example

September 30, 2022

1 Kernel Density Inference: Example

1.1 Load Module

[1]:
```
include("src/main.jl")
using .KernelTests
```

1.2 Kernel Density

1.2.1 Univariate

[2]:
```
x = randn(1000) .* 5 .+ 3 # Normal random sample N = 1000,   = 3,   = 5
kd = KernelDensity(x) # Kernel Density
kd |> display
```

```
KernelDensity:
    n = 1000
    Bandwidth (h) = 1.3196004252724247
    domain = (-22.804777916778914, 31.972903880045113)
```

You can spacify the bandwith, other wise the optimum one will be used.

[3]:
```
KernelDensity(x, h = 1.1) |> display
```

```
KernelDensity:
    n = 1000
    Bandwidth (h) = 1.1
    domain = (-22.804777916778914, 31.972903880045113)
```

[4]:
```
# Evaluate at one point
KernelTests.density(0,kd)
```

[4]:
```
0.06566020195770363
```

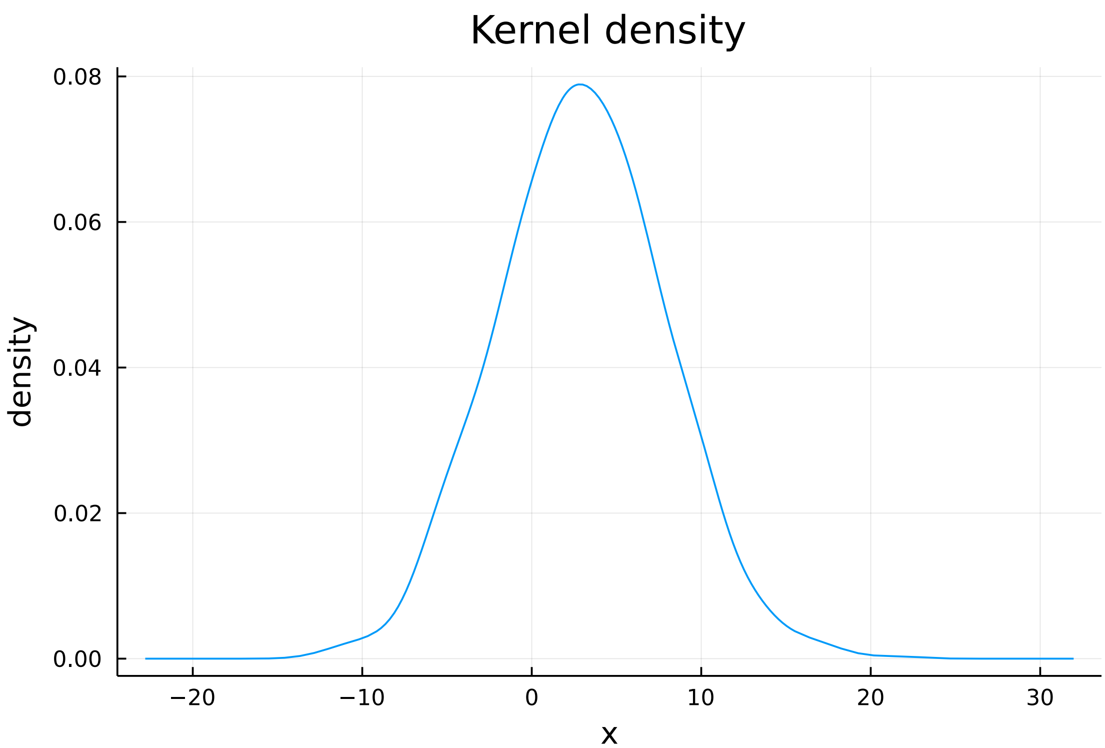
[5]:
```
# Evaluate at several points
KernelTests.density([-10,0,10],kd)
```

[5]: 3-element Vector{Float64}:
```
 0.0028059246932455807
 0.0656602019577 0363
 0.030563081099 70754
```

Plot

[6]:
```julia
plot(kd,
    title="Kernel density",
    legend = false,
    xlabel = "x",
    ylabel = "density"
)
```

[6]:



Another way to plot it is to discretize it and then use the plot function from Plots package

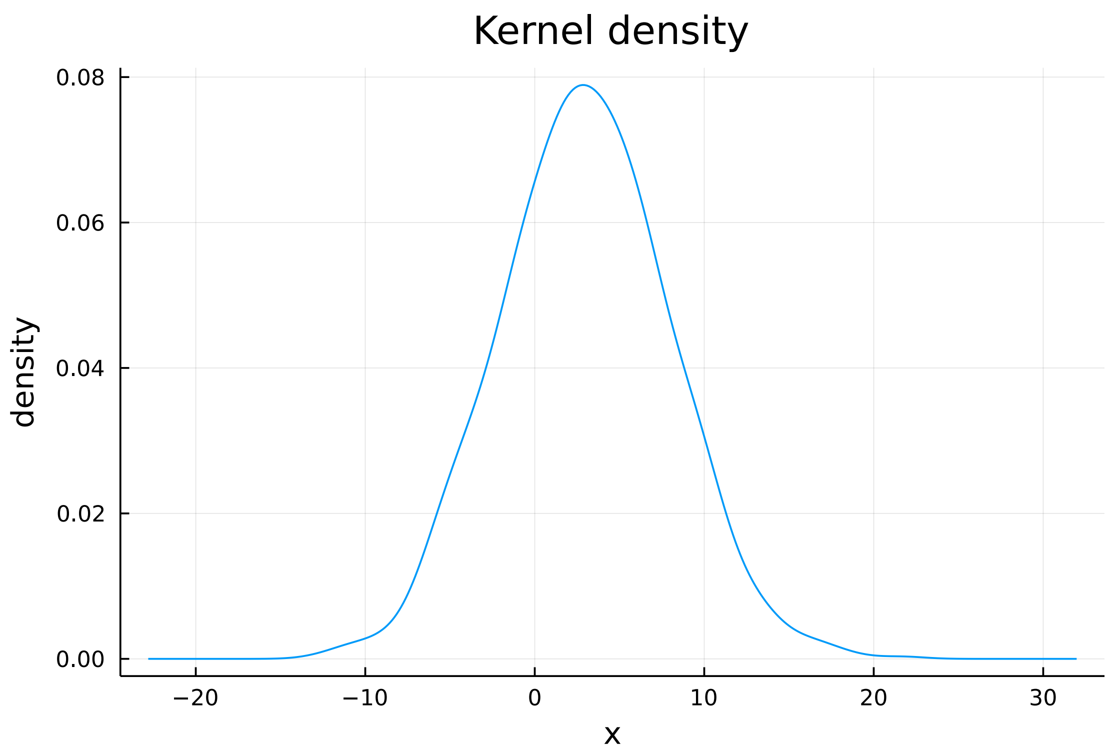
[7]:
```julia
using Plots
my_x,my_y = discretize(kd)
# my_x,my_y = discretize(kd,nGrid = 1000)
[my_x my_y]
```

[7]:
```
401×2 Matrix{Float64}:
 -22.8048  2.11907e-17
 -22.6678  4.72748e-17
 -22.5309  1.04338e-16
```

```
-22.3939  2.27813e-16
-22.257   4.92087e-16
-22.1201  1.05156e-15
-21.9831  2.22306e-15
-21.8462  4.64942e-15
-21.7092  9.61998e-15
-21.5723  1.96915e-14
-21.4353  3.98762e-14
-21.2984  7.98877e-14
-21.1614  1.58335e-13
 ⋮
 30.4665  7.97158e-14
 30.6035  3.97983e-14
 30.7404  1.96566e-14
 30.8774  9.60446e-15
 31.0143  4.6426e-15
 31.1512  2.2201e-15
 31.2882  1.05028e-15
 31.4251  4.91545e-16
 31.5621  2.27585e-16
 31.699   1.04243e-16
 31.836   4.72357e-17
 31.9729  2.11748e-17
```

[8]:
```
plot(my_x,my_y,
    title="Kernel density",
    legend = false,
    xlabel = "x",
    ylabel = "density"
)
```

[8]:



1.2.2 Bivariate

[9]:
```
x = rand(100) # 100 samples from uniform(0,1) distribution
y = -(x .- 1).*(x .+ 1) .+ 0.1 .*randn(100)
bkd = BivariateKernelDensity(x,y)
# you can also specify x and y bandwidths with arguments hx and hy.
bkd |> display
```

```
KernelDensity:
    n = 100
    Bandwidths (hx,hy) = (0.12526769848607014, 0.1400254508296314)
    domainx = (-0.2983285461070769, 1.2940439551018128)
    domainy = (-0.4437317141641693, 1.5686070505864542)
```

[10]:
```
# evaluate it at one point
KernelTests.density((0,0),bkd)
```

[10]: 6.453062924176445e-8

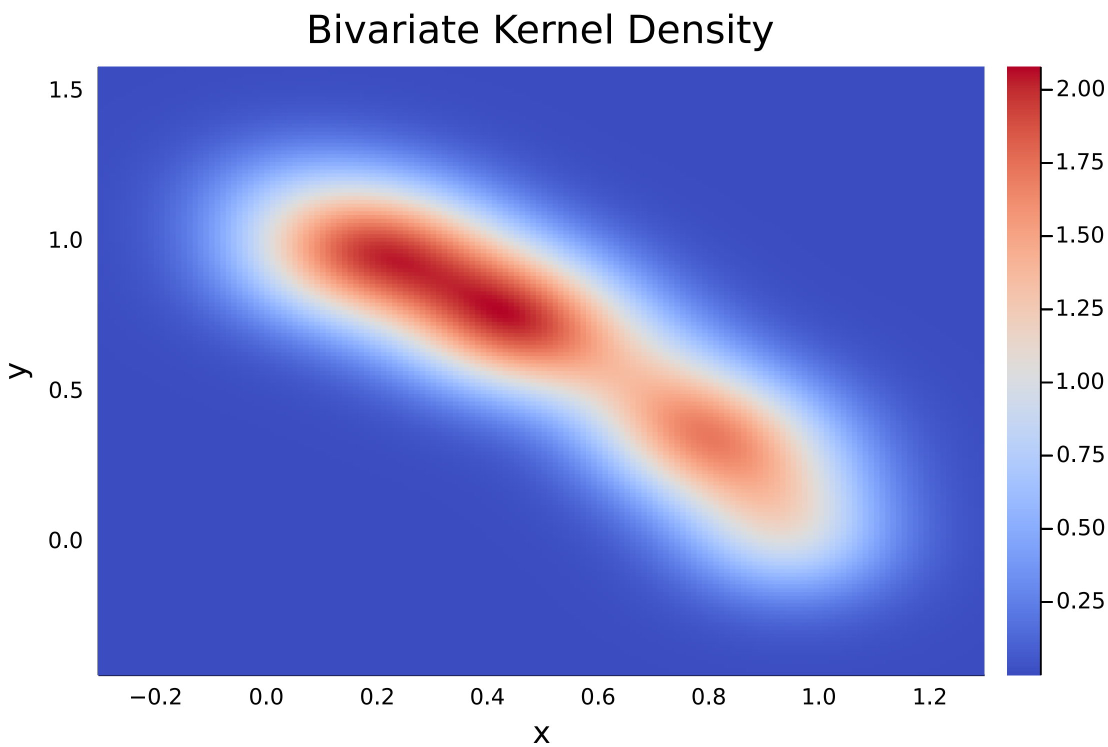
[11]:
```
# evaluate it at several points
KernelTests.density([(0,0),(-0.2,1.0)],bkd)
```

[11]: 2-element Vector{Float64}:
```
 6.453062924176445e-8
 0.09828016165838052
```

Plot

[12]:
```julia
plot(bkd,
    title = "Bivariate Kernel Density",
    xlabel = "x",
    ylabel = "y"
)
```

[12]:



You can discretize bivariate kernels too:

[13]:
```julia
my_pts,my_dty = discretize(bkd)
# my_pts,my_dty = discretize(bkd, nGrid = 100)
[my_pts my_dty]
```

[13]:
```
160000×2 Matrix{Any}:
 (-0.298329, -0.443732)  5.84408e-21
 (-0.298329, -0.438688)  7.78644e-21
 (-0.298329, -0.433645)  1.03686e-20
 (-0.298329, -0.428601)  1.37988e-20
 (-0.298329, -0.423558)  1.83526e-20
 (-0.298329, -0.418514)  2.43936e-20
```

```
(-0.298329, -0.413471)  3.24015e-20
(-0.298329, -0.408428)  4.30088e-20
(-0.298329, -0.403384)  5.70487e-20
(-0.298329, -0.398341)  7.56174e-20
(-0.298329, -0.393297)  1.00156e-19
(-0.298329, -0.388254)  1.3256e-19
(-0.298329, -0.38321)   1.75313e-19

(1.29404, 1.51313)      1.88747e-14
(1.29404, 1.51817)      1.5466e-14
(1.29404, 1.52322)      1.26603e-14
(1.29404, 1.52826)      1.03534e-14
(1.29404, 1.5333)       8.45844e-15
(1.29404, 1.53835)      6.90348e-15
(1.29404, 1.54339)      5.62879e-15
(1.29404, 1.54843)      4.58494e-15
(1.29404, 1.55348)      3.73097e-15
(1.29404, 1.55852)      3.03305e-15
(1.29404, 1.56356)      2.46325e-15
(1.29404, 1.56861)      1.99852e-15
```

1.3 Hypothesys Tests

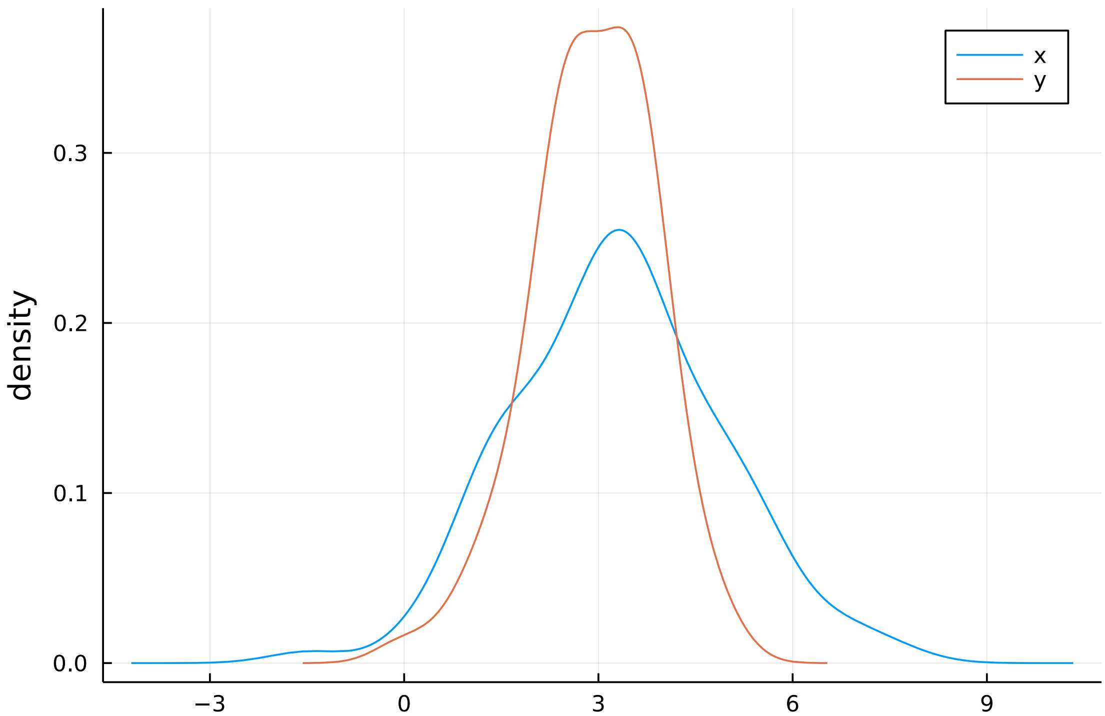
1.3.1 Distributions equality test

[14]:
```
# Generate samples
x = randn(100) .* 1.5 .+ 3
y = randn(80) .+ 3
length(x),length(y)
```

[14]: (100, 80)

[15]:
```
# plot their kernel density estimations
plot(KernelDensity(x),
    label = "x",
    ylabel = "density"
)
plot!(KernelDensity(y),
    label = "y"
)
```

[15]:

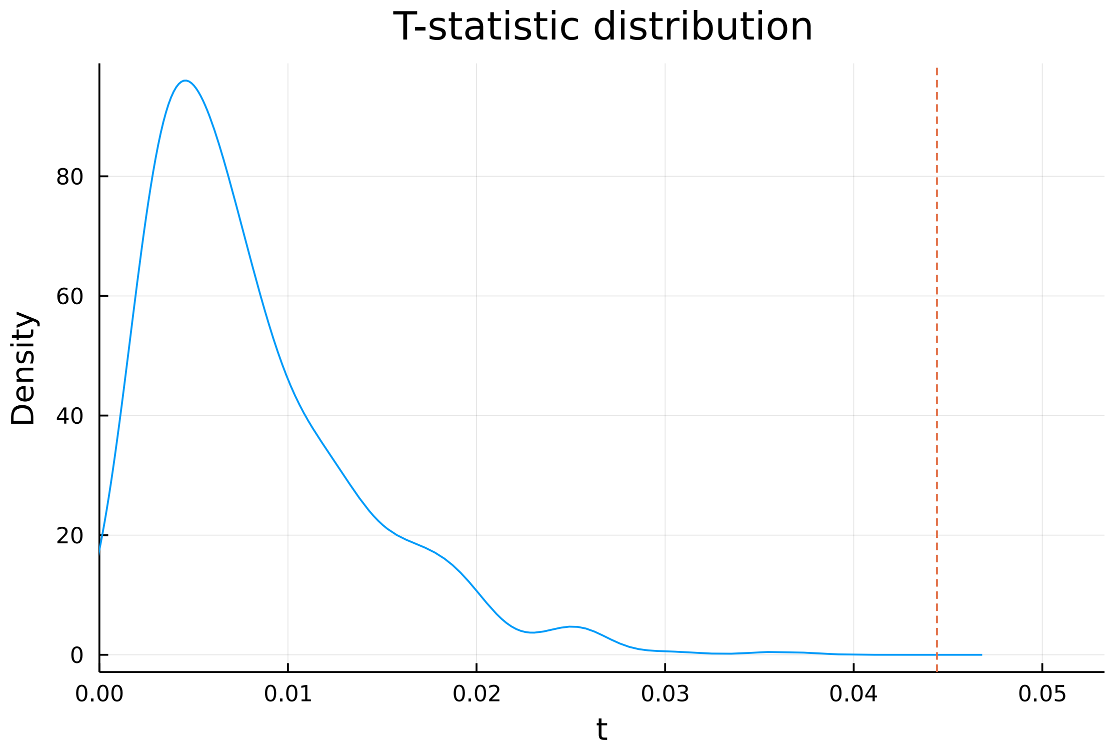


[16]:
```
# perform test
my_test = sameDistributionTest(x,y)
# my_test = sameDistributionTest(x,y, NIter = 1000)
my_test |> display
```

```
Equal distributions test:
    Samples length:
        n1 = 100
        n2 = 80
    Null Hypothesys:
        Both samples have the same probability density function.
    Test Statistic:
        Tc = 0.044415554696254314
        p-value = 0.0
            (aproximated using 500 iterations).
```

[17]:
```
# plot statistic distribution and the realization (red dotted line)
plot(my_test)
```

[17]:

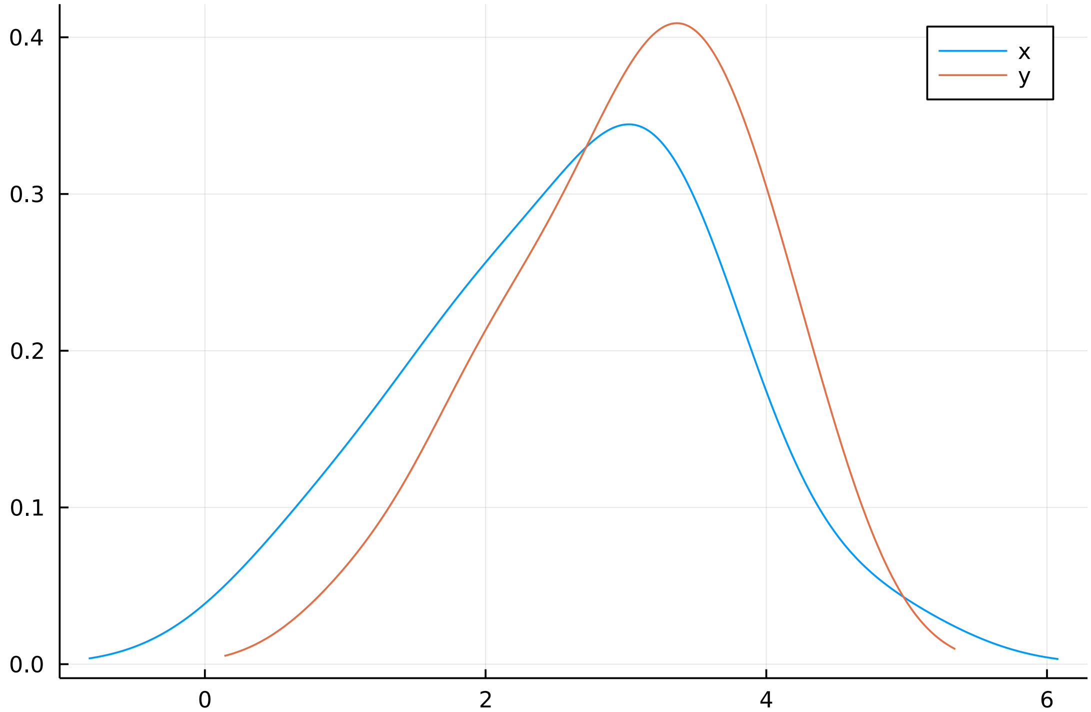


Another example:

[18]:
```
x = randn(20) .+ 2.7
y = randn(20) .+ 3

plot(discretize(KernelDensity(x))...,legend = :topright,label = "x")
plot!(discretize(KernelDensity(y))...,label = "y")
```

[18]:

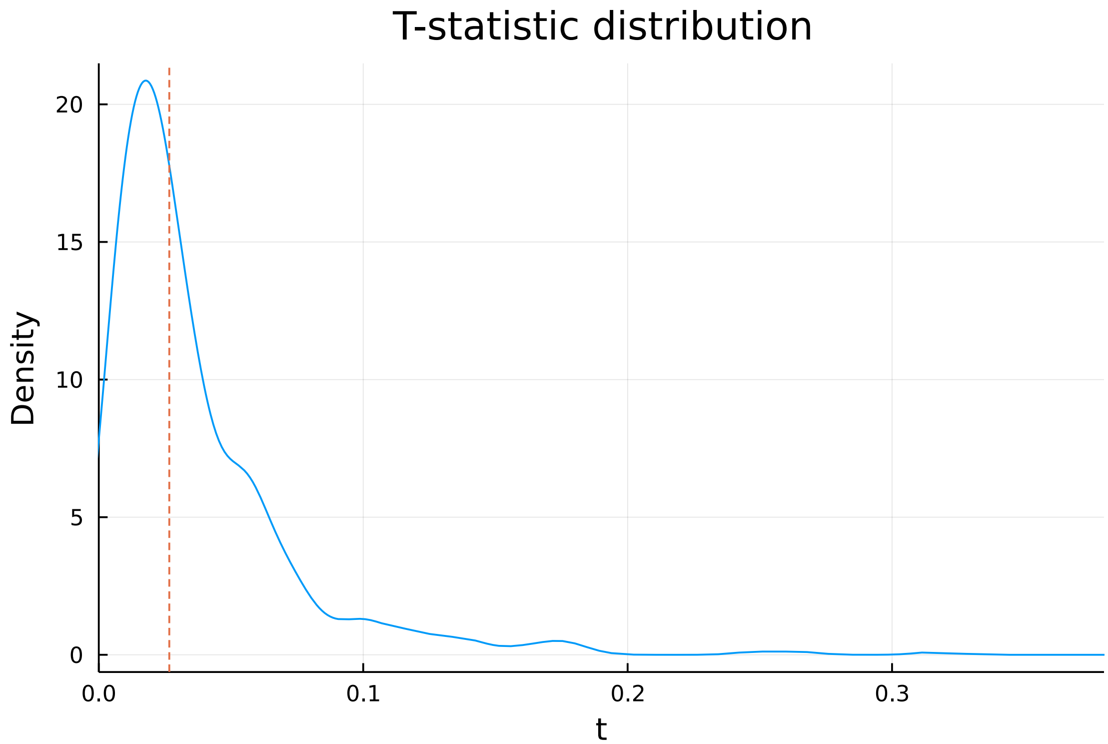


[19]:
```
my_test = sameDistributionTest(x,y)
my_test |> display
```

```
Equal distributions test:
    Samples length:
        n1 = 20
        n2 = 20
    Null Hypothesys:
        Both samples have the same probability density function.
    Test Statistic:
        Tc = 0.02666591759191063
        p-value = 0.456
            (aproximated using 500 iterations).
```

[20]:
```
plot(my_test)
```

[20]:



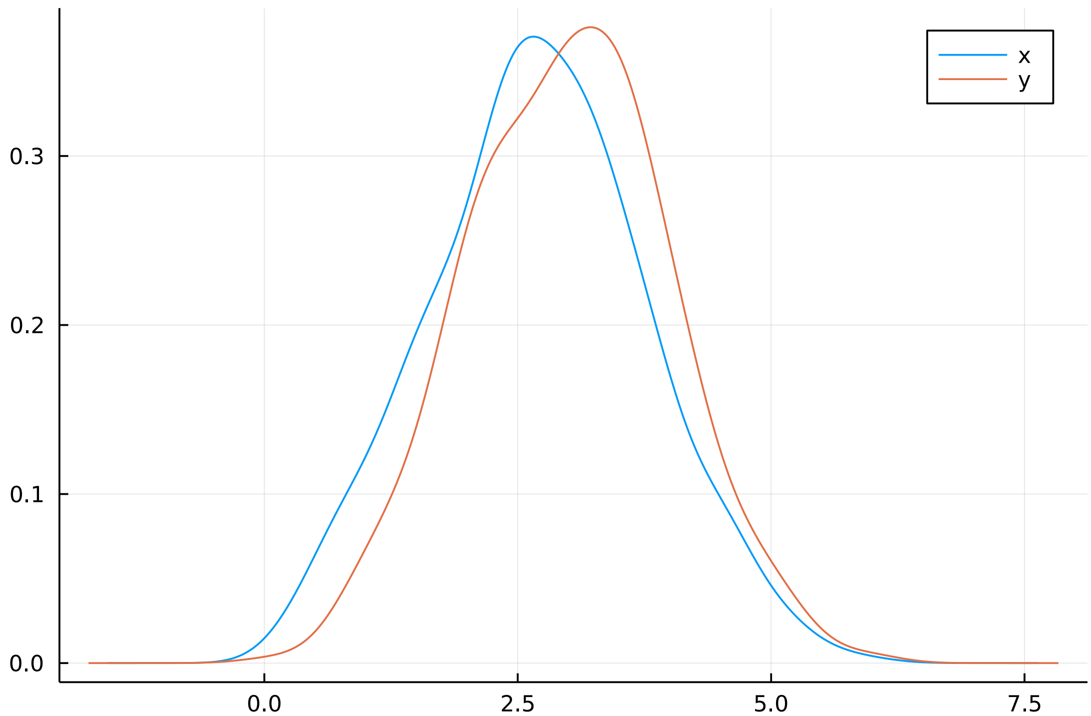
Now we increase the sample size

[21]:

```
x = randn(500) .+ 2.7
y = randn(500) .+ 3

plot(discretize(KernelDensity(x))...,legend = :topright,label = "x")
plot!(discretize(KernelDensity(y))...,label = "y")
```

[21]:

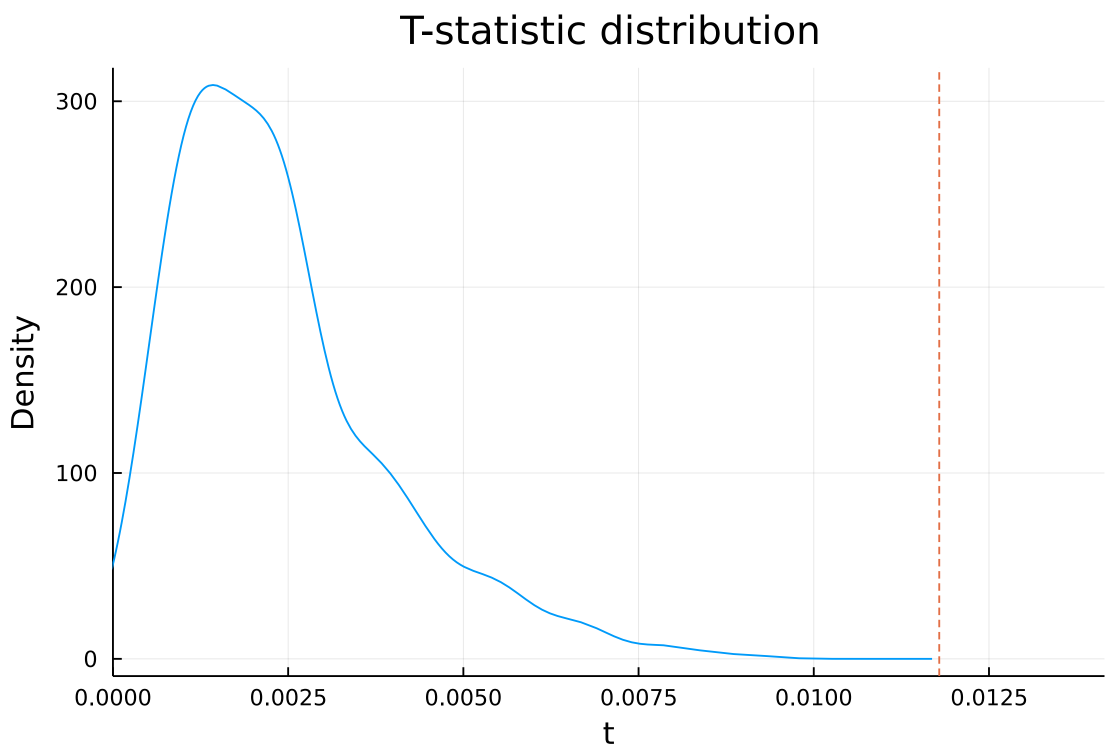


[22]:
```
my_test = sameDistributionTest(x,y)
my_test |> display
```

```
Equal distributions test:
    Samples length:
        n1 = 500
        n2 = 500
    Null Hypothesys:
        Both samples have the same probability density function.
    Test Statistic:
        Tc = 0.011790236111444684
        p-value = 0.0
            (aproximated using 500 iterations).
```

[23]:
```
plot(my_test)
```

[23]:



1.3.2 Independet populations test

Independent samples:

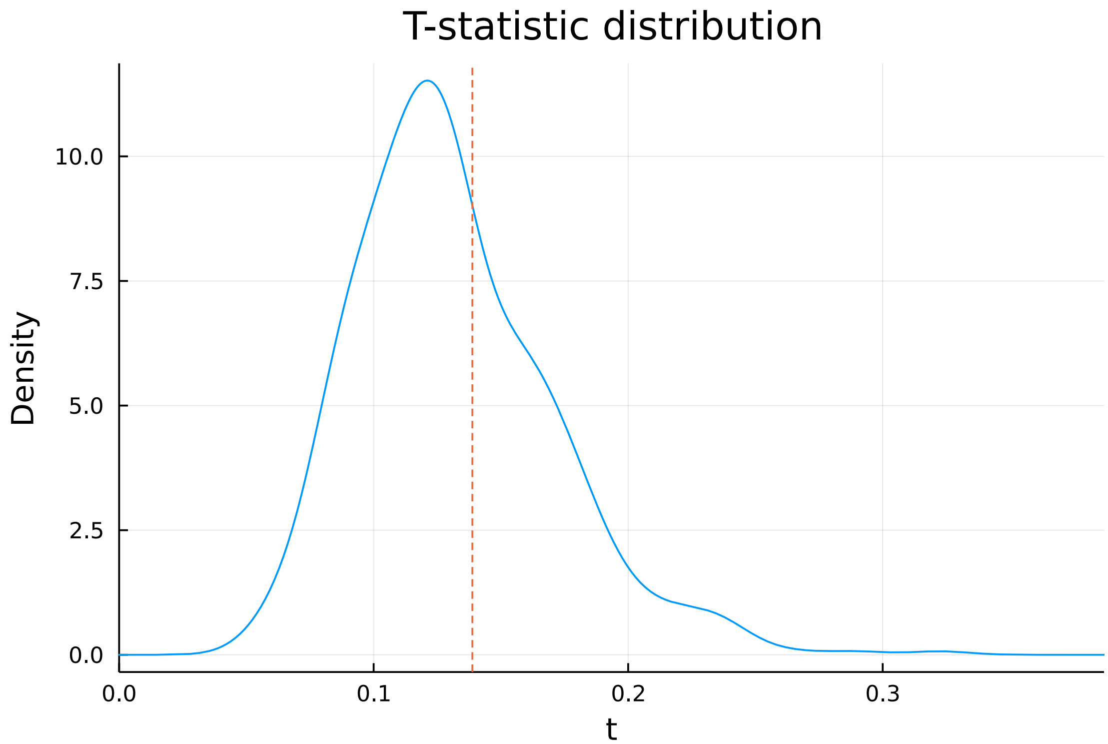
[24]:
```
x = randn(30)
y = exp.(2 .+ rand(30))

# perform test
my_test = independencyTest(x,y)
my_test |> display
```

```
Independency test:
    Sample length:
        n = 30
    Null Hypothesys:
        Variables x and y are independent.
    Test Statistic:
        Tc = 0.1387902994190565
        p-value = 0.348
            (aproximated using 500 iterations).
```

[25]:
```
plot(my_test)
```

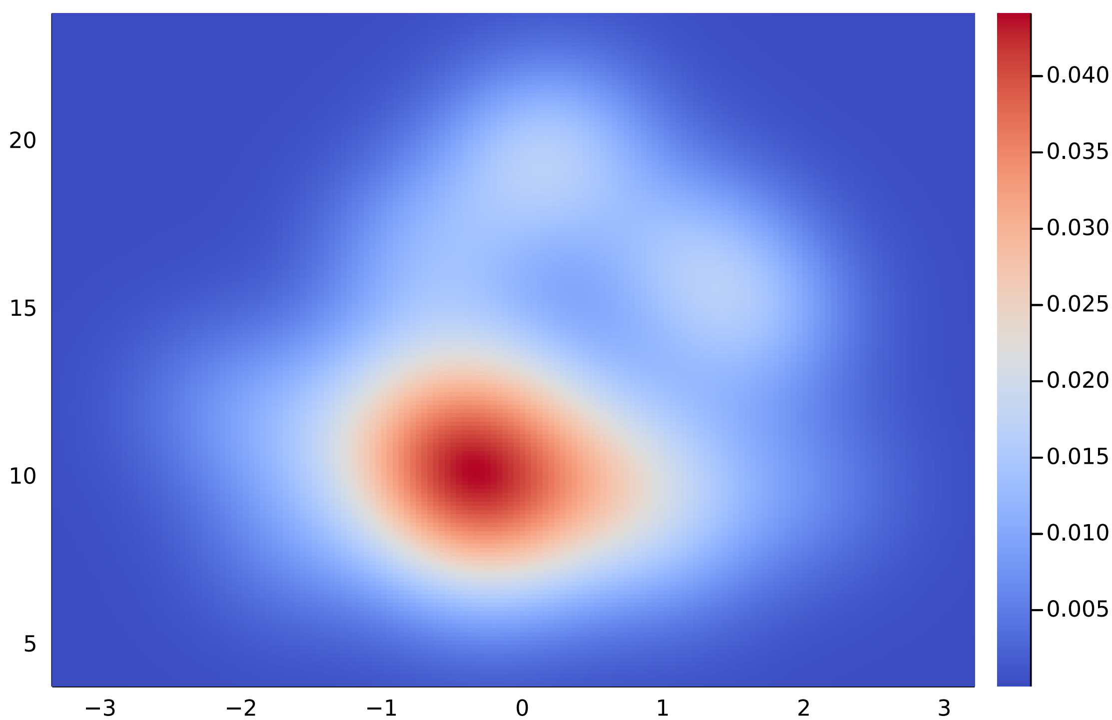
[25]:



[26]:

```
# their bivariate kernel density
BivariateKernelDensity(x,y) |> plot
```

[26]:



Dependent samples:

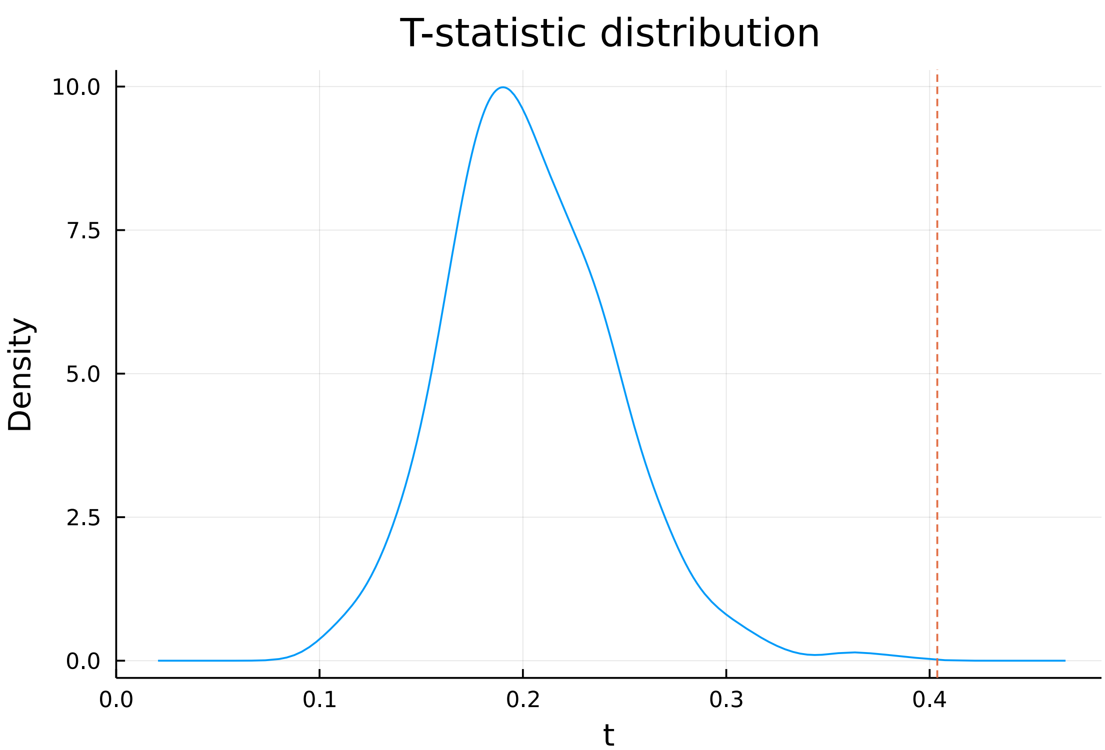
[27]:
```
x = randn(30)
y = exp.(2 .+ rand(30)) .+  4x

my_test = independencyTest(x,y)
my_test |> display
```

```
Independency test:
    Sample length:
        n = 30
    Null Hypothesys:
        Variables x and y are independent.
    Test Statistic:
        Tc = 0.4037591691854353
        p-value = 0.0
            (aproximated using 500 iterations).
```

[28]:
```
plot(my_test)
```

[28]:



[29]:
```
# their bivariate kernel density
BivariateKernelDensity(x,y) |> plot
```

[29]: