Question 1: DiPasquale and Wheaton Model (15 P.)

a) Define and explain the long-run steady state condition.

In the long-run steady state condition, the DiPasquale & Wheaton model spans a rectangle where each of the four vertices are located on a ray in one of the four quadrants of the model. Alternatively, the long-run steady state is defined as the intersection between the demand function for housing and the long-run space supply function. In such a condition, rent is consistent with price, price with construction, construction with depreciated stock (no net investment) so that the capital stock remains constant over time ($\Delta S = 0$). Furthermore, in the long run, the asset market should equate market prices with replacement costs.

b) Determine the long-run space supply function in the space market (Hint: The space market is captured in Quadrant I of the DiPasquale and Wheaton 4 Quadrant Model)

The long-run supply curve can be determined by drawing two different rectangles each with vertices touching the lines in three of the quadrants: northwest, southwest, and southeast. Subsequently, the two vertices in the northeast quadrant can be connected by a line, which signifies the long-run space supply function. In the point where the long-run supply line crosses the demand line is the long-run equilibrium.

Using the construction function $P = \beta_0 + \beta_1 \cdot C$, the LRS can is determined as $-\beta_0 / (\beta_1 \cdot \delta) + 1 / (i \cdot \beta_1 \cdot \delta) \cdot R$, by substituting C for $\delta \cdot S$ and P for R / i and then rearranging the formula to isolate S. Alternatively, it can be calculated as $c_0 / \delta + (c_1 / (\delta \cdot i)) \cdot R$ using $C = c_0 + c_1 \cdot P$. Note that the intercept is negative, because c_0, the C intercept of the construction equation, is negative (there is a price threshold below which no construction will occur). The slope of the long-run supply curve is the slope of the short-run construction supply curve divided by the *depreciation* and the *cap rate i*.

c) Determine the long-run equilibrium values of the amount of space S and the rent R. Given a positive change in parameter α_0, i.e., due to an increase in the economic activity in the economy, what are the effects on the long-run equilibrium values of S and R? Show the comparative static of both variables.

The long-run equilibrium amount of space S and rent R is the intersection between the two functions and can be determined as follows.

1.) Substituting R / i for P and $\delta \cdot S$ for C (as $\Delta S = 0$) the construction function becomes $R / i = \beta_0 + \beta_1 \cdot \delta \cdot S$
2.) Taking i over to the other side and setting it equal to the space demand we receive $\beta_0 \cdot i + \beta_1 \cdot \delta \cdot S \cdot i = a_0 - a_1 \cdot S$
3.) Isolating S we receive $S = (a_o - \beta_0 \cdot i) / (\beta_1 \cdot \delta \cdot i + a_1)$
4.) To obtain the equilibrium for the rent R, isolate S in point 1 to receive $S = R / (\beta_1 \cdot \delta \cdot i) - \beta_o / (\beta_1 \cdot \delta)$
5.) Inserting point 4 into the demand function we obtain $R = a_o + a_1 \cdot R / (\beta_1 \cdot \delta \cdot i) - \beta_o / (\beta_1 \cdot \delta)$
6.) Isolating R we receive $R = (a_0 - (\beta_o / (\beta_1 \cdot \delta)) / (1 - a_1 / (\beta_1 \cdot \delta \cdot i))$

A positive increase in α_0 will shift the demand curve upwards and thus increase the demand for current space on the market. When demand shifts, the long-run supply curve on the space market stays fixed and a new long-run equilibrium is found where the new demand intersects the long-run supply curve. Systematically going through the DiPasquale & Wheaton four quadrant model, an upward shift in the demand curve will increase rents R in the short-run, as stock S is assumed to be inelastic in the short-run. An increase in rents R will trigger an increase in prices P, which can be obtained from the *cap rate i*. Higher prices in the asset market will in turn trigger more construction, as it becomes more profitable to build. However, construction takes a couple of years to complete and thus creates cyclical effects at the real estate markets. The net construction (after depreciation) will in the first period will be larger than in the previous period, thus reducing the rent R from the short-run equilibrium.

After, potentially, a couple of periods, the long-run equilibrium will be found where the upward shifted space demand curve intersects the fixed long-run space supply curve, which will be at a higher rent R as well as an increased stock supply S. The increase in rent and stock after an increase in a_0 can also be seen looking at the functions in point 3 and 6.

Question 2: Extended DiPasquale and Wheaton Model (15 P.)

a) Explain the extensions compared to the original DiPasquale and Wheaton model.

The extension splits up the price and rent into constituent parts, namely the construction rent or structure rent and the land rent. This extension allows us to determine comparative statics of land scarcity, making the model more realistic. Furthermore, splitting up price and rent into its constituents allows us to differentiate between capital market condition for land and the capital market condition of construction. These differ, as land is not depreciable and thus does not incorporate the depreciation factor, in contrast to the capital market condition of construction. Lastly, the variables of the space demand function was altered, now using R_0 and β, representing the willingness to pay given a certain stock/supply.

b) Determine the long-run space supply function. Explain the difference compared to the equation in part b of question 1. Interpret the effect of parameter a.

1.) $P = \beta_0 + \beta_1 \cdot C$ where $C = \delta \cdot S$ as $\Delta S = 0$ in the long-run steady state.
2.) $P = P_C + P_L$ where $P_C = R_C / (i + \delta)$ and $P_L = R_L / i = a \cdot S$
3.) Transforming P_L we receive $R_L = S \cdot a \cdot i$
4.) Setting 1 and 2 equal we receive $\beta_0 + \beta_1 \cdot \delta \cdot S = R_C / (i + \delta) + a \cdot S$
5.) Isolating S we receive $S = -\beta_0 / (\beta_1 \cdot \delta - a) + (R_C / (i + \delta)) / (\beta_1 \cdot \delta - a)$ *or* $S = -\beta_0 / (\beta_1 \cdot \delta - a) + P_C / (\beta_1 \cdot \delta - a)$

Compared to the equation in part b of question 1, this long-run space supply function is also dependent on the land scarcity, where more scarce land leads to higher land prices and therefore higher house prices, which in turn means that there will be more construction as it is more profitable for real estate developers. An increase in a shifts the intercept up as well as increases the slope of the long-run space supply function and vice versa. The parameter a could artificially increase due to new regulation decreasing the area of developable land, or high a's could be given naturally due to the topography, such as water or mountains.

c) Calculate the long-run equilibrium values of the amount of space S and the rent R. Show the comparative statics for a change in the willingness to pay, R_0, as well as an increase in the expected capital market price for construction, i.e., P_C.

The long-run equilibrium amount of space S and rent R is the intersection between the two functions and can be determined as follows.

1.) Substituting S in the space demand function with the *LRS* function from b5 we receive $R = R_0 - \beta \cdot (-\beta_0 / (\beta_1 \cdot \delta - a) + P_C / (\beta_1 \cdot \delta - a))$
2.) $R = R_C + R_L$ where $R_C = P_C \cdot (i + \delta)$ and $R_L = P_L \cdot i$ where $P_L = a \cdot S$
3.) Aggregating the above we receive $R = P_C \cdot (i + \delta) + a \cdot S \cdot i$
4.) Inserting point 3 into the demand function $R = R_0 - \beta \cdot S$ we receive $P_C \cdot (i + \delta) + a \cdot S \cdot i = R_0 - \beta \cdot S$
5.) Isolating S we receive $S = R_0 / (a \cdot i + \beta) - P_C (i + \delta) / (a \cdot i + \beta)$

An increase in the willingness to pay R_0 will result in a shift of the demand function to the right, leading to an equilibrium in which the stock S and the rent R is higher, which can also been seen in the functions in point 1 and 5. The dynamics will follow the same pattern as in exercise 1. A decreased willingness to pay will have an opposite effect. An increase in the expected capital market price for construction will mean that construction is more expensive everything else held constant. This means that the long-run supply curve will shift to the left, resulting in higher rents due to lower supply. The comparative statics also become clear when looking at the functions $S = R_0 / (a \cdot i + \beta) - P_C (i + \delta) / (a \cdot i + \beta)$ and $R = R_0 - \beta \cdot (-\beta_0 / (\beta_1 \cdot \delta - a) + P_C / (\beta_1 \cdot \delta - a))$.