Algorithmen und Wahrscheinlichkeit Programming Exercises

FS 2021

Exercise - Painting Stones

You have a set of beautiful stones, out of which b are painted in black and w are painted in white. However, your younger brother thinks they will be more beautiful if they are all of the same colour, so he decides to start repainting them. As he is too young to understand concepts like efficiency, he follows the following procedure to achieve his goal. At each step he picks a stone uniformly at random, paints it black (even if the stone was already black), and puts it back with the rest of the stones

You are amazed by your brother's strategy and want to understand it further. For that, you want to answer the following three questions:

- (1) After one step of his repainting procedure you choose a stone uniformly at random. What is the probability that the stone you drew is black?
- (2) After one step of his repainting procedure you choose a stone uniformly at random. Observing that the stone you drew is black, what is the probability that the stone your brother repainted was originally white?
- (3) After w + 1 steps of his repainting the procedure, what is the probability that all the stones are black?

Input The first line of the input contains the number $t \leq 30$ of test cases. Each of the t test cases is described as follows.

- It starts with a line that contains two integers b w, separated by a space, denoting the number of black $(1 \le b \le 20)$ and white $(1 \le w \le 20)$ stones you initially posses.
- The following line contains a single integer x, denoting the type of question you are supposed to answer $(1 \le x \le 3)$.

Output For each test case output a single line with the answer to one of the questions stated above. The x value of each test set determines which question you should output the answer to. For example, if x=3 you should output the probability that your brother manages to paint all stones black after w+1 steps. Your solution is going to be accepted if it has an absolute or relative error of at most 10^{-3} .

Points There are three groups of test sets, worth 100 points in total.

- For the first group of test sets, worth 40 points, you may assume that x = 1.
- For the second group of test sets, worth 30 points, you may assume that and $x \in \{1, 2\}$.
- For the third group of test sets, worth 30 points, there are no additional assumptions.

Sample Input

3

1 1

1

1 1 2

1 1

3

Sample Output

0.75

0.6666667

0.75