Analysis of evaluation heuristics.

The following evaluation heuristics were implemented:

Heuristic name	Formula/pseudocode	Rationale
AB custom MORE_MOVES_WITH_NO_NEG	if own_moves > opp_moves then own_moves else 0	It is good to have more legal moves than your opponent, but if your opponent has more legal moves than you then there is no point searching for a better move. Also, there's no positional consideration.
AB custom 2 ABSOLUTE MOVE DIFF	ABS(own_moves - opp_moves)	It is good to have more legal moves than your opponent, but if your opponent has more than you then maximise search such that may favour your opponent. No positional consideration. I don't expect this one to go well.
AB custom 3 CENTER_ WEIGHTED	if move_count >0 and move_count < 10 then centre else 0.3 * centre	If you are within 10 moves into the game then it is better for you to be positioned close to the centre of the board otherwise it is not as good to be close to the centre.
AB custom 4 MORE_MOVES_WITH_INVERSE	if (own_moves - opp_moves) >0 then 1.0/(own_moves - opp_moves) else 0.0	It is probably good to have more legal moves than your opponent and to consider an experimental twist that is the inverse of your advantage.
AB custom 5 CENTER_WEIGHTED_INVERSE	if move_count >0 and move_count < 10 then 1.0/centre else 0.3 * 1.0/centre	If you are within 10 moves into the game then it is better for you to be positioned close to the centre of the board otherwise it is not as good to be close to the centre. However, consider an inverse of the distance as an experiment.
AB custom 6 CENTER_INVERSE	1.0/centre	Experimenting with just considering the inverse of our distance from the centre of the board.

Result

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                         Playing Matches
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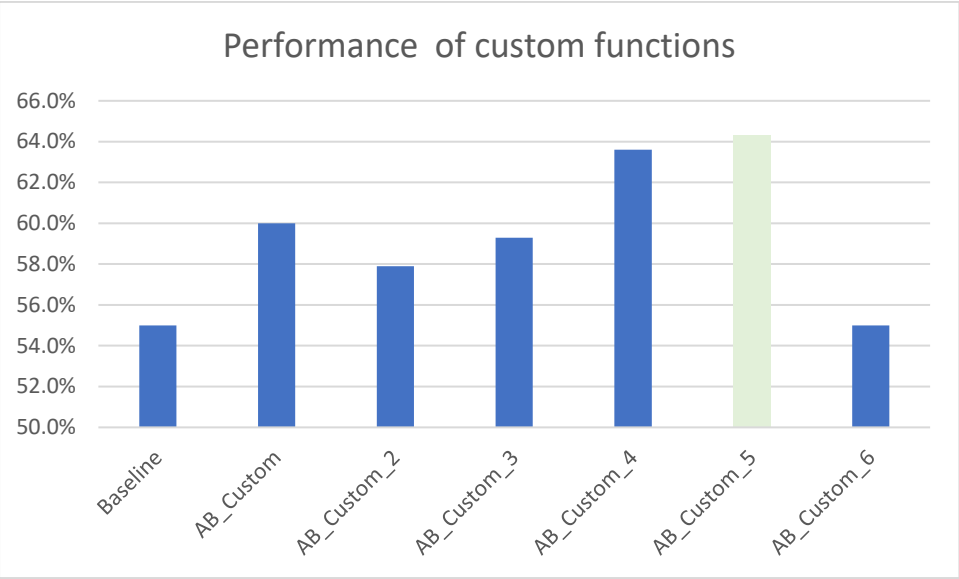
Match #   Opponent    AB_Improved   AB_Custom   AB_Custom_2  AB_Custom_3  AB_Custom_4  AB_Custom_5  AB_Custom_6
                      Won | Lost   Won | Lost   Won | Lost   Won | Lost   Won | Lost   Won | Lost   Won | Lost
   1       Random      19  |   1    16  |   4    15  |   5    17  |   3    18  |   2    19  |   1    15  |   5
   2       MM_Open     12  |   8    14  |   6    14  |   6    12  |   8    10  |  10    15  |   5    12  |   8
   3      MM_Center    12  |   8    14  |   6    12  |   8    14  |   6    13  |   7    16  |   4    14  |   6
   4     MM_Improved    8  |  12    13  |   7    12  |   8    10  |  10    12  |   8    11  |   9    10  |  10
   5       AB_Open      9  |  11     7  |  13    10  |  10     8  |  12    13  |   7     9  |  11    10  |  10
   6      AB_Center     9  |  11    11  |   9     8  |  12    10  |  10    11  |   9    10  |  10     9  |  11
   7     AB_Improved    8  |  12     9  |  11    10  |  10    12  |   8    12  |   8    10  |  10     7  |  13
--------------------------------------------------------------------------------------------------------------
          Win Rate:      55.0%        60.0%        57.9%        59.3%        63.6%        64.3%        55.0%
```

In the above table, the performance of the custom_score evaluation functions is shown against a baseline agent using alpha-beta search and iterative deepening (ID) called `AB_Improved`. The five `AB_Custom` agents use ID and alpha-beta search with the custom_score functions.

Analysis



Custom_score function/ Heuristic name	**Analysis**
AB custom (Baseline) MORE_MOVES_WITH_NO_NEG	Overall score 60.0%. This has performed only slightly overall better than the baseline. As it is a very simplistic heuristic without positional considerations, it comes as no big surprise to me.
AB custom 2 ABSOLUTE MOVE DIFF	Overall score 57.9%. Another simplistic heuristic function without positional consideration. I expected this to perform the worst because it as absolute value of player/opponent available legal moves, it ignores opponent disadvantage.
AB custom 3 CENTER_ WEIGHTED	Overall score 59.3%. I had some hope for this simple positional heuristic but it now is clear that on its own it cannot make profound impact.
AB custom 4 MORE_MOVES_WITH_INVERSE	Overall score 63.6%. The use of invers seems to be a good thing. Even without positional considerations this heuristic has come 2nd place.
AB custom 5 CENTER_WEIGHTED_INVERSE	Overall score 64.3%. This is the winner. With positional consideration and inverse values of distance from the centre, with diminishing significance.
AB custom 6 CENTER_INVERSE	Overall score 55.0%. This was an experiment gone not quite so well.

Conclusions/Recommendation

The recommended heuristic function is one that combines positional considerations with available number of legal moves and which also used the inverse of the distance of the player from a reference point. However other considerations have not been tried out, and even in the ones tried here there is still a wide scope for varying the parameters. For instance, in considering that the influence of the distance from the centre should be weighed, we used an arbitrary value of 10 for the threshold, this could be varied and tiered over more than one boundary. It should also be possible to include known winning features/strategies into the heuristics.