no cointegration was found, a vector autoregression (VAR) model was estimated instead of a vector error correction model (VECM). We also conduct diagnostic tests, including lag selection criteria, to ensure model robustness. In Part II, we introduce interest rate differentials as an additional explanatory variable to assess whether its inclusion alters our previous findings, providing a sensitivity analysis. This follows the approach of Devereux & Smith (2021), who highlight the role of interest rate differentials in exchange rate dynamics.

This paper adds to the literature by refining the specification of exchange rate-commodity price relationships, and other macroeconomic fundamentals, and imposing stringent econometrics to investigate short- and long-run dynamics. The results offer evidence on the degree to which exchange rate fluctuations can be explained by movements in commodity prices, whether or not the variables share a long-run stable relationship, and how well interest rate differentials perform in accounting for exchange rate movements.

The rest of the paper is organized as follows: Section 2 outlines the theory with particular focus on the PV approach and exchange rate and commodity price dynamic implications. Section 3 outlines data sources and empirical methodology, i.e., unit root tests, cointegration analysis, and model estimation. Section 4 reports empirical findings, while Section 5 concludes with policy implications and suggestions for future research.

Economic Theory and Model Specification

2.1 Theoretical Framework

The Present Value Model (PVM) of exchange rates provides a theoretical foundation for understanding the relationship between exchange rates and fundamental macroeconomic variables, including commodity prices. The PVM posits that exchange rates are forward-looking and should reflect the expected future values of these fundamentals under rational expectations (Engel & West, 2005). This implies that exchange rates and commodity prices may share a long-run equilibrium relationship, leading to potential cointegration (Chen, Rogoff, & Rossi, 2010).
Mathematically, the PVM expresses the exchange rate as:

$$e_t = \sum_{j=0}^{\infty} \beta^j E_t[f_{t+j}]$$

Where: e_t is the exchange rate at time t, β is a discount factor ($0 < \beta < 1$), Et is the expectation operator, $ft+j$ represents future fundamental variables, including commodity prices.
Under this framework, if commodity prices are an essential determinant of exchange rates, then the two series should exhibit a cointegrating relationship. This aligns with the empirical findings of Chen et al. (2010), who demonstrated that exchange rates can serve as predictors of future commodity prices.

2.2 Cointegration and Error Correction Model

Given the forward-looking nature of exchange rates and their possible link to commodity prices, we apply the Johansen (1991) system approach to cointegration to test for a long-run equilibrium relationship. If cointegration exists, the appropriate econometric model is the Vector Error Correction Model (VECM), which accounts for both short-run dynamics and long-run equilibrium adjustments. However, if no cointegration is found, a Vector Autoregression (VAR) model is more appropriate to capture short-term dynamics and analyze the interactions between the variables without imposing long-run equilibrium restrictions. The choice between these models will depend on the results of the cointegration test, which will determine whether the series share a stable long-run relationship.

The VECM representation for the exchange rate (Δe_t) and commodity prices (Δc_t) is given by:

$$\Delta e_t = \alpha_1(e_{t-1} - \beta c_{t-1}) + \sum_{i=1}^{p} \gamma_{1i} \Delta e_{t-i} + \sum_{i=1}^{p} \delta_{1i} \Delta c_{t-i} + \varepsilon_{1t}$$

$$\Delta c_t = \alpha_2(e_{t-1} - \beta c_{t-1}) + \sum_{i=1}^{p} \gamma_{2i} \Delta e_{t-i} + \sum_{i=1}^{p} \delta_{2i} \Delta c_{t-i} + \varepsilon_{2t}$$

Where: α_1 and α_2 represent the error correction terms, measuring the speed of adjustment to deviations from long-run equilibrium, β is the cointegration coefficient, γ and δ capture short-run dynamics, ε_{1t} and ε_{2t} are error terms.

2.3 Sensitivity Analysis: The Role of Interest Rate Differentials

Following the project instructions, We take the natural logarithm of exchange rates and commodity prices before estimation. This transformation is necessary because it stabilizes variance by reducing heteroskedasticity, enhances interpretability by allowing coefficients to be expressed in percentage terms, and helps achieve normality, given that many economic time series exhibit skewed distributions (Hamilton, 1994).

One of the key refinements in this project is testing whether the missing fundamental in our previous analysis is the interest rate differential between Canada and the U.S. This follows the work of Devereux and Smith (2021), who argue that incorporating interest rate differentials improves the explanatory power of exchange rate models. If a long-run relationship exists, a VECM would be the appropriate framework. However, if no cointegration is found, a VAR model will be used to analyze the short-run interactions among exchange rates, commodity prices, and interest rate differentials.
To investigate this, we extend our VECM framework to include the interest rate differential IRD_t:

$$\Delta e_t = \alpha_1(e_{t-1} - \beta_1 c_{t-1} - \beta_2 IRD_{t-1}) + \sum_{i=1}^{p} \gamma_{1i}\Delta e_{t-i} + \sum_{i=1}^{p} \delta_{1i}\Delta c_{t-i} + \sum_{i=1}^{p} \theta_{1i}\Delta IRD_{t-i} + \varepsilon_{1t}$$

$$\Delta c_t = \alpha_2(e_{t-1} - \beta_1 c_{t-1} - \beta_2 IRD_{t-1}) + \sum_{i=1}^{p} \gamma_{2i}\Delta e_{t-i} + \sum_{i=1}^{p} \delta_{2i}\Delta c_{t-i} + \sum_{i=1}^{p} \theta_{2i}\Delta IRD_{t-i} + \varepsilon_{2t}$$

where IRD_t is the interest rate differential between Canada and the U.S., and captures its dynamic effects on exchange rates and commodity prices.

2.4 Implications for Granger Causality

The Present Value Model suggests that commodity prices should Granger-cause exchange rates if exchange rates reflect expectations about future fundamentals. However, when interest rate differentials are included, the causal relationship may shift, suggesting that interest rates may also play a predictive role in exchange rate movements (Devereux & Smith, 2021). To formally test these relationships, we conduct conduct Granger causality tests to examine whether past values of one variable provide predictive information about another.
By incorporating these refinements, our empirical strategy improves upon the initial analysis in Project 1, ensuring a more comprehensive examination of the Canada-U.S. exchange rate and commodity price relationship. The next section outlines the data sources and methodology used in the empirical estimation.

3. Methodology and Empirical Analysis

This chapter presents the empirical strategy employed to analyze the relationship between the Canada-US exchange rate, commodity prices, and interest rate differentials. The dataset used in this study consists of monthly time series data spanning from January 1997 to April 2017. This period was selected to maintain consistency with Project 1, ensuring comparability of results and building upon the prior analysis. The data sources include Statistics Canada, the Bureau of Labor Statistics (BLS) of the United States, and the shadow short rates constructed by Krippner. The commodity price index and the Canada-US exchange rate data were obtained from Statistics Canada, while the Consumer Price Index (CPI) for the United States was retrieved from the BLS database. The shadow short rates (SSR) for Canada and the United States, which serve as proxies for monetary policy stance when interest rates approach the zero lower bound, were sourced from Krippner's publicly available dataset.

The choice of variables follows the economic theory underlying exchange rate determination. The commodity price index represents the value of Canada's commodity exports, which is in line with the present value model (PVM) of exchange rates, is expected to influence the exchange rate. The Canada-US nominal exchange rate (CAD/USD) is used, reflecting the value of the Canadian dollar relative to the US dollar. The US CPI is included to adjust for the real value of commodity prices, as inflation in the United States affects the purchasing power of the US dollar. Additionally, the shadow short rates (SSR) for Canada and the United States are incorporated to capture monetary policy effects. The interest rate differential between Canada and the US will be derived from these SSR values, as specified in Part II of the project requirements.

To ensure consistency in interpretation and to stabilize variance in the data, the natural logarithm was applied to key economic variables. Specifically, the natural logs of commodity prices, the Canada-US exchange rate, and the US CPI were taken before proceeding with the econometric analysis. This transformation serves two main purposes: first, it allows for direct elasticity interpretation in regression models, and second, it improves the stationarity properties of the data, which is crucial for time series analysis.

Before proceeding to formal econometric testing, it is useful to visually inspect the data. Figures 1 to 6 display the time series plots of the key variables. These graphs provide an initial understanding of the trends, volatility, and potential structural breaks that may exist in the data.

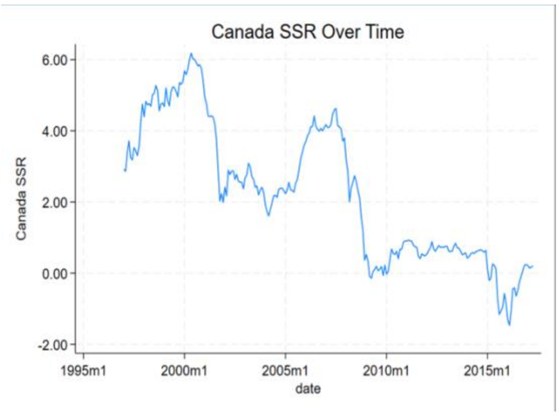


Fig 1: Line graph of Canada SSR

Fig 2: Line graph of commodity prices

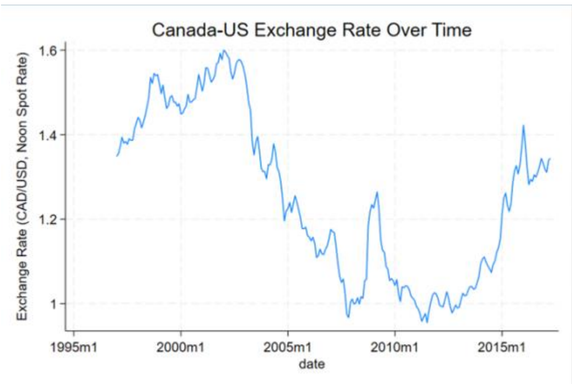


Fig 3: Line graph of Canada-US Exchange Rate

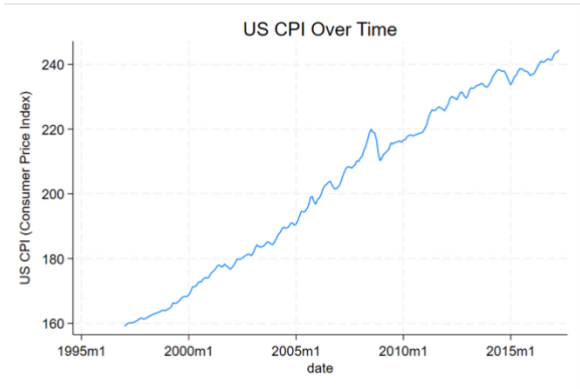


Fig 4: Line graph of US CPI

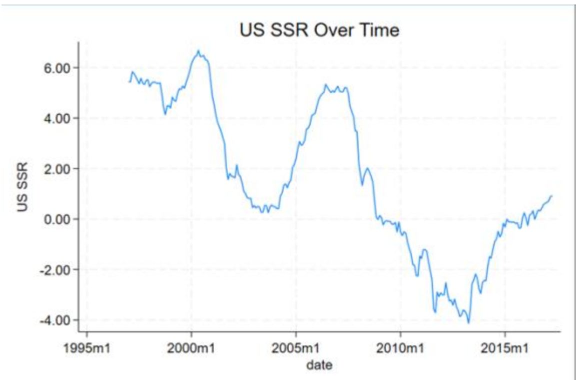


Fig 5: Line graph US SSR

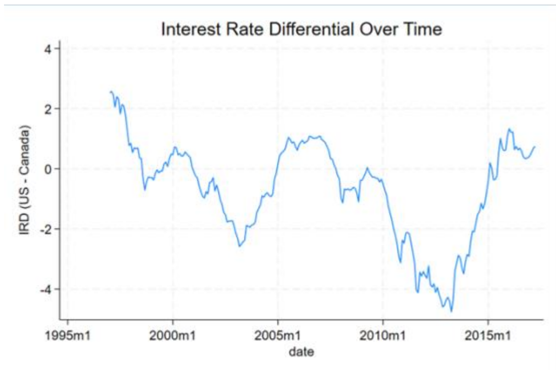


Fig 6: Line graph of IRD

The graphs reveal several important patterns. Commodity prices exhibit cyclical fluctuations, with noticeable spikes around the 2008 financial crisis and the subsequent commodity boom. The Canada-US exchange rate also follows a cyclical pattern, suggesting a possible co-movement with commodity prices. The US CPI demonstrates a persistent upward trend, reflecting inflation over the two-decade period. The shadow short rates for both Canada and the US indicate distinct phases of monetary policy, including periods of near-zero interest rates following the financial crisis. The interest rate differential (IRD) between the US and Canada shows fluctuations over time, reflecting shifts in monetary policy and economic conditions in both countries. The IRD graph highlights periods of divergence, notably during the early 2000s and the post-2008 financial crisis era, when interest rate policies differed significantly between the two economies. This variable will be critical in assessing its relationship with exchange rates and commodity prices.

Having established the descriptive properties of the data, the next step is to conduct stationarity tests**.** Given that most macroeconomic time series tend to exhibit unit roots, it is essential to formally test for stationarity using Augmented Dickey-Fuller (ADF) tests**.** These tests will determine whether the variables are integrated of order zero (I(0)) or require differencing to become stationary. The results of the stationarity tests will guide the subsequent choice of econometric models, particularly whether a cointegration analysis is warranted.

3.1 Stationarity Tests: Augmented Dickey-Fuller (ADF) Test

From the Augmented Dickey-Fuller (ADF) test results for stationarity, we obtained the following findings:

Exchange Rate (log_exrate): The test statistic is -1.242, which is greater than the critical values at the 1%, 5%, and 10% significance levels (-3.994, -3.431, and -3.131, respectively). The p-value is 0.9015, indicating that we fail to reject the null hypothesis of a unit root. This suggests that the exchange rate is non-stationary.

```
. dfuller log_exrate, lags(4) trend

Augmented Dickey-Fuller test for unit root

Variable: log_exrate                      Number of obs  = 239
                                          Number of lags =   4

H0: Random walk with or without drift

                                       Dickey-Fuller
                   Test          ------- critical value -------
                statistic           1%           5%          10%
----------------------------------------------------------------
 Z(t)              -1.242       -3.994       -3.431       -3.131
----------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.9015.
```

Commodity Price (log_commprice): The test statistic is -2.114, which is also greater than the critical values at all significance levels. The p-value is 0.5381, indicating that we fail to reject the null hypothesis, meaning that commodity prices are non-stationary.

```
. dfuller log_commprice, lags(4) trend

Augmented Dickey-Fuller test for unit root

Variable: log_commprice                         Number of obs  = 239
                                                Number of lags =   4

H0: Random walk with or without drift

                                       Dickey-Fuller
                   Test          ─────── critical value ───────
              statistic           1%           5%           10%
──────────────────────────────────────────────────────────────
 Z(t)            -2.114       -3.994       -3.431        -3.131
──────────────────────────────────────────────────────────────
MacKinnon approximate p-value for Z(t) = 0.5381.
```

Interest Rate Differential (IRD): The test statistic is -1.527, which again is greater than all the critical values. The p-value is 0.8198, meaning we fail to reject the null hypothesis, implying that IRD is non-stationary.

```
. dfuller IRD, lags(4) trend

Augmented Dickey-Fuller test for unit root

Variable: IRD                                   Number of obs  = 239
                                                Number of lags =   4

H0: Random walk with or without drift

                                       Dickey-Fuller
                   Test          ─────── critical value ───────
              statistic           1%           5%           10%
──────────────────────────────────────────────────────────────
 Z(t)            -1.527       -3.994       -3.431        -3.131
──────────────────────────────────────────────────────────────
MacKinnon approximate p-value for Z(t) = 0.8198.
```

Since all three variables log_exrate, log_commprice, and IRD fail to reject the null hypothesis of a unit root, they are non-stationary in levels. This suggests that we need to difference the variables to test for stationarity at the first difference.

Upon conducting the stationarity tests on the first-differenced series, the results showed a substantial shift in statistical significance. The test statistic for the first difference of the log exchange rate *(d_log_exrate)* was -5.845, which is significantly lower than the 1% critical value of -3.994. The corresponding p-value of 0.0000 strongly rejects the null hypothesis of a unit root, confirming that the series is now stationary. Similarly, the first-differenced log of commodity prices *(d_log_commprice)* had a test statistic of -6.790, again well below the 1% critical value, demonstrating stationarity. The first-differenced interest rate differential *(d_IRD)* also displayed stationarity, with a test statistic of -5.745, further rejecting the presence of a unit root.

```
. dfuller d_log_commprice, lags(4) trend

Augmented Dickey-Fuller test for unit root

Variable: d_log_commprice                    Number of obs  = 238
                                             Number of lags =   4

H0: Random walk with or without drift

                                       Dickey-Fuller
                   Test         ────── critical value ──────
              statistic           1%           5%          10%
──────────────────────────────────────────────────────────────
 Z(t)            -6.790       -3.994       -3.432       -3.132
──────────────────────────────────────────────────────────────
MacKinnon approximate p-value for Z(t) = 0.0000.
```

```
. dfuller d_log_exrate, lags(4) trend

Augmented Dickey-Fuller test for unit root

Variable: d_log_exrate                       Number of obs  = 238
                                             Number of lags =   4

H0: Random walk with or without drift

                                       Dickey-Fuller
                   Test         ────── critical value ──────
              statistic           1%           5%          10%
──────────────────────────────────────────────────────────────
 Z(t)            -5.845       -3.994       -3.432       -3.132
──────────────────────────────────────────────────────────────
MacKinnon approximate p-value for Z(t) = 0.0000.
```

```
. dfuller d_IRD, lags(4) trend

Augmented Dickey-Fuller test for unit root

Variable: d_IRD                              Number of obs  = 238
                                             Number of lags =   4

H0: Random walk with or without drift

                                       Dickey-Fuller
                   Test         ────── critical value ──────
              statistic           1%           5%          10%
──────────────────────────────────────────────────────────────
 Z(t)            -5.745       -3.994       -3.432       -3.132
──────────────────────────────────────────────────────────────
MacKinnon approximate p-value for Z(t) = 0.0000.
```

These results confirm that all three variables are integrated of order one, I(1), meaning they become stationary after taking the first difference. This finding is crucial for the subsequent econometric analysis. Since the original variables were non-stationary, direct regression analysis in levels could have led to spurious results. However, given that all variables are I(1), there is now a basis to examine potential long-run relationships through cointegration analysis.

The number of lags for the ADF tests was chosen based on model selection criteria that help balance accuracy and complexity. Specifically, the Akaike Information Criterion (AIC) and Schwarz Information Criterion (SIC) were used to evaluate different lag lengths, and the optimal choice was the one that minimized these criteria. A lag length of four was selected to account for autocorrelation while ensuring sufficient observations for reliable estimation. This choice helps improve the accuracy of the stationarity test by capturing short-term dynamics without overfitting the model.

With all variables confirmed to be I(1), the next step in the analysis is to conduct cointegration tests to determine whether a stable long-run equilibrium exists between the exchange rate, commodity prices, and interest rate differentials. If a cointegrating relationship is present, it would indicate that despite short-term fluctuations, these variables move together in the long run, reinforcing the economic intuition underlying their interactions.

3.2 Cointegration Test: Johansen's Cointegration Test

The Johansen cointegration test results indicate that there is no cointegrating relationship among log_exrate, log_commprice, and IRD. The null hypothesis at rank 0 states that there is no cointegration among the variables. The trace statistic of 24.4295 is lower than the 5% critical value of 29.68, meaning we fail to reject the null hypothesis of no cointegration. Further tests at rank 1 and beyond also fail to reject their respective null hypotheses, confirming that there are no cointegrating equations in the system.

```
. vecrank log_exrate log_commprice IRD, lag(4) trend(constant)

Johansen tests for cointegration
Trend: Constant                               Number of obs  = 240
Sample: 1997m5 thru 2017m4                    Number of lags =   4
------------------------------------------------------------------
                                                         Critical
Maximum                                          Trace      value
   rank  Params          LL  Eigenvalue      statistic         5%
      0      30    1039.244           .       24.4295*      29.68
      1      35   1048.5621     0.07471        5.7933       15.41
      2      38   1050.7156     0.01779        1.4863        3.76
      3      39   1051.4587     0.00617
------------------------------------------------------------------
* selected rank

.
end of do-file
```

These findings suggest that the variables do not exhibit a stable long-run relationship. Since no cointegration is detected, a Vector Error Correction Model (VECM) is not appropriate. Instead, a Vector Autoregression (VAR) model will be estimated using the first-differenced series to analyze the short-run dynamics.

The next step is to determine the optimal lag length for the VAR model before proceeding with the estimation.

3.3 Determining Optimal Lag

To determine the optimal lag length for the VAR model, we will use lag selection criteria such as the Akaike Information Criterion (AIC) and Schwarz Information Criterion (SIC). These criteria help identify the lag length that balances model complexity and goodness of fit, ensuring that we do not overfit or underfit the data. The goal is to choose the lag length that minimizes these criteria.

```
. varsoc d_log_exrate d_log_commprice d_IRD, maxlag(6)

Lag-order selection criteria

   Sample: 1997m8 thru 2017m4                            Number of obs = 237
```

Lag	LL	LR	df	p	FPE	AIC	HQIC	SBIC
0	994.356				4.7e-08	-8.36587	-8.34818	-8.32197
1	1020.19	51.669*	9	0.000	4.1e-08*	-8.50793*	-8.43716*	-8.33234*
2	1023.77	7.1565	9	0.621	4.2e-08	-8.46218	-8.33832	-8.15488
3	1025.74	3.9406	9	0.915	4.5e-08	-8.40286	-8.22592	-7.96386
4	1034.09	16.706	9	0.054	4.5e-08	-8.3974	-8.16737	-7.82671
5	1041.87	15.549	9	0.077	4.6e-08	-8.38706	-8.10395	-7.68467
6	1049.98	16.225	9	0.062	4.6e-08	-8.37957	-8.04338	-7.54548

```
   * optimal lag
   Endogenous: d_log_exrate d_log_commprice d_IRD
    Exogenous: _cons

.
end of do-file
```

The lag selection results indicate that the optimal lag length for the VAR model is one (1) based on the Akaike Information Criterion (AIC), Schwarz Information Criterion (SIC), and Hannan-Quinn Information Criterion (HQIC). These criteria are used to balance model complexity and accuracy, with lower values indicating better model fit.

The AIC, which prioritizes goodness of fit, selects lag 1 as the optimal lag length since it has the lowest value of -8.50793. Similarly, the HQIC and SBIC also favor lag 1, confirming its suitability. Since all three criteria agree on lag 1, we proceed with this selection for the VAR and cointegration analysis.

Now that we have determined the optimal lag length, the next step is to estimate the Vector Autoregression (VAR) model using the first-differenced variables.

3.4 Estimate VAR Model

Given that the Johansen cointegration test did not indicate a cointegrating relationship, a Vector Error Correction Model (VECM) is not appropriate. Instead, we proceed with a VAR model in first differences to capture the short-run dynamics between exchange rates, commodity prices, and interest rate differentials.

Estimating the VAR model in first differences allows us to analyze the interdependencies among the variables while ensuring stationarity. The VAR framework treats all included variables as endogenous, allowing each variable to be expressed as a function of its own past values and the past values of the other variables in the system.

```
. var d_log_exrate d_log_commprice d_IRD, lags(1)

Vector autoregression

Sample: 1997m3 thru 2017m4                      Number of obs     =        242
Log likelihood =   1042.758                     AIC               = -8.518665
FPE            =   4.01e-08                     HQIC              = -8.448972
Det(Sigma_ml)  =   3.63e-08                     SBIC              =  -8.34566

Equation           Parms      RMSE     R-sq      chi2     P>chi2
----------------------------------------------------------------
d_log_exrate          4     .017784   0.1132   30.87867   0.0000
d_log_commprice       4     .050684   0.0924   24.65105   0.0000
d_IRD                 4     .249949   0.0475   12.05802   0.0072
----------------------------------------------------------------
```

	Coefficient	Std. err.	z	P>|z|	[95% conf.	interval]
d_log_exrate						
d_log_exrate L1.	.2544729	.0715502	3.56	0.000	.1142372	.3947087
d_log_commprice L1.	-.0412401	.0252381	-1.63	0.102	-.0907059	.0082258
d_IRD L1.	.002371	.0045004	0.53	0.598	-.0064496	.0111915
_cons	.0000313	.0011344	0.03	0.978	-.0021922	.0022547
d_log_commprice						
d_log_exrate L1.	-.2021419	.2039146	-0.99	0.322	-.6018072	.1975235
d_log_commprice L1.	.2606952	.0719275	3.62	0.000	.1197199	.4016704
d_IRD L1.	.0099088	.0128258	0.77	0.440	-.0152293	.035047
_cons	.0011717	.0032331	0.36	0.717	-.005165	.0075084
d_IRD						
d_log_exrate L1.	.0883553	1.005613	0.09	0.930	-1.88261	2.05932
d_log_commprice L1.	-.3190192	.3547131	-0.90	0.368	-1.014244	.3762057
d_IRD L1.	.2002949	.0632511	3.17	0.002	.0763251	.3242647
_cons	-.0058147	.015944	-0.36	0.715	-.0370644	.0254349

The VAR model was estimated using the first differences of the log-transformed exchange rate and commodity prices, along with the first difference of the interest rate differential. The results indicate the short-run interactions between these variables. The chi-square test statistics and p-values confirm that the variables included in the model are significantly related at least in some equations.

From the coefficient estimates, the lagged difference of the exchange rate *(d_log_exrate)* has a significant positive impact on its own value at the next period, while the lagged difference of commodity prices *(d_log_commprice)* does not significantly influence the exchange rate. The interest rate differential *(d_IRD)* also does not appear to significantly impact the exchange rate in the short run.

For the commodity price equation, the lagged difference of commodity prices significantly affects its own next-period value, suggesting some degree of persistence in commodity price fluctuations. The exchange rate and interest rate differential, however, do not significantly affect commodity prices in the short run.

In the equation for the interest rate differential, the lagged value of the interest rate differential is significant, indicating some degree of persistence. However, neither exchange rate changes nor commodity price changes appear to have a significant short-run impact on the interest rate differential.

These findings suggest that the short-run dynamics among these variables are weak, but further analysis is needed to determine their long-run relationships. The next step is to conduct impulse response function (IRF) analysis to examine how shocks to one variable propagate through the system over time.

3.5 Impulse Response Functions (IRFs)

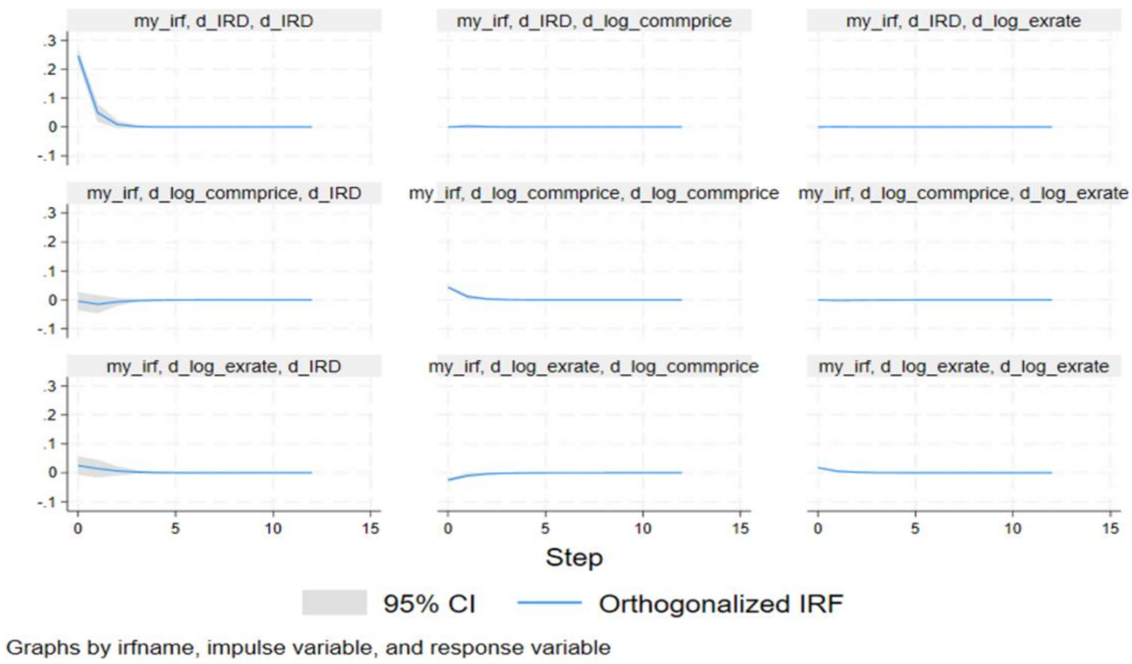
Impulse response functions (IRFs) are used to examine how a shock to one variable in the system affects the other variables over time. In this analysis, the IRFs illustrate how exchange rates, commodity prices, and interest rate differentials respond to shocks in each of these variables within a vector autoregression (VAR) framework. The response paths are presented for a 12-period horizon to capture both short-term and medium-term dynamics.

The results indicate that an interest rate differential (IRD) shock has a significant initial effect on itself, but this impact quickly diminishes and stabilizes around zero after a few periods. This pattern suggests that IRD shocks do not have persistent long-term effects within the system. The exchange rate and commodity prices show minimal responses to an IRD shock, implying that changes in interest rate differentials do not exert strong immediate or prolonged influence on these variables in the dataset.

A shock to commodity prices does not appear to have a substantial impact on exchange rates or IRD. The response functions remain close to zero, indicating that commodity price movements may not be a primary driver of exchange rate fluctuations in this framework. This finding aligns with the possibility that other macroeconomic variables, such as capital flows and broader monetary policies, play a more dominant role in exchange rate determination.

Similarly, a shock to the exchange rate does not produce strong responses in commodity prices or IRD. The response functions remain largely flat, suggesting limited feedback effects within the system. This may indicate that while exchange rates fluctuate due to various external factors, their direct influence on commodity prices and IRD is not statistically pronounced over the given time horizon.

The 12-step horizon was chosen to provide sufficient time for the system to adjust to shocks while avoiding excessive forecasting uncertainty. This allows for a comprehensive view of how responses evolve over time while maintaining reliability in the results.



The impulse response functions are presented in the screenshot below, with the x-axis representing the number of periods after the shock and the y-axis showing the magnitude of the response. The shaded areas represent 95% confidence intervals, providing a measure of statistical uncertainty around the estimated responses. The results confirm that, within this framework, interest rate differentials, commodity prices, and exchange rates exhibit weak dynamic interactions over time.

3.6 Granger causality tests

The Granger causality tests reveal no statistically significant causal relationships among the exchange rate, commodity prices, and interest rate differentials. The p-values for all tests exceed conventional significance thresholds, indicating that past values of one variable do not systematically predict future values of the others. This suggests that while these variables may be theoretically linked, their short-run predictive relationships are not robust in this dataset. Possible explanations include omitted variables, external shocks, or structural shifts over time. The results imply that the dynamic interactions among these macroeconomic variables are more complex than a simple Granger causality framework can capture.

The table below presents the Granger causality results.

```
. vargranger

  Granger causality Wald tests
```

Equation	Excluded	chi2	df	Prob > chi2
d_log_exrate	d_log_commprice	2.6701	1	0.102
d_log_exrate	d_IRD	.27756	1	0.598
d_log_exrate	ALL	2.9738	2	0.226
d_log_commprice	d_log_exrate	.98269	1	0.322
d_log_commprice	d_IRD	.59686	1	0.440
d_log_commprice	ALL	1.4403	2	0.487
d_IRD	d_log_exrate	.00772	1	0.930
d_IRD	d_log_commprice	.80887	1	0.368
d_IRD	ALL	1.2359	2	0.539

```
.
end of do-file
```

3.7 Summary of Empirical Methodology

This chapter presented the methodology and empirical analysis conducted to examine the relationship between the Canada-US exchange rate, commodity prices, and interest rate differentials. The data was transformed using logarithms where appropriate, and stationarity tests confirmed that all variables were integrated of order one, I(1). Given this finding, the Johansen cointegration test was performed to assess the presence of a long-run equilibrium relationship. The results indicated no cointegrating vectors, suggesting that the variables do not share a stable long-term relationship.

In light of the cointegration results, a Vector Autoregression (VAR) model was estimated using the optimal lag length selected based on information criteria. The Impulse Response Functions (IRFs) were analyzed to trace the dynamic effects of shocks to each variable. The responses were largely transitory, further supporting the absence of a long-run relationship. Additionally, the Granger causality tests showed no statistically significant predictive causality among the variables at conventional significance levels.

Overall, the empirical findings suggest that while short-term interactions exist, the Canada-US exchange rate, commodity prices, and interest rate differentials do not exhibit a stable long-run equilibrium. These results align with prior literature indicating that additional macroeconomic factors may be necessary to fully explain the movements in exchange rates and commodity prices. The next chapter will provide a broader discussion on these findings in relation to theoretical expectations and policy implications.

4. Discussion and Interpretation of Results

The empirical findings from the previous chapter provide critical insights into the relationship between the Canada-US exchange rate, commodity prices, and interest rate differentials. This chapter interprets the results in the context of theoretical expectations, particularly the present-value (PV) approach, and discusses their broader implications for exchange rate behavior.

4.1 Stationarity and Cointegration Analysis

The stationarity tests confirmed that all variables were non-stationary in levels but became stationary after first differencing, establishing that they are integrated of order one. This property justified the use of cointegration analysis to assess whether a stable long-run relationship exists among the exchange rate, commodity prices, and interest rate differentials.

The Johansen cointegration test results indicated no evidence of a long-run equilibrium relationship among the variables. This finding implies that while these economic fundamentals may exhibit short-term dependencies, they do not move together in a predictable and stable manner over time. The absence of cointegration challenges theoretical expectations that exchange rates should reflect fundamental macroeconomic relationships in the long run.

4.2 Implications for the Present-Value (PV) Approach

The lack of cointegration carries significant implications when evaluated in the context of the PV approach. The PV model suggests that exchange rates, as asset prices, should reflect the discounted expected future values of economic fundamentals, particularly commodity prices. If this theory held empirically, a cointegrating relationship would exist between exchange rates and their key determinants. However, the results indicate otherwise, suggesting that additional macroeconomic factors or market inefficiencies may drive deviations from theoretical expectations.

The findings suggest that exchange rate movements may be influenced by speculative forces, external shocks, and country-specific policies that prevent a stable long-term relationship with commodity prices and interest rate differentials. This aligns with previous research indicating that financial markets often deviate from fundamentals due to investor sentiment, liquidity constraints, and global risk conditions.

4.3 Short-Run Dynamics and VAR Model Interpretation

In the absence of cointegration, a Vector Autoregression (VAR) model was estimated to analyze the short-term interactions among the variables. The Impulse Response Functions (IRFs) revealed that shocks to commodity prices and interest rate differentials had only transitory effects on the Canada-US exchange rate. The responses tended to dissipate quickly, suggesting that while these variables may momentarily influence exchange rate fluctuations, they do not exert persistent or structural effects.

These results align with existing literature that posits exchange rate fluctuations are often driven by short-term market adjustments rather than long-term macroeconomic fundamentals. Factors such as speculation, short-term capital flows, and policy announcements likely contribute to exchange rate volatility in ways that are not fully captured by commodity prices and interest rate differentials alone.

4.4 Granger Causality and Predictive Relationships

The Granger causality tests provided further insights into the predictive relationships among the variables. The results indicated no statistically significant causality between commodity prices, interest rate differentials, and the exchange rate. This implies that past values of these economic fundamentals do not systematically predict future exchange rate movements.

This outcome is consistent with studies suggesting that exchange rate fluctuations are influenced by a broader set of factors, including monetary policy expectations, global capital flows, investor sentiment, and geopolitical risks. The lack of causal relationships underscores the complexity of exchange rate determination and challenges the assumption that macroeconomic fundamentals alone dictate currency movements.

4.5 Broader Implications

The combined findings suggest that the Canada-US exchange rate operates within a flexible regime where short-term market forces rather than long-term economic fundamentals dominate its movements. The absence of cointegration and Granger causality further supports the notion that external factors, such as central bank policies, speculative trading, and global economic shocks, significantly shape exchange rate behavior.

From a policy perspective, these results highlight the need for exchange rate forecasting models to incorporate a wider range of economic and financial variables. Reliance on commodity prices and interest rate differentials as sole predictors of exchange rate movements may be insufficient given the evidence of short-lived effects and lack of long-term equilibrium relationships.

4.6 Conclusion

The empirical analysis suggests that while commodity prices and interest rate differentials are important macroeconomic indicators, they do not singularly determine the Canada-US exchange rate in a long-run equilibrium framework. This underscores the complexity of exchange rate movements and emphasizes the role of short-term financial market dynamics.

The implications of these findings extend beyond academia to policymakers, investors, and financial analysts, who must consider a broader set of factors when assessing exchange rate behavior. Future research could explore the impact of global capital flows, investor sentiment, and macroeconomic policy changes on exchange rate fluctuations to provide a more comprehensive understanding of currency dynamics.

5. Conclusion

5.1 Summary of Findings

This study examined the relationship between the Canada-US exchange rate, commodity prices, and interest rate differentials using time-series econometric methods. The analysis followed a structured approach, beginning with stationarity tests, followed by cointegration analysis, estimation of a vector autoregression (VAR) model, impulse response functions (IRFs), and Granger causality tests.

The results from the Augmented Dickey-Fuller (ADF) tests confirmed that all variables were non-stationary in levels but became stationary after first differencing, indicating that they are integrated of order one. The Johansen cointegration test provided no evidence of a long-run equilibrium relationship among the exchange rate, commodity prices, and interest rate differentials. This suggests that these variables do not move together in a stable manner over time, challenging the fundamental-based prediction of the present-value (PV) approach.

Given the lack of cointegration, a VAR model was estimated to analyze the short-run interactions among the variables. The impulse response functions (IRFs) revealed that shocks to commodity prices and interest rate differentials had only short-lived effects on exchange rate movements. These responses dissipated quickly, suggesting that exchange rate fluctuations are primarily driven by short-term market adjustments rather than persistent structural forces.

The Granger causality tests further supported these findings by showing no statistically significant predictive relationships among the variables. Neither commodity prices nor interest rate differentials were found to Granger-cause the exchange rate, indicating that changes in these economic fundamentals do not systematically predict future exchange rate movements.

5.2 Policy Implications

The empirical results have important policy implications. The lack of a long-run relationship between the exchange rate, commodity prices, and interest rate differentials suggests that policymakers should be cautious when using these factors to guide exchange rate predictions. While commodity prices and interest rate differentials influence exchange rates in the short run, other variables such as monetary policy decisions, capital flows, geopolitical risks, and investor sentiment may play a more dominant role in determining exchange rate dynamics.

For firms and investors, these findings emphasize the importance of considering a broader range of macroeconomic and financial indicators when assessing exchange rate risks. Strategies that rely solely on commodity price fluctuations or interest rate differentials to forecast exchange rate movements may not be effective, given the absence of a stable long-run relationship between these variables.

5.3 Directions for Future Research

Future research could extend this analysis by incorporating additional explanatory variables, such as global liquidity conditions, fiscal policy indicators, or trade balances, to capture a more comprehensive picture of exchange rate determination. Alternative econometric approaches, including nonlinear models or regime-switching frameworks, could also be explored to assess whether different exchange rate regimes exhibit distinct relationships with commodity prices and interest rates.

Additionally, examining how exchange rate dynamics evolve in response to macroeconomic shocks in different economic cycles could provide further insights into the underlying mechanisms driving currency fluctuations. Expanding the analysis to a panel setting that includes multiple commodity-exporting countries could also offer a broader perspective on the role of commodity prices in exchange rate determination.

5.4 Concluding Remarks

Overall, this study contributes to the ongoing discussion on exchange rate determination by providing empirical evidence that challenges the assumption of a stable long-run relationship between the Canada-US exchange rate, commodity prices, and interest rate differentials. The findings highlight the complexity of exchange rate movements and underscore the need for a broader analytical perspective that goes beyond traditional fundamental-based models.

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